MATHS

BOOKS - KC SINHA ENGLISH

BINOMIAL THEOREM - FOR COMPETITION

Solved Examples

1. Find the coefficient of $x^{-1} \in (1+3x^2+x^4)\left(1+\frac{1}{x}\right)^8$

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3. If the greatest term in the expansion of $(1+x)^{2n}$ has the greatest coefficient if and only if $x \in \left(\frac{10}{11}, \frac{11}{10}\right)$ and the fourth term in the expansion of $\left(\lambda x + \frac{1}{x}\right)^{m}$ is $\frac{n}{4}$, the value of $m\lambda$ is .

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4. If $p+q=1,$ then show that $\sum_{r=0}^{n} r^2 \hat{} nC_r p^r q^{n-r} = npq + n^2p^2.$

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5. If $(1+x)^n = C_0 + C_1x + C_2x^2$

$+C_3x^3 + \ldots + C_nx^n$, prove that

$C_1 - 2C_2 + 3C_3 - \ldots + (-1)^{n-1} nC_n = 0$.

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6. Find the sum $2C_0 + \frac{2^3}{2}C_1 + \frac{2^3}{3}C_2 + \frac{2^4}{4}C_3 + \ + \frac{2^{11}}{11}C_{10}.$

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7. If $(1+x)^n = C_0 + C_1x + C_2x^2$

$+C_3x^3 + \ldots + C_nx^n$, prove that

$C_1 - 2C_2 + 3C_3 - \ldots + (-1)^{n-1}nC_n = 0$.

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9. Find the greatest term in the expansion oif $(7-5x)^{11}, where x = \frac{2}{3}.$

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10. Let $R=(5\sqrt{5}+11)^{2n+1} and f = R-[R] where []$ denotes the greatest integer function, prove that $Rf = 4^{2n+1}$

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11. The coefficient of $x^2y^4z^2$ in the expansion of $(2x-3y+4z)^9$ is

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12. Show that the roots of the equation $ax^2+2bx+c=0$ are real and unequal whre a,b,c are the three consecutive coefficients in any binomial expansion with positive integral index.

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13. Find the coefficient of $x^9 is (1+3x+3x^2+x^3)^{15}$

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14. Find the the term of x in $(1+x)^{m}\left(1+\frac{1}{x}\right)^{n}$

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15. If the sum of the binomial coefficients in the expansion of $\left(x+\frac{1}{x}\right)^{n}$ is 64, then the term independent of x is equal to (A) 10 (B) 20 (C) 30 (D) 40

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16. The sum of the coefficients in the expansion of $\left(2+5x^{2}-7x^{3}\right)^{2000}=$ (A) 0 (B) 1 (C) 2 (D) none of these

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17. $(115)^{96}-(96)^{115}$ is divisible by (A) 17 (B) 19 (C) 21 (D) 23

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18. If {x} denotes the fractional part of x, then $\left\{\frac{3^{2n}}{8}\right\}, n \in N,$ is

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19. If a is the remainder when 5^{40} is divided by 11 and b is the remainder when 2^{2003} is divided by 17 then the value of b-a is (A) 1 (B) 8 (C) 7 (D) 6

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20. The sum of the series $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots.. + \frac{1}{(n-1)!1!}$ is = (A) $\frac{1}{n!2^n}$ (B) $\frac{2^n}{n}!$ (C) $\frac{2^{n-1}}{n}!$ (D) $\frac{1}{n!2^{n-1}}$

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21. the digit at the units place of the number $(32)^{32} =$ (A) 0 (B) 2 (C) 4

(D) 6

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22. In the expansion of $(x^2 + 2x + 2)^n, n \in N$

(A) coefficient of $x = n.2^n$

(B) coefficient of $x^2 = n^2 2^{n-1}$

(C) coefficient of $x^3 = n^2 2^{n-2}$

(D) none of these

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23. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots + a_{20} x^{20}$ then (A) $a_1 = 20$ (B) $a_2 = 210$ (C) $a_3 = 1500$ (D) $a_{20} = 2^{3.3}$ ^ 7

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24. Which of the following holds true?

(A) $101^{50} - 100^{50} > 99^{50}$

(B) $101^{50} - 99^{50} < 100^{50}$

(C) $(1000)^{1000} > (1000)^{999}$

(D) $(10001)^{999} < (1000)^{1000}$ `

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25. Given that $(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \ldots. + a_{2n} x^{2n}$ find

i) $a_0 + a_1 + a_2 \ldots . + a_{2n}$ ii) $a_0 - a_1 + a_2 - a_3 \ldots + a_{2n}$ iii) $(a_0)^2 - (a_1)^2 \ldots . + (a_{2n})^2$

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26. If n is a positive integer such that $(1 + x)^n = {}^n C_0 + {}^n C_1 + {}^n C_2 x^2 + \ldots\ldots + {}^n C_n x^n$, for εR. Also $.{}^n C_r = C_r$ On the basis of the above information answer the following questions The value of the series $\sum_{r=1}^{n} r^2 . C_r =$

(A) 1

(B) $(-1)^{\frac{n}{2}} \cdot \dfrac{n!}{\left(\frac{n}{2!}\right)^2}$

(C) $(n-1).^{2n}C_n + 2(2n)$

(D) $n(n+1)2^{n-2}$

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27. If n is a positive integer such that $(1+x)^n = ^nC_0 + ^nC_1 + ^nC_2x^2 + \ldots\ldots + ^nC_nx^n,$ for $\varepsilon R.$ Also $.^nC_r = C_r$ On the basis of the above information answer the following questions The value of the expression $a-(a-1)C_1+(a-2)C_2-(a-3)C_3+\ldots\ldots+(1)^n(a-n)C_n=$

(A) 0

(B) $a^n.(-1)^n.^{2n}C_n$

(C) $[2a-n(n+1)].^{2n}C_n$

(D) none of these

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28. If n is a positive integer such that $(1+x)^n = ^n C_0 + ^n C_1 + ^n C_2 x^2 + \ldots\ldots + ^n C_n x^n$, for εR. Also $.^n C_r = C_r$ On the basis of the above information answer the following questions the value of $.^m C_r . ^n C_0 + ^m C_{r-1} . ^n C_1 + ^m C_{r-2} . ^n C_2 + \ldots. + ^m C_1 . ^n C_{r-1} + ^n C_0^n C_r$ where m,n, r are positive interges and $r < m, r < n =$

(A) $.^{mn} C_r$

(B) $.^{m+n} C_r$

(C) 0

(D) 1

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29. If in a positive integer such that If a number a=p+f whre p is an integer and $0 < f < 1$. Here p is called the integral part of a and f its fractional part. Let $n \in N$ and $\left(\sqrt{3}+1\right)^{2n} = p + f$, where p is the integral part and $0 < f < 1$. On the basis of bove informationi answer teh following question:

The integral part p of $\left(\sqrt{3}+1\right)^{2n}$ is

(A) an even number for al n epsilon N

(B) an odd number for all nepsilon N

(C) anodd or even number according as n is odd or even

(D) an even or odd nuber according as n is odd or even

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30. If in a positive integer such that If a number a=p+f whre p is an integer and $0 < f < 1$. Here p is called the integral part of a and f its fractional part. Let $n \in N$ and $\left(\sqrt{3}+1\right)^{2n} = p + f$, where p is the integral part and $0 < f < 1$. On the basis of bove informationi answer teh following question:

$f^2 + (p-1)f + 4^n =$

(A) p (B) $-p$ (C) 2p (D) $-2p$

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31. If ((x)) represents the least integer greater then x, prove that $\left(\left(\left\{\left(\sqrt{3}+1\right)^{2n}\right\}\right)\right), n \in N$ is divisible by 2^{n+1}

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32. In the expansion of $\left[2-2x+x^2\right]^9$ (A) Number of distinct terms is 10 (C) Sum of coefficients is 1 (B) Number of distinct terms is 55 (D) Coefficient of x^4 is 97

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33. The number of terms free from radical sign in the expansion of $\left(1+3^{\frac{1}{3}}+7^{\frac{1}{7}}\right)^{10}$ is

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34. IF $(1+x)^p = 3+\frac{8}{3}+\frac{80}{3^3}+\frac{240}{3^4}+\ldots\ldots\infty, then(1+x)^p =$

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Exercise

1. Find the value of $\frac{1}{81^n}-\frac{10}{81^n}C_1+\frac{10^2}{81^n}C_2-\frac{10^3}{81^n}C_3++\frac{10^{2n}}{81^n}$.

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2. With the notation $C_r=^n C_2=\frac{n!}{r!(n-r)!}$ when n is positive inteer let

$$S_n=C_n-\left(\frac{2}{3}\right)C_{n-1}+\left(\frac{2}{3}\right)^2 C_{n-2}\pm\ldots\ldots..+(-1)^n\left(\frac{2}{3}\right)^n.C_0$$

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3. If $kandn$ are positive integers and $s_k=1^k+2^k+3^k++n^k,$ then prove that $\sum_{r=1}^{m}\hat{}(m+1)C_r s_r=(n+1)^{m+1}-(n+1)\cdot$

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4. Let $\left(1+x^2\right)^2(1+x)^n = \sum_{k=0}^{n+4} a_k x^k$. If a_1, a_2 and a_3 are in arithmetic progression, then the possible value/values of n is/are a. 5 b. 4 c. 3 d. 2

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5. Findthe coefficient of $x^2 \in \left(x+\frac{1}{x}\right)^{10}.\left(1-x+2x^2\right)$

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6. Findthe coefficient of x^4 in te expansion of $\left(1+x-2x^2\right)^6$

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7. If $\left(1-x^3\right)^n = \sum_{r=0}^{n} a_r x^r (1-x)^{3n-2r}$, then the value of a_r, where $n \in N$ is

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8. Find the consecutive terms in the binomial expansion oif $(3+2x)^7$ whose coefficients are equal

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9. If $a_0, a_1, a_2, \ldots\ldots a_n$ be the successive coefficients in the expnsion of $(1+x)^n$ show that $(a_0 - a_2 + a_4 \ldots\ldots\ldots)^2 + (a_1 - a_3 + a_5 \ldots\ldots\ldots)^2 = a_0 + a_1 + a_2 + \ldots\ldots$

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10. If n is positive integer show that $9^n + 7$ is divisible 8

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11. If n is a positive integer, then show tha $3^{2n+1} + 2^{n+2}$ is divisible by 7 .

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12. If $\frac{nC_0}{2^n} + 2.\ \frac{nC_1}{2^n} + 3.\ \frac{nC_2}{2^n} + ...\ (n+1)\frac{nC_n}{2^n} = 16$ then the value of 'n' is

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13. If $a_1, a_2, a_3, \ldots a_{n+1}$ are in arithmetic progression, then $\sum_{k=0}^{n} .^n C_{k.a_{k+1}}$ is equal to (a)$2^n(a_1 + a_{n+1})$ (b)$2^{n-1}(a_1 + a_{n+1})$ (c) $2^{n+1}(a_1 + a_{n+1})$ (d)$(a_1 + a_{n+1})$

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14. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots.. + C_nx^n$, show that $3.\ C_0 + 3^2.\ \frac{C_1}{2} + 3^3.\ \frac{C_2}{2} + .\ . + 3^{n+1}.\ \frac{C_n}{n+1} = \frac{4^{n+1} - 1}{n+1}$

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15. Deduce that: $\sum_{r=0}^{n} .^n C_r(-1)^r \frac{1}{(r+1)(r+2)} = \frac{1}{n+2}$

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16. If P_4 denote the produt of the binomial coefficients in the expansion of $(1+x)^n$, then $\frac{P_{n+1}}{P_n}$ equals

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17. If n is a positive integer, prove that

$$\sum_{r=1}^{n} r^3 \left(\frac{{}^nC_r}{{}^nC_{r-1}}\right)^2 = \frac{(n)(n+1)^2(n+2)}{12}$$

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18. Find the coefficients of x^4 in the expansion of $\left(1+2x+x^2\right)^3$

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19. Find the number of terms in the expansion of $(a+b+c+d+e)^{100}$

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20. If in the expansion of $\left(2a-\frac{a^2}{4}\right)^9$ the sum of middle tem sis S, then the following is (are) thrue (A) $S=\left(\frac{63}{32}\right)a^{14}(a+8)$ (B) $S=\left(\frac{63}{32}\right)a^{14}(a-8)$ (C) $S=\left(\frac{63}{32}\right)a^{13}(a-8)$ (D) $S=\left(\frac{63}{32}\right)a^{13}(8-a)$

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21. In the expansion of $(1+x)^{50}$ the sum of the coefficients of odd power of x is (A) 0 (B) 2^{50} (C) 2^{49} (D) 2^{51}

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22. If the coefficients of $x^2 and\ x^3$ in the expansion o $(3+ax)^9$ are the same, then the value of a is $-\frac{7}{9}$ b. $-\frac{9}{7}$ c. $\frac{7}{9}$ d. $\frac{9}{7}$

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23. if the rth term in the expansion of $\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$ contains x^4 then r is equal to

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24. I the expansin of $\left(x^2+\frac{2}{x}\right)^n$ for positive integer n has 13th term independent of x, then the sum of divisors of n is (A) 36 (B) 38 (C) 39 (D) 32

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25. The expansion $\left\{x+\left(x^3-1\right)^{1/2}\right\}^5+\left\{x-\left(x^3-1\right)^{1/2}\right\}^5$ is a polynomial of degree

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26. The coefficient of x^4 in the expansion of $\left(1+x+x^2+x^3\right)^n$

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27. If $\left(1-x+x^2\right)^n=a_0+a_1x+a_2x^2+\ldots\ldots..+a_{2n}x^{2n}$, find the value of $a_0+a_2+a_4+\ldots\ldots.+a_{2n}$.

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28. If $a_n=\sum_{r=0}^{n}\frac{1}{\hat{}\,nC_r}$, then $\sum_{r=0}^{n}\frac{r}{\hat{}\,nC_r}$ equals $(n-1)a_n$ b. na_n c. $(1/2)na_n$ d. none of these

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29. Find the term independent of x in the expansion of

$$\left(\sqrt{\frac{x}{3}}+\left(\frac{\sqrt{3}}{2x^2}\right)\right)^{10}$$

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30. For integer $n > 1$, the digit at unit's place in the number

$$\sum_{r=0}^{100} r! + 2^{2^n} \text{ I}$$

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31. If in the expansion of $(1+x)^m(1-x)^n$, the coefficients of x and x^2 are 3 and - 6 respectively, the value of m and n are

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32. The coefficient of term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is

(A) $\frac{9}{4}$ (B) $\frac{3}{4}$

(C) $\frac{5}{4}$ (D) $\frac{7}{4}$

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33. Prove that $\frac{C_1}{2}+\frac{C_3}{3}+\frac{C_5}{6}+ \;=\frac{2^{n+1}-1}{n+1}.$

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34. Find the coefficient of $x^k \in 1+(1+x)+(1+x)^2+ \;+(1+x)^n(0\leq k\leq n).$

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35. The term independent of x in the expansion of $\left(1+x+2x^2\right)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is (A) $\frac{7}{18}$ (B) $\frac{2}{27}$ (C) $\frac{7}{18}+\frac{2}{27}$ (D) $\left(\frac{7}{18}\right)-\left(\frac{2}{27}\right)$

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36. If the largest interval to which x belongs so that the greatest therm in $(1+x)^{2n}$ has the greatest coefficient is $\left(\frac{10}{11}, \frac{11}{10}\right)$ then n= (A) 9 (B) 10 (C) 11 (D) none of these

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37. The number of terms in te expansion of $\left(1+5x+10x^2+10x^3+x^5\right)^{20}$ is (A) 100 (B) 101 (C) 120 (D) none of these

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38. The number of terms in the expansion $\left(x^2+\frac{1}{x^2}+2\right)^{100}$ is (A) 3200

(B) 102 (C) 201 (D) none of these

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39. The number of terms ins $\left(x^3+1+\frac{1}{x^3}\right)^{100}$ is (A) 300 (B) 200 (C) 100

(D) 201

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40. The number of terms in the expansion of $\left(x+\frac{1}{x}+1\right)^n$ is (A) 2n (B)

$2n+1$ (C) $2n-1$ (D) none of these

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42. The coefficient of a^3b^6c in the expansionof $(2a-b+c)^{10}$ is (A) 6720

(B) 840 (C) 10 (D) none of these

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43. Thenumber of distinct terms in the expansion of

$(x_1+x_2+\ldots\ldots.+x_p)^n$ is

(A) $.^{n+p}C_n$

(B) $.^nC_1$

(C) $n+1$

(D) $.^{n+p-1}C_{p-1}$

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44. If $\{x\}$ denotes the fractional part of 'x' , then $82\left\{\frac{3^{1001}}{82}\right\}=$

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45. The digit at unit's place in $2^{9^{100}}$ is

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46. If $(1+ax)^n = 1 + 8x + 24x^2 + \ldots$, then a =... and n= ....

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47. The sum of the coefficients in $(1 + x - 3x^2)^{2143}$ is

A. (a) 2^{2143}

B. (b)0

C. (c)1

D. (d)-1

Answer: (d)

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48. The coefficients fo x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ are in the ratio

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49. If $\dfrac{1}{1+2x+x^2}=1+a_1x+a_2x^2+\ldots..$ then the value of a_r is (A) 2 (B) $r+1$ (C) r (D) $2r$

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50. The coefficients of x^7 in the expansion of $\left(1-x^4\right)(1+x)^9$ is (A) 27 (B) -24 (C) 48 (D) -48

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51. If $\left(1+x+x^2+x^3\right)^n=\sum_{r=0}^{3n} a_r x^r \text{and} \sum_{r=0}^{3n} a_r=k$ and if $\sum_{r=0}^{3n} r a_r=\frac{\lambda n k}{2}$, the value of λ is

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52. If the number of terms in $\left(x+1+\frac{1}{x}\right)^n$ n being a natural number is 301 the n=

A. (A)300

B. (B) 100

C. (C) 149

D. (D) 150

Answer: null

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53. Find the coefficient of x^5 in the expansion of $\left(1+x^2\right)^5(1+x)^4$.

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54. Let the co-efficients of x' in $(1+x)^{2n}$ & $(1+x)^{2n-1}$ be P & Q respectively then $\left(\frac{P+Q}{Q}\right)^5 =$

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55. The sum of the coefficients of powers of x int eh expansion of the polynomial $\left(x-3x^2+x^3\right)^{99}$ is (A) 0 (B) 1 (C) 2 (D) -1

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56. The sixth term in the expansion of

$\left[\sqrt{\left\{2^{\log(10-3^x)}\right\}}+5\sqrt{\left\{2^{(x-2)\log 3}\right\}}\right]^m$ is equal to 21, if it is know that the binomial coefficient of the 2^{nd}, 3^{nd} and 4^{th} terms in the expansion represents, respectively, the first, third and fifth terms of an A.P. (the symbol log stand for logarithm tothe base 10). The minimum of expansion is

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57. A student wrote $(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots f$ or $-2 < x < 2$ and got zero marks because

(A) x was allowed to be 0

(B) x was allowed to be -ve

(C) x was allowed to have negative as well as positive values

(D) $|x|$ was greater than 1 for some values of x

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58. If the coefficient of the rth, $(r+1)th$ and $(r+2)th$ terms in the expansion of $(1+x)^n$ are in A.P., prove that $n^2 - n(4r+1) + 4r^2 - 2 = 0$.

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59. The two consecutive term in the expansion of $(3+2x)^{74}$ which have equal cofficients are

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60. If the 21st and 22nd terms in the expansin of $(1+x)^{44}$ are equal then x is equal to $(A)\frac{21}{20}$ (B) $\frac{23}{24}$ (C) $\frac{8}{7}$ (D) $\frac{7}{8}$

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61. If C_r stands for $.^n C_r$ and $\sum_{r=1}^{n}\frac{r.\,C_r}{C_{r-1}}=210$ then $n=$

(A) 19

(B) 20

(C) 21

(D) none of these

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62. If $(1+x)^n = \sum_{r=0}^{n} a_2 x^r, b_r = 1 + \frac{a_r}{a_r - 1}$

and $\prod_{r=1}^{n} b_r = \frac{(101)^{100}}{100!}$, then the value of $\frac{n}{20}$ is

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63. If P_n denotes the product of all the coefficients of $(1+x)^n$ and $8! P_{n+1} = 9^8 P_n$ then n is equal to

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64. If the coefficient of x^{100} is in $1 + (1+x) + (1+x)^2 + (1+x)^3 + \ldots\ldots\ldots + (1+x)^n, (n \geq 100) is^{201} ($

then n (A) 100 (B) 200 (C) 101 (D) none of these

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65. If $\hat{}\, n\frac{C_0}{2} -^n \frac{C_1}{3} +^n \frac{C_2}{4} - \ldots\ldots + (-1)^n \hat{}\, n\frac{C_n}{n+2}$ = $\frac{1}{1999x2000} thenn$= (A) 1998 (B) 1999 (C) 2000 (D) 2001

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66. Statement-1: If $\sum_{r=1}^{n} r^3 \left(\frac{.^n C_r}{.^n C_{r-1}}\right)^2 = 196,$ then the sum of the coeficients of powerr of xin the expansion of the polynomial $(x - 3x^2 + x^3)^n$ is -1.

Statement-2: $\frac{.^n C_r}{.^n C_{r-1}} = \frac{n-r+1}{r} \forall n \in N \text{ and } r \in W.$

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67. If $(1 + x + x^2 + x^3)^n = \sum_{r=0}^{3n} a_r x^r \text{and} \sum_{r=0}^{3n} a_r = k$ and if $\sum_{r=0}^{3n} r a_r = \frac{\lambda n k}{2}$, the value of λ is

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68. If in the expansion of $(1+x)^n$ the coefficients of 14th, 15th and 16th terms are in A.P. then n=` (A) 12 (B) 23 (C) 27 (D) 34

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69. If the four consecutive coefficients in any binomial expansion be a, b, c and d then (A) $\frac{a+b}{a}, \frac{b+c}{b}, \frac{c+d}{c}$ are in H.P. (B) $(bc+ad)(b-c) = 2\left(ac^2 - b^2 d\right)$ (C) $\frac{b}{a}, \frac{c}{b}, \frac{d}{c}$ are in A.P. (D) none of these

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70. In the expansion of $(a+b+c)^{10}$ (A) total number of terms in 66 (B) coefficient of $a^8 bcis 90$ (C) coefficient of $a^4 b^5 c^3$ is 0 (D) none of these

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71. Let $a_n = \frac{1000^n}{n!}$ for $n \in N$, then a_n is greatest, when value of n is

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72. If in the expansion of $(a+b)^n, n\varepsilon N$ sum of odd and even terms be α and β respectively, then (A) $\left(a^2-b^2\right)^n = \alpha^2 - \beta^2 (B)$ $\left(a^2-b^2\right)^n = (\alpha-\beta)^n$ (C) $(a+b)^n - (a-b)^n = 4\alpha\beta (D)$ $(a+b)^{2n} - (a-b)^{2n} = 4\alpha\beta$

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73. If the 4th term in the expansion of $(ax + 1/x)^n$ is 5/2, then a. $a = \frac{1}{2}$ b. $n = 8$ c. $a = \frac{2}{3}$ d. $n = 6$

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74. If in the expansion of $(1+x)^m(1-x)^n$, the coefficients of x and x^2 are 3 and - 6 respectively, the value of m and n are

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75. In the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n, n \in N,$ number of terms is $2n+1$

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76. If n is a positive integer then $\hat{} \; nC_r +^n C_{r+1} =^{n+1} \; \hat{} \; C_{r+1}$ Also coefficient of x^r in the expansion of $(1+x)^n =^n C_r$. In an identity in x, coefficient of similar powers of x on the two sides re equal. On the basis of above information answer the following question: If n is a positive integer then $\hat{} \; nC_n +^{n+1} C_n +^{n+2} C_n + \ldots . . +^{n+k} C_n =$ (A) $\hat{} \; (n+k+1)C_{n+2}$ (B) $\hat{} \; (n+k+1)C_{n+1}$ (C) $\hat{} \; (n+k+1)C_k$ (D) $\hat{} \; (n+k+1)C_{n-2}$

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77. If n is a positive integer then $\hat{} \ nC_r +^n C_{r+1} =^{n+1} \ \hat{} \ C_{r+1}$ Also coefficient of x^r in the expansion of $(1+x)^n =^n C_r$. answer the following question: If n is a positive integer then $\hat{} \ nC_n +^{n+1} C_n +^{n+2} C_n + \ldots. +^{n+k} C_n =$ (A) $\hat{} \ (n+k+1)C_{n+2}$ (B) $\hat{} \ (n+k+1)C_{n+1}$ (C) $\hat{} \ (n+k+1)C_k$ (D) $\hat{} \ (n+k+1)C_{n-2}$

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78. If n is a positive integer then $\hat{} \ nC_r +^n C_{r+1} =^{n+1} \ \hat{} \ C_{r+1}$ Also coefficient of x^r in the expansion of $(1+x)^n =^n C_r$. answer the following question: If n is a positive integer then $\hat{} \ nC_n +^{n+1} C_n +^{n+2} C_n + \ldots. +^{n+k} C_n =$ (A) $\hat{} \ (n+k+1)C_{n+2}$ (B) $\hat{} \ (n+k+1)C_{n+1}$ (C) $\hat{} \ (n+k+1)C_k$ (D) $\hat{} \ (n+k+1)C_{n-2}$

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79. If n is a positive integer then

$$(1+x)^n = ^n C_0 x^0 + ^n C_1 x^1 + ^n C_2^2 + \ldots\ldots\ldots + ^n C_r x^r = \sum_{r=0}^{n} \hat{} \; nC_r x^r \text{ an}$$

$\hat{} \; nC_r x^r$

On the basis of above information answer the following question: If n is a positive integer then $\frac{1}{(49)^n}$-

$$\frac{8}{(49)^n}(\hat{}\;(2n)C_1) + \frac{8^2}{(49)^n}(\hat{}\;(2n)C_2) - \frac{8^3}{(49)^n}(\hat{}\;(2n)c_3) + \ldots\ldots + \frac{8^{2n}}{(49)}$$

(A) -1 (B) 1 (C) $\left(\frac{64}{49}\right)^n$ (D) none of these

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80.

$$\sum_{r=0}^{n}(-1)^r \;\hat{}\; nC_r\left[\frac{1}{2^r} + \frac{3}{2^{2r}} + \frac{7}{2^{3r}} + \frac{15}{2^{4r}} + up \to mterms\right] = \frac{2^{mn} -}{2^{mn}(2^n -}$$

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81. Sum of the series $a^n + a^{n-1}b + ^{n-2}b^2 + \ldots\ldots\ldots\ldots + ab^n$ can be obtained by taking outt a^n or b^n comon and using the forumula of sum of $(n+1)$ terms of G.P. N the basis of above information answer the following question: Coefficient of $x^{50} \in (1+x)^{1000} + x(1+x)^{999} + \ldots\ldots. + x^{999}(1+x) + x^{1000}$ is (A) $\hat{} \ 1000C_{50}$ (B) $\hat{} \ 1002C_{50}$ (C) $\hat{} \ 1001C_{50}$ (D) $\hat{} \ 1001C_{49}$

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82. Sum of the series $a^n + a^{n-1}b +^{n-2} b^2 + \ldots\ldots\ldots\ldots + ab^n$ can be obtained by taking outt a^n or b^n comon and using the forumula of sum of $(n+1)$ terms of G.P. answer the following question:Um of coeficients of x^{50} `(1+x)^1000 +x(1+x)^999+....

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83. Sum of the series $a^n + a^{n-1}b +^{n-2} b^2 + \ldots\ldots\ldots\ldots + ab^n$ can be obtained by taking outt a^n or b^n comon and using the forumula of sum of $(n+1)$ terms of G.P. N the basis of above information answer the following question: Coefficient of

$x^{50} \in (1+x)^{1000} + x(1+x)^{999} + \ldots\ldots. + x^{999}(1+x) + x^{1000}$ is (A) $\hat{}\ 1000C_{50}$ (B) $\hat{}\ 1002C_{50}$ (C) $\hat{}\ 1001C_{50}$ (D) $\hat{}\ 1001C_{49}$

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84. In a binomial expansion $(x+y)^n$ gretest term means numericaly greatest term and therefore greatest term in $(x-y)^n$ and $(x+y)^n$ are ame. I frth therm t_r be the greatest term in the expansion of $(x+y)^n$ whose therms are all ositive, then $t_r \geq t_{r+1}$ and $t_r \geq t_=(r-1)i.\ e.\ \frac{t_r}{t_m} \geq 1$ and $\frac{t_r}{t_{r-}} \geq 1$ ON the basis of above information answer the following question: Greatest term in the expansion of $\left(2+3x0^{10}, whernx = \frac{3}{5}\right.$ is (A) $\hat{}\ 10C_5\left(\frac{18}{5}\right)^5$ (B) $\hat{}\ 10C_6\left(\frac{18}{5}\right)^6$ (C) $\hat{}\ 10C_4\left(\frac{18}{5}\right)^4$ (D) none of these

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85. In a binomial expansion $(x_y)^n$ gretest term means numericaly greatest term and therefore greatest term in $(x-y)^n$ and $(x+y)^n$ are ame. I frth therm t_r be the greatest term in the expansion of $(x+y)^n$ whose therms are all ositive, then $t_r \geq t_{r+1}$ and $t_r \geq t_=(r-1)i.e.\ \frac{t_r}{t_m} \geq 1$ and $\frac{t_r}{t_{r-}} \geq 1$ On the basis of above information answer the following question:If rth term is the greatest term in the expansion f $\left(2-3x0^{10}\right.$ then r= (A) 5 (B) 6 (C) 7 (D) none of these

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86. In a binomial expansion $\left(x_y\right)^n$ gretest term means numericaly greatest term and therefore greatest term in $(x-y)^n$ and $(x+y)^n$ are ame. I frth therm t_r be the greatest term in the expansion of $(x+y)^n$ whose therms are all ositive, then $t_r \geq t_{r+1}$ and $t_r \geq t_=(r-1)i.e.\ \frac{t_r}{t_m} \geq 1$ and $\frac{t_r}{t_{r-}} \geq 1$ On the basis of above information answer the following question:The set al values of x for which thegreatest term in teh expnsionof $(1+x)^{30}$ may have the greatest coefficient is (A) $\left(\frac{14}{15}, \frac{15}{14}\right)$ (B) $\left[\frac{15}{16}, \frac{16}{15}\right]$ (C) $\left(\frac{15}{16}, \frac{16}{15}\right)$ (D) none of these

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87. Let a,b,c,d be the four consecutive coefficients of the binomial expansion $(1+x)^n$ On the basis of above information answer the following question: $\frac{a}{a+b}, \frac{b}{b+c}, \frac{c}{c+d}$ are in (A) A.P. (B) G.P. (C) H.P. (D) none of these

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88. If the four consecutive coefficients in the any binomial expansion be a,b,c,d then prove that: $\frac{a+b}{a}, \frac{b+c}{b}, \frac{c+d}{c}$ are in H.P.

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90. Assertion: $\sum_{r=0}^{n}(r+1).^{n}C_r=(n+2)2^{n-1}$, Reason: $\sum_{r=0}^{n}(r+1).^{n}C_r x^r=(1+x)^n+nx(1+x)^{n-1}$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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91. Assertion: Number of rational terms in the expansion of $\left(2^{\frac{1}{3}}+3^{\frac{1}{2}}\right)^{630}$ is 6, Reason: If p is a prime number then p^k is rational only when k is an integer (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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92. Assertion: If $(1+ax)^n = 1+8x+24^2+\ldots\ldots\ldots$ then vaues of a and n are 2 and 4 Reason IN an identity in x the coefficients of similar powers of x on the two sides are equal. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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93. Assertion: Sum of coefficient of the polynomiasl `(1+3x^2-5x^3)^2001 is -1. Reason: Sum of coefficients of a polynomial in x can be obtained by putting x=1 in the polynomial. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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94. Assertion: $\sum_{r=0}^{n}(r+1).^{n}C_r=(n+2)2^{n-1}$, Reason: $\sum_{r=0}^{n}(r+1).^{n}C_r x^r=(1+x)^n+nx(1+x)^{n-1}$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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95. Assertion: If n is an even positive integer n then $\sum_{r=1}^{n-1}\frac{1}{\lfloor r\lfloor n-r}=\frac{2^{n-1}}{\lfloor n}$. Reason: $\hat{}\ nC_1+^{n}C_3+\ldots\ldots..+^{n}C_{n-1}=2^{n-1}$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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96. Assertion: If n is an even positive integer n then $\sum_{r=0}^{n} \frac{{}^nC_r}{r+1} = \frac{2^{n+1}-1}{n+1}$, Reason : $\sum_{r=0}^{n} \frac{{}^nC_r}{r+1} x^r = \frac{(1+x)^{n+1}-1}{n+1}$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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97. Assertion: The coefficient of x^4 in $\left(1+x+x^2+x^3\right)^n$ is $\hat{}\ nC_4 +^n C_2 +^n C_1 .^n C_2$, Reason: $\left(1+x+x^2+x^3\right)^n = (1+x)^n\left(1+x^2\right)^n$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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98. Let the coefficients of (2r+1)th and (r+2)th terms be equal. Assertion: $r = 15$ Reason : $\hat{} \; nC_x =^I nC_y \rightarrow x + y = n$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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99. Let n $\in$ N and $f(n) = 2^{5n+5} - 31n - 32$ Assertion: $f(n)$ is divisible by 961, Reason : $2^{5n} = (1 + 31)^n$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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100. If $a_n = \sum_{r=0}^{n} \frac{1}{\hat{} \; nC_r}$, then $\sum_{r=0}^{n} \frac{r}{\hat{} \; nC_r}$ equals $(n-1)a_n$ b. na_n c. $(1/2)na_n$ d. none of these

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101. If in the expansion of $(1+x)^m(1-x)^n$, the coefficients of x and x^2 are 3 and - 6 respectively, the value of m and n are

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102. For any positive integer (m,n) (with $n \geq m$), Let $\binom{n}{m} = .^n C_m$ Prove that

$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + ... + (n-m+1)\binom{m}{m}$=

(n+2,m+2)

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103. Prove that

$$\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} = \binom{n+2}{r}$$

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104. In the binomial expansion of $(a-b)^{\cap} \geq 5,$ the sum of the 5th and 6th term is zero. Then a/b equals $(n-5)/6$ b. $(n-4)/5$ c. $n/(n-4)$ d. $6/(n-5)$

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105. The sum $\sum_{i=0}^{m} ((10)c, (i))\binom{20}{m-1}$, where $\binom{p}{q} = 0$ if $p < q$, is maximum when m is equal to (A) 5 (B) 10 (C) 15 (D) 20

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106. The coefficient of x^{24} in

$(1+x^2)^{12}(1+x^{12})(1+x^{24})$ is

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107. If n and k are positive integers, show that $2^k(nC0)(nk) - 2^{k-1}(nC1)(n-1Ck-1) + 2^{k-2}(nC2)((n-2k-2)) \_ ...$ stands for $\hat{} \ nC_k.$

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108. If $\hat{} \ n-1C_r = (k^2-3)^n C_{r+1}, then k \in \ (-\infty, -2]$ b. $[2, \infty)$ c. $[-\sqrt{3}, \sqrt{3}]$ d. $(\sqrt{3}, 2]$

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109. The value of $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + (302)(3012) + + (3020)(3030) =$ a. $\hat{} \ 60C20$ b. $\hat{} \ 30C10$ c. $\hat{} \ 60C30$ d. $\hat{} \ 40C30$

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