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India's Number 1 Education App

MATHS

BOOKS - KC SINHA ENGLISH

COMPLEX NUMBERS - FOR COMPETITION

Solved Examples

1. Express $\dfrac{1}{1+\cos\theta - I\sin\theta}$ in the form of a +ib .

Hint : $\sin^2\theta + \cos^2\theta = 1$

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2. If $z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$ and $z_1 = \dfrac{i(z_2+1)}{z_2-1}$, prove that $x_1^2 + y_1^2 - x_1 = \dfrac{x_2^2 + y_2^2 + 2x_2 - 2y_2 + 1}{(x_2-1)^2 + y_2^2}$

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3. Find the complex number z such that $z^2 + |z| = 0$

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4. Show that the equation $A^2/(x-a) + B^2/(x-b) + C^2/(x-c) + \quad + H^2/(x-h) = k$ has no imaginary root, where $A, B, C, , Handa, b, c, handk \in R\cdot$

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5. If α be a root of equation $x^2 + x + 1 = 0$ then find the vlaue of $\left(\alpha + \frac{1}{\alpha}\right) + \left(\alpha^2 + \frac{1}{\alpha^2}\right)^2 + \left(\alpha^3 + \frac{1}{\alpha^3}\right)^2 + \ldots + \left(\alpha^6 + \frac{1}{\alpha^6}\right)^2$

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6. If n is anodd integer greter than 3 but not a multiple of 3 prove that $\left[(x+y)^n - x^n - y^n\right]$ is divisible by $xy(x+y)\left(x^2+xy+y^2\right)$.

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7. Prove that $\left|\frac{z_1 - z_2}{1 - \bar{z}_1 z_2}\right| < 1 \text{ if } |z_1| < 1, |z_2| < 1$

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8. If $|z_1 = 1, |z_2| = 2, |z_3| = 3 \text{ and } |z_1 + z_2 + z_3| = 1, then|9z_1z_2 + 4z_3z_1 + z$

is equal to

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9. Let z_1, z_2 and z_3 be three distinct complex numbers , satisfying $|z_1| = |z_2| = |z_3| = 1$. Which of the following is/are true :

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10. If $\frac{3}{2} + \cos\theta + i\sin\theta = a + ib$, prove that $a^2 + b^2 = 4a - 3$

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11. If z_1, z_2, z_3 are distinct nonzero complex numbers and $a, b, c \in R^+$ such that $\frac{a}{|z_1 - z_2|} = \frac{b}{|z_2 - z_3|} = \frac{c}{|z_3 - z_1|}$ Then find the value of $\frac{a^2}{|z_1 - z_2|} + \frac{b^2}{|z_2 - z_3|} + \frac{c^2}{|z_3 - z_1|}$

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12. If $1, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{n-1}$ are n, nth roots of unity, then $(1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3)\ldots(1 - \alpha_{n-1})$ equals to

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13. If the argument of $(z-a)(\bar{z}-b)$ is equal to that $\left((\sqrt{3}+i)\dfrac{1+\sqrt{3}i}{1+i}\right)$ where a,b,c are two real number and z is the complex conjugate o the complex number z find the locus of z in the rgand diagram. Find the value of a and b so that locus becomes a circle having its centre at $\dfrac{1}{2}(3+i)$

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14. Find the locus of z if arg $\left(\dfrac{z-1}{z+1}\right)=\dfrac{\pi}{4}$

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15. If z_1, z_2, z_3 are the roots of cubic $3z^3+3az^2=a^2z+b=0$ then find the value of $\dfrac{1}{z_1-z_2}+\dfrac{1}{z_2-z_3}+\dfrac{1}{z_3-z_1}$ $3a+b$ b. $a+b$ c. 6 d. 0 e. b

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16. If z_1, z_2 and z_3 are the vertices of ΔABC, which is not right angled triangle taken in anti-clock wise direction and z_0 is the circumcentre, then $\left(\frac{z_0 - z_1}{z_0 - z_2}\right)\frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right)\frac{\sin 2C}{\sin 2B}$ is equal to

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17. Let the complex numbers z_1, z_2 and z_3 be the vertices of an equilateral triangle let z_0 be the circumcentre of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$

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18. Two different non-parallel lines cut the circle $|z| = r$ at points a, b, c and d, respectively. Prove that these lines meet at the point z given by $\frac{a^{-1} + b^{-1} - c^{-1} - d^{-1}}{a^{-1}b^{-1} - c^{-1}d^{-1}}$

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19. If $\sqrt[3]{a+ib}=\xi y$ then prove that $\dfrac{a}{x}+\dfrac{b}{y}=4()x^2-y^2\Big)$

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20. Point P represents the complex num,ber $z=x+iy$ and point Q the complex num,ber $z+\dfrac{1}{z}$. Show that if P mioves on the circle $|z|=2$ then Q oves on the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{9}=\dfrac{1}{9}$. If z is a complex such that $|z|=2$ show that the locus of $z+\dfrac{1}{z}$ is an ellipse.

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21. Let

$$f(\theta)=\left(\cos\theta-\cos\frac{\pi}{8}\right)\left(\cos\theta-\cos\frac{3\pi}{8}\right)\left(\cos\theta-\cos\frac{5\pi}{8}\right)\left(\cos\theta-\cos\frac{7\pi}{8}\right.$$

then :

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22. The points, $z_1, z_2, z_3, z_4,$ in the complex plane are the vertices of a parallelogram taken in order, if and only if (a)$z_1 + z_4 = z_2 + z_3$ (b) $z_1 + z_3 = z_2 + z_4$ (c)$z_1 + z_2 = z_3 + z_4$ (d) None of these

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23. State true or false for the following.

For any complex number z, the minimum value of |z| + |z-1| is 1 .

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24. The least positive integer n for which $\left(\frac{1-i}{1-i}\right)^n = \frac{2}{\pi}\sin^{-1}\frac{1+x^2}{2x},$ where x > 0 and $i = \sqrt{-1}$ is :

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25. Find the point of intersection of the curves $arg(z-3i)=\frac{3\pi}{4}andarg(2z+1-2i)=\pi/4.$

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27. If all the roots of $z^3+az^2+bz+c=0$ are of unit modulus, then (A) $|a|\leq 3$ (B) $|b|\leq 3$ (C) $|c|=1$ (D) none of these

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28. Let z_1, z_2 and origin represent vertices A,B,O respectively of an isosceles triangel OAB, where OA=OB and $\angle AOB=2\theta.$ If z_1, z_2 are the roots of the equation $z^2 + 2az + b = 0$ where a,b re comlex numbers then $\cos^2\theta =$ (A) $\frac{a}{b}$ (B) $\frac{a^2}{b^2}$ (C) $\frac{a}{b^2}$ (D) $\frac{a^2}{b}$

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30. If $k > 1, |z_1|, k$ and $\left|\frac{k - z_1\bar{z}_2}{z_1 - kz_2}\right| = 1$, then (A) $z_2 = 0$ (B) $|z_2| = 1$ (C) $|z_2| = 4$ (D) $|z_2| < k$

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31. Show that the area of the triangle on the Argand diagram formed by the complex number $z, izandz + iz$ is $\frac{1}{2}|z|^2$

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32. Let $z_1 = 6 + i$ and $z_2 = 4 - 3i$. If z is a complex number such thar $\arg\left(\frac{z - z_1}{z_2 - z}\right) = \frac{\pi}{2}$ then (A) $|z - (5 - i) = \sqrt{5}$ (B) $|z - (5 + i) = \sqrt{5}$ (C) $|z - (5 - i)| = 5$ (D) $|z - (5 + i)| = 5$

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33. If z and $\bar{z}$ represent adjacent vertices of a regular polygon of n sides where centre is origin and if $\frac{Im(z)}{Re(z)} = \sqrt{2} - 1$, then n is equal to:

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34. Let $z_1 and z_2$ be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1 + z_2}{z_1 - z_2}$ may be (a) zero (b) real and positive (c) real and negative (d) purely imaginary

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35. Let the complex numbers z of the form $x+iy$ satisfy arg $\left(\frac{3z-6-3i}{2z-8-6i}\right)=\frac{\pi}{4}$ and $|z-3+i|=3$. Then the ordered pairs (x,y) are (A) $\left(4-\frac{4}{\sqrt{5}},1+\frac{2}{\sqrt{5}}\right)$ (B) $\left(4+\frac{5}{\sqrt{5}},1-\frac{2}{\sqrt{5}}\right)$ (C) $(6-1)$ (D) $(0,1)$

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36. If $z_1=a+ib$ and $z_2=c+id$ are complex numbers such that $|z_1|=|z_2|=1$ and $Re(z_1\bar{z}_2)=0$, then the pair ofcomplex nunmbers $\omega_1=a+ic$ and $\omega_2=b+id$ satisfies

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37. If z_1, z_2, z_3 are non zero non collinear complex number such that $\frac{2}{z_1}=\frac{1}{z_2}+\frac{1}{z_3}, then$ (A) ponts z_1, z_2, z_3 form and equilateral triangle (B) points z_1, z_2, z_3lies on a circle (C) z_1, z_2, z_3 and origin are concylic (D) $z_1+z_2+z_3=0$

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38. If $\cos\alpha + \cos\beta + \cos\gamma = 0 and also \sin\alpha + \sin\beta + \sin\gamma = 0$, then prove that $\cos 2\alpha + \cos 2\beta + \cos 2\gamma \quad = \sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$ $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$ $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

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39. let $z1, z2, z3$ be vertices of $\triangle ABC$ in an anticlockwise order and $\angle ACB = \theta$ then $z_2 - z_3 = \frac{CB}{CA}(z_1 - z3)e^i\theta$. let p point on a circle with op diameter 2 points Q & R taken on a circle such that $\angle POQ\&QOR = \theta$ if O be origin and PQR are complex no. $z1, z2, z3$ respectively then $\frac{z_2}{z_1} =$ (A) $e^{i\theta}\cos\theta$ (B) $e^{i\theta}\cos 2\theta$ (C) $e^{-i\theta}\cos\theta$ (D) $e^{2i\theta}\cos 2\theta$

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40.

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41. let $z1, z2, z3$ be vertices of $\triangle ABC$ in an anticlockwise order and $\angle ACB = \theta$ then $z_2 - z_3 = C\frac{B}{C}A(z_1 - z3)e^{i}\theta$ .let p point on a circle with op diameter 2points Q & R taken on a circle such that $\angle POQ\&QOR = \theta$ if O be origin and PQR are complex no. $z1, z2, z3$ respectively then $\frac{z_3^2}{z_1 . z_2} =$ (A) $\sec^2\theta . \cos 2\theta$ (B) $\cos\theta . \sec^2(2\theta)$ (C) $\cos^2\theta . \sec 2\theta$ (D) $\sec\theta . \sec^2(2\theta)$

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42. Which of the following is (are) correct? (A) $\bar{a}z_1 + a\bar{z}_1 - \bar{a}z_2 - a\bar{z}_2 = 0$ (B) $\bar{a}z_1 + a\bar{z}_1 + \bar{a}z_2 + a\bar{z}_2 = -b$ (C) $\bar{a}z_1 + a\bar{z}_1 + \bar{a}z_2 + a\bar{z}_2 = 2b$ (D) $\bar{a}z_1 + a\bar{z}_1 + \bar{a}z_2 + a\bar{z}_2 = -2b$

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43. Which of the following is (are) correct? (A) $\overline{z_1 - z_2} - a(\bar{z}_1 - \bar{z}_2) = 0$ (B) $\overline{z_1 - z_2} + a(\bar{z}_1 - \bar{z}_2) = 0$ (C) $\overline{z_1 - z_2} + a(\bar{z}_1 - \bar{z}_2) = -b$ (D) $\overline{z_1 - z_2} + a(\bar{z}_1 - \bar{z}_2) = -b$

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44. Which of the following is (are) correct? (A) $\bar{z}_1 + a\bar{z}_2 = 2b$ (B) $\bar{z}_1 + a\bar{z}_2 = b$ (C) $\bar{z}_1 + a\bar{z}_2 = -b$ (D) $\bar{z}_1 + a\bar{z}_2 = -2b$

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45. If $2 + z + z^4 = 0, where z$ is a complex number then (A) $\frac{1}{2} < |z| < 1$ (B) $\frac{1}{2} < |z| < \frac{1}{3}$ (C) $|z| \geq 1$ (D) none of these

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46. If $|a_n| < 1f$ or $n = 1, 2, 3, \ldots$ and $1 + a_1z + a_2z^2 + \ldots + a_nz^n = 0$ then z lies (A) on the circle $|z| = \frac{1}{2}$ (B) inside the circle $|z| = \frac{1}{2}$ (C) outside the circle $|z| = \frac{1}{2}$ (D) on the chord of the circle $|z| = \frac{1}{2}$ cut off by the line $Re[(1+i)z] = 0$

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47. zo is one of the roots of the equation $z^n \cos\theta_0 + z^{n-1}\cos\theta_2 + \ldots\ldots + z\cos\theta_{n-1} + \cos\theta_n = 2$, where $\theta \in R$, then

(A) $|z_0| < \frac{1}{2}$

(B) $|z_0| > \frac{1}{2}$

(C) $|z_0| = \frac{1}{2}$

(D)None of these

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48. If ω and ω^2 are the nonreal cube roots of unity and $[1/(a+\omega)]+[1/(b+\omega)]+(1/(c+\omega)]=2\omega^2$ and $[1/(a+\omega^2)]+[1/(b+\omega^2)]+[(1/(c+\omega^2)]+[1/(c+\omega^2)]=2\omega$, then find the value of $[1/(a+1)]+[1/(b+1)]+[1/(c+1)]$

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49. Given that the complex numbers which satisfy the equation $|z\bar{z}^3|+|\bar{z}z^3|=350$ form a rectangle in the Argand plane with the length of its diagonal having an integral number of units, then area of rectangle is 48 sq. units if z_1, z_2, z_3, z_4 are vertices of rectangle, then $z_1+z_2+z_3+z_4=0$ rectangle is symmetrical about the real axis $arg(z_1-z_3)=\frac{\pi}{4}$ or $\frac{3\pi}{4}$

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Exercise

1. Put the following in the form $A + iB$: $\dfrac{(\cos x + i\sin x)(\cos y + i\sin y)}{(\cot u + i)(1 + i\tan v)}$

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2. IF $a \geq 1,$ find all complex numbers z satisfying the equation $z + a|z + 1| + i = 0$

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3. ω is an imaginary root of unity. Prove that If $a + b + c = 0,$ then prove that $(a + b\omega + c\omega^2)^3 + (a + b\omega^2 + c\omega^{\square})^3 = 27ab \cdot$

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4. Find the integral solutions of the following equation: $(3 + 4i)^x = 5^{\frac{x}{2}}$

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5. Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$.

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6. Find the integral solutions of the following equation: $(1-i)^x = (1+i)^x$

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7. Let $\left|\left((z)_1 - 2(z)_2\right) / \left(2 - z_1(z)_2\right)\right| = 1 and |z_2| \neq 1, where z_1 and z_2$ are complex numbers. Show that $|z_1| = 2$.

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8. if a,b,c are complex numbers such that $a+b+c=0$ and $|a| = |b| = |c| = 1$ find the value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

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9. Show that for any two non zero complex numbers z_1, z_2 $(|z_1| + |z_2|)|z_1|z_1| + z_2|z_2|| \leq 2|z_1 + z_2|$

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10. Prove that $\left|\frac{z-1}{1-\bar{z}}\right| = 1$ where z is as complex number.

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11. Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$ given that one of roots is $2 + \sqrt{-3}$

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12. If z_1, z_2, z_3 are the roots of cubic $3z^3 + 3az^2 = a^2z + b = 0$ then find the value of $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1}$ $3a + b$ b. $a + b$ c. 6 d. 0 e. b

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14. If $\left|z^2 - 1\right| = |z|^2 + 1$ show that the locus of z is as straight line.

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15. If $\log_{\sqrt{3}}\left|\frac{|z|^2 - |z| + 1}{|z| + 2}\right| < 2$ then locus of z is

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16. Three points represented by the complex numbers a,b,c lie on a circle with centre O and rdius r. The tangent at C cuts the chord joining the points a,b and z. Show that $z = \dfrac{a^{-1} + b^{-1} - 2c^{-1}}{a^{-1}b^{-1} - c^{-2}}$

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17. Show that $\left(\dfrac{1 + \cos\phi + i\sin\phi}{1 + \cos\phi - i\sin\phi}\right)^n = \cos n\phi + i\sin n\phi$

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18. Show that the roots of equation $(1+z)^n = (1-z)^n are - i\tan\left(r\dfrac{\pi}{n}\right), r = 0, 1, 2, , \ldots\ldots\ldots\ldots, (n-1)$ excluding the vlaue when n is even and $r = \dfrac{n}{2}$

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19. Find the least positive integer n for which $\left(\dfrac{1+i}{1-i}\right)^n$

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21. For any integer n, the argument of $\frac{\left(\sqrt{3}+i\right)^{4n+1}}{\left(1-i\sqrt{3}\right)^{4n}}$

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22. Values of $\left(1-i\sqrt{3}\right)^{\frac{1}{3}}$ is (are) (A) $2^{\frac{1}{3}}\left(\cos 20^0+i\sin 20^0\right)$ (B) $2^{\frac{1}{3}}\left(\cos 20^0-i\sin 20^0\right)$ (C) $2^{\frac{1}{3}}\left(\cos 100^0+i\sin 100^0\right)$ (D) $2^{\frac{1}{3}}\left(\cos 220^0+i\sin 220^0\right)$

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23. The complex numbers z_1, z_2 and the origin form an equilateral triangle only if (A) $z_1^2 + z_2^2 - z_1 z_2 = 0$ (B) $z_1 + z_2 = z_1 z_2$ (C) $z_1^2 - z_2^2 = z_1 z_2$ (D) none of these

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24. for any complex nuber z maximum value of $|z| - |z - 1|$ is (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$

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25. $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$ is equal to

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26. The argument of $\frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$ is 60^0 b. 120^0 c. 210^0 d. 240^0

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27. Which of the following is not correct? (A) $|7+i|>|5+i|$ (B) $|7+i|>|7-i|$ (C) $|7+2i|>|7+i|$ (D) none of these

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28. If Z is a complex number the radius of $z\bar{z}-(2+3i)z-(2-3i)\bar{z}+9=0$ is equal to

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29. The polynomial $x^6+4x^5+3x^4+2x^3+x+1$ is divisible by______ where ω is one of the imaginary cube roots of unity. (a) $x+\omega$ (b) $x+\omega^2$ (c) $(x+\omega)(x+\omega^2)$ (d) $(x-\omega)(x-\omega^2)$

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30. In Argand diagram, O, P, Q represent the origin, z and $z + iz$ respectively then $\angle OPQ$=

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32. The value of $(\sin\theta + i\cos\theta)^n$ is (A) $\sin n\theta + i\cos n\theta$ (B) $\cos n\theta - i\sin n\theta$ (C) $\cos\left(\frac{n\pi}{2} - n\theta\right) + is\sin\left(\frac{n\pi}{2} - n\theta\right)$ (D) none of these

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33. If $x = 2 + 5i$ $(where$i^2 =-1) and 2(1/(1!9!)+1/(3!7!))+1/(5!5!)=2^a/(b!), $then the value of$(x^3 - 5x^2 + 33x - 19)` is equal to

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34. $|z-i| < |z+i|$ represents the region (A) $Re(z) > 0$ (B) $Re(z) < 0$ (C) $Im(z) > 0$ (D) $Im(z) < 0$

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35. The points representing complex numbers z for which $|z-3| = |z-5|$ lie on the locus given by (A) circle (B) ellipse (C) straight line (D) none of these

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37. if $1, \omega, \omega^2, \ldots\ldots\ldots\ldots \omega^{n-1}$ are nth roots of unity , then $(1-\omega)(1-\omega^2)\ldots\ldots. (1-\omega^{n-1})$ equal to

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38. If $1, \alpha_1, \alpha_2, \ldots \alpha_{n-1}$ be nth roots of unity then $(1+\alpha_1)(1+\alpha_2)\ldots\ldots\ldots (1+\alpha_{n-1}) =$ (A) 0 or 1 according as n is even or odd (B) 0 or 1 according as n is odd or even (C) n (D) $-n$

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39. If ω be a nth root of unity, then $1+\omega+\omega^2+\ldots\ldots+\omega^{n-1}$ is (a)0(B) 1 (C) -1 (D) 2

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40. If $|z|=2$ and locus of $5z-1$ is the circle having radius a and $z_1^2+z_2^2-2z_1z_2\cos\theta=0, then |z_1|:|z_2|=$ (A) a (B) 2a (C) $\frac{a}{10}$ (D) none of these

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41. If $|z-4+3i|\leq 1$ and m and n be the least and greatest values of $|z|$ and K be the least value of $\frac{x^4+x^2+4}{x}$ on the interval $(0,\infty)$, then $K=$

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42. If $a\hat{i}+b\hat{j}+c\hat{k}$ be a unit vector and z is a complex number such that $(1+a)z=b+ic, then \frac{1-iz}{1+z}$ (A) $\frac{a+ib}{1+z}$ (B) $\frac{1+c}{a+ib}$ (C) $(a+ib)(1+c)$ (D) none of these

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43. If for the complex numbers z_1 and z_2, $|z_1 + z_2| = |z_1 - z_2|$ then $Argz_1 - Argz_2$ is equal

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44. Number of solutions of $Re(z^2) = 0$ and $|z| = r\sqrt{2}$ where z is a complex number and $r > 0$ is (A) 2 (B) 4 (C) 5 (D) none of these

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45. If ω is an imaginary fifth root of uinty, the find the value of $\log_2|1 + \omega + \omega^2 + \omega^3 - 1/\omega|$.

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46. If z is a unimodular number $(\neq \pm i)$ then $\dfrac{z+i}{z-i}$ is (A) purely real (B) purely imaginary (C) an imaginary number which is not purely imaginary (D) both purely real and purely imaginary

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47. The locus of the complex number z satisfying the inequaliyt $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z-1|+6}{2|z-1|-1}\right) > 1\left(2where|z-1| \neq \frac{1}{2}\right)$ is (A) a circle (B) interior of a circle (C) exterior of circle (D) none of these

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48. The number of complex numbers z satisfying $|z-3-i| = |z-9-i| \text{ and } |z-3+3i| =$ are

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49. If |z|= maximum $\{|z+2|, |z-2|\}$, then(A)|z-$\bar{z}|$ $=$ 1/2(B)|z+$\bar{z}$|=4(C) |z+$\bar{z}$|= 1/2(D)|$z-\bar{z}$=2

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50. If z_1 and z_2 are complex numbers such that $|z_1 - z_2| = |z_1 + z_2|$ and A and B re the points representing z_1 and z_2 then the orthocentre of $\triangle OAB$, where O is the origin is (A) $\frac{z_1 + z_2}{2}$ (B) 0 (C) $\frac{z_1 - z_2}{2}$ (D) none of these

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51. If α is an imaginary root of $z^n - 1 = 0$ then $1 + \alpha + \alpha^2 + \ldots\ldots\ldots + \alpha^{n-1} =$ (A) 1 (B) -1 (C) 0 (D) 2

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52. If $|z^2 - 3| = 3|z|$, then the maximum value of $|z|$ is 1 b. $\frac{3 + \sqrt{21}}{2}$ c. $\frac{\sqrt{21} - 3}{2}$ d. none of these

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53. If $|z+2-i|$=5 then the maximum value of $|3z+9-7i|$ is K, then find k

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54. Let $P \equiv \sqrt{3}e^{i\frac{\pi}{3}}, Q \equiv \sqrt{3}e^{-\frac{\pi}{3}}$ and $R \equiv \sqrt{3}e^{-i\pi}$. If P,Q,R form a triangle PQR in the Argand plane, then $\triangle PQR$ is (A) isosceles (B) equilateral (C) scalene (D) none of these

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55. If $|z| \geq 5$ then the least value of $\left|z+\frac{2}{z}\right|$ is (A) $\frac{23}{5}$ (B) $\frac{24}{5}$ (C) 5 (D) none of these

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56. If $Re\left(\frac{2z+1}{iz+1}\right) = 1$, the the locus of the point representing z in the complex plane is a (A) straight line (B) circle (C) parabola (D) none of these

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57. $|z-4| + |z+4| = 16$ where z is as complex number ,tehn locus of z is (A) a circle (B) a straight line (C) a parabola (D) none of these

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58. A,B and C are the points respectively the complex numbers z_1, z_2 and z_3 respectivley, on the complex plane and the circumcentre of $\triangle ABC$ lies at the origin. If the altitude of the triangle through the vertex. A meets the circumcircle again at P, prove that P represents the complex number $\left(-\frac{z_2 z_3}{z_1}\right)$.

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59. The points, $z_1, z_2, z_3, z_4,$ in the complex plane are the vertices of a parallelogram taken in order, if and only if (a)$z_1 + z_4 = z_2 + z_3$ (b) $z_1 + z_3 = z_2 + z_4$ (c)$z_1 + z_2 = z_3 + z_4$ (d) None of these

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60. If all the roots of $z^3 + az^2 + bz + c = 0$ are of unit modulus, then (A) $|a| \leq 3$ (B) $|b| \leq 3$ (C) $|c| = 1$ (D) none of these

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61. If $a = z_1 + z_2 + z_3, b = z_1 + \omega z_2 + \omega^2 z_3, c = z_1 + \omega^2 z_2 + \omega z_3 (1, \omega, \omega^2$ are cube roots of unity), then the value of z_2 in terms of a,b, and c is (A) $\frac{a\omega^2 + b\omega + c}{3}$ (B) $\frac{a\omega^2 + b\omega^2 + c}{3}$ (C) $\frac{a + b + c}{3}$ (D) $\frac{a + b\omega^2 + c\omega}{3}$

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62. $z = x + iy$ satisfies $arg(z + 2) = arg(z + i)$ then

(A) $x + 2y + 1 = 0$

(B) $x + 2y + 2 = 0$

(C) $x - 2y + 1 = 0$

(D) $x - 2y - 2 = 0$

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63. The points $A(z_1), B(z_2)$ and $C(z_3)$ form an isosceles triangle in the Argand plane right angled at B, then $\frac{z_1 - z_2}{z_3 - z_2}$ can be (A) 1 (B) -1 (C) $-i$ (D) none of these

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64. The number of solutions of $\sqrt{2}|z - 1| = z - i,$ where $z = x + iy$ is

(A) 0 (B) 1 (C) 2 (D) 3

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65. If $|2z-1| = |z-2| and z_1, z_2, z_3$ are complex numbers such that

`|z_1-alpha||z|$d.$ >2|z|`

A. <|z|

B. null

C. null

D. null

Answer: null

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66. If $1, \alpha_1, \alpha_2, ...\alpha_{3n}$ be the roots of equation $z^{3n+1} - 1 = 0$ and ω be an imaginary cube root of unity , then

$$\frac{(\omega^2 - \alpha_1)(\omega^2 - \alpha_2)......(\omega^2 - \alpha_{3n})}{(\omega - \alpha_1)(\omega - \alpha_2)......(\omega - \alpha_{3n})}$$

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67. If α and β are two fixed complex numbers, then the equation $z = a\alpha + (1-a)\beta, where a \varepsilon R$ represents in the Argand plane (A) a straight line passsing through α and β (B) a straight line passing through α but not through β (C) a striaght line passing through β but not throughα (D) a straight line passing neighter through α not or through β

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68. If $\begin{vmatrix} x^2+x & x-1 & x+1 \\ x & 2x & 3x-1 \\ 4x+1 & x-2 & x+2 \end{vmatrix} = px^4 + qx^3 + rx^2 + sx + t$ be n identity in x and ω be an imaginary cube root of unity, $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} + \frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} =$ (A) p (B) $2p$ (C) $-2p$ (D) $-p$

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69. z_1, z_2, z_3, z_4 are distinct complex number representing the vertices of a quadrilateral ABCD taken in order. If $z_1 - z_4 = z_2 - z_3$ and

$ar\frac{g(z_4 - z_1)}{z_2 - z_1} = \frac{\pi}{2}$, then the quadrilateral is rectangle (2) rhombus trapezium (4) parallelogram square

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70. If z_1, z_2, z_3 be the vertices A,B,C respectively of triangle ABC such that $|z_1| = |z_2| = |z_3|$ and $|z_1 + z_2| = |z_1 - z_2|$ then C=

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

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71. If z_1, z_2, z_3 be the vertices of a triangle ABC such that $|z_1| = |z_2| = |z_3|$ and $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then $\left|arg, \left(\frac{z_3 - z_1}{z_3 - z_2}\right)\right| =$ (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

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72. If $\sec^{-1}\left(\frac{z-2}{i}\right)$ lies between 0 and $\frac{\pi}{2}$, where $z = x + iy$ then (A) $x > 2, y > 1$ (B) $x = 2, y > 1$ (C) $x = 2, y = 1$ (D) $x < 2, y = 1$

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73. The system of equation $|z - 1 - i| = \sqrt{2}$ and $|z| = 2$ has

(A) one solutions

(B) two solution

(C) three solutions

(D) none of these

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74. If $z_1, z_2, z_3, \ldots\ldots\ldots\ldots z_{n-1}$ are the roots of the equation $1 + z + z^2 + \ldots\ldots + z^{n-1} = 0, where n \varepsilon N, n > 2$ then (A) $z_1, z_2, \ldots z_{n-1}$ are terms of a G.P. (B) $z_1, z_2, \ldots\ldots, z_{n-1}$ are terms of an A.P. (C) $|z_1| = |z_2| = |z_3| = .|z_{n-1}| \neq 1$ (D) none of these

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75. If the greatest valueof $|z|$ such that $|z-3-4i| \leq a$ is equal to the least value of $x^4+x+\frac{5}{x} \in (0, \infty) then a =$ (A) 1 (B) 4 (C) 3 (D) 2

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76. $|z-4|+|z+4|=16$ where z is as complex number ,then locus of z is (A) a circle (B) a straight line (C) a parabola (D) none of these

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77. Let z_1, z_2, z_3 be three distinct non zero complex numbers which form an equilateral triangle in the Argand pland. Then the complex number associated with the circumcentre of the tirangle is (A) $\frac{z_1 z_2}{z_3}$ (B) $\frac{z_1 z_3}{z_2}$ (C) $\frac{z_1+z_2}{z_3} (D) \frac{z_1+z_2+z_3}{3}$

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78. If $\sqrt{5-12i}+\sqrt{-5-12i}=z$, then principal value of $argz$ can be

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79. If $z+\sqrt{2}|z+1|+i=0$, then z= (A) $2+i$ (B) $2-i$ (C) $-2-i$ (D) $-2+i$

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80. If A and B represent the complex numbers z_1 and z_2 such that $|z_1-z_2|=|z_1+z_2|$, then circumcentre of $\triangle AOB$, O being the origin is (A) $\frac{z_1+2z_2}{3}$ (B) $\frac{z_1+z_2}{3}$ (C) $\frac{z_1+z_2}{2}$ (D) $\frac{z_1-z_2}{3}$

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81. If α, β are complex numbers then the maximum value of $\frac{\alpha\bar{\beta}+\bar{\alpha}\beta}{|\alpha\beta|}$ is equal to :

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83. If z_1, z_2, z_3 be the vertices A,B,C respectively of an equilateral trilangle on the Argand plane and $|z_1| = |z_2| = |z_3|$ then (A) Centroid oif the triangle ABC is the complex number 0 (B) Distance between centroid and orthocentre of the triangle ABC is 0 (C) Centroid of the tirangle ABC divides the line segment joining circumcentre and orthcentre in the ratio 1:2 (D) Complex number representing the incentre of the triangle ABC is a non zero complex number

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84. If $|z - 4 + 3i| \leq 3,$ then the least value of $|z| =$

(A) 2

(B) 3

(C) 4

(D) 5

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85. If $|z| = 5,$ then the locus of $-1 + 2z$ is (A) a circle having center (2,0) (B) a circle having center $(-1, 0)$ (C) a circle having radius 5 (D) a circle having radius 9

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86. $|z + 3| \leq 3$, then the greatest and least value of $|z|$ are

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87. If z_1 and z_2 are two complex numbers such that $|(\bar{z}_1 - 2\bar{z}_2)(2 - z_1\bar{z}_2)| = 1$ then (A) $|z_1| = 1,$ if $|z_2| \neq 1$ (B)

$|z_1| = 2, \quad \text{if} \quad |z_2| \neq 1$ (C) $|z_2| = 2, \quad \text{if} \quad |z_1| \neq 1$ (D) $|z_2| = 1, \quad \text{if} \quad |z_1| \neq 2$

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88. If $\left|z - \frac{4}{z}\right| = 2$ then the greatest value of $|z|$ is:

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89. If z is a complex number different form $\frac{i}{3}$ then locus of z if $\left|\frac{3z}{3z - i}\right| = 1$ is (A) a straightline paralel to x axis (B) a straight line having slope undefined (C) as straight line having slope 0 (D) a straight line passing through the point $\left(2, \frac{1}{6}\right)$

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90. If z_1 and z_2 two non zero complex numbers such that $|z_1 + z_2| = |z_1|$ then which of the following may be true (A)

$argz_1 - argz_2 = 0$ (B) $argz_1 - argz_2 = \pi$ (C) $|z_1 - z_2| = ||z_1| - |z_2||$

(D) $argz_1 - argz_2 = 4\pi$

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91. The complex numbers $z_1 = 1 + 2i, z_2 = 4 - 2i$ and $z_3 = 1 - 6i$ form the vertices of a (A) a right angled triangle (B) isosceles triangle (C) equilateral triangle (D) triangle whose one of the sides is of length 8

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92. If the vertices of an equilateral triangle are situated at $z = 0, z = z_1$ and $z = z_2$ then which of the following is(are) true?

(A) $|z_1| = |z_2|$

(B) $|z_1 + z_2| = |z_1| + |z_2|$

(C) $|z_1 - z_2| = |z_1|$

(D) |argz_1-argz_2|= $\frac{\pi}{3}$

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93. If z_1 and z_2 are two complex numbers for which $|(z_1 - z_2)(1 - z_1 z_2)| = 1$ and $|z_2| \neq 1 then$ (A) $|z_2| = 2$ (B) $|z_1| = 1$ (C) $z_1 = e^{i\theta}$ (D) $z_2 = e^{i\theta}$

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94. If $\sin x + \sin y + \sin z = 0 = \cos x + \cos y + \cos z$, then find the value of $\cos(\theta - x) + \cos(\theta - y) + \cos(\theta - z)$

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95. Find the complex number z satisfying the equation $\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}, \left|\frac{z-4}{z-8}\right| = 1$

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96. Which of the following is correct for any two complex numbers $z_1 \text{ and} z_2$?

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97. Values $(s)(-i)^{1/3}$ is/are $\frac{\sqrt{3}-i}{2}$ b. $\frac{\sqrt{3}+i}{2}$ c. $\frac{-\sqrt{3}-i}{2}$ d. $\frac{-\sqrt{3}+i}{2}$

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98. The modulus and the principal asrgumentof the complex nuber $\frac{1-i}{3+i}+4i$ are (A) modulus $=\sqrt{3}$ (B) modulus $=6$ (C) $arg=\tan^{-1}(18)$ (D) $arg=tn^{-1}\left(\frac{3}{4}\right)$

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99. If a and b are two real number lying between 0 and 1 such that $z_1 = a + i$, $z_2 = 1 + bi$ and $z_3 = 0$ form anequilateral trilangle , then (A) $a = 2 + \sqrt{3}$ (B) $b = 4 - \sqrt{3}$ (C) $a = b = 2 - \sqrt{3}$ (D) $a = 2, b = \sqrt{3}$

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100. If z_1, z_2, z_3, z_4 be the vertices of a parallelogram taken in anticlockwise direction and $|z_1 - z_2| = |z_1 - z_4|$, then $\sum_{r=1}^{4} (-1)^r z_r = 0$ (b) $z_1 + z_2 - z_3 - z_4 = 0$ $ar\frac{g(z_4 - z_2)}{z_3 - z_1} = \frac{\pi}{2}$ (d) None of these

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101. If $|z_1 + z_2| = |z_1 - z_2|$ and $|z_1| = |z_2|$, then (A) $z_1 = \pm iz_2$ (B) $z_1 = z_2$ (C) $z_= - z_2$ (D) $z_2 = \pm iz_1$

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102. If $|z| = \min(|z-1|, |z+1|\}$, where z is the complex number and f be a one -one function from $\{a, b, c\} \to \{1, 2, 3\}$ and $f(a) = 1$ is false, $f(b) \neq 1$ is false and $f(c) \neq 2$ is true then $|z+\bar{z}| =$ (A) $f(a)$ (B) $f(c)$ (C) $\frac{1}{2}f(a)$ (D) $f(b)$

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104. If $|z_1 = 1, |z_2| = 2, |z_3| = 3$ and $|z_1+z_2+z_3| = 1, then|9z_1z_2 + 4z_3z_1 + z$

is equal to

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105. If $|z_1| = |z_2| = ... = |z_n| = 1$, then $|z_1 + z_2 + ... + z_n|$ is equal to :

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106. If $\left|z-\frac{4}{z}\right|=2$, then the maximum value of $|Z|$ is equal to (1) $\sqrt{3}+1$ (2) $\sqrt{5}+1$ (3) 2 (4) $2+\sqrt{2}$

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107. If $\left|z-\frac{4}{2}z\right|=2$ then the least of $|z|$ is (A) $\sqrt{5}-1$ (B) $\sqrt{5}-2$ (C) $\sqrt{5}$ (D) 2

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108. If $|z-4+3i| \leq -2,$ then the least value of $|z| =$ (A) 2 (B) 3 (C) 4 (D) 5

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109. let A & B be two set of complex number defined by $A = \{z : |z| = 12\}$ and $B = \{z : |z - 3 - 4i| = 5\}$. Which of the given statement(s) is (are) true? (A) $A \subseteq B$ (B) $A = B = \phi$ (C) $A \cap B \neq \phi$ (D) $B \subseteq A$

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110. let A & B be two set of complex number defined by $A = \{z : |z| = 12\}$ and $B = \{z : |z - 3 - 4i| = 5\}$. Let $z_1 \varepsilon A$ and $z_2 \varepsilon B$ then the value of $|z_1 - z_2|$ necessarily lies between (A) 3 and 15 (B) 0 and 22 (C) 2 and 22 (D) 4 and 14

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111. If $B : \{z : |z - 3 - 4i|\} = 5$ and C={z:Re[(3+4i)z]=0} then the number of elements in the set B intesection C `is (A) 0 (B) 1 (C) 2 (D) none of these

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112. If $|z-4+3i| \leq 3,$ then the least value of $|z| =$ (A) 2 (B) 3 (C) 4 (D) 5

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113. If $|z-25i| \leq 15$ then least positive value of $argz =$ (A) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$ (B) $\tan^{-1}\left(\frac{3}{4}\right)$ (C) $\tan^{-1}\left(\frac{4}{3}\right)$ (D) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

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114. If $|z| < 1, then 1 + 2z$ lies (A) on or inside circle having center at origin and radius 2 (B) outside the circle having center at origin and radius 2 (C) inside the circle having center at (1,0) and radius 2 (D) outside the circle having center at (1,0) and radius 2.

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115. If the complex numbers z_1, z_2, z_3 represents the vertices of a triangle ABC, where z_1, z_2, z_3 are the roots of equation $z^3 + 3\alpha z^2 + 3\alpha z^2 + 3\beta z + \gamma = 0, \alpha, \beta, \gamma$ beng complex numbers and $\alpha^2 = \beta then \triangle ABC$ is (A) equilateral (B) right angled (C) isosceles but not equilateral (D) scalene

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116. If a and b are two real number lying between 0 and 1 such that $z_1 = a + i, z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle , then (A) $a = 2 + \sqrt{3}$ (B) $b = 4 - \sqrt{3}$ (C) $a = b = 2 - \sqrt{3}$ (D) $a = 2, b = \sqrt{3}$

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117. Let the complex numbers z_1, z_2 and z_3 be the vertices of a equilateral triangle. Let z_0 be the circumcentre of the tringel ,then $z_1^2 + z_2^2 + z_3^2 =$ (A) z_0^2 (B) $3z_0^2$ (C) $9z_0^2$ (D) 0

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118. If the complex number z_1, z_2 and z_3 represent the vertices of an equilateral triangle inscribed in the circle $|z| = 2$ and $z_1 = 1 + i\sqrt{3}$ then (A) $z_2 = 1, z_3 = 1 - i\sqrt{3}$ (B) $z_2 = 1 - i\sqrt{3}, z_3 = -i\sqrt{3}$ (C) $z_2 = 1 - i\sqrt{3}, z_3 = -1 + i\sqrt{3}$ (D) $z_2 = -2, z_3 = 1 - i\sqrt{3}$

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119. The locus of the centre of a variable circle touching circle $|z| = 2$ internally and circle $|z - 4| = 1$ externally is (A) a parabola (B) a hyperbola (C) a ellipse (D) none of these

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120. Locus of the centre of the circle touching circles $|z| = 3$ and $|z - 4| = 1$ externally is (A) a parabola (B) a hyperbola (C) an ellipse (D) none of these

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121. Locus the centre of the variable circle touching $|z-4|=1$ and the line $Re(z)=0$ when the two circles on the same side of the line is (A) a parabola (B) an ellipse (C) a hyperbola (D) none of these

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122. If $|z-1|+|z+3|\leq 8,$ then the maximum, value of $|z-4| is =$

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123. If z_1, z_2, z_3 are three points lying on the circle $|z|=2$ then the minimum value of the expression $|z_1+z_2|^2+|z_2+z_3|^2+|z_3+z_1|^2=$

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124. If z and $\bar{z}$ represent adjacent vertices of a regular polygon of n sides where centre is origin and if $\frac{Im(z)}{Re(z)} = \sqrt{2} - 1$, then n is equal to:

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125. The value of the expression $2^{199}\sin\left(\frac{\pi}{199}\right)\sin\left(\frac{2\pi}{199}\right)\sin\left(\frac{3\pi}{199}\right)\ldots\ldots\ldots\sin\left(\frac{198\pi}{199}\right)\Big) =$

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126. $\frac{1}{a+\omega} + \frac{1}{b+\omega} + \frac{1}{c+\omega} + \frac{1}{d+\omega} = \frac{1}{\omega}$ where, a,b,c,d, $\in$ R and ω is a complex cube root of unity then find the value of $\sum \frac{1}{a^2 - a + 1}$

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127. If $x = 2 + 5i$ (where $i^2 = -1\Big)$ and $2\left(\frac{1}{1!9!} + \frac{1}{3!7!}\right) + \frac{1}{5!5!} = \frac{2^a}{b!}$, then the value of

$(x^3 - 5x^2 + 33x - 19)$ is equal to

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128. Let z be a complex number lying on a circle centred at the origin having radius r. If the area of the triangle having vertices as $z, z\omega$ and $z + z\omega$, where omega is an imaginary cube root of unity is $12\sqrt{3}$ sq. units, then the radius of the circle r=

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129. Number of solutions of $Re(z^2) = 0$ and $|z| = r\sqrt{2}$ where z is a complex number and $r > 0$ is (A) 2 (B) 4 (C) 5 (D) none of these

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130. Let z_1, z_2 and origin be the vertices A,B,O respectively of an isosceles triangle OAB, where OA=OB and $\angle AOB = 2\theta. If z_1, z_2$ are the roots of equation $z^2 + z + 9 = 0$ then $\sec^2 \theta =$

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131. Let the center of the circle represented by $z\bar{z} - (2+3i)z - (2-3i)\bar{z} + 9 = 0' be(x, y)$, then the value of $x^2 + y^2 + xy$ is

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132. Assertion (A): If $1, \omega, \omega^2$ are the cube roots of unity, then roots of equation $(x-2)^3 - 27 = 0$ are 5,$2 + 3\omega$, $2 + 3\omega^2$, Reason (R): If α be one cube root of a number, then its other two cube roots are $\alpha\omega \text{ and } \alpha\omega^2$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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133. Assertion (A): $arg z_1 - arg z_2 = 0$, Reason: If $|z_1 + z_2| = |z_1| + |z_2|$, then origin z_1, z_2 are colinear and z_1, z_2 lie on the same side of the origin. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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134. Assertion (A): Circumcentre of $\triangle POQ$ is $\dfrac{z_1 + z_2}{2}$, Reason (R): Circumcentre of a right triangle is the middle point of the hypotenuse. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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135. If α, β are complex numbers, then maximum value of $\dfrac{\alpha\bar{\beta} + \overline{\alpha}\beta}{|\alpha\beta|}$ is 2. Reason (R): For any two complex numbers

z_1 and $z_2, |z_1 - z_2| \geq |z_1| - |z_2|$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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136. Assertion (A): z_1, z_2 and origin form an equilateral triangle if $p^2 = 6q$ for the equation $z^2 + pz + q = 0$, Reason (R): Triangle having vertices z_1, z_2, z_3 in the Argand plane is equilateral if $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z3z_1$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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137. Assertion (A): Points representing z_1, z_2, z_3 are collinear. Reason (R): Three numbers a,b,c are in A.P., if $b - a = c - b$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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138. Assertion (A): $arg z_1 - arg z_2 = 0$, Reason: If $|z_1 + z_2| = |z_1| + |z_2|$, then origin z_1, z_2 are colinear and z_1, z_2 lie on the same side of the origin. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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139. Assertion (A): $\frac{z}{4 - z^2}$ lies on y-axis. $Reason(R):$ |z|^2=$z\bar{z}$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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140. If α, β are complex numbers, then maximum value of $\dfrac{\alpha\bar{\beta}+\overline{\alpha}\beta}{|\alpha\beta|}$ is 2.

Reason (R): For any two complex numbers z_1 and z_2, $|z_1 - z_2| \geq |z_1| - |z_2|$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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141. Assertion (A): $\dfrac{z}{4-z^2}$ lies on y-axis. $Reason(R):$ |z|^2=zbarz` (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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142. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals

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143. Let z and ω be two non zero complex numbers such that $|z| = |\omega|$ and $arg z + arg \omega = \pi$, then z equals

(A) ω

(B) $-\omega$

(C) $\overline{\omega}$

(D) $-\overline{\omega}$

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144. Let $z and \omega$ be two complex numbers such that $|z| \leq 1, |\omega| \leq 1 and |z - i\omega| = |z - i\omega| = 2, then z$ equals $1 \text{ or } i$ b. $i \text{ or } -i$ c. $1 \text{ or } -1$ d. $i \text{ or } -1$

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146. If $|z| \leq 1$ and $|\omega| \leq 1$, show that

$|z-\omega|^2 \leq \left(|z|-|\omega|^2\right)+\{arg(z)-arg(\omega)\}^2.$

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147. Find the sum

$1 \times (2-\omega) \times \left(2-\omega^2\right)+2 \times (-3-\omega) \times \left(3-\omega^2\right)+\ldots.+(n-1) \times ($

, where ω is an imaginary cube root of unity.

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148. For positive integer n_1, n_2 the value of the expression $(1+i)^{n1}+\left(1+i^3\right)^{n1}\left(1+i^5\right)^{n2}\left(1+i^7\right)^{n_{20}}$, where $i=\sqrt{-1}$, is a real number if and only if (a) $n_1=n_2+1$ (b) $n_1=n_2-1$ (c) $n_1=n_2$ (d) $n_1>0, n_2>0$

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149. find all nonzero complex number z satisfying $\bar{z} = iz^2$.

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150. Let z_1 and z_2 be the roots of the equation $z^2 + pz + q = 0$, where the coefficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane, respectively. If $\angle AOB = \theta \neq 0$ and $OA = OB, where\, O$ is the origin, prove that $p^2 = 4q\cos^2(\theta/2)$.

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151. Let $\bar{b}z + b(\bar{z}) = c, b \neq 0$ be a line the complex plane, where $\bar{b}$ is the complex conjugate of b. If a point z_1 i the reflection of the point z_2 through the line then show that $c = \bar{z}_1 b + z_2 \bar{b}$

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152. If ω is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ is equal to

128ω (b) -128ω $128\omega^2$ (d) $-128\omega^2$

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153. The value of $sum\sum_{n=1}^{13}\left(i^n+i^{n+1}\right)$, $w h e r e i=\sqrt{-1} e q u a l s\ i$ (b) $i-1$ (c) $-i$ (d) 0

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154. $x+iy=\begin{vmatrix}6i & -3i & 1\\ 4 & 3i & -1\\ 20 & 3 & i\end{vmatrix}$, find x and y.

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157. If $arg(z) < 0,$ then $arg(-z) - \arg(z)$ equals π (b) $-\pi$ (d) $-\frac{\pi}{2}$

(d) $\frac{\pi}{2}$

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160. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of triangle which is (1) of area zero (2) right angled isosceles(3) equilateral (4) obtuse angled isosceles

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161. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\left|1111 - 1 - \omega^2\omega^2 1\omega^2\omega^4\right|$ is 3ω b. $3\omega(\omega - 1)$ c. $3\omega^2$ d. $3\omega(1 - \omega)$

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163. Let a complex number $\alpha, \alpha \neq 1,$ be a rootof hte euation $z^{p+q} - z^p - z^q + 1 = 0, wherep, q$ are distinct primes. Show that either

$1+\alpha+\alpha^2+ \quad +\alpha^{p-1}=0$ or $1+\alpha+\alpha^2+ \quad +\alpha^{q-1}=0$, but not both together.

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164. If $|z|=1$ and $w=\frac{z-1}{z+1}$ (where $z\neq -1$), then $Re(w)$ is 0 (b) $\frac{1}{|z+1|^2}\left|\frac{1}{z+1}\right|, \frac{1}{|z+1|^2}$ (d) $\frac{\sqrt{2}}{|z|1|^2}$

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165. If z_1 and z_2 are two complex numbers such that $|z_1|<1<|z_2|$ then prove that $\left|\frac{1-z_1\bar{z}_2}{z_1-z_2}\right|<1$

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167. If $\omega(\neq 1)$ be an imaginary cube root of unity and $(1+\omega^2)^n = (1+\omega^4)^n$, then the least positive value of n is (a) 2 (b) 3 (c) 5 (d) 6

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168. Find the centre and radius of the circle formed by all the points represented by $z = x + iy$ satisfying the relation $\left|\frac{z-\alpha}{z-\beta}\right| = k(k \neq 1)$, where α and β are the constant complex numbers given by $\alpha = \alpha_1 + i\alpha_2, \beta = \beta_1 + i\beta_2$.

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169. a, b, c are integers, not all simultaneously equal, and ω is cube root of unity $(\omega \neq 1)$, then minimum value of $|a + b\omega + c\omega^2|$ is 0 b. 1 c. $\frac{\sqrt{3}}{2}$ d. $\frac{1}{2}$

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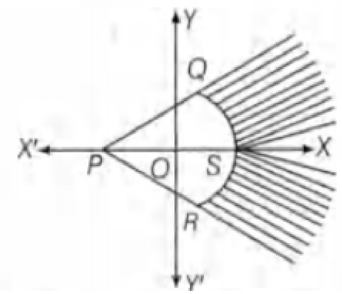
170. The shaded region, where $P = (-1, 0), Q = \left(-1+\sqrt{2}, \sqrt{2}\right) R = \left(-1+\sqrt{2}, -\sqrt{2}\right), S = (1, 0)$ is represented by Figure



$|z+1| > 2, \ |arg(z+1) < \dfrac{\pi}{4}$

$|z+1| < 2, \ |arg(z+1) < \dfrac{\pi}{2}$

$|z+1| > 2, \ |arg(z+1) > \dfrac{\pi}{4}$

$|z+1| < 2, \ |arg(z+1) > \dfrac{\pi}{2}$

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171. If one of the vertices of the square circumscribing the circle $|z-1| = \sqrt{2}$ is $2+\sqrt{3}i$, where $i = \sqrt{-1}$. Find the other vertices of the square.

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172. If $w = \alpha + i\beta,$ where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w - \overline{w}z}{1 - z}\right)$ is a purely real, then the set of values of z is $|z| = 1, z \neq 2$ (b) $|z| = 1 and z \neq 1$ (c)$z = \bar{z}$ (d) None of these

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173. A man walks a distance of 3 units from the origin towards the North-East $(N45^0 E)$ direction.From there, he walks a distance of 4 units towards the North-West $(N45^0 W)$ direction to reach a point P. Then, the position of P in the Argand plane is (a) $3e^{\frac{i\pi}{4}} + 4i$ (b) $(3 - 4i)e^{\frac{i\pi}{4}}$ $(4 + 3i)e^{\frac{i\pi}{4}}$ (d) $(3 + 4i)e^{\frac{i\pi}{4}}$

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174. If $|z| = 1 and z \neq \pm 1,$ then all the values of $\frac{z}{1 - z^2}$ lie on a line not passing through the origin $|z| = \sqrt{2}$ the x-axis (d) the y-axis

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175. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by $6 + 7i$ (b) $-7 + 6i$ $7 + 6i$ (d) $-6 + 7i$

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176. Let $z = \cos\theta + i\sin\theta$, where $i = \sqrt{-1}$. Then the value of $\sum_{m=1}^{15} Im\left(z^{2m-1}\right)$ at $\theta = 2^\circ$ is

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177. Let $z = x + iy$ be a complex number where x and y are integers. Then ther area of the rectangle whose vertices are the roots of the equaiton $\bar{z}z^3 + z\bar{z}^3 = 350$.

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