



# MATHS

## **BOOKS - KC SINHA ENGLISH**

# APPLICATIONS OF INTEGRALS - FOR BOARDS

Solved Examples

**1.** Find the area enclosed by the circle of radius a.

2. Find the area of the region bounded by the parabola  $y^2 = 4ax$ , its axis and two ordinates x = 4 and x = 9

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**3.** Using integration, find the area of the region bounded by the parabola  $y^2 = 16x$  and the line

x = 4

4. Find the area of the region bounded by the

ellipse 
$$rac{x^2}{a^2}+rac{y^2}{b^2}=1.$$

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5. Find the area of the smaller portion of the circle

$$x^2+y^2=4$$
 cut off by the line  $x^2=1$ 

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6. Find the area bounded by the ellipse $rac{x^2}{a^2}+rac{y^2}{b^2}=1$  and the ordinates



**8.** Find the area bounded by the curve  $y = \sin x$ between x = 0 and  $x = 2\pi$ . **Watch Video Solution** 

**9.** Find the area of the quadrilateral formed by the lines y = 2x + 3, y = 0, x = 2 and x = 4 using integration.



10. Using integration, find the area of the region bounded by the line 2y + x = 8, the x-axis and the lines x = 2 and x = 4. 11. Find the area of the triangle formed by the lines

y=x+1, 3y=x+5 and y=-x+7 by

method of integration.



12. Find the area of the region bounded by the curve  $y=x^2$  and the line y=4 .

13. Find the area bounded by the curve |x|+y=1

and axis of x.

14. Find the area of the region bounded by  $y^2 = 4x, x = 1, x = 4$  and the x-axis in the first

quadrant.



15. Sketch the graph y=|x+3|. Evaluate $\int_{-6}^{0}|x+3|dx$ . What does the value of this

integral represent on the graph?



16. Using integration, find the area of the region bounded by the following curves after making a rough sketch:

$$y=1+|x+1|, x=\ -2, x=3, y=0.$$

17. Find the area included between the line y = x

and the parabola  $x^2 = 4y$ .

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**18.** Find the area bounded by the curve  $x^2 = 4y$ 

and the straight line x = 4y - 2.

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19. Find the area enclosed by the parabola  $4y = 3x^2$  and the line 2y = 3x + 12.



**20.** Using integration, find the area of the region bounded by the triangle ABC whose vertices A, B, C are (-1, 1), (0,5) and (3,2) respectively.

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**21.** Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A(2, 0), B(4, 5) and C(6, 3).

22. Find the area of the smaller region bounded by

the ellipse  $rac{x^2}{a^2}+rac{y^2}{b^2}=1$  and the straight line  $rac{x}{a}+rac{y}{b}=1.$ 



23. Find the area enclosed between first quadrant of a circle  $x^2 + y^2 = 16$  and line y = x.

24. Find the area of the region bounded by the

curves 
$$x=2y-y^2$$
 and  $y=2+x$ .

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25. Find the area of the region bounded by y

$$x=x^2+1$$
,y=x ,x=0 and y=2

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**26.** Find the area of the region enclosed by the parabola  $y^2 = 4ax$  and the line y = mx.





**28.** Draw a rough sketch of the curves  $y = \sin x$ and  $y = \cos x$  as x varies from 0 to  $\frac{\pi}{2}$ . Find the area of the region enclosed by the curves and the y-axis. 29. Find the area of the region bounded by the two

parabolas 
$$y = x^2$$
 and  $y^2 = x$ .

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**30.** The area included between the parabolas  $y^2 = 4x$  and  $x^2 = 4y$  is (in square units) a. 4/3 b. 1/3 c. 16/3 d. 8/3

**31.** Find the area of the region included between the parabolas

$$y^2 = 4axandx^2 = 4ay, wherea > 0.$$

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**32.** Draw a rough sketch of the curves 
$$y^2 = x + 1$$
 and  $y^2 = -x + 1$  and find the area

enclosed between them,

**33.** find the area common to the circle  $x^2y^2 = 16a^2$ and the parabola  $y^2 = 6ax$ . Or Find the area of the region

$$ig\{(x,y)\!:\!y^2\leq 6a\ ig\}andig\{(x,y)\!:\!x^2+y^2\geq 16a^2ig\}$$
 .



**34.** Find the area of the circle  $x^2 + y^2 = 16$ 

exterior to the parabola  $y^2=6x$ .

**35.** Find the area, lying above the x=axis and included between the circle  $x^2 + y^2 = 8x$  and the parabola  $y^2 = 4x$ .

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**36.** Prove that the curves  $y^2 = 4x$  and  $x^2 = 4y$  divide the area of square bounded by x = 0, x = 4, y = 4 and y = 0 into three equal

parts.

**37.** Find the area bounded by the curves  $y = xandy = x^3$ . Watch Video Solution **38.** Find the area of the region enclosed between circles the two  $x^{2} + y^{2} = 1$  and  $(x - 1)^{2} + y^{2} = 1$ .

**39.** Find the area of region : $\{(x,y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$ 



**42.** Prove that the area in the first quadrant enclosed by the axis, the line  $x=\sqrt{3}y$  and the circle  $x^2+y^2=4~is~\pi/3$ .



### Exercise

1. Find the area bounded by the curve

$$y=x^2, x=1, x=2$$
 and x-axis

**2.** Find the area bounded by the line y = x, the x-

axis and the ordinates  $x=\ -1$  and x=2



3. Find the area of the region bounded by the line

- $y=3x\ +2$  , the x-axis and the ordinates
- x = -1 and x = 1.

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4. Using integration, find the area of the region bounded by the line y-1=x, thex-a xi s



6. Find the area of the region bounded by the parabola  $y^2 = 4ax$ , its axis and two ordinates x = a and x = 2a.

7. Find the area of the region bounded by the curve

 $y^2=x$ and the lines x=1, x=4and the x-axis.

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8. Find the area of the region bounded by  $y^2 = 4x, x = 1, x = 4$  and x-axis in the first quadrant.

9. Find the area of the region bounded by  $y^2 = 9x, x = 2, x = 4$ and the x-axis in the first quadrant.



10. Find the area of the region bounded by  $x^2 = 4y$ , y = 2, y = 4 and the y-axis in the first quadrant.

11. Find the area bounded by the parabola  $y^2 = 4ax$  and its latus rectum. Watch Video Solution

12. Using integration, find the area of the region bounded by the parabola  $y^2 = 16x$  and the line x = 4



**13.** Find the area bounded by the curve  $y^2 = 4ax$ 

and the lines y = 2a and y-axis.



14. The area between  $x = y^2$  and x = 4 is divided into two equal parts by the line x = a, find the value of a.



15. Find the area of the region bounded by the

ellipse 
$$rac{x^2}{4}+rac{y^2}{9}=1$$

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17. Draw a rough sketch of the graph of the curve

 $rac{x^2}{4}+rac{y^2}{9}=1$  and evaluate the area of the region

under the curve and above the x-axis.



**19.** Using integration find the area of region bounded by the triangle whose vertices are (1, 0), (2, 2) and (3, 1).



20. Using the method of integration find the area

of the region bounded by lines:

2x + y = 4, 3x - 2y = 6and x - 3y + 5 = 0

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**21.** Find the area of that region of the ellipse  $4x^2 + 9y^2 = 36$  which lies in first quardrant.

**22.** Find the area of the smaller part of the circle  $x^2 + y^2 = a^2$  cut off by the line  $x = \frac{a}{\sqrt{2}}$ . **Vatch Video Solution** 

**23.** Find the area of the triangle formed by the straight lines y = 2x, x = 0 and y = 2 by integration.



24. Find the area between x-axis and the curve

 $y = \sin x$ , from x = 0 to  $x = \pi$ .





**26.** The area bounded by the curve y = x|x|, x-axis

and the ordinates x = -1 & x = 1 is:

27. Find the area bounded by the curve  $y=4x-x^2$ , the x-axis and the ordinates x=1 and x=3.

28. Find the area of the region boounded by the

curve  $y = 2x - x^2$  and the X-axis.

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**29.** Find the area of the region enclosed by the parabola  $x^2 = y$ , the line y = x + 2 and the X-axis.

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**30.** Find the area of the region enclosed by the parabola  $x^2 = y$ , the line y = x + 2 and the X-axis.



**31.** Find the area of the region bounded by the parabola  $y = x^2$  and y = |x| .

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32. Find the area of the smaller region bounded by

the ellipse  $rac{x^2}{9}+rac{y^2}{4}=1$  and the line  $rac{x}{3}+rac{y}{2}=1.$ 

33. Find the area of the portion of the parabola

 $y^2 = 4x$  cut off by the line y = x.



**35.** Find the area enclosed between the circle  $x^2 + y^2 = 25$  and the straight line x + y = 5.



**36.** Find the area enclosed by the curves  $y = 4x^2$ 

and  $y^2 = 2x$ .



**37.** Find the area of the region in the first quadrant enclosed by the y-axis, the line y = x and the circle

 $x^2 + y^2 = 32$ , using integration.

**38.** Find the area of circle  $4x^2 + 4y^2 = 9$  which is

interior to the parabola  $x^2=4y$ 



**39.** Using integration, find the area of the region enclosed between the two circles  $x^2 + y^2 = 4$  and  $(x-2)^2 + y^2 = 4.$ 

**40.** Sketch the graph y = |x - 5|. Evaluate  $\int_0^1 |x - 5| dx$  what does this value f the integral

represent on the graph.



**41.** Sketch the graph y = |x + 1|. Evaluate  $\int_0^1 |x + 1| dx$ . What does this value of the integral

represent on the graph?

42. Using the metal of integration find the area

bounded by the curve |x|+|y|=1.



 $\{ ig(x,y) \colon \! x^2 + y^2 \leq 2ax, y^2 \geq ax, x, y \geq 0 ig\}.$ 



