MATHS

BOOKS - KC SINHA ENGLISH

APPLICATIONS OF DIFFERENTIAL EQUATIONS - FOR BOARDS

Solved Examples

1. Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

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2. The slope of the tangent any point on a curve is λ times the slope of the joining the point of contact to the origin. Formulate the differential equation and hence find the equation of the curve.

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3. Find the equation of the curve passing through the point (0, -2) given that at any point (x, y) on the curve the product of the slope of its tangent and y coordinate of the point is equal to the x-coordinate of the point.

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4. At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4, -3)$. Find the equation of the curve given that it passes through $(2, -1)$.

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5. Find the equation of the curve passing through the point $(1, -1)$ whose differential equation is $xy\frac{dy}{dx} = (x+2)(y+2)$

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6. Find the equation of a curve passing through the point (0, 1). If the slope of the tangent to the curve at any point (x, y) is equal to the sum of the x coordinate (abscissa) and the product of the x coordinate and y coordinate (ordinate) of t

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7. Show that the equation of the curve whose slope at any point is equal to $y + 2x$ and which passes through the origin is $y + 2(x+1) = 2e^x$.

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8. The normal to a given curve at each point (x, y) on the curve passes through the point $(3, 0)$. If the curve contains the point $(3, 4)$, find its equation.

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9. Show that the curve for which the normal at every point passes through a fixed point is a circle.

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10. The normal lines to a given curve at each point pass through $(2, 0)$. The curve pass through $(2, 3)$. Formulate the differntioal equation and hence find out the equation of the curve.

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11. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20, 000 in 1999 and 25000 in the year 2004, what will be the population of the

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12. The surface are of a balloon being inflated changes at a constant rate. If initially, its radius is3 units sand after 2 seconds, it s 5 units, find the radius after t seconds.

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13. In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs 1000 double itself?

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14. Assume that a spherical rain drop evaporates at a rate proportional to its surface area .Radius originally is 3 mm and 1 hour later has been reduced to 2 mm, find an expression for the radius of the rain drop at any time.

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16. The rate of increase of the bacteria in a culture is proportional to the number of bacteria present and it is found that the number doubles in 6 hours. Prove that the bacteria becomes times at the end of 18 hours.

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17. A wet porous substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet hung in the wind loses half of its moisture during the first hour, when will it have lost $95\,\%$ moisture, if weather conditions remain the same?

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18. A radioactive substance has a half-life of h days. Find a formula for its mass m in terms of time t if the initial mass is m_0. What is its initial decay rate?

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19. A steamboat is moving at velocity v_0 when steam is shut off. It is given that the retardation at any subsequent time is equal to the magnitude of the velocity at that time. Find the velocity and distance travelled in time t after the steam is shut off.

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20. Water at temperature 100^0C coos in 10 minutes 80^0C in a room of temperature 25^0C . Find The temperature of water after 20 minutes The time when the temperature is 40^0C [Given: $(\log)_e \frac{11}{15} = -0.3101, \log 5 = 1.6094$]

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21. Solve: $(x + \log y)dy + ydx = 0$

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22. $ydx - xdy + y^2 \cos xdx = 0$

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23. The solution of $(1 + xy)ydx + (1 - xy)xdy = 0$ is

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24. Solve: $x\frac{dy}{dx} = y + \cos\left(\frac{1}{x}\right)$

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25. Solve the differential equation $\frac{d^2y}{dx^2} = x^2 + e^{2x}$

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26. Solve the differential equation $x\frac{d^2y}{dx^2} = 1$, given that $y = 1$, $\frac{dy}{dx} = 0$ when $x = 1$

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27. Solve the following differential equation: $\frac{d^2y}{dx^2} = \sec^2 x + xe^x$

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28. Solve the following differential equation: $\frac{d^2y}{dx^2} = e^x + \cos x$, given that $\frac{dy}{dx} = 1 = y$, when $x = 0$

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29. Solve the differential equation: $\frac{d^2y}{dx^2} = x^2 + \sin 3x$, given that $\frac{dy}{dx} = 1 = y$, when $x = 0$

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Exercise

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2. A curve passes through the point $(5, 3)$ and at any point (x, y) on it, the product of its slope and the ordinate is equal to its abscissa. Find the equation of the curve and identify it.

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3. Show that the equation of the curve passing through the point $(1, 0)$ and satisfying the differential equation $\left(1 + y^2\right)dx - xydy = 0$ is $x^2 - y^2 = 1$

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4. The tangents to a curve at a point on it is perpendicular to the line joining the point with the origin. Find the equation of the curve.

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5. Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\frac{x^2+y^2}{2xy}$, is given by $x^2 \quad y^2 = \quad cx$.

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6. Find the equation of the curve whose gradient at (x, y) is $\frac{y}{2x}$ and which passes through the point $(a, 2a)$.

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7. Find the curve for which the intercept cut off by any tangent on y-axis is proportional to the square of the ordinate of the point of tangency.

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8. The equation of the curve which is such that the portion of the axis of x cut off between the origin and tangent at any point is proportional to

the ordinate of that point is (a) $(b)(c)x = y(a - b\log x)(d)$ (e) (f) $(g)(h)\log x = b(i)y^{(j)2(k)}(l) + a(m)$ (n) (o) $(p)(q)(r)x^{(s)2(t)}(u) = y(a - b\log y)(v)$ (w) (d) None of these

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9. Let C be a curve which has the property that length of the perpendicular from origin upon any of its tangent is equal to the abscissa of the point of tangency. If the curves passes through $(2, 2)$ and the area enclosed by the curve above the x-axis is $k\pi$, then find the value of k .

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10. A wet porous substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet hung in the wind loses half of its moisture during the first hour, when will it have lost $95\,\%$ moisture, if weather conditions remain the same?

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11. In a certain culture the number of bacteria at any instant increases at a rate proportional to the cube root of the number present at that instant. If the number becomes 8 times in 3 hours, when will the number be 64 times?

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12. The population of a town is decreasing at a rate proportional to its size. In 1975, the population was 50,000 and in 1985, it was 44,000. What was the expected population in 1995?

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13. The surface area of a spherical balloon being inflated, changes at a rate proportional to time t. If initially its radius is 1 unit and after 3 secons it is 2 units, find the radius after t seconds.

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14. The temperature T of a cooling object drops at a rate proportional to the difference $T - S$ where S is constant temperature of surrounding medium. If initially $T = 150C$; find the temperature of the cooling object at any time t.

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15. The solution of the differential equation $xdx + ydy = \frac{xdy - ydx}{x^2 + y^2}$ is $\tan(f(x, y) - C) = \frac{y}{x}$ (where, C is an arbitrary constant). If $f(1, 1) = 1$, then $f(\pi, \pi)$ is equal to

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16. Solve: $y(1 + xy)dx - xdy = 0$

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17. $y(2xy + e^x)dx - e^x dy = 0$

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18. Solution of $(xy\cos(xy) + \sin(xy))dx + x^2\cos(xy)dy = 0$is

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19. Solve: $ydx - xdy = xydy$

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20. The solution of the differential equation $\left\{1 + x\sqrt{\left(x^2 + y^2\right)}\right\}dx + \left\{\sqrt{\left(x^2 + y^2\right)} - 1\right\}ydy = 0$ is equal to (a) $(b)(c)(d)x^{(e)2(f)}(g) + (h)\frac{(i)(j)y^{(k)2(l)}(m)}{n}2(o)(p) + (q)\frac{1}{r}3(s)(t)(u)(v)$ (qq) (rr) *[Math Processing Error]* (dddd) (eeee) *[Math Processing Error]* (qqqqq)

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21. Solve: $\frac{d^2y}{dx^2} = x$

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22. Solve: $\frac{d^2y}{dx^2} = \sin^2 x + xe^x$

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23. Solve: $\frac{d^2y}{dx^2} = 6x + \sec^2 x$

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24. Solve: $\frac{d^2y}{dx^2} = \cos x - \sin x$

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25. Solve: $\frac{d^2y}{dx^2} = \log x$, given that $y = 1$ and $\frac{dy}{dx} = -1$ when $x = 1$

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26. Solve: $\frac{d^2y}{dx^2} = e^x(\sin x + \cos x)$, given that $y = 1$ and $\frac{dy}{dx} = 0$, when $x = 0$

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27. Solve: $\frac{d^2y}{dx^2} = x + \sin x$, given that $y = 0$ and $\frac{dy}{dx} = -1$ when $x = 0$

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