

MATHS

BOOKS - KC SINHA ENGLISH

BINOMIAL THEOREM - FOR BOARDS

Solved Examples

1. Expand of the expression
$$:\left(x+rac{1}{x}
ight)^6$$

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2. Expand:
$$\left(x-rac{1}{y}
ight)^{11}, y
eq 0$$

3. Expand $(2x - 3y)^4$ by binomial theorem.



7. Find the value of $\left(\sqrt{2}+1
ight)^6-\left(\sqrt{2}-1
ight)^6$.

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8. Evaluate the following:
$$\left(x+\sqrt{x^2-1}
ight)^6+\left(x-\sqrt{x^2-1}
ight)^6$$

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9. Find the expansion of $\left(3x^2-2ax+3a^2
ight)^3$ using binomial

theorem.



10. If A be the sum of odd terms and B be the sum of even terms in

the expansion of $(x+a)^n$, prove that $A^2-B^2=\left(x^2-a^2
ight)^n$

11. If *o* be the sum of odd terms and *E* that of even terms in the expansion of $(x + a)^n$ prove that: (i) $O^2 - E^2 = (x^2 - a^2)^n$ (ii) $4OE = (x + a)^{2n} - (x - a)^{2n}$ (iii) $2(O^2 + E^2) = (x + a)^{2n} + (x - a)^{2n}$

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12. The first three terms in the expansion of a binomial are 1, 10 and 40. Find the expansion.

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13. Using binomial theorem compute the following: $(99)^5$

14. Using binomial theorem compute the following: $(10.\ 1)65$



18. Prove that
$$\sum_{r=0}^{n} 3^{r} \cap nC_{r} = 4^{n}$$
.
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19. Find the 4^{th} term in the expansion of $(x - 2y)^{12}$.
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20. Find the 13^{th} term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$
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21. Find the 5 th term from the end in the expansion of $\left(\frac{x^{3}}{2} - \frac{2}{x^{2}}\right)^{9}$

22. Write the general term in the expansion of $\left(x^2-y
ight)^6$.



25. In the binomial expansion of $(a-b)^n, n \geq 5$, the sum of 5th and 6th terms is zero, then $\frac{a}{b}$ equals Watch Video Solution **26.** The number of rational terms in the expansion $\left(2^{rac{1}{5}}+3^{rac{1}{10}}
ight)^{45}$ is Watch Video Solution **27.** Find the middle term in the expansion of : $\left(\frac{x}{2} + 9y\right)^{10}$ Watch Video Solution **28.** Find the middle term in the expansion of $\left(x - \frac{1}{2x}\right)^{12}$ Watch Video Solution

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31. Prove that the term independent of x in the expansion of $\left(x+\frac{1}{x}\right)^{2n}is\frac{1\cdot 3\cdot 5(2n-1)}{n!}\cdot 2^n$

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32. Prove that the term independent of x in the expansin of $\left(x+\frac{1}{x}\right)^{2n}is\frac{1.3.5(2n-1)}{n!}.2^n$.

33. Show that the coefficient of the middle term in the expansion of $(1 + x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{2n-1}$.

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34. Find the coefficient of
$$rac{1}{y^2}$$
 in $\left(y+rac{c^3}{y^2}
ight)^{10}$

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35. Find the coefficient of x^9 in $\left(1+3x+3x^2+x^3\right)^{15}$.







coefficient of $a^m and a^n$ are equal.

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43. Find a if the coefficients of x^2 and x^3 in the expansion of $\left(3+ax\right)^9$ are equal.

44. If in the expansion of $\left(1+x
ight)^m \left(1-x
ight)^n$, the coefficients

of x and x^2 are 3 and - 6 respectively, the value of m and n are

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45. If the coefficients of a^{r-1} , $a^r and a^{r+1}$ in the binomial expansion of $(1+a)^n$ are in A.P., prove that $n^2 - on(4r+1) + 4r^2 - 2 = 0.$

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46. Find the coefficient of x^5 in the expansion of the product $(1+2x)^6(1-x)^7$.





49. Show that there will be no term containing x^{2r} in the expansion of $(x + x^{-2})^{n-3}$ if n - 2r is a positive integer but not a multiple of 3.



50. Show that there will ve a term indepandent of x in the

expansion of $\left(x^{a}+a^{-b}
ight)$ only , if an is multiple of (a+b) .



53. The value of m, for which the coefficients of the (2m + 1) th terms in the expansion of $(1 + x)^{10}$ are equal is 3 b. 1 c. 5 d. 8





56. If n is a positive integer, then show that $6^n - 5n - 1$ is divisible

by 25

57. Show that: $2^{4n} - 2^n(7n+1)$ is some multiple of the square of

14, where n is a positive integer.



60. Write down the binomial expansion of $(1+x)^{n+1}$, when x=8. deduce that $9^{n+1}-8n-9$ is divisible

64 where n is a positive integer.



61. If a and b are distinct integers, prove that a - b is a factor of

 $a^n - b^n$, whenever n is a positive integer.



62. If the coefficients of three consecutive terms in the expansion of

 $\left(1+x
ight)^n$ are 165,330 and 462 respectively , the value of n is is

63. If a_1, a_2, a_3, a_4 be the coefficient of four consecutive terms in

the expansion of $(1+x)^n,$ then prove that: $rac{a_1}{a_1+a_2}+rac{a_3}{a_3+a_4}=rac{2a_2}{a_2+a_3}.$

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64. The 2nd, 3rd and 4th terms in the expansion of $(x + y)^n$ are 240, 720 and 1080 respectively, find the values of x, y and n.

65. The coefficient of three consecutive terms in the expansion of $(1 + x)^k$. Are in the ratio 1: 7: 42 find the value of k.

66. I three consecutive coefficients in the expansion of $\left(1+x
ight)^n$ are

in the ratio 6:33:110, find n and r.



can happen.





9. Expand the following by binomial theorem: $\left(1-2x
ight)^5$



13. Evaluate the following:
$$\left(x+\sqrt{x^2-1}
ight)^6+\left(x-\sqrt{x^2-1}
ight)^6$$

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14. Find the value of
$$\left(a^2+\sqrt{a^2-1}
ight)^4+\left(a^2-\sqrt{a^2-1}
ight)^4$$

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15. Using binomial theorem, expand $\left\{ (x+y)^5 + (x-y)^5 \right\}$ and hence find the value of $\left\{ \left(\sqrt{2}+1\right)^5 + \left(\sqrt{2}-1\right)^5 \right\}$.

16. Find
$$(a+b)^4 - (a-b)^4$$
. Hence evaluate $\left(\sqrt{3}+\sqrt{2}
ight)^4 - \left(\sqrt{3}-\sqrt{2}
ight)^4$



21. Find the number of terms in the following expansion: $\left(1-2x+x^2
ight)^{30}$

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22. Find the number of terms in the expansions of the following:

$$\left(\sqrt{x}+\sqrt{y}
ight)^{10}+\left(\sqrt{x}-\sqrt{y}
ight)^{10}$$

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23. If P be the sum of odd terms and Q be the sum of even terms in the expansion of $(x + a)^n$, then $(x + a)^{2n} + (x - a)^{2n}$ is (i) $P^2 - Q^2$ (ii) $P^2 + Q^2$



(iv) 4PQ

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24. Find the value of $(0.99)^{10}$ correct of 4 places of decimal.

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25. Using Binomial theorem, evaluate $(0.99)^{15}$ correct to four places

of decimal.



26. Using binomial theorem, evaluate : $\left(101
ight)^4$















42. Find the fifth expansion of $\left(\frac{a}{3} - 3b\right)^7$



43. Find the fifth expansion of
$$\left(2x^2-rac{1}{3x^2}
ight)^{10}$$

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44. Find a if the 7th and 18th terms of the expansion $\left(2+a
ight)^{50}$ are

equal.



45. Find the r^{th} term from the end in the expansion of $\left(x+a
ight)^n$.



49. Write the general term in the following expansion: $\left(x-\frac{3}{x^2}
ight)^{10}$

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50. Write the general term in the following expansion: $\left(x^2-rac{1}{x}
ight)^{12}, x
eq 0$

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Find the middle term in
$$\left(rac{2x}{3}-rac{3y}{2}
ight)^{20}$$

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51.

52. Find the middle term in the expansion of $\left(\frac{2x}{3}\right)$

$$\left(-\frac{3}{2x}\right)^6$$



57. Find the general and middle term in the expansion of $\left(\frac{x}{y} + \frac{y}{x}\right)^{2n+1}$; n being a positive integer and show that there is no term free from $\frac{x}{y}$.

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58. Show that the middle term in the expansion of $(1+x)^{2n}is\frac{(1.3.5(2n-1))}{n!}2^nx^n$, where n is a positive integer.

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59. Show that the middle term in the expansion $\left(x - \frac{1}{x}\right)^{2n}$ is $\frac{1 \cdot 3 \cdot 5(2n-1)}{n}(-2)^n$.



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68. Find the coefficient of $a^5b^7\in \left(a-2b
ight)^{12}$



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71. Prove that the coefficients of x^n in $(1+x)^{2n}$ is twice the coefficient of x^n in $(1+x)^{2n-1}$.





between aandb so that coefficients are equal.

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73. If the coefficients of second, third and fourth terms in the expansion of $(1+x)^{2n}$ are in A.P., show that $2n^2 - 9n + 7 = 0$.

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74. Find the term independent of x in
$$\left(x+rac{1}{x}
ight)^{2n}$$

75. Find the term independent of x in the expression $\left(x-rac{1}{x}
ight)^{14}$



76. Find the term independent of x in the following binomial

expansions
$$(x
eq 0)$$
 : $\left(2x^2+rac{1}{x}
ight)^{13}$

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77. Find the term independent of x in the following binomial

expansions
$$(x
eq 0)$$
 : $\left(x^2+rac{1}{x}
ight)^{12}$

78. Find the term independent of x in the following binomial

expansions
$$(x
eq 0)$$
 : $\left(\sqrt{rac{x}{3}}+rac{3}{2x^2}
ight)^{10}$



79. Find the term independent of x in the expansion of the following binomials:

$$\left(2x^2-rac{1}{x}
ight)^{12}$$
 What is its value?

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80. Find the term independent of x in the expansion of

$$\left(2x^2-rac{3}{x^3}
ight)^{25}$$

81. Find the term independent of x in the expansion off the

following expression:
$$\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^6$$

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82. Which term in the expansion of
$$\left(x^3 - \frac{3}{x^2}\right)^{15}$$
 is independent of x?

83. Find the term independent of x in the following binomial

expansions
$$(x
eq 0)$$
 : $\left(x^2-rac{3}{x^3}
ight)^{10}$

84. Find the term independent of x in the expansion of the

and

following expressions: $\left(rac{1}{2}x^{rac{1}{3}}+x^{-rac{1}{5}}
ight)^8$ $(1+x+2x^3)igg(rac{3}{2}x^2-rac{1}{3x}igg)^9$

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85. Find the term independent of x in the expansion of: $\left(x-\frac{1}{x}\right)^{12}$.

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86. Find the term independent of x in the expansion of the

following expression:
$$\left(\sqrt[3]{x}+rac{1}{2\sqrt[3]{x}}
ight)^{18},\;x>2$$

87. If there is a term independent of x in $\left(x+\frac{1}{x^2}\right)^n$, show that it is equal to $\frac{n!}{(x+1)!}$

$$\left(\frac{n}{3}\right)!\left(\frac{2n}{3}\right)!$$

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88. If x^p occurs in the expansion of $\left(x^2+1/x
ight)^{2n}$, prove that its

coefficient is
$$\frac{(2n)!}{\left[\frac{1}{3}(4n-p)\right]!\left[\frac{1}{3}(2n+p)\right]!}$$

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89. Find the coefficient of a^4 in the product $\left(1+a
ight)^4 \left(2-a
ight)^5$ using

binomial theorem.



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92. If the coefficient of (r-5) th and (2r-1) th term in the expansion of $\left(1+x
ight)^{34}$ are equal , find r.

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93. If the coefficient of (2r + 5) th term and (r - 6) th term in the expansion $(1 + x)^{39}$ are equal, find r

94. If the integers r>1, n>2 and coefficients of (3r)th and (r+1)th terms in the Binomial expansion of $(1+x)^{2n}$ are equal, then

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95. If the coefficient of (p+1) th term in the expansion of $(1+x)^{2n}$ be equal to that of the (p+3) th term, show that p=n-1

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96. Find the two consecutive coefficients in the expansion of $\left(3x-2
ight)^{75}$ whose values are equal



100. Find the greatest term in the expansion of $(a+x)^{13}, whena = 5, x = 2$

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101. Find the limits between which x must lie in order that the greatest term in the expansion of $(1 + x)^{30}$ may have the greatest coefficient.



102. If n is a positive integer, show that: $4^n - 3n - 1$ is divisible by

9

103. If p is a natural number, then prove that $p^{n+1} + (p+1)^{2n-1}$ is divisible by $p^2 + p + 1$ for every positive integer n.



105. If p is a natural number, then prove that $p^{n+1} + \left(p+1
ight)^{2n-1}$ is

divisible by $p^2 + p + 1$ for every positive integer n.

106. If n is a positive integer, show that: $3^{2n} - 1 + 24n - 32n^2$ is divisible by 512 if n > 2Watch Video Solution

107. If three consecutive coefficients in the expansion of $\left(1+x
ight)^n$

be 56, 70 and 56, find n and the position of the coefficients.

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108. If three successive coefficients in the expansion of $\left(1+x
ight)^n$ be

220,495 and 972, find n.



109. If 3rd, 4th, 5th terms in the expansion of $(x + a)^n$ be 84, 280 and 560, Find x, a and n.



expansion of $(x + 1)^n$ are in the ratio 1:3:5 .Find n and r.



$$(bc+ad)(b-c)=2ig(ac^2-b^2dig)$$

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115. If the four consecutive coefficients in any binomial expansion be a, b, c, d, then prove that (i) $\frac{a+b}{a}$, $\frac{b+c}{b}$, $\frac{c+d}{c}$ are in H.P. (ii) $(bc+ad)(b-c) = 2(ac^2 - b^2d)$

116. If the coefficients of flour consecutive terms in the expansion of

 $(1+x)^n$ are a, b, c, d respectively then prove that:

$$rac{a}{a+b}+rac{C}{c+d}=rac{2b}{b+c}$$

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117. If the coefficients of 5th, 6th , and 7th terms in the expansion of

 $\left(1+x
ight)^n$ are in A.P., then $n=\,$ a. 7 only b. 14 only c. 7 or 14 d. none

of these

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118. If the coefficient of 2nd, 3rd and 4th terms in the expansion of

$$\left(1+x
ight)^{2n}$$
 are in A.P. , show that $2n^2-9n+7=0.$

119. If the coefficient of rth, $(r+1)^{th}$, and (r+2)th terms in the binamial expansion of $\left(1+y\right)^m$ are in A.P. then prove that $m^2 - m(4r+1) + 4r^2 - 2 = 0.$

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120. The coefficients of three consecutive terms in the expansion of

 $\left(1+x
ight)^n$ are in the ratio $182\!:\!84\!:\!30$. prove that n=18

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121. The sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^m$, $x \neq 0$, m being a natural number, is 559. Find the term of the expansion containing x^3 .