MATHS

BOOKS - KC SINHA ENGLISH

DEFINITE INTEGRALS AND PROPERTIES OF DEFINITE INTEGRALS - FOR COMPETITION

Solved Examples

1. Determine a positive integer $n \leq 5$ such that $\int_0^1 e^x (x-1)^n = 16 - 6e.$

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2. Prove that $\int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx = 2\int_0^1 \tan^{-1} x dx$. Hence or otherwise, evaluate the integral $\int_0^1 \tan^{-1}(1-x+x^2)dx$

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3. Let $f(x), x \geq 0$, be a non-negative continous function,and let $F(x) = \int_0^x dt, x \geq 0$, if for some $c > 0$, $f(x) \leq cF(x)$ for all $x \geq 0$, then show that $f(x) = 0$ for all $x \geq 0$.

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4. If f(x) $= \frac{x-1}{x+1}, f^2(x) = f(f(x)), \ldots\ldots, \ldots\ldots f^{k+1}(x) = f\left(f^k(x)\right)$,k=1,2,3,......and $g(x) = f^{1998}(x)$ then $\int_{1/e}^{1} g(x)dx$ is equal to

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5. If $I_n = \int_0^\infty e^{-x} x^{n-1} \log_e x dx$, then prove that $I_{n+2} - (2n+1)I_{n+1} + n^2 I_n = 0$

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6. If $f: R \to R, f(x) = x + \sin x$, then value of $\int_0^{\pi} \left(f^{-1}(x)\right) dx$ equals

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7. If $n > 1$, evaluate: $\int_0^{\infty} \frac{1}{\left(x + \sqrt{1 + x^2}\right)^n} dx$

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8. Evaluate: $\int_{-1}^{1} \frac{x^3 + 4x^2 - 10}{(x+3)^2 (x+2)^3} dx$

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9. If $\int_0^{\pi} \left(\frac{x}{1 + \sin x}\right)^2 dx = A$, then the value for $\int_0^{\pi} \frac{2x^2 . \cos^2 x / 2}{\left(1 + \sin x^2\right)} dx$

is equal to

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10. Evaluate $\int_{-4}^{-5} e^{(x+5)^2} dx + 3\int_{1/3}^{2/3} e^{9\left(x-\frac{2}{3}\right)^2} dx.$

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11. Evaluate $\int_0^{3\pi/2} (\log|\sin x|)(\cos(2nx))dx, n \in N.$

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12. Find a function $g: R \to R,$ continous in $[0, \infty)$ and positive in $(0, \infty)$ satisfying $g(0) = 1$ and $\frac{1}{2}\int_0^x g^2(t)dt = \frac{1}{x}\left(\int_0^x g(t)dt\right)^2$

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13. Evaluate: $\int_0^{\frac{\pi}{2}} \ln(1 + \sin\alpha \sin^2 x)\cos ec^2 x dx$ (where $\alpha > 0$)

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14. Evaluate: $\int_0^1 \frac{x^\alpha - 1}{\log_e x} dx$

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15. Prove that $\int_0^{\pi/2} \frac{dx}{\left(a^2 \sin^2 x + b^2 \cos^2 x\right)^2} = \frac{\pi\left(a^2 + b^2\right)}{4a^3 b^3}$.

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16. The value of $\int_0^\infty \left[2e^{-x}\right] dx$ (where ,[.] denotes the greatest integer function of x) is equal to

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17. Evaluate: $\int_{-\left(\frac{\pi}{4}\right)}^{\frac{3\pi}{4}} \frac{\sqrt{2}\left[1 + \sin\left(x - \frac{\pi}{4}\right)\right]}{\sqrt{2} - \cos|x| + \sin|x|} dx$

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18. If $\int_{-1}^{1} \left(\sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right] \right) dx = k\pi$ where $[x]$ denotes the integral part of x, then the value of k is ...

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19. Evaluate the integral:

$$\int_0^4 \frac{\{\sqrt{x}\}}{(1+\sin\{[x]\})^5} dx + \int_0^{\frac{\pi}{4}} \sin(x-[x]) d\left(x-[x]^5\right)$$

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20. Evaluate $\int_0^{\pi} \frac{x \sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi} dx.$

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21. Evaluate $\int_0^{\pi} \frac{x \sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi} dx.$

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22. Evaluate $\int_0^{\pi/4} \frac{x^2(\sin 2x - \cos 2x)dx}{(1+\sin 2x)\cos^2 x}$

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23. Evaluate: $\int_{\frac{1}{2}}^{2} \frac{1}{x}\tan^7\left(x - \frac{1}{x}\right)dx$

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24. Evaluate : $\int_{-\sqrt{2}}^{\sqrt{2}} \frac{2x^7 + 3x^6 - 10x^5 - 7x^3 - 12x^2 + x + 1}{x^2 + 2}dx$

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25. Prove that $\int_0^x e^{zx}e^{-z^2}dz = e^{\frac{x^2}{4}}\int_0^x e^{-\frac{z^2}{4}}dz$

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26. Evaluate the definite integral: $\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}}\left(\frac{x^4}{1-x^4}\right)\cos^{01}\left(\frac{2x}{1+x^2}\right)dx\cdot$

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27. The value of $\int_{-1/\sqrt{3}}^{1/\sqrt{3}}\frac{\cos^{-1}\left(\frac{2x}{1+x^2}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)dx}{1+e^x}$ is equal to

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28. Find the value of $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{\pi+4x^3}{2-\cos\left(|x|\frac{\pi}{3}\right)}dx$

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29. Evaluate: $\int_0^{\pi}e^{|\cos x|}\left(2s\in\left(\frac{1}{2}\cos x\right)+3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x dx\cdot$

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30. Given a function $f(x)$ such that It is integrable over every interval on the real line, and $f(t+x)=f(x)$, for every x and a real t. Then show that the integral $\int_a^{a+t} f(x)dx$ is independent of a.

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31. It is known that $f(x)$ is an odd function in the interval $\left[\frac{p}{2}, \frac{p}{2}\right]$ and has a period p, Prove that $\int_q^x (t)dt$ is also periodic function with the same period.

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32. Show that $\int_0^{n\pi+v} |\sin x|dx = 2n+1-\cos v$, where n is a positive integer and `,lt=v

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33. If $f(x)$ is a function satisfying $f(x+a)+f(x)=0$ for all $x \varepsilon R$ and positive constant a such that $\int_b^{c+b} f(x)dx$ is independent to b, then find the least positive value of c.

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34. Estimate the absolute value of the integral $\int_{10}^{19} \frac{\sin x}{1+x^8} dx$

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35. Prove that $\int_0^1 \sqrt{(1+x)(1+x^3)}dx$ cannot exceed $\sqrt{15/8}$.

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36. Prove that $\frac{\pi}{6} \leq \int_0^1 \frac{dx}{\sqrt{4-x^2-x^3}} \leq \frac{\pi}{4\sqrt{2}}$

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37. Prove that $4 \leq \int_1^3 \sqrt{3+x^3}dx \leq 2\sqrt{30}$.

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38. prove it $2e^{-\frac{1}{4}} < \int_0^2 e^{x^2-x}dx < 2e^2$

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39. If $f(x) = \cos\left(\tan^{-1}x\right)$, then the value of the integral $\int_0^1 xf''(x)dx$ i s (a) $\frac{3-\sqrt{2}}{2}$ (b) $\frac{3+\sqrt{2}}{2}$ 1 (d) $1 - \frac{3}{2\sqrt{2}}$

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40. The value of the definite integral $\int_0^{3\pi/4} [(1+x)\sin x + (1-x)\cos x]dx$ is

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41. The equation of the curve is $y = f(x)$. The tangents at $[1, f(1), [2, f(2)], and [3, f(3)]$ make angles $\frac{\pi}{6}, \frac{\pi}{3}, and \frac{\pi}{4}$, respectively, with the positive direction of x-axis. Then the value of $\int_2^3 f'(x) f^x dx + \int_1^3 f^x dx$ is equal to $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (e) 0 (d) none of these

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42. Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt, 0 \leq x \leq 1, and\ f(0) = 0$, then

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43. If $\int_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1 + \pi^x) \sin x} dx, n = 0, 1, 2, \ldots\ldots\ldots$ then

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44. Let the function f satisfies f(x).f ' (−x)=f(−x).f ' (x) for all x and f(0)=3 The value of $f(x).\ f(-x)$ for all x is

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45. Let f be a function such that $f(x).\ f'(-x) = f(-x).\ f'(x)$ for all x and $f(0) = 3.$

Now answer the question:Number of roots of equation $f(x) = 0$ in interval $[-2, 2]$ is

(A) 0 (B) 2 (C) 4 (D) 1

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46. Suppose f and g are differentiabel functions such that $xg(f(x))f'(g(x))g'(x) = (g(x))g'(f(x))f'(x)\ \forall x \in R$ and f is positive $\forall n \in R.$ Also $\int_0^x f(g(t))dt = \frac{1}{2}(1 - e^{-2x})\ \forall x \in R, g(f(0)) = 1$ and $h(x) = \frac{g(f(x))}{f(g(x))}$

The value of $f(g(0)) + g(f(0))$ is equal to:

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47. Suppose f and g are differentiabel functions such that $xg(f(x))f'(g(x))g'(x) = (g(x))g'(f(x))f'(x)\ \forall x \in R$ and f is positive $\forall n \in R.$ Also $\int_0^x f(g(t))dt = \frac{1}{2}(1 - e^{-2x})\ \forall x \in R, g(f(0)) = 1$ and $h(x) = \frac{g(f(x))}{f(g(x))}$ \
The value of $f(g(0)) + g(f(0))$ is equal to:

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48. Suppose f and g are differentiabel functions such that $xg(f(x))f'(g(x))g'(x) = (g(x))g'(f(x))f'(x)\ \forall x \in R$ and f is positive $\forall n \in R.$ Also $\int_0^x f(g(t))dt = \frac{1}{2}(1 - e^{-2x})\ \forall x \in R, g(f(0)) = 1$ and $h(x) = \frac{g(f(x))}{f(g(x))}$ \
The value of $f(g(0)) + g(f(0))$ is equal to:

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49. Evaluate: $5050\frac{\int 01\left(1-x^{50}\right)^{100}dx}{\int 01(1-x^{50})^{101}dx}$

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50. For any real number x, let $[x]$ denote the largest integer less than or equal to x, Let f be a real-valued function defined on the interval $[-10, 10]$ be $f(x) = \{x-[x], \quad \text{if} \quad [x]$ is odd, $1+[x]-x, \quad \text{if} \quad [x]$ is even Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x dx$ is___

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51. Let $f: R \to R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of $f(1n5)$ is_____

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Exercise

1. Let

$f(x) = |\sec x \cos x sec^2 x + \cot x \cos ec\, x \cos^2 x \cos^2 x \cos ec^2 x 1 \cos^2 x \cos^2 x$

. Prove that $\int_0^{\pi/2} f(x) dx = -\frac{\pi}{4} - \frac{8}{15}$.

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2. $\int_0^{\frac{\pi}{4}} \frac{2 \sin \theta \cos \theta}{\sin^4 \theta + \cos^4 \theta} d\theta$

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3. $\int_0^1 x^5 \sqrt{\frac{1+x^2}{1-x^2}} dx$

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4. $\int_1^2 \frac{x-1}{x+1} \cdot \frac{1}{\sqrt{x^3+x^2+x}} dx$

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5. Evaluate : $\int_1^{16} \tan^{-1}\sqrt{\sqrt{x}-1}dx$

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6. Evaluate: $\int_0^1 \cot^{-1}(1-x+x^2)dx$

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7. If $U_n = \int_0^{\frac{\pi}{2}} \frac{\sin^2 nx}{\sin^2 x}dx$, then show that $U_1, U_2, U_3 \ldots\ldots U_n$ constitute an AP. Hence or otherwise find the value of U_n.

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8. If n is a positive integer, prove that: $\int_0^{2\pi} \frac{\cos(n-1)x - \cos nx}{1-\cos x}dx = 2\pi$, hence or otherwise, show that $\int_0^{2\pi}\left(\frac{\sin\left(\frac{nx}{2}\right)}{\sin\left(\frac{x}{2}\right)}\right)^2 dx = 2n\pi.$

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9. Let $f(x) = \int_0^{\sin^2 x} \sin^{-1}(\sqrt{t})\,dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t})\,dt$, then (A) $f(x)$ is a constant function (B) $f\left(\frac{\pi}{4}\right) = 0$ (C) $f\left(\frac{\pi}{3}\right) = \frac{\pi}{4}$ (D) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

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10. Prove that for $n > 1$.

$$\int_0^1 \left(\cos^{-1} x\right)^n dx = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)\int_0^1 \left(\cos^{-1} x\right)^{n-2} dx$$

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11. Prove that the value of : $\int_0^{\pi} \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)} dx = \pi$

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12. $$If I_n = \int_0^1 x^n (\tan^{-1} x) dx, then prove that$$

$$(n+1)I_n + (n-1)I_{n-2} = -\frac{1}{n} + \frac{\pi}{2}$$

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13. Evaluate: $\int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2+1)(x^4-1)} dx$

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14. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sec x}{1 + 2\sin^2 x} dx$

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15. Evaluate the following : $\int_0^1 \frac{2 - x^2}{(1+x)\sqrt{1-x^2}} dx$

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16. Let $I_m = \int_0^{\pi} \frac{1 - \cos mx}{1 - \cos x} dx$. Show that $I_m = m\pi$.

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17. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{2 + \sin x + \cos x} dx$

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18. Show that: $\int_0^1 \frac{\tan^{-1} x}{x} dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} y \cos ecy dy$

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19. Find $\int_0^1 \frac{\log\left(2 + x^{\frac{1}{3}}\right)}{x^{\frac{1}{3}}} dx$

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20. If $[x]$ denotes the integral part of x, find $\int_0^x [t+1]^3 dt.$

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21. Prove that $\int_0^x [t]dt = \frac{[x]([x]-1)}{2} + [x](x-[x]),$ where [.] denotes the greatest integer function.

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22. If $[x]$ denotes the integral part of $[x]$, show that: $\int_0^{(2n-1)\pi} [\sin x]dx = (1-n)\pi, n \in N$

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23. If $[x]$ denotes the integral part of x and $f(x) = \min(x-[x], -x-[-x])$ show that: $\int_{-2}^{2} f(x)dx = 1$

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24. Prove that $\int_0^x [t]dt = \frac{[x]([x]-1)}{2} + [x](x-[x])$, where [.] denotes the greatest integer function.

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25. Evaluate: $\int_{-100}^{100} Sgn(x-[x])dx$, where $[x]$ denotes the integral part of x.

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26. Show that: $\int_0^{[x]} (x-[x])dx = \frac{[x]}{2}$, where $[x]$ denotes the integral part of x.

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27. Show that: $\left(\int_0^x\right.$ [x]dx/(int_0^[x] {x}dx)=[x]-1, $where$[x] denotes the integral part of x and $\{x\} = x - [x]$.

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28. The value of $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$ where [.] denotes greatest integer function , is

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29. Evaluate $\int_0^{\pi/2} \frac{\sin^2 x dx}{\sin x + \cos x}$

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30. Evaluate: $\int_0^{\pi} \sin^m \cos^{2m+1} x dx$

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31. about to only mathematics

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32. about to only mathematics

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33. Evaluate: $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$

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34. Evaluate: $\int_0^1 \frac{\sin^{-1} x}{x} dx$

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35. Evaluate $\int_0^{\pi/2} x \cot x dx$

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36. Find the value of the integral $\int_0^{\pi} \log(1 + \cos x) dx$.

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37. Evaluate: $\int_0^{\infty} \log\left(x + \frac{1}{x}\right) \frac{dx}{1 + x^2}$

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38. Evaluate: $\int_0^{\pi} x \log \sin x dx$

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39. $\int_0^{\pi/2} \left(\frac{\theta}{\sin \theta}\right)^2 d\theta =$

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40. Evaluate: $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\varphi}{1-\sin\varphi} d\varphi$

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41. Evaluate: $\int_0^2 \frac{dx}{(17+8x-4x^2)\left[e^{6(1-x)}+1\right)}$

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42. If f is an odd function, then evaluate $I = \int_{-a}^{a} \frac{f(\sin x)dx}{f(\cos x)+f(\sin^2 x)}$

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43. If $f(x)$ is a continuous function and attains only rational values in $[-3, 3]$ and its greatest value in $[-3, 3]$ is 5, then $\int_{-3}^{3} f(x)dx =$ (A) 5 (B) 10 (C) 20 (D) 30

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44. If $g(x)=\int_0^x \cos^4 t dt,$ then $g(x+\pi)$ equals $g(x)+g(\pi)$ (b) $g(x)-g(\pi)$ $g(x)g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

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45. $IfA_n=\int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x}{\sin x} dx, b_n=\int_0^{\frac{\pi}{2}} \left(\frac{\sin nx}{\sin x}\right)^2 dx f \text{ or } n \in N,$

Then $A_{n+1}=A_n$ (b) $B_{n+1}=B_n$ $A_{n+1}-A_n=B_{n+1}$ (d) $B_{n+1}-B_n=A_{n+1}$

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46. If $n \in N$ and $\int_0^1 e^x (x-1)^n dx = 2e-5$, then $n=$ (A) 1 (B) 2 (C) 3 (D) none of these

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47. If $\int_0^1 \frac{e^t}{t+1} dt = a$, then $\int_{b-1}^{b} \frac{e^{-t}}{t-b-1} dt$=

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48. $If \int_0^x f(t)dt = x + \int_x^1 tf(t)dt,$ then the value of $f(1)$

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49. If $n \neq 1, \int_0^{\frac{\pi}{4}} (\tan^n x + \tan^{n-2} x) d(x - [x]) =$ (A) $\frac{1}{n-1}$ (B) $\frac{1}{n+1}$ (C) $\frac{1}{n}$ (D) $\frac{2}{n-1}$

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50. The equation $\int_0^x (t^2 - 8t + 13) dt = x \sin\left(\frac{a}{x}\right)$ has a solution if $\sin\left(\frac{a}{6}\right)$ is

(A) zero (B) -1 (C) 1 (D) none of these

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51. If $f(\alpha) = f(\beta)$ and $n \in N$, then the value of $\int_{\alpha}^{\beta} (g(f(x)))^n g'(f(x)) \cdot f'(x) dx =$

(A) 1 (B) 0 (C) $\frac{\beta^{n+1} - \alpha^{n+1}}{n+1}$ (D) none of these

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52. If $l_1 = \int_{e}^{e^2} \frac{dx}{\log x}$ and $l_2 = \int_{1}^{2} \frac{e^x}{x} dx$, then (A) $l_1 = 2l_2$ (B) $l_1 + l_2 = 0$ (C) $2l_1 = l_2$ (D) $l_1 = l_2$

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53. Let $f: (0, \infty) \to R$ and $F(x) = \int_{0}^{x} f(t) dt. If F(x^2) = x^2(1+x)$, then f (4) is equal to

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54. $\int_0^{\frac{4}{\pi}} \left(3x^2 \sin\left(\frac{1}{x}\right) - x\cos\left(\frac{1}{x}\right)\right)dx =$

(A) $\frac{8\sqrt{2}}{\pi^3}$

(B) $\frac{32\sqrt{2}}{\pi^3}$

(C) $\frac{24\sqrt{2}}{\pi^3}$

(D) $\frac{\sqrt{2048}}{\pi^3}$

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55. $\int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2}, a < 1$ is equal to

(A) $\frac{\pi a \log 2}{4}$

(B) $\frac{4\pi}{2 - a^2}$

(C) $\frac{\pi}{1 - a^2}$

(D) none of these

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56. $\int_0^1 \log\left(\sqrt{1+x}+\sqrt{1-x}\right)dx =$

(A) $\frac{1}{2}\left(\log 2 - \frac{\pi}{2} + 1\right)$

(B) $\frac{1}{2}\left(\log 2 + \frac{\pi}{2} + 1\right)$

(C) $\frac{1}{2}\left(\log 2 + \frac{\pi}{2} - 1\right)$

(D) none of these

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57. If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x dx$, then $[I_4 + 12I_2]$ is equal to (A) 4π (B) $3\left(\frac{\pi}{2}\right)^3$
(C) $\left(\frac{\pi}{2}\right)^2$ (D) $4\left(\frac{\pi}{2}\right)^3$

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58. If $f(x) = ae^{2x} + be^x + cx$ satisfies the conditions $f(0) = -1, f'\log(2) = 28, \int_0^{\log 4} [f(x) - cx]dx = \frac{39}{2}$, then

(A) $a = 5, b = 6, c = 3$

(B) $a = 5, b = -6, c = 0$

(C) $a = -5, b = 6, c = 3$

(D) none of these

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59. $\int_0^{\pi} \frac{x}{1 + \sin x}$

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60. All the value of d for which

$\int_1^2 \{a^2 + (4 - 4a)x + 4x^3\}dx \leq 12$ are given by

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61. Let f (x) be a function satisfying $f(x) = f(x)$ with f(0) = 1 and g be the function satisfying $f(x) + g(x) = x^2$

The value of integral $\int f(x)g(x)dx$ is

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62. If $\int_0^1 \frac{\sin t}{1+t} dt = \alpha$, then the value of the integral $\int_{4\pi-2}^{4\pi} \frac{\sin\left(\frac{t}{2}\right)}{4\pi+2-t} dt$ is (1)2α (2) -2α (3) α (4) $-\alpha$

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63. If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\tan x)^{-n} dx (n > 1)$, then $I_n + I_{n+2} =$ (A) $\frac{1}{n-1}$ (B) $\frac{1}{n+1}$ (C) $-\frac{1}{n+1}$ (D) $\frac{1}{n} - 1$

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64. $\int_0^{\frac{\pi}{2}} \frac{3+4\cos x}{(4+3\cos x)^2} dx =$ (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{4}$

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65. If a$f(x)$+b$f\left(\frac{1}{x}\right)=\frac{1}{x}-5, x\neq 0, a\neq b$, then $\int_1^2 f(x)dx$ equals

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66. $\int_0^{\pi}\frac{\sin\left(\frac{n+1}{2}\right)x}{\sin x}dx=$ (A) 0 (B) $\frac{\pi}{2}$ (C) π (D) none of these

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67. If $f(x)=\int_0^{\sin x}\cos^{-1}tdt+\int_0^{\cos x}\sin^{-1}tdt, 0<x<\frac{\pi}{2}$, then $f\left(\frac{\pi}{4}\right)=$ (A) 0 (B) $\frac{\pi}{\sqrt{2}}$ (C) 1 (D) none of these

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68. Let $f(x)=\int_0^{\sin^2 x}\sin^{-1}(\sqrt{t})dt+\int_0^{\cos^2 x}\cos^{-1}(\sqrt{t})dt$, then (A) $f(x)$ is a constant function (B) $f\left(\frac{\pi}{4}\right)=0$ (C) $f\left(\frac{\pi}{3}\right)=\frac{\pi}{4}$ (D) $f\left(\frac{\pi}{4}\right)=\frac{\pi}{4}$

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69. If $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, then $\int_0^{\infty} e^{-ax^2} dx$ where $a > 0$ is:

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70. Evalaute $\lim_{n \to \infty} \left(\frac{1}{n}\right) \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$

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71. $Lt_{n \to \infty} \sum_{r=1}^{6n} \frac{1}{n+r} =$ (A) $\log 6$ (B) $\log 7$ (C) $\log 5$ (D) 0

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72. $Lt_{n \to \infty} \sum_{r=1}^{n} \frac{(2r)^k}{n^{k+1}}, k \neq -1$, is equal to (A) $\frac{2^k}{k-1}$ (B) $\frac{2^k}{k}$ (C) $\frac{1}{k+1}$ (D) $\frac{2^k}{k+1}$

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73. $Lt_{n \to \infty}\left(\frac{n!}{n^n}\right)^{\frac{1}{n}}$ = (A) e^{-2} (B) e^{-1} (C) e^3 (D) e

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74. If $k \neq 0$ is a constant and $n \in N$ then, $\lim_{n \to \propto}\left\{\frac{n!}{(kn)^n}\right\}$ is equal to

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75. The value of $(\lim)_{n \xrightarrow{\infty}} \sum_{r=1}^{4n} \frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+\sqrt{n})^2}$ is equal to $\frac{1}{35}$ (b) $\frac{1}{4}$ (c) $\frac{1}{10}$ (d) $\frac{1}{5}$

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76. $l_n = \int_0^{\pi/4} \tan^n x dx$, then $\lim_{n \to \infty} n[l_n + l_{n-2}]$ equals

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77. Let $\frac{d}{dx}F(x)=\left(\frac{e^{\sin x}}{x}\right), x>0 If \int_1^4 \frac{3}{x}e^{\sin x^3}dx=F(k)-F(1)$,

then one of the possible values of k, is:

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78. $\lim_{n\to\infty}\frac{1+2^4+3^4+\ldots..+n^4}{n^5}-\lim_{n\to\infty}\frac{1+2^2+3^3+\ldots..+n^3}{n^5}$

is:

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79. If $f(y)=e^y, g(y)=y, y>0$ and $F(t)=\int_0^1 f(t-y)g(y)dt$ then

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80. The value of $Lt_{x\to 0}\left\{\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}\right\}$ is (A) 0 (B) 3 (C) 2 (D) 1

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81. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}}$ is (A) $1-e$ (B) $e-1$ (C) e (D) $e+1$

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82. The value of $I=\int_0^{\frac{\pi}{2}}\frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$ is (A) 2 (B) 1 (C) 0 (D) 3

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83. Evaluate the following limit:

$$\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+\ldots\ldots\ldots\ldots+\frac{1}{n}\sec^2 1\right]$$

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84. Let $f: R\overrightarrow{R}$ be a differentiable function having $f(2) = 6, f'(2) = \frac{1}{48}$.

Then evaluate $(\lim)_{x \overrightarrow{2}} \int_6^{f(x)} \frac{4t^3}{x-2} dt$

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85. If $I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2^{x^3} dx, I_3 = \int_1^2 2^{x^2} dx$

and $I_4 = \int_1^2 2^{x^3} dx$ then

A. I1 > I2

B. I2 > I1

C. I4 > I3

D. I1>I3

Answer: null

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86. Let $F(x) = f(x) + f\left(\frac{1}{x}\right), \text{w h e r e} f(x) = \int_1^x \frac{\log t}{1+t} dt.$ Then $2F(x) =$ (1)1/2` (2) 0 (3) 1 (4) 2

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87. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$ is

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88. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx \text{and} J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$ Then which one of the following is true? (1) $I > \frac{2}{3} \text{and} j > 2$ (2) $I < \frac{2}{3} \text{and} j < 2$ (3) $I < \frac{2}{3} \text{and} j > 2$ (4) $I > \frac{2}{3} \text{and} j < 2$

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89. $\int_0^{\pi} [\cot x] dx,$ where [.] denotes the greatest integer function, is equal to

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90. Let $f(x)=\int_1^x \sqrt{2-t^2}dt\cdot$ Then the real roots of the equation $x^2-f'(x)=0$ are ± 1 (b) $\pm\frac{1}{\sqrt{2}}$ $\pm\frac{1}{2}$ (d) 0 and 1

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91. If $I(m,n)=\int_0^1 t^m(1+t)^n.dt$, then the expression for $I(m,n)$ in terms of $I(m+1,n-1)$ is:

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92. If $f(x)$ is differentiable and $\int_0^{t^2} xf(x)dx=\frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals (a)$\frac{2}{5}$ (b) $-\frac{5}{2}$ (c)1 (d) $\frac{5}{2}$

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93. $\int_{-2}^{0}\{x^3+3x^2+3x+3+(x+1)\cos(x+1)\}dx$ is equal to

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94. If $\int_{\sin x}^{1} t^2(f(t))dt=(1-\sin x)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is

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95. Consider the function $f:(-\infty,\infty)\to(-\infty,\infty)$ defined by $f(x)=\frac{x^2-ax+1}{x^2+ax+1}, 0<a<2$, and let $g(x)=\int_0^{e^x}\frac{f'(t)dt}{1+t^2}$. Which of the following is true? (A) $g'(x)$ is positive on $(-\infty,0)$ and negative on $(0,\infty)$ (B) $g'(x)$ is negative on $(-\infty,0)$ and positive on $(0,\infty)$ (C) $g'(x)$ changes sign on both $(-\infty,0)$ and $(0,\infty)$ (D) $g'(x)$ does not change sign on $(-\infty,\infty)$

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96. The value of integral $\int_0^{\log 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} dx$ is

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98. $\int_0^a [f(x) + f(-x)]dx =$ (A) 0 (B) $2\int_0^a f(x)dx$ (C) $\int_{-a}^a f(x)dx$ (D) none of these

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99. If $f(x) = \begin{cases} e^{\cos x} \sin x & |x| \leq 2 \\ 2 & otherwise \end{cases}$. Then $\int_{-2}^3 f(x)dx =$ _____

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100. $\sqrt{3}\int_0^{\pi} \frac{dx}{1+2\sin^2 x} =$ (A) $-\pi$ (B) 0 (C) π (D) none of these

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101. The value of the integral $\int_0^{\infty} \frac{x\log x}{(1+x^2)^2}dx$ (a) 0 (b) log 7 (c) 5 log 13

(d) none of these

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102. $\int_{-\left(\frac{\pi}{3}\right)}^{\frac{\pi}{3}} \frac{x^3\cos x}{\sin^2 x}dx =$ (A) 0 (B) 1 (C) -1 (D) none of these

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103. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{e^x.\sec^2 x dx}{e^{2x}-1}$ is equal to

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105. If $\int_{0}^{100\pi+a}|\sin x|dx = k - \cos\alpha$, where $0 < \alpha < \pi$, then k= (A) 101 (B) 100 (C) 201 (D) none of these

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106. $\int_{-\pi}^{3\pi}\cot^{-1}(\cot x)dx =$ (A) π^2 (B) $2\pi^2$ (C) $3\pi^2$ (D) none of these

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107. The value of $\int_{-3}^{3}\log\left(\frac{30-x^3}{30+x^3}\right)dx$ is (A) $2\int_{0}^{3}\log\left(\frac{30-x^3}{30+x^3}\right)dx$ (B) $\log\left(\frac{3}{57}\right)$ (C) $\log\left(\frac{57}{3}\right)$ (D) 0

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108. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is

(A) $\frac{\pi}{2}$

(B) π

(C) 0

(D) 2π

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109. The integral $\int_{-a}^{a} \frac{\sin^2 x}{1-x^2} dx, 0 < a < 1,$ is equal to (A) $\int_{-a}^{a} \frac{\sin^2 x}{1+x^2} dx$ (B) $2\int_0^{a} \frac{\sin^2 x}{1-x^2} dx$ (C) $\int_a^{0} \frac{\sin^2 x}{1+x^2} dx$ (D) 0

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110. Evaluate: $\int_{-\pi}^{\pi} (1-x^2) \sin x \cos^2 x dx$

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111. $\int_{-1}^{1} \frac{\sin x + x^2}{3 - |x|} dx =$ (A) 0 (B) $2\int_0^1 \frac{\sin x}{3 - |x|} dx$ (C) $2\int_0^1 \frac{x^2}{3 - |x|} dx$ (D) $2\int_0^1 \frac{\sin x + x^2}{3 - |x|} dx$

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112. $\int_{-a}^{a} \left(1 + x^3\right)^{-1} dx =$ (A) 0 (B) $2\int_0^a \left(1 - x^6\right)^{-1} dx$ (C) $2\int_0^a \left(1 + x^3\right)^{-1} dx$ (D) $2\int_0^a \left[1 + \left(a - x^3\right)\right]^{-1} dx$

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113. Q. $\int_0^{\pi} e^{\cos^2 x} \left(\cos^3 (2n + 1) x dx, n \in I\right.$

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114. Evaluate the following integral: $\int_0^{\pi} \frac{x}{1 + \cos \alpha \sin x} dx$

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115. Write the coefficient a, b, c of which the value of the integral $\int_{-3}^{3}(ax^2+bx+c)dx$ is independent.

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116. The value of the integral $\int_0^{\frac{3}{2}}[x^2]dx$, where $[]$ denotes the greatest integer function, is (A) $2+\sqrt{2}$ (B) $2-\sqrt{2}$ (C) $4+2\sqrt{2}$ (D) $4-2\sqrt{2}$

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117. The value of the integral $\int_{-2}^{2}|1-x^2|dx$ is a.4 b. 2 c. -2 d. 0

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118. Let f be a function defined by $f(x)=\frac{4^x}{4^x+2}$ $I_1=\int_{f(1-a)}^{f(a)}xf\{x(1-x)\}dx$ and $I_2=\int_{f(1-a)}^{f(a)}f\{x(1-x)\}dx$ where

$2a-1>0$ then $I_1 : I_2$ is

(A) 2 (B) k (C) $\frac{1}{2}$ (D) 1

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119. If $[.]$ represents the greatest integer function, then $\int_4^{10} \frac{[x^2]}{[x^2-28x+196]+[x^2]} dxisequa < o$ 0 (b) 1 (b) 3 (d) none of these

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120. The value of $\int_{-1}^{1} [x[1+\sin \pi x]+1] dx$ where [.] is greatest integer factor, is (A) 3 (B) 2 (C) 8 (D) 1

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121. The value of $\sum_{n=1}^{1000} \int_{n-1}^{n} e^{x-[x]}\,dx$, where $[x]$ is the greatest integer function, is (A) $\frac{e^{1000}-1}{1000}$ (B) $\frac{e-1}{1000}$ (C) $\frac{e^{1000}-1}{e-1}$ (D) $1000(e-1)$

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122. If $a = \int_{\frac{1}{2}}^{2} \frac{1}{x}\cot^7\left(x - \frac{1}{x}\right)dx$, then (A) $a = 0$ (B) $0 < a < 1$ (C) $a > 0$ (D) none of these

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123. $\int_0^2 x^3\left[1 + \cos\left(\frac{\pi x}{2}\right)\right]dx$, where $[x]$ denotes the integral part of x, is equal to (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) none of these

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124. $\int_0^1 [x^2 - x + 1] dx$, where $[x]$ denotes the integral part of x, is (A) 1 (B) 0 (C) 2 (D) none of these

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125. If $[x]$ denotes the integral part of x and $a = \int_{-1}^{0} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx, b = \int_0^1 \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} d(x - [x])$, then (A) $a = b$ (B) $a = -b$ (C) $a = 2b$ (D) none of these

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126. $\int_0^{\pi} \frac{dx}{1 + 10^{\cos x}} + \int_{-1}^{1} \log\left(\frac{2 - x}{2 + x}\right) dx =$ (A) $\frac{\pi}{2}$ (B) $-\pi$ (C) 0 (D) none of these

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127. Let $f(x) = \int_0^x \frac{\sin^{100} t}{\sin^{100} t + \cos^{100} t} dt$, then $f(2\pi) =$

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128. Let $[x]$ denote the greatest integer less than or equal to x, then $\int_0^{\frac{\pi}{4}} \sin x d(x - [x]) =$

(A) $\frac{1}{2}$ (B) $1 - \frac{1}{\sqrt{2}}$ (C) 1 (D) none of these

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129. The value of $\int_{\frac{1}{e} \to \tan x} \frac{tdt}{1+t^2} + \int_{\frac{1}{e} \to \cot x} \frac{dt}{t \cdot (1+t^2)} =$

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130. The value of $\int_{-1}^{1} \frac{\sin^2 x}{\left[\frac{x}{\sqrt{2}}\right] + \frac{1}{2}} dx$, where $[x]$=greatest integer less than or equal to x, is (A) 1 (B) 0 (C) $4 - \sin 4$ (D) none of these

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131. If $I = \int\limits_0^1 \frac{x}{8+x^3} dx$ then the smallest interval is which I less is

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132. The value of $\int_0^1 e^{x^2} dx$ is

(A) less than e (B) greater than 1 (C) less than e but greater than 1 (D) all of these

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133. The value of $\int_0^x \frac{2^x}{2^{[x]} dx}$ is equal to (where , [.] denotes the greatest integer function)

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134. $\int_0^1 \frac{\tan^{-1} x}{x} dx - \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{t}{\sin t} dt$ has the value (A) -1 (B) 1 (C) 2 (D) 0

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135. $\int_0^{\pi} \cos 2x \log \sin x dx =$ (A) π (B) $-\frac{\pi}{2}$ (C) $\frac{\pi}{2}$ (D) none of these

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136. If $a, b(a < b)$ be the points of discontinuity of function $f(f(f(x)))$, where $f(x) = \frac{1}{1-x}, x \neq 1$, then $\int_a^b \frac{f(x)}{f(x) + f(1-x)} dx =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

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137. Evaluate $\int_{-1}^{3} \left(\tan^{-1} \frac{x}{x^2+1} + \tan^{-1} \frac{x^2+1}{x} \right) dx.$

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139. If f be decreasing continuous function satisfying $f(x+y)=f(x)+f(y)-f(x)f(y)Yx, y\varepsilon R, f'(0)=1$ then $\int_0^1 f(x)dx$ is

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140. $\int_0^x [\sin t]dt, where x \in (2n\pi, (2n+1)\pi), n \in N, and [.]$ denotes the greatest integer function is equal to $-n\pi$ (b) $-(n+1)\pi$ $2n\pi$ (d) $-(2n+1)\pi$

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141. Find the value of : $\int_0^{10} e^{2x-[2x]} d(x-[x]) where [.]$ denotes the greatest integer function).

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142. If 1 lies between the roots of the equation $y^2 - my + 1 = 0$ and [x] denotes the greatest integer less than or equal to x, then the values of $\left[\left(\frac{4|x|}{|x|^2 + 16}\right)^m\right]$, is

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143. $\int_{-10}^{0} \frac{\left|\frac{2\{x\}}{3x - [x]}\right|}{\frac{2[x]}{3x - [x]}} dx$ is equal to (where $[\,*\,]$ denotes greatest integer function.)

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144. $\int_0^{2\pi} e^{\sin^2 nx} \tan nx\,dx =$ (A) 1 (B) π (C) 2π (D) 0

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145. $\int_a^b \frac{|x|}{x} dx$, $a < b$, is equal to (A) $b - a$ (B) $b + a$ (C) $|b| - |a|$ (D) $|b| + |a|$

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146. The equation $\int_{-\pi/4}^{\pi/4} \left\{ a|\sin x| + \frac{b \sin x}{1 + \cos^2 + c} dx = 0, \right.$ where a, b, c, are constants gives a relation between

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147. Let $f(x) = \max . \{2 - x, 2, 1 + x\}$ then $\int_{-1}^{1} f(x) dx =$ (A) 0 (B) 2 (C) $\frac{9}{2}$ (D) none of these

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148. Let f(x) be a continuous function such that $f(a - x) + f(x) = 0$ for all $x \neq [0, a]$. Then $\int_0^a \frac{dx}{1 + e^{f(x)}}$ is equal to :

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149. Let $f: R\overrightarrow{a}ndg: R\overrightarrow{R}$ be continuous function. Then the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[f(x)+f(-x)][g(x)-g(-x)]dxis$ (a)π (b) 1 (c) -1 (d) 0

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150. $\int_0^{\pi}\frac{x}{1+\sin x}$

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151. $\int_{\log\left(\frac{1}{3}\right)}^{\log 3}\sin\left(\frac{e^x-1}{e^x+1}\right)dx=$ (A) $\log 3$ (B) $\sin 3$ (C) $2\sin 3$ (D) 0

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152. Let $f(x)$ be a continuous function in R such that $f(x) + f(y) = f(x + y)$, then $\int_{-2}^{2} f(x)dx =$ (A) $2\int_{0}^{2} f(x)dx$ (B) 0 (C) $2f(2)$ (D) none of these

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153. If $fandg$ are continuous function on $[0, a]$ satisfying $f(x) = f(a - x)andg(x)(a - x) = 2,$ then show that $\int_{0}^{a} f(x)g(x)dx = \int_{0}^{a} f(x)dx.$

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154. Let $f(x)$ be a continuous function in R such that $f(x)$ does not vanish for all $x \in R$. If $\int_{1}^{5} f(x)dx = \int_{-1}^{5} f(x)dx$, then in R, $f(x)$ is (A) an even function (B) an odd function (C) a periodic function with period 5 (D) none of these

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155. Let $f(x)$ be an integrable odd function in $[-5, 5]$ such that $f(10+x) = f(x)$, then $\int_x^{10+x} f(t)dt =$ (A) 0 (B) $2\int_0^5 f(x)dx$ (C) > 0

(D) none of these

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156. If $\int_0^1 xe^{x^2} dx = k\int_0^1 e^{x^2} dx$, then (A) $k > 1$ (B) $0 < k < 1$ (C) $k = 1$

(D) none of these

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157. If $\int_0^1 e^{x^2}(x - \alpha)dx = 0$, then

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158. Let $a = \int_0^{\frac{\pi}{2}} \frac{\sin x}{x} dx$, then (A) $0 < a < 1$ (B) $a > 2$ (C) $1 < a < \frac{\pi}{2}$

(D) none of these

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159. If $f(x) = \int_0^x \frac{e^{\cos t}}{e^{\cos t} + e^{-(\cos t)}} dt$, then $2f(\pi) =$ (A) 0 (B) π (C) $-\pi$

(D) none of these

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160. $\int_0^{\sqrt{2}} [x^2] dx$ is equal to (A) $2 - \sqrt{2}$ (B) $2 + \sqrt{2}$ (C) $\sqrt{2} - 1$ (D) $\sqrt{2} - 2$

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161. If $f(a + b - x) = f(x)$, then $\int_a^b x f(x) dx$ is equal to

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162. The value of $\int_{-2}^{3} |1 - x^2| dx$ is (A) $\frac{7}{3}$ (B) $\frac{14}{3}$ (C) $\frac{28}{3}$ (D) $\frac{1}{3}$

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163. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is

(A) $\frac{\pi}{2}$

(B) π

(C) 0

(D) 2π

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164. If $f(x) = \frac{e^x}{1 + e^x}, I_1 = \int_{f(-a)}^{f(a)} x g(x(1 - x)) dx,$ and $I_2 = \int_{f(-a)}^{f(a)} g(x(1 - x)) dx,$ then the value of $\frac{I_2}{I_1}$ is

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165. Find the value of integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx.$

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166. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is

(A) $\frac{\pi}{2}$

(B) π

(C) 0

(D) 2π

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167. Evaluate the following: $\int_{-3\pi/2}^{-\pi/2} \left[(x+\pi)^3 + \cos^2(x+3\pi)\right] dx$

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168. The value of $\int_1^a [x]f'(x)dx f'(x)dx, where a > 1, and [x]$ denotes the greatest integer not exceeding x, is

$af(a) - \{f(1)f(2) + \ldots + f([a])\}$

$[a]f(a) - \{f(1) + f(2) + \ldots + f([a])\}$

$[a]f(a) - \{f(1) + f(2) + \ldots + fA\}$

$af([a]) - \{f(1) + f(2) + \ldots + fA\}$

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169. The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{(\log)_e x}{x}\right| dx is \frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 5

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170. If $f(x) = \begin{cases} e^{\cos x} \sin x & |x| \leq 2 \\ 2 & otherwise \end{cases}$. Then $\int_{-2}^{3} f(x)dx =$ ______

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171. Find the value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0.$

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172. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([x] + 1n\left(\frac{1+x}{1-x}\right)\right) dx$ is equal to (where [.] represents the greatest integer function)

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173. Let $T > 0$ be a fixed real number. Suppose f is continous function such that for all $x \varepsilon R, f(x+T) = f(x)$. If $I = \int_0^T f(x) dx$, then the value of $\int_3^{3+3T} f(2x) dx$ is

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174. $\int_{\frac{1}{n}}^{\frac{an-1}{n}} \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}} dx =$ (A) $\frac{a}{2}$ (B) $\frac{na+2}{2n}$ (C) $\frac{na-2}{2n}$ (D) none of these

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175. $\int_{-1}^{1} \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2+1} dx$ is equal to
(A) $\frac{4}{5}$ (B) $\frac{5}{4}$ (C) $\frac{4}{3}$ (D) $\frac{3}{4}$

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176. $\int_{0}^{4014} \frac{2^x}{2^x + 2^{4014-x}} dx =$ (A) 2^{2007} (B) 2^{4014} (C) 4014 (D) 2007

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177. $\int_{0}^{3} x(3-x)^{\frac{3}{2}} dx =$ (A) $\frac{108\sqrt{3}}{35}$ (B) $-\frac{108\sqrt{3}}{35}$ (C) $\frac{54\sqrt{3}}{35}$ (D) none of these

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178. $\int_2^4 \log[x]dx$ is (A) $\log 2$ (B) $\log 3$ (C) $\log 5$ (D) none of these

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179. If $I = \int_0^{3\pi} f(\cos^2 x)dx$ and $J = \int_0^{\pi} f(\cos^2 x)dx$, then (A) $I = 5J$ (B) $I = J$ (C) $I = 3J$ (D) none of these

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180. $\int_{-\pi}^{\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx$ is equal to

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181. The value of the integral $\int_{-a}^{a} \frac{xe^{x^2}}{1+x^2} dx$ is (A) e^{a^2} (B) 0 (C) e^{-a^2} (D) a

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182. The function f is continuous and has the property $f(f(x)) = 1 - x$ for all $x \in [0, 1]$ and $f = \int_0^1 f(x)dx$, then

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183. A function f is defined by $f(x) = \int_0^{\pi} \cos t \cos(x - t)dt, 0 \leq x \leq 2\pi$.

Then which of the following hold(s) good ?

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184. Let $L = \lim_{n \to \infty} \int_a^{\infty} \frac{ndx}{1 + n^2x^2}$ where $a \in R$ then $\cos L$ can be (A) -1 (B) 0 (C) 1 (D) $\frac{1}{2}$

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185. $\int_0^{\infty} \frac{xdx}{(1 + x)(1 + x^2)}$

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186. Let $f(x)=\int_1^x \frac{3^t}{1+t^2}dt$, where $x>0$, Then

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187. Let $\phi(x,t)=\begin{cases}x(t-1) & x\le t\\ t(x-1) & t<x\end{cases}$, where t is a continuous function of x in $[0,1]$. Let $g(x)=\int_0^1 f(t)\phi(x,t)dt$, then $g''(x)$= (A) $g(0)+g(1)=1$ (B) $g(0)=0$ (C) $g(1)=1$ (D) $g''(x)=f(x)$

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188. Evaluate: $\int_0^{\sin^2 x}\sin^{-1}\sqrt{t}dt+\int_0^{\cos^2 x}\cos^{-1}\sqrt{t}dt$

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189. The value of the integral $\int_0^{\frac{\pi}{4}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ (A) $\frac{1}{ab} \tan^{-1}\left(\frac{b}{a}\right)(a, b > 0)$ (B) $\frac{1}{ab} \tan^{-1}\left(\frac{b}{a}\right)(a, b < 0)$ (C) $\frac{\pi}{4}(a = 1, b = 1)$ (D) none of these

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190. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is

(A) $\frac{\pi}{2}$

(B) π

(C) 0

(D) 2π

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191. Let $\int_a^b \frac{f(a+b-x)}{f(a+b-x)+f(a+b-(a+b-x))} dx = 4$, then (A) $a = -1, b = 7$ (B) $a = 0, b = 8$ (C) $a = -10, b = 2$ (D) $a = 10, b = 18$

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192. Statement-1: If $\int_0^{\infty} e^{-ax} dx = \frac{1}{a}$, then $\int_0^{\infty} x^m e^{-ax} dx = \frac{\lfloor m}{a^{m+1}}$,Statement-2: $\frac{d^n}{dx^n}(e^{kx}) = k^n e^{kx}$ and $\frac{d^n}{dx^n}\left(\frac{1}{x}\right) = \frac{(-1)^n \lfloor n}{x^{n+1}}$ (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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193. Statement-1: $\int_0^{[x]} 4^{x-[x]} dx = \frac{3[x]}{2\log 2}$,Statement-2: $\int_0^{[x]} a^{x-[x]} dx = [x]\int_0^1 a^{x-[x]} dx$ (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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194. Statement-1: Let a, b, c be non zero real numbers and $f(x) = ax^2 + bx + c$ satisfying $\int_0^1 (1+\cos^8 x) f(x) dx = \int_0^2 (1+\cos^8 x) f(x) dx$ then the equation $f(x) = 0$ has at least one root in $(0, 2)$.Statement-2: If $\int_a^b g(x) dx$ vanishes and $g(x)$ is continuous then the equation $g(x) = 0$ has at least one real root in (a, b). (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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195. Let $I_1 = \int_0^1 \frac{e^x}{1+x} dx$ and $I_2 = \int_0^1 \frac{x^2 e^{x^2}}{2-x^3} dx$Statement-1: $\frac{I_1}{I_2} = 3e$Statement-2: $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$ (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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196. Statement-1: $\int_{-3}^{3} x^8\{x^9\}dx = 2 \times 3^7$, where $\{x\}$ denotes the fractional part of x.Statement-2: $[x] + [-x] = -1$, if x is not an integer, where $[x]$ denotes the integral part of x. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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197. Let a function f be even and integrable everywhere and periodic with period 2. Let $g(x) = \int_0^x f(t)dt$ and $g(t) = k$The value of $g(x+2) - g(x)$ is equal to (A) $g(1)$ (B) 0 (C) $g(2)$ (D) $g(3)$

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198. Let a function f be even and integrable everywhere and periodic with period 2. Let $g(x) = \int_0^x f(t)dt$ and $g(t) = k$The value of $g(2)$ in terms of k is equal to (A) k (B) $2k$ (C) $3k$ (D) $5k$

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199. If $m > 0, n > 0$, the definite integral $I = \int_0^1 x^{m-1}(1-x)^{n-1}dx$ depends upon the values of m and n is denoted by $\beta(m, n)$, called the beta function.Obviously, $\beta(n, m) = \beta(m, n)$.Now answer the question:The integral $\int_0^{\frac{\pi}{2}} \cos^{2m}\theta \sin^{2n}\theta d\theta =$ (A) $\frac{1}{2}\beta\left(m+\frac{1}{2}, n+\frac{1}{2}\right)$ (B) $2\beta(2m, 2n)$ (C) $\beta(2m+1, 2n+1)$ (D) none of these

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200. If $m > 0, n > 0$, the definite integral $I = \int_0^1 x^{m-1}(1-x)^{n-1}dx$ depends upon the values of m and n is denoted by $\beta(m, n)$, called the beta function.Obviously, $\beta(n, m) = \beta(m, n)$.Now answer the question:If $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}dx = k\int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}}dx$, then k is equal to (A) $\frac{m}{n}$ (B) 1 (C) $\frac{n}{m}$ (D) none of these

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201. If $m > 0, n > 0$, the definite integral $I = \int_0^1 x^{m-1}(1-x)^{n-1} dx$ depends upon the values of m and n is denoted by $\beta(m, n)$, called the beta function.Obviously, $\beta(n, m) = \beta(m, n)$.Now answer the question:If $\int_0^2 (8 - x^3)^{\frac{-1}{3}} dx = k\beta\left(\frac{1}{3}, \frac{2}{3}\right)$, then k equals to (A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

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202. Comprehension # 1 (For Q. No. 39 to 40) Consider the integral

$$I = \int_0^{10\pi} \frac{\cos 4x \cos 5x \cos 6x \cos 7x}{1 + e^{2 \sin 2x}} dx$$

$IfI = k\int_0^{\frac{\pi}{2}} \cos 4x \cos 5x \cos 6x \cos 7x dx,$ then $'k'$ is equal to 5 (b) 10 (c) 1 (d) 20

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203. Comprehension # 1 (For Q. No. 39 to 40) Consider the integral

$$I = \int_0^{10\pi} \frac{\cos 4x \cos 5x \cos 6x \cos 7x}{1 + e^{2 \sin 2x}} dx$$

$IfI = k\int_0^{\frac{\pi}{2}} \cos 4x \cos 5x \cos 6x \cos 7x dx,$ then $'k'$ is equal to 5 (b) 10

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204. Comprehension # 1 (For Q. No. 39 to 40) Consider the integral

$$I = \int_0^{10\pi} \frac{\cos 4x \cos 5x \cos 6x \cos 7x}{1 + e^{2\sin 2x}} dx$$

$IfI = k\int_0^{\frac{\pi}{2}} \cos 4x \cos 5x \cos 6x \cos 7x dx,$ then $'k'$ is equal to 5 (b) 10

(c) 1 (d) 20

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205. Evaluate: If $\int f(x)dx = g(x), then \int f^{-1}(x)dx$

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206. If $A = \int_0^1 x^{50}(2-2x)^{50}dx,\, B = \int_0^1 x^{50}(1-x)^{50}dx$, which of the following is true? (A) $A = 2^{50}B$ (B) $A = 2^{-50}B$ (C) $A = 2^{100}B$ (D) $A = 2^{-100}B$

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207. Let $u = \int_0^1 \frac{\ln(x+1)}{x^2+1}dxandv = \int_0^{\frac{\pi}{2}} \ln(\sin 2x)dx, then$ (a) $u = -\frac{\pi}{2}\ln 2$ (b) $4u + v = 0$ (c)$u + 4v = 0$ (d) $u = \frac{\pi}{8}\ln 2$

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208. If $f(4-x) = f(4+x)$ and $f(8-x) = f(8+x)$ and $f(x)$ is a function for which $\int_0^8 f(x)dx = 5$. Then $\int_0^{200} f(x)dx$ is equal to

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209. Let $f(x) = \int_1^x \frac{dt}{t^3(1+t^3)^{\frac{1}{3}}}$ and $\lim_{x \to \infty} f(x) = \frac{a^2-1}{a^3}$, then a^{30}=

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210. Let $a = \int_0^{\log 2} \frac{2e^{3x} + e^{2x} - 1}{e^{3x} + e^{2x} - e^x + 1} dx$, then $4e^a$=

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211. If $\int_0^1 \frac{1 + x + \sqrt{x + x^2}}{\sqrt{x} + \sqrt{1 + x}} dx = \frac{1}{a}(2^a - 1)$, then the value of $4a^2$ is ...

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212. $\alpha = \frac{\pi}{4020}$ and $\beta = \frac{2009}{4020}\pi$ and $\int_\alpha^\beta \frac{dx}{1 + \tan x} = \frac{k\pi}{8040}$, then k is equal to .....

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213. If $\int_0^{\frac{\pi^2}{4}} \left(2 \sin \sqrt{x} + \sqrt{x} \cos \sqrt{x}\right) dx = \frac{\pi^2}{n}$, then n= ....

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214. If $\int_0^{4+2\sqrt{3}} \frac{16}{4+x^2} dx = \frac{3+m\pi}{12}$, then m= ...

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215. The value of $\int_{-2\pi}^{5\pi} \cot^{-1}(\tan x) dx$ is equal to

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216. $\int_1^{10\pi} \left(\left[\sec^{-1} x\right] + \left[\cot^{-1} x\right] dx\right.$ where [.] denotes the greatest integer function is (A) $10\pi - \sec 1$ (B) $10\pi + \sec 1$ (C) $10\pi - \sec 1 + \cot 1$ (D) $\sec 1 + \cot 1$

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217. $f(x)$ is a continuous function for all real values of x and satisfies $\int_{n}^{n+1} f(x)dx = \frac{n^2}{2} \forall n \varepsilon I$. Then $\int_{-3}^{5} (|x|)dx$ is equal to

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218. For $x \in R$ and a continuous function f, let $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} x f\{x(2-x)\}dx$ and $I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f\{x(2-x)\}dx$.Then $\frac{I_1}{I_2}$ is (a) -1 (b) 1 (c) 2 (d) 3

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