MATHS

BOOKS - KC SINHA ENGLISH

CONDITIONAL TRIGONOMETRIC IDENTITIES - FOR BOARDS

Solved Examples

1. Given $\alpha + \beta + \gamma = \pi$, prove that $\sin^2\alpha + \sin^2\beta - \sin^2\gamma = 2\sin\alpha\sin\beta\cos\gamma$.

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2. If $A + B + C = 180^0$, Prove that :

$$\sin^2\left(\frac{A}{2}\right) + \sin^2\left(\frac{B}{2}\right) + \sin^2\left(\frac{C}{2}\right) = 1 - 2\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$$

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3. Prove that $\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta\cos(\alpha+\beta) = \sin^2(\alpha+\beta)$.

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4. In a ΔABC, prove that : $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

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5. If $A + B + C = \pi$, prove that :

$$\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right) + \tan\left(\frac{B}{2}\right)\tan\left(\frac{C}{2}\right) + \tan\left(\frac{C}{2}\right)\tan\left(\frac{A}{2}\right) = 1$$

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6. If $\alpha + \beta + \gamma = \pi$, show that :

$$\tan(\beta+\gamma-\alpha) + \tan(\gamma+\alpha-\beta) + \tan(\alpha+\beta-\gamma) =$$

$$\tan(\beta+\gamma-\alpha)\tan(\gamma+\alpha-\beta)\tan(\alpha+\beta-\gamma)$$

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7. If $A+B+C=\pi$, prove that: $\cot B\cot C+\cot C\cot A+\cot A\cot B=1$.

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8. If in a ΔABC, $\cot A+\cot B+\cot C=\sqrt{3}$. Prove that triangle is equilateral

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9. If A, B, C, D be the angles of a quadrilateral, prove that : $\frac{\tan A+\tan B+\tan C+\tan D}{\cot A+\cot B+\cot C+\cot D}=\tan A\tan B\tan C\tan D$

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10. If $A+B+C=\pi$, prove that :

$$\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$$

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11. If $A+B+C=\pi$ then prove that

$$\cos A+\cos B+\cos C=1+4\sin\left(\frac{A}{2}\right).\sin\left(\frac{B}{2}\right).\sin\left(\frac{C}{2}\right)$$

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12. Prove that :

$$\frac{\sin 2A+\sin 2B+\sin 2C}{\cos A+\cos B+\cos C-1}=8\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)$$

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13. In triangle ABC, prove that

$$\cos\left(\frac{A}{2}\right)+\cos\left(\frac{B}{2}\right)+\cos\left(\frac{C}{2}\right)=4\frac{\cos(\pi-A)}{4}\frac{\cos(\pi-B)}{4}\frac{\cos(\pi-C)}{4}$$

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14. If $A+B+C=\pi$, Prove that :

$$\sin\left(\frac{A}{2}\right)+\sin\left(\frac{B}{2}\right)+\sin\left(\frac{C}{2}\right)=1+4\sin\left(\frac{B+C}{4}\right)\sin\left(\frac{C+A}{4}\right)\sin$$

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15. If $A+B+C=\pi$, prove that

$$\frac{\sin^2 A}{2}+\frac{\sin^2 B}{2}-\frac{\sin^2 C}{2}=1-2\frac{\cos A}{2}\frac{\cos B}{2}\frac{\sin C}{2}.$$

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16. $1+\cos 56^0+\cos 58^0-\cos 66^0=4\cos 28^0\cos 29^0\sin 33^0.$

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17. x,y,z are real numbers satisfying $xy + yz + zx = 1$, then prove that

$$\frac{x}{1-x^2} + \frac{y}{1-y^2} + \frac{z}{1-z^2} = \frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}$$

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18. If $x + y + z = xyz$, prove that:

a) $$\frac{3x-x^3}{1-3x^2} + \frac{3y-y^3}{1-3y^2} + \frac{3z-z^3}{1-3z^2} = \frac{3x-x^3}{1-3x^2} \cdot \frac{3y-y^3}{1-3y^2} \cdot \frac{3z-z^3}{1-3z^2}$$

b) $$\frac{x+y}{1-xy} + \frac{y+z}{1-yz} + \frac{z+x}{1-zx} = \frac{x+y}{1-xy} \cdot \frac{y+z}{1-yz} \cdot \frac{z+x}{1-zx}$$

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19. If $A + B + C + D = 2\pi$, then cos A + cos B + cos C + cos D =

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20. If $A + B + C = 2S$, prove that :

$\cos^2 S + \cos^2(S-A) + \cos^2(S-B) + \cos^2(S-C) = 2 + 2\cos A \cos B \cos$

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21. If A +B+C=π, prove that $\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}\geq 1$

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22. If $A+B+C=\pi$, prove that :

$(\tan A+\tan B+\tan C)(\cot A+\cot B+\cot C)=1+\sec A\sec B\sec C$

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23. If $A+B+C=\pi$, prove that :

$(\cot B+\cot C)(\cot C+\cot A)(\cot A+\cot B)=\cos ecA\cos ecB\cos ecC$

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24. If $A+B+C=\pi$, show that :

$$\frac{1}{2}\sum \sin^2 A(\sin 2B+\sin 2C)=3\sin A\sin B\sin C.$$

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Exercise

1. If $A+B+C=180^\circ$, then prove that

$$\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C.$$

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2. If $A+B+C=180^0$, prove that :

$$\sin^2 A+\sin^2 B+\sin^2 C=2(1+\cos A\cos B\cos C)$$

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3. If $A+B+C=180^\circ$, then prove that

$\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C.$

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4. If $A+B+C=180^0$, prove that :

$$\cos^2\left(\frac{A}{2}\right)+\cos^2\left(\frac{B}{2}\right)-\cos^2\left(\frac{C}{2}\right)=2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$$

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5. If $A+B+C=180^0$, prove that :

$$\cos^2\left(\frac{A}{2}\right)+\cos^2\left(\frac{B}{2}\right)+\cos^2\left(\frac{C}{2}\right)=2+2\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$$

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6. If $A+B+C=\pi/2$, show that

(a) $\sin^2 A+\sin^2 B+\sin^2 C=1-2\sin A\sin B\sin C$

(b) $\cos^2 A + \cos^2 B + \cos^2 C = 2 + 2\sin A \sin B \sin C$

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7. If $A + B + C = \frac{\pi}{2}$, show that :

$\cos^2 A + \cos^2 B + \cos^2 C = 2 + 2\sin A \sin B \sin C.$

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8. If $\alpha + \beta + \gamma = \pi$, prove that :

$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + 2\cos \alpha \cos \beta \cos \gamma = 1.$

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9. If $\alpha + \beta = 60^0$, prove that $\cos^2 \alpha + \cos^2 \beta - \cos \alpha \cos \beta = \frac{3}{4}.$

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10. Show that $\cos^2\theta + \cos^2\theta(\alpha+\theta) - 2\cos\alpha\cos\theta\cos(\alpha+\theta)$ is independent of θ.

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11. If $A+B+C=\pi$ and $A+B=2C$, prove that : $4\left(\sin^2 A + \sin^2 B - \sin A \sin B\right) = 3.$

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12. If $A+B+C=2\pi$, prove that : $\cos^2 B + \cos^2 C - \sin^2 A - 2\cos A\cos B\cos C = 0.$

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13. If $A+B+C=0$, Prove : $\cos^2 A + \cos^2 B + \cos^2 C = 1 + 2\cos A\cos B\cos C.$

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14. Prove that $\cos^2(\beta-\gamma)+\cos^2(\gamma-\alpha)+\cos^2(\alpha-\beta)$

$=1+2\cos(\beta-\alpha)\cos(\gamma-\alpha)\cos(\alpha-\beta)$

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15. If $A+B+C=\pi$, prove that :

$\sin A\cos B\cos C+\sin B\cos C\cos A+\sin C\cos A\cos B=\sin A\sin B\sin C$

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16. If $A+B+C=\pi/2$, show that

(a) $\cot A+\cot B+\cot C=\cot A\cot B\cot C$

(b) $\tan A\tan B+\tan B\tan C+\tan C\tan A=1.$

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17. If $A+B+C=\frac{\pi}{2}$, show that :

$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

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18. If $A+B+C=\pi$, prove that

$\tan 3A + \tan 3B + \tan 3C = \tan 3A \cdot \tan 3B \cdot \tan 3C$

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19. If $A+B+C=\pi$, prove that

(a) $\tan 3A + \tan 3B + \tan 3C = \tan 3A \tan 3B \tan 3C$

(b) $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}$

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20. If $A+B+C=\pi$, prove that :

$$\frac{\cot A+\cot B}{\tan A+\tan B}+\frac{\cot B+\cot C}{\tan B+\tan C}+\frac{\cot C+\cot A}{\tan C+\tan A}=1$$

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21. Prove that:

$$(\tan(\alpha-\beta)+\tan(\beta-\lambda)+\tan(\lambda-\alpha)=\tan(\alpha-\beta)\tan(\beta-\lambda)\tan(\lambda-$$

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22. If $x+y+z=0$, show that :

$$\cot(x+y-z)\cot(z+x-y)+(\cot(x+y-z)\cot(y+z-x)+$$

$$\cot(y+z-x)\cot(z+x-y)=1$$

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23. If $\tan\alpha+\tan B+\tan C=\tan A\tan B\tan C$, then

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24. If $A+B+C=\pi$, prove that :

$\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$

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25. Prove that sin2A + sin2B + sin2C = 4sinA · sinB · sin C

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26. In ΔABC, prove that:

a) $\sin A+\sin B+\sin C=4\frac{\cos A}{2}\frac{\cos B}{2}\frac{\cos C}{2}$

b) $\sin A-\sin B+\sin C=4\frac{\sin A}{2}\frac{\cos B}{2}\frac{\sin C}{2}$

c) $\cos A+\cos B-\cos C=4\frac{\cos A}{2}\frac{\cos B}{2}\frac{\sin C}{2}-1$

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27. If $A+B+C=\pi$, prove that :

$$\cos A+\cos B-\cos C=4\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)-1$$

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28. In triangle ABC, prove that

$$\sin(B+C-A)\sin(C+A-B)+\sin(A+B-C)=4s\in As\in Bs\in C$$

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29. If $A+B+C=\pi$, prove that

$$\frac{\cos A}{\sin B\sin C}+\frac{\cos B}{\sin C\sin A}+\frac{\cos C}{\sin A\sin B}=2.$$

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30. In ΔABC, prove that: a)

$$\frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}=8\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$$

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31. If $x+y+z=\frac{\pi}{2}$, prove that :

$$\cos(x-y-z)+\cos(y-z-x)+\cos(z-x-y)-4\cos x\cos y\cos z=0$$

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32. Show that

$$\sin(x-y)+\sin(y-z)+\sin(z-x)+4\sin\left(\frac{x-y}{2}\right)\sin\left(\frac{y-z}{2}\right)\sin\left(\frac{z}{}\right.$$

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33. In a triangle ABC

$$\sin(B+2C)+\sin(C+2A)+\sin(A+2B)=4\sin\left(\frac{B-C}{2}\right)\sin\left(\frac{C-A}{2}\right.$$

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34. If $A+B+C=\pi$, prove that : $\left(\sin\left(\frac{B+C}{2}\right)+\sin\left(\frac{C+A}{2}\right)+\sin\left(\frac{A+B}{2}\right)\right)$ equals $\left(4\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\cos\left(\frac{\pi-C}{4}\right)\right)$.

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35. If $x+y+z=xyz$ prove that $\frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2}=\frac{2x}{1-x^2}\frac{2y}{1-y^2}\frac{2z}{1-z^2}$.

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36. If $x+y+z=xzy$, prove that : $x\left(1-y^2\right)\left(1-z^2\right)+y\left(1-z^2\right)\left(1-x^2\right)+z\left(1-x^2\right)\left(1-y^2\right)=4xyz.$

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37. If $A+B+C+D=2\pi$, show that :

$\cos A-\cos B+\cos C-\cos D=4\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A+D}{2}\right)\cos\left(\frac{A+C}{2}\right)$

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38. If A, B, C, D are angles of a cyclic quadrilateral, then prove that $\cos A+\cos B+\cos C+\cos D=0$.

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39. If $A+B+C=\pi$, prove that :

$\cos A\sin B\sin C+\cos B\sin C\sin A+\cos C\sin A\sin B=1+\cos A\cos B\cos C$

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40. If $A + B + C = \pi$, prove that: $\cot^2 A + \cot^2 B + \cot^2 C \geq 1$

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41. If cos, (A)/2 cos, (B-C)/2 + cos, B/2 cos, (C-A)/2 + cos, C/2 cos, (A-B)/2 = sinA +sinB+sinC`

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42. In any triangle ABC, prove that $\sin^3 A \cos(B - C) + \sin^3 B \cos(C - A) + \sin^3 C \cos(A - B)$= $3 \sin A \sin B \sin C$

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