MATHS

BOOKS - KC SINHA ENGLISH

ALGEBRA - PREVIOUS YEAR QUESTIONS - FOR COMPETITION

Exercise

1. Let p and q be real numbers such that $p \neq 0, p^3 \neq q,$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having α/β and β/α as its roots is

A. $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

B. $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

C. $(p^3 + q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

D. $(p^3 + q)x^2 - (5p^3 + 2q)x + (p^3 + q) = 0$

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2. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$ has exactly two distinct solution is a. 0 b. $2^9 - 1$ c. 168 d. 2

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4. Let $S_k, k = 1, 2, \ldots . 100$ denote the sum of the infinite geometric series whose first term is $\frac{k-1}{K!}$ and the common ration is $\frac{1}{k}$ then the value of $\frac{(100)^2}{100!} + \sum_{k=1}^{100}$ |(k^2-3k+1)S_k| is __________`

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5. Let $S = \{1, , 2, 34\}$. The total number of unordered pairs of disjoint subsets of S is equal a.25 b. 34 c. 42 d. 41

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6. For $r = 0, 1, \ldots.., 10$, let $A_r, B_r,$ and C_r denote, respectively, the coefficient of x^r in the expansion of $(1+x)^{10}, (1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to

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7. Let $(a_1, a_2, a_3 \ldots, a_{11})$ be real numbers satsfying $a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4 \ldots, 11,$ If

$$\frac{a_1^2 + a_2^2 + \ldots\ldots + a_{11}^2}{11} = 90 \text{ then the value of } \frac{a_1 + a_2 + \ldots\ldots + a_1}{11}$$

is equal to ________.

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8. Let P be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_P = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, ..., p-1\} \right\}$$

The number of A in T_P such that det (A) is not divisible by p is

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11. The number of 3 x 3 non-singular matrices, with four entries as 1 and all other entries as 0, is:- (1) 5 (2) 6 (3) at least 7 (4) less than 4

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12. Let a be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is a 2×2 identity matrix. Define Tr(A)= sum of diagonal elements of A and |A| = determinant of matrix A.

Statement 1 : Tr $(A) = 0$

Statement 2 : $|A| = 1$

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13. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts is the n^{th} minute .If $a_1 = a_2$ =...... $= a_{10}$= 150 and $a_{10}, a_{11}. . ,$ are in A.P with common difference -2, then the time to count all notes

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14. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn, two balls are taken out at random and then transferred to the other. The number of ways in which this can be done. Is

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15. consider the system of linear equations

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3,$

$3x_1 + 5x_2 + 2x_3 = 1$

the system has

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16. The number of complex numbersd z, such that $|z-1| = |z+1| = |z-i|$, where $i = \sqrt{-1}$ equals to

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17. A polynomial of degree 2 which takes values y_0, y_1, y_2 at points x_0, x_1, x_2 respectively , is given by $p(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x}{(x_2-x_0)(x_2-}$ A polynomial of degree 2 which takes values y_0, y_0, y_1 at points x_0, x_{0+t}, x_1 $t \neq 0$ is given by

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18. Let α and β be the roots of the equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$. For $n \geq 1$, then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is :

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19. Let M and N be two 3×3 nonsingular skew-symmetric matrices such that $Mn = NM$. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

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20. Let $\omega \neq 1$ be cube root of unity and S be the set of all non-singular matrices of the form $[1ab\omega 1c\omega^2\omega 1]$, $where$ each of $a, b, andc$ is either ω or ω^2. Then the number of distinct matrices in the set S is a. 2 b. 6 c. 4 d. 8

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21. Let a, b, and c be three real numbers satifying $\begin{bmatrix} a & b & c \end{bmatrix}$

$$\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

If the point P(a, b, c) with reference to (E) lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

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22. Let a, b, and c be three real numbers satifying $\begin{bmatrix} a & b & c \end{bmatrix}$

$$\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

Let ω be a solution of $x^3 - 1 = 0$ with $Im(\omega) > 0$. If $a = 2$ with b and c satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to

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23. Let a, b, and c be three real numbers satifying $\begin{bmatrix} a & b & c \end{bmatrix}$

$$\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

Let $b = 6$, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty} \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is

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24. Let $a_1, a_2, a_3, , a_{100}$ be an arithmetic progression with $a_1 = 3 and s_p = \sum_{i=1}^{p} a_i, 1 \leq p \leq 100.$ For any integer n with $1 \leq n \leq 20,$ let$m = 5n\cdot$ If $\frac{S_m}{S_n}$ does not depend on $n,$ then a_2 is________.

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26. The minimum value of the sum of real number $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8, and a^{10} with a > 0$ is

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28. The number of distinct real roots of $x^4 - 4x^3 + 12x62 + x - 1 = 0is\_\_\_\_\_\_$.

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29. Let M be a 3×3 matrix satisfying $M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$,

$M\begin{bmatrix}1\\-1\\1\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$ and $M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}$

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30. Find the coefficient of x^7 in the expansion of $\left(1 - x - x^2 + x^3\right)^6$.

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31. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that : (1) $b \in (0, 1)$ (2) $b \in (- 1, 0)$ (3) $|b| = 1$ (4) $b \in (1, \infty)$

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32. 38. Assertion (A): The area of a rectangle is 630 sq.cm and its breadth is 15cm then its lengthis 55 cm

Reason (R): The area of a rectangle is given by A = length x breadtha)

(A) Both A and R are true and R is correct explanation of A

(B) Both A and R are true and R is not correct explanation of A

(C) A is false and R is true.

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33. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after how many months

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34. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals

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35. The number of values of k for which the linear equations

$4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

possess a non-zero solution is

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36. Statement-1 : The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $\hat{}\ 9C_3$. Statement-2 : The number of ways of choosing any 3 places from 9 different places is $\hat{}\ 9C_3$. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1. Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. Statement-1 is true, Statement-2 is false. Statement-1 is false, Statement-2 is true.

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37. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

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38. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5anda_{20} = 25$. The least positive integer n for which $a_n < 0$ 22 b. 23 c. 24 d. 25

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39. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value (s) of the determinant of P is (are)

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40. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + z + 1$ is real. Then a cannot take the value.

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41. The total number of ways in which 5 balls of different colours can be distributed among 3 persons, so that each person gets atleast one ball is

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42. The value of

$$6+\log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)$$ is ________.

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43. Let a_n denotes the number of all n-digits positive integer formed by the digits 0,1 or both such that no consecutive diigits in them are 0. let b_n be the number of such n-digit integers ending with digit 1 and c_n be the number of such n digits integers ending with digit 0.

Q. The value of b_6, is

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44. Let a_n denotes the number of all n-digits positive integer formed by the digits 0,1 or both such that no consecutive diigits in them are 0. let b_n be the number of such n-digit integers ending with digit 1 and c_n be the number of such n digits integers ending with digit 0.

Q. The value of b_6, is

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45. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

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46. Statement 1 :

The sum of the series 1+(1+2+4)+(4+6+9)+(9+12+16)+....+(361 +380 +400) is 8000

Statement 1:

$\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$, for any natural number n.

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47. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to :

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48. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is (1) an irrational number (2) an odd positive integer (3) an even positive integer (4) a rational number other than positive integers

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49. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is (1) 150 (2) 150 times its 50^{th} term (3) 150 (4) zero

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50. Out of 10 white, 9 black, and 7 red balls, find the number of ways in which selection of one or more balls can be made (balls of the same color are identical).

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51. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies (1) either on the real axis or on a circle passing through the origin (2) on a circle with centre at the origin (3) either on the real axis or on a circle not passing through the origin (4) on the imaginary axis

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52. Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :

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