MATHS

BOOKS - KC SINHA ENGLISH

DIFFERENTIAL EQUATIONS - FOR COMPETITION

Solved Examples

1. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of (a) order 1 (b) order 2 (c) degree 3 (d) degree 4

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2. From the differential equation having $y = (\sin^{-1} x)^2 + A\cos^{-1} x + B$, where A A and B are arbitary constants as its general solutions.

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3. Find the order of the differential equation of the family of curves. $y = a\sin x + b\cos(x + c)$, where a, b, c are parameters.

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4. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where $c_1 \text{and} c_2$ are arbitrary constants, is (1) $y' = y^2$ (2) $y'' = y'y$ (3) $yy'' = y'$ (4) $yy'' = (y')^2$

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5. Solve the following differential equation: $\sqrt{1 + x^2 + y^2 + x^2 y^2} + xy\frac{dy}{dx} = 0$

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6. Solve the differential equation $\frac{dy}{dx} = \frac{2}{x+y}$

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7. Solve $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$

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8. Solve the differential equation $\left(x^2 + 4y^2 + 4xy\right)dy = (2x + 4y + 1)dx$

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9. Solve: $xdx + ydy = xdy - ydx$

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10. Solve $\dfrac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \dfrac{y^4}{x^2}$

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11. Solve $\left(1 + 2e^{x/y}\right)dx + 2e^{x/y}(1 - x/y)dy = 0.$

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12. Solve: $\dfrac{dy}{dx} = \dfrac{\sin y + x}{\sin 2y - x\cos y}$

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13. Solve $\dfrac{dy}{dx} = \dfrac{y}{2y \ \ln \ y + y - x}$

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14. Solve $\frac{dy}{dx} + xy = xy^2$

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15. Solve $\frac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1.$

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16. Solve $\sin y. \frac{dy}{dx} = \cos y(1 - x\cos y).$

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17. Solve:

$$\frac{dy}{dx} = \frac{yf'(x) - y^2}{f(x)}$$

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18. If $\phi(x)$ is a differentiable function, then the solution of the different equation $dy+\{y\phi'(x)-\phi(x)\phi'(x)\}dx=0,$ is

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19. Let u(x) and v(x) be two continous functions satisfying the differential equations $(du)(dx)+p(x)u=f(x)$ and $\frac{dv}{dx}+p(x)v=g(x),$ respectively. If $u(x_1)>v(x_1)$ for some x_1 and $f(x)>g(x)$ for all $x>x_1$, prove that any point (x,y),where $x>x_1$, does not satisfy the equations $y=u(x)$ and $y=v(x)$ simultaneously.

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20. Solve $\frac{y+\sin x\cos^2(xy)}{\cos^2(xy)}dx+\left(\frac{x}{\cos^2(xy)}+\sin y\right)dy=0$

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21. Solve $(2x \log y)dx + \left(\frac{x^2}{y^2} + 3y^2\right)dy = 0.$

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22. The solution of

$e^{x\frac{(y^2-1)}{y}}\{xy^2dy + y^3dx\} + \{ydx - xdy\} = 0$, is

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23. Solve: $x^2dy - y^2dx + xy^2(x - y)dy = 0$

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24. The solution of differential equation

$xdy\left(y^2e^{xy} + e^{x/y}\right) = ydx\left(e^{x/y} - y^2e^{xy}\right)$, is

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25. Let $y = f(x)$ be a curve passing through $(1, 1)$ such that the triangle formed by the coordinate axes and the tangent at any point of the curve lies in the first quadrant and has area 2. Form the differential equation and determine all such possible curves.

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26. A curve $y = f(x)$ passes through the point $P(1, 1)$. The normal to the curve at P is $a(y - 1) + (x - 1) = 0$. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point. Determine the equation of the curve

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27. Find all the curves $y = f(x)$ such that the length of tangent intercepted between the point of contact and the x-axis is unity.

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28. A curve passing through the point (1,1) has the porperty that the perpendicular distance of the normal at any point P on the curve from the origin is equal to the distance of P from x-axis Determine the equation of the curve.

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29. A country has a food deficit of 10%. Its population grows continously at a rate of 3% per year. Its annual food production every year is 4% more than that of the last year. Assuming that the average food requirement per person remains constant, prove that the country will become self-sufficient in food after n years, where n is the smallest integer bigger than or equal to $\dfrac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$

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30. A hemi-spherical tank of radius 2 m is initially full of water and has an outlet of $12cm^2$ cross-sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $v(t) = 0.6\sqrt{2gh(t)}$, where $v(t)$ and $h(t)$ are, respectively, the velocity of the flow through the outlet and the height of water level above the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank.

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31. At any point (x,y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (−4,−3). Find the equation of the curve given that it passes through (−2,1)

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32. The ordinate and the normal at any point P on the curve meet the x-axis at points A and B respectively. Find the equation of the family of curves satisfying the condition , The product of abscissa of P and AB =arithmetic mean of the square of abscissa and ordinate of P.

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33. Consider a curve y=f(x) in xy-plane. The curve passes through (0,0) and has the property that a segment of tangent drawn at any point P(x,f(x) and the line y=3 gets bisected by the line x+y=1. then the equation of curve, is

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34. If the velocity of flow of water through a small hole is $0.6\sqrt{2gy}$, where g is the acceleration due to gravity and y is the height of water level above the hole, find the time required to empty a tank having the shape of a right circular cone of base radius a and height h filled completely with water and having a hole of area A_0 in the base.

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35. An inverted cone of height H, and radius R is pointed at bottom. It is completely filled with a volatile liquid. If the rate of evaporation is directly proportional to the surface area of the liquid in contact with air (constant of proportionality k $>$ 0). Find the time in which whole liquid evaporates.

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36. The tangent at a point P of a curve meets the axis of y in N, the line through P parallel to the axis of y meets the axis of x at M, O is the origin. If the area of $\triangle MON$ is constant. Show that the curve is a hyperbola.

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37. Let the $f(x)$ be differentiabe function on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x}\left(\frac{t^2 f(x) - x^2 f(t)}{t^2 - x^2}\right) = \frac{1}{2} \forall x > 0$, then $f(x)$

is:

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38. The differential equation of the family of curves whose equation is $(x-h)^2+(y-k)^2=a^2$, where a is a constant, is (A) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=a^2\frac{d^2y}{dx^2}$ (B) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=a^2\left(\frac{d^2y}{dx^2}\right)^2$ (C) $\left[1+\left(\frac{dy}{dx}\right)\right]^3=a^2\left(\frac{d^2y}{dx^2}\right)^2$ (D) none of these

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39. Let y_1 and y_2 be two different solutions of the equation $\frac{dy}{dx}+P(x).\,y=Q(x)$. Then $\alpha y_1+\beta y_2$ will be solution of the given equation if $\alpha+\beta=\ldots\ldots\ldots\ldots\ldots\ldots$

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41. Suppose we define integral using the following formula $\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b))$, for more accurate result for $c \in (a, b), F(c) = \frac{c-a}{2}(f(a)+f(c)) + \frac{b-c}{2}(f(b)+f(c))$.

When $c = \frac{a+b}{2}$, then $\int_a^b f(x)dx = \frac{b-a}{4}(f(a)+f(b)+2f(c))$.

$\lim_{t \to a} \frac{\int_a^t f(x)dx - \frac{(t-a)}{2}(f(t)+f(a))}{(t-a)^3} = 0 \forall a$ Then the degree of $f(x)$ can at most be

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42. Let f be a real-valued differentiable function on R (the set of all real numbers) such that $f(1) = 1$. If the $y - \in tercept$ of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to the cube of the abscissa of P, then the value of $f(-3)$ is equal to_______

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Exercise

1. Find the degree of the differential equation: $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}}=\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$

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2. Find order and degree: $\cos\left(\frac{dy}{dx}\right)=x+y$

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3. Find order and degree: $e^{\frac{dy}{dx}}=x^2+1$

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4. Find order and degree: $e^{\frac{dy}{dx}} = \left(1 + \frac{d^2y}{dx^2}\right)$

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5. Find order and degree: $\log_e\left(1 + \frac{d^2y}{dx^2}\right) = x$

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6. Form the differential equation of the family of curves $y = a\sin(bx + c),\ a\ and\ c$ being parameters.

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7. Find the differential equation whose solution represents the family $xy = ae^x + be^{-x}$

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8. Find the differential equation of all straight lines touching the circle $x^2 + y^2 = a^2$

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9. Form the differential equation of the family of circles touching the y-axis at origin.

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10. Form the differential equation of the family of circles touching the x-axis at origin.

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11. Solve: $(1 - x^2y^2)dx = ydx + xdy$

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12. Reduce the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$ using the transformation $y = v(x).\, e^x$. Hence solve the equation when $y = 1, \frac{dy}{dx} = 0$, for $x = 0$

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13. Solve: $2y\frac{dy}{dx} = e^{\frac{x^2+y^2}{x}} + \frac{x^2+y^2}{x} - 2x$

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14. Solve: $x\left(\frac{dy}{dx}\right)^2 + (y - x)\frac{dy}{dx} - y = 0$

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16. A curve passes through the point $(5, 3)$ and at any point (x, y) on it, the product of its slope and the ordinate is equal to its abscissa. Find the equation of the curve and identify it.

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17. Show that the equation of the curve passing through the point $(1, 0)$ and satisfying the differential equation $(1 + y^2)dx - xydy = 0$ is $x^2 - y^2 = 1$

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18. If $p = 9, v = 3,$ then find p is terms of v from the equation $\frac{dp}{dv} = v + \frac{1}{v^2}$

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19. Solve the equation $e^x dx + e^y(y+1)dy = 0$

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20. Solve: $\left(\frac{dy}{dx}\right) + \frac{\sqrt{(x^2-1)(y^2-1)}}{xy} = 0$

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21. Solve the differential equation $\cos^2 x \frac{d^2y}{dx^2} = 1$

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22. Solve: $\tan^{-1}\left(\frac{dy}{dx}\right) = x + y$

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23. Determine the equation of the curve passing through the origin, in the form $y = f(x)$, which satisfies the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$.

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24. Solution of $\left(\frac{x+y-a}{x+y-b}\right)\left(\frac{dy}{dx}\right) = \left(\frac{x+y+a}{x+y+b}\right)$

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25. The solution of the differential equation $ydx - xdy + xy^2dx = 0$, is

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26. Solve the differential equation

$xdy - ydx = \sqrt{x^2 + y^2}dx.$

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27. Solve $(x + y + 1)(dy/dx) + 2xy = x\sqrt{1 - x^2}$

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28. Solve: $x + y\frac{dy}{dx} = \left(a^2 \frac{\left(x\frac{dy}{dx} - y\right)}{x^2 + y^2}\right)$

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29. $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$

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30. The solution of $\frac{xdx + ydy}{xdy - ydx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}}$, is given by

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31. The solution of $\frac{xdy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right)dx$, is given by

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32. Solve the differential equation $y + x\frac{dy}{dx} = x$

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33. Solve $\frac{dy}{dx} = \frac{(x+y)^2}{(x+2)(y-2)}$

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34. $\left(xe^{\frac{y}{x}} - y\sin\left(\frac{y}{x}\right)\right)dx + x\sin\left(\frac{y}{x}\right)dy = 0$

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35. Solve the differential equation $(xdy - ydx)y\sin\left(\frac{y}{x}\right) = (ydx + xdy)x\cos\left(\frac{y}{x}\right)$.

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36. $x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + 2y^2 - x^2 = 0$

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37. Show that the differential equation $y^3dy + (x + y^2)dx = 0$ can be reduced to a homogeneous equation.

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38. Prove that the equation of a curve whose slope at (x, y) is $-\frac{x+y}{x}$ and which passes through the point $(2, 1)$ is $x^2 + 2xy = 8$

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39. Find the equation of the curve which passes through $(1, 0)$ and the slope of whose tangent at (x, y) is $\frac{x^2 + y^2}{2xy}$

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40. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solutions of the equation is

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41. Solve $xdy = \left(y + x\frac{f\left(\frac{y}{x}\right)}{f'\left(\frac{y}{x}\right)}\right)dx$

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42. The general solution of the differential equation $(1 + \tan y)(dx - dy) + 2xdy = 0$ is

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43. The solution of $(1+x^2)\frac{dy}{dx}+y=e^{\tan-1x}$, is given by

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44. If y_1 and y_2 are the solutions of the differential equation $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x alond and $y_2=y_1z$, then move that $z=1+ce^{-\int\left(\frac{Q}{y_1}\right)dx}$, where c is an arbitrary constant.

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45. If `y+d/(dx)(x y)=x(sinx+logx),find y.

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46. $\frac{dy}{dx}=x^3y^3-xy$

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47. If $\frac{dy}{dx} + 2y\tan x = \sin x$ and $y = 0$, when $x = \frac{\pi}{3}$, show that the maximum value of y is `1/8

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48. $\sec^2 y \frac{dy}{dx} + 2x\tan y = x^3$.

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49. $x\frac{dy}{dx} + y = y^2 \log x$

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50. $(x^2y^3 + xy)dy = dx$

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51. Solve $\frac{dy}{dx} + 2.\ \frac{y}{x}\frac{y^3}{x^3}$

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52. The solution of $\frac{dy}{dx} + \frac{y}{x}\log = \frac{y}{x^2}(\log y)^2$, is

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53. Solve: $(dy/dx) = e^{x-y}(e^x - e^y)$.

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54. $(y^2 e^x + 2xy)dx - x^2 dy = 0$

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55. General solution of $\left(2x - 10y^3\right)\dfrac{dy}{dx} + y = 0$, is

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56. $xy - \dfrac{dy}{dx} = y^3 e^{-x^2}$

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57. $\dfrac{dy}{dx} = \dfrac{2xy}{x^2 - 1 - 2y}$

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58. The solution of $\dfrac{dy}{dx} + yf'(x) - f(x).\, f'(x) = 0,\, y \neq f(x)$ is

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59. $y \sin x \dfrac{dy}{dx} = \left(\sin x - y^2\right)\cos x$

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60. Solution of the equation $\cos^2 x \frac{dy}{dx} - (\tan 2x)y = \cos^4 x,$ where $|x| < \frac{\pi}{4}$ and $y\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{8}$ is

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61. Solve $\left(x^2 - ay\right)dx + \left(y^2 - ax\right)dy = 0.$

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62. The solution of $ydx - xdy + \left(1 + x^2\right)dx + x^2 \sin ydy = 0,$ is given by

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63. The solution of the differential equation $\left\{1+x\sqrt{(x^2+y^2)}\right\}dx+\left\{\sqrt{(x^2+y^2)}-1\right\}ydy=0$ is equal to (a) $(b)(c)(d)x^{(e)2(f)}(g)+(h)\frac{(i)(j)y^{(k)2(l)}(m)}{n}2(o)(p)+(q)\frac{1}{r}3(s)(t)(u)(v$

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64. Solve: $(x+\log y)dy+ydx=0$

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65. The tangents to a curve at a point on it is perpendicular to the line joining the point with the origin. Find the equation of the curve.

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66. The tangent at a point 'P' of a curve meets the axis of 'y' in N, the parallel through 'P' to the axis of 'y' meets the axis of X at M, O is the origin of the area of ΔMON is constant then the curve is (A) circle C) ellipse (D) hyperbola (B) parabola

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67. Find the curve for which the intercept cut off by a tangent on x-axis is equal to four times the ordinate of the point of contact.

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68. Show that equation to the curve such that the y-intercept cut off by the tangent at an arbitrary point is proportional to the square of the ordinate of the point of tangency is of the form $\frac{a}{x} + \frac{b}{y} = 1.$

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69. Find the equation of the curve which is such that the area of the rectangle constructed on the abscissa of and the initial ordinate of the tangent at this point is a constanta $= a^2$.

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70. Which of the following is not the differential equation of family of curves whose tangent form an angle of $\frac{\pi}{4}$ with the hyperbola $xy = c^2$?

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71. Find the family of curves for which subnormal is a constant in a parabola.

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72. Find the equation of the curve whose slope at $x = 0$ is 3 and which passes through the point $(0, 1)$ satisfying the differential equation

$$\left(x^2+1\right)\frac{d^2y}{dx^2}=2x\frac{dy}{dx}.$$

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73. The curve in the first quadrant for which the normal at any point (x, y) and the line joining the origin to that point form an isosceles triangle with the x-axis as base is (a) an ellipse (b) a rectangular hyperbola (c) a circle (d) None of these

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74. Show that the curve for which the normal at every point passes through a fixed point is a circle.

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75. Find the curve in which the subtangent is always bisected at the origin.

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76. The curve is such that the length of the perpendicular from the origin on the tangent at any point P of the curve is equal to the abscissa of P. Prove that the differential equation of the curve is $y^2 - 2xy\frac{dy}{dx} - x^2 = 0$, and hence find the curve.

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77. The normal PG to a curve meets the x-axis in G. If the distance of G from the origin is twice the abscissa of P, prove that the curve is a rectangular hyperbola.

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78. Find the curve for which the area of the triangle formed by the x-axis tangent drawn at any point on the curve and radius vector of the point of tangency is constant equal to a^2

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79. Determine all curve for which the ratios of the linght of the sagment intercepted by tangent on the y-axis to the length of the radius vector is a constant.

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80. Given two curves: $y = f(x)$ passing through the point $(0, 1)$ and $g(x) = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{n}\right)$. The tangents drawn to both the curves at the points with equal abscissas intersect on the x-axis. Find the curve $y = f(x)$.

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81. A student studying a foreign language has 50 verbs to meemorize, the rate at which the student can memorize these verbs is proportional to the number of verbs remaining to be memorized, that is, if the student memorizes y verbs in t minutes, $\frac{dy}{dt} = k(50 - y)$Assume that initially no verbs are memorized, and suppose that 20 verbs are memorized in the first minutes. How many verbs will the student memorize in t min.

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82. Find the degree of the differential equation satisfying the relation $\sqrt{1+x^2} + \sqrt{1+y^2} = \lambda\left(x\sqrt{1+y^2} - y\sqrt{1+x^2}\right)$

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83. $x = f(t)$ satisfies $\frac{d^2x}{dt^2} = 2t + 3$ and for $t = 0, x = 0, \frac{dx}{dt} = 0$, then $f(t)$ is given by (A) $t^3 + \frac{t^2}{2} + t$ (B) $\frac{2t^3}{3} + \frac{3t^2}{2} + t$ (C) $\frac{t^3}{3} + \frac{3t^2}{2}$ (D) none of these

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84. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$ is

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85. If c_1, c_2 are arbitrary constants then general solution of the differential equation $\frac{d^2y}{dx^2}=e^{-3x}$ can be expressed as $y=9e^{-3x}+c_1x+c_2$ $y=-3e^{-3x}+c_1x+c_2$ $y=3e^{-3x}+c_1x+c_2$ $y=\frac{e^{-3x}}{9}+c_1x+c-2$

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86. The general solution of the differential equation $x^2(1+y^3)dx=y^2(1+x^3)dy$ is (A) $(1+x^2)(1+y^2)=C$ (B) $1+x^3=C(1+y^3)$ (C) $(x+y)(1+x^2+x^3)=C$ (D) $x(1+y^2)=Cy(1+x^2)$

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87. The degree of the differential equation of all tangent lines to the parabola $y^2 = 4ax$ is

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88. The equation of the curve passing through origin, whose slope at any point is $\frac{x(1+y)}{1+x^2}$, is (A) $(1+y)^2 - x^2 = 1$ (B) $x^2 + (y+1)^2 = 1$ (C) $(x+y)y = 1 - x^2$ (D) $x = ye^{(1+y)}$

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89. The solution of $\cos(x+y)dy = dx$ is (A) $y = \cos^{-1}\left(\frac{y}{x}\right) + C$ (B) $y = x\sec\left(\frac{y}{x}\right) + C$ (C) $y = \tan\left(\frac{x+y}{2}\right) + C$ (D) none of these

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90. If $\frac{dx}{dy} = 2^{\tan y} \sec^2 y$, then x is equal to (A) $\frac{2^{\tan y}}{\log 2} + C$ (B) $2^{\tan y} + C$ (C) $\tan y + C$ (D) none of these

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91. The differential equation of the family of curves $y = A(x+B)^2$ after eliminating A and B is (A) $yy'' = y'^2$ (B) $2yy'' = y' - y$ (C) $2yy'' = y' + y$ (D) $2yy'' = y'^2$

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92. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

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93. The general solution of differential equation $(e^x + 1)ydy = (y+1)(e^x)dx\text{is}$

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94. Show that the curve for which the normal at every point passes through a fixed point is a circle.

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95. The function $f(\theta) = \frac{d}{dth\eta}\int_0^\theta \frac{dx}{1-\cos\theta\cos x}$ satisfies the differential equation (a) $(b)(c)(d)\frac{(e)df((f)\theta(g))}{h}((i)dth\eta)(j)(k) + 2f((l)\theta(m)) = 0(n)$ (o) (p) $(q)(r)(s)\frac{(t)df}{u}((v)dth\eta)(w)(x) - 2f((y)\theta(z))\cot\theta = 0(aa)$ (bb) (cc) $(dd)(ee)(ff)\frac{(gg)df}{hh}((ii)dth\eta)(jj)(kk) + 2f((ll)\theta(mm)) = 0(nn)$ (oo) (d)

$(pp)(qq)(rr)\frac{(ss)df}{tt}((uu)dth\eta)(vv)(ww) - 2((xx)\theta(yy)) = 0(zz)$

(aaa)

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96. If the family of curves $y = ax^2 + b$ cuts the family of curves $x^2 + 2y^2 - y = a$ orthogonally, then the value of b= (A) 1 (B) $\frac{2}{3}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$

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97. If $f(x)$ is a differentiable real valued function such that $f(0) = 0$ and $f'(x) + 2f(x) \leq 1$, then (A) $f(x) > \frac{1}{2}$ (B) $f(x) \geq 0$ (C) $f(x) \leq \frac{1}{2}$ (D) none of these

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98. Solution of the equation $\cos^2 x \frac{dy}{dx} - (\tan 2x)y = \cos^4 x$, where $|x| < \frac{\pi}{4}$ and $y\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{8}$ is

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99. Given two curves: $y = f(x)$ passing through the point $(0, 1)$ and $g(x) = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{n}\right)$. The tangents drawn to both the curves at the points with equal abscissas intersect on the x-axis. Find the curve $y = f(x)$.

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100. The orthogonal trajectories of the family of curves $y = a^n x^n$ are given by (A) $n^2 x^2 + y^2$ = constant (B) $n^2 y^2 + x^2$ = constant (C) $a^n x^2 + n^2 y^2$ = constant (D) none of these

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101. Find the curve for which the length of normal is equal to the radius vector.

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102. The solution of the equation $\frac{dy}{dx} = x^3y^2 + xy$ is (A) $x^2y - 2y + 1 = cye^{-\frac{x^2}{2}}$ (B) $xy^2 + 2x - y = ce^{-\frac{y}{2}}$ (C) $x^2y - 2y + x = cxe^{-\frac{y}{2}}$ (D) none of these

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103. The solution of the equation $ydx - xdy = x^2ydx$ is (A) $y^2e^{-\frac{x^2}{2}} = C^2x^2$ (B) $y = Cxe^{\frac{x^2}{2}}$ (C) $x^2 = C^2y^2e^{x^2}$ (D) $ye^{x^2} = x$

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104. The solution of $\frac{dy}{dx} + yf'(x) - f(x).\ f'(x) = 0, y \neq f(x)$ is

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105. IF $y' = \frac{y}{x}(\log y - \log x + 1)$, then the solution of the equation is :

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106. The solution of $(x^2 + xy)dy = (x^2 + y^2)dx$ is

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107. The solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is

(a) $(b)(c)y = 2(d)$ (e) (b) $(f)(g)y = 2x(h)$ (i) (c) $(d)(e)y = 2x - 4(f)$

(g) (d) $(h)(i)y = 2(j)x^{(k)2(l)}(m) - 4(n)$ (o)

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108. The solution of the equation $xdy - ydx = \sqrt{x^2 - y^2}dx$ subject to the condition $y(1) = 0$ is (A) $y = x\sin(\log x)$ (B) $y = x^2\sin(\log x)$ (C)

$y = x^2(x-1)$ (D) none of these

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109. The differential equation of family of curves whose tangents form an angle of $\frac{\pi}{4}$ with the hyperbola $xy = k$ is (A) $\frac{dy}{dx} = \frac{x^2 + ky}{x^2 - ky}$ (B) $\frac{dy}{dx} = \frac{x+k}{x-k}$ (C) $\frac{dy}{dx} = -\frac{k}{x^2}$ (D) $\frac{dy}{dx} = \frac{x^2 - k}{x^2 + k}$

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110. The solution of $x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right)$ is (A) $\tan\left(\frac{y}{2x}\right) = C - \frac{1}{2x^2}$ (B) $\sec\left(\frac{y}{x}\right) = 1 + \frac{C}{y}$ (C) $\sin\left(\frac{y}{x}\right) = C + \frac{1}{y}$ (D) $y^2 = (C + x^2)\tan\left(\frac{y}{x}\right)$

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111. The solution of $xdy = (2y + 2x^4 + x^2)dx,$ is (A) $y = x^4 + x\log x + C$ (B) $y = x^2 + x\log x + C$ (C)

$y = x^4 + x^2 \log x + C$ (D) none of these

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112. The solution of differential equation $xy^2\left(y_1^2 + 2\right) = 2y_1 y^3 + x^3$, is

(A) $(x + y - a)\left(x^2 - y^2 - bx^2\right) = 0$ (B) $\left(x^2 - y^2 - a\right)\left(x^2 - y^2 + bx^4\right) = 0$ (C) $\left(x^2 + y^2 - a\right)\left(x^2 + y^2 - bx^4\right) = 0$ (D) none of these

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113. The solution of $\dfrac{dy}{dx} + \dfrac{xy^2 - x^2y^3}{x^2y + 2x^3y^2} = 0$, is (A) $\log\left(\dfrac{y^2}{x}\right) - \dfrac{1}{xy} = C$ (B) $\log\left(\dfrac{x}{y}\right) + \dfrac{y^2}{x} = C$ (C) $\log\left(x^2 y\right) + \dfrac{y^2}{x} = C$ (D) none of these

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114. The solution of $y^2 dx + \left(x^2 - xy + y^2\right) dy = 0$ is (A) $y = Ce^{\tan^{-1} x}$

(B) $y = Ce^{\tan^{-1} y}$ (C) $y = Ce^{\tan^{-1}\left(\frac{y}{x}\right)}$ (D)

$y = C\left[\tan^{-1}\left(\frac{y}{x}\right) + e^{x^2} + y^2\right]$

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115. The solution of $(1 - x^2)\frac{dy}{dx} + 2xy - x\sqrt{1 - x^2} = 0,$ is (A) $\frac{y}{(1 - x^2)} = \frac{1}{\sqrt{1 - x^2}} + C$ (B) $y(1 - x^2) = \sqrt{1 - x^2} + C$ (C) $y(1 - x^2)^{\frac{3}{2}} = \sqrt{1 - x^2} + C$ (D) none of these

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116. The solution of $y(2xy + e^x)dx - e^x dy = 0$ is (A) $x^2 + ye^{-x} = C$ (B) $xy^2 + e^{-x} = C$ (C) $\frac{x}{y} + \frac{e^{-x}}{x^2} = C$ (D) $x^2 + \frac{e^x}{y} = C$

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117. The largest value of c such that there exists a differentiable function $f(x)$ for $-c < x < c$ that satisfies the equation $y_1 = 1 + y^2$ with $f(0) = 0$ is (A) 1 (B) π (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

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118. If the slope of tangent to a curve $y = f(x)$ is maximum at $x = 1$ and minimum at $x = 0$, then equation of the curve which also satisfies $\frac{d^3y}{dx^3} = 4x - 3$, is (A) $y = \frac{x^4}{6} - \frac{x^3}{2} + \frac{x^2}{2} + 1$ (B) $y = \frac{x^4}{4} + x^3 - \frac{x^2}{3} + 1$ (C) $y = \frac{x^4}{4} - \frac{x^3}{7} + \frac{x^2}{3} + 3$ (D) none of these

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119. The degree and order of the differential equation of the family of all parabolas whose axis is x-axs are respectively

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120. Solve the differential equation: $(1 + y^2) + \left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$

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121. The differential equation of the family of curves of $x^2 + y^2 - 2ay = 0$ where a is arbitary constant, is

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122. IF $y' = \frac{y}{x}(\log y - \log x + 1)$, then the solution of the equation is :

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123. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of (a) order 1 (b) order 2 (c) degree 3 (d) degree 4

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124. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants, is of (a) second order and second degree (b) first order and second degree (c) first order and first degree (d) second order and first degree

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125. The differential equation of all circles passing through the origin and having their centres on the x-axis is (1) $x^2 = y^2 + xy\frac{dy}{dx}$ (2) $x^2 = y^2 + 3xy\frac{dy}{dx}$ (3) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (4) $y^2 = x^2 - 2xy\frac{dy}{dx}$

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126. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is (1) $(x2)y'^2 = 25(y2)^2$ (2) $(y2)y'^2 = 25(y2)^2$ (3) $(y2)2y'^2 = 25(y2)^2$ (4) $(x2)2y'^2 = 25(y2)^2$

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127. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is (1) $y = \ln x + x$ (2) $y = x\ln x + x^2$ (3) $y = xe(x-1)$ (4) $y = x\ln x + x$

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128. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt} - ty = 1 and y(0) = -1$ then show that $y(1) = -\frac{1}{2}$.

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129. If $y = y(x)$ and $\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x, y(0) = 1,$ then $y(\pi/2)$ equals

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130. If $xdy = ydx + y^2 dy$ and $y(1) = 1$, then $y(-3)$ is equal to (A) 1 (B) 5 (C) 4 (D) 3

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131. $(x^2 + y^2)dy = xydx. Ify(x_o) = e, y(1) = 1,$ then the value of x_o is equal to :

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132. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determinea a family of circles with :

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133. Solution of the following equation

cos x dy =y(sinx-y)dx,$0 < x < \frac{\pi}{2}$ is

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134. A curve $y = f(x)$ passes through $(1, 1)$ and tangent at $P(x, y)$ cuts the x-axis and y-axis at A and B , respectively, such that $BP : AP = 3$, then (a) equation of curve is $xy' - 3y = 0$ (b) normal at $(1, 1)$ is $x + 3y = 4$ (c) curve passes through $2, \frac{1}{8}$ (d) equation of curve is $xy' + 3y = 0$

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135. For the differential equation $(x^2 + y^2)dx - 2xydy = 0$, which of the following are true. (A) solution is $x^2 + y^2 = cx$ (B) $x^2 - y^2 = cx$ (C) $x^2 - y^2 = x + c$ (D) $y(0) = 0$

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136. The curve represented by the differential equation $(x^2 + y^2 + 1)dx - 2xydy = 0$ satisfying $y(1) = 1$ is (A)

$x^2 - y^2 + x - 1 = 0$ (B) $(x-1)^2 + (y-2)^2 = 1$ (C) a hyperbola (D) a circle

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137. Which of the following are true for the differential equation $\frac{dy}{dx} - \frac{y}{x} + \frac{5x}{(x+2)(x-3)} = 0$, if the curve represented by it passes through the point $\left(5, a\log\left(\frac{7}{12}\right)\right)$ (A) Integrating factor is $\frac{1}{x}$ (B) $a = 5$ (C) $a = 4$ (D) solution is $y = x\log\left(\frac{x+2}{6(x-3)}\right)$

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138. Which of the following are true for the curve represented by the differential equation $\sec^2 y\frac{dy}{dx} + 2x\tan y = x^3$ satisfying $y(1) = 0$ (A) equation of curve is $2\tan y = x^2 - 1$ (B) equation of curve is $y^2 = x^3 - 1$ (C) curve is a parabola (D) curve is not a conic

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139. Consider the differential equation of the family of curves $y^2 = 2a\left(x + \sqrt{a}\right)$, where a is a positive parameter.Statement 1: Order of the differential equation of the family of curves is 1.Statement 2: Degree of the differential equation of the family of curves is 2. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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140. Statement 1: Order of the differential equation of the family of curves $y = a\sin x + b\cos(x + c)$ is 3.Statement 2: Order of the differential equation of a family of curves is equal to the number of independent arbitrary constants in the equation of family of curves.`. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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141. Statement-1: Curve satisfying the differential equation $\frac{dy}{dx} = \frac{y}{2x}$ and passing through the point $(2, 1)$ is a parabola having focus $\left(\frac{1}{2}, 0\right)$ Statement-2: The differential equation $\frac{dy}{dx} = \frac{y}{2x}$ is homogeneous. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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142. Statement-1: The solution of the differential equation $(x^2 + y^2)dx = 2xydy$ satisfying $y(1) = 0$ is $x^2 - y^2 = x$.Statement-2: The differential equation $(x^2 + y^2)dx = 2xydy$ can be solved by putting $y = vx$. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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143. Statement-1: Solution of the differential equation $\tan y \cdot \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$ is $\sec y + 2\cos x = c$

.Statement-2: The differential equation $\tan y \cdot \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$ is homogenous

(A) Both 1 and 2 are true and 2 is the correct explanation of 1

(B) Both 1 and 2 are true and 2 is not correct explanation of 1

(C) 1 is true but 2 is false

(D) 1 is false but 2 is true

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144. Statement-1: The differential equation of all circles in a plane must be of order 3.Statement-2: The differential equation of family of curve $y = a\sin x + b\cos(x+c)$, where a, b, c are parameters is 2. (A) Both 1 and 2 are true and 2 is the correct explanation of 1 (B) Both 1 and 2 are true and 2 is not correct explanation of 1 (C) 1 is true but 2 is false (D) 1 is false but 2 is true

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145. The solution of differential equation $(1+x^2)y' + 2xy = 4x^2$, $y(0) = 0$ is :

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146. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at Q such that PQ is of constant length k. Answer the question:The differential equation describing such a curve is (A) $y\frac{dy}{dx} = \pm\sqrt{k^2 - x^2}$ (B) $x\frac{dy}{dx} = \pm\sqrt{k^2 - x^2}$ (C) $y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$ (D) $x\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$

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147. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at Q such that PQ is of constant length k. Answer the question:If the curve passes through the point $(0, k)$, then its equation is (A) $x^2 - y^2 = k^2$ (B) $x^2 + y^2 = k^2$ (C) $x^2 - y^2 = 2k^2$ (D) $x^2 + y^2 = 2k^2$

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148. A tangent drawn to the curve $y = f(x)$ at $P(x, y)$ cuts the x-axis and y-axis at A and B respectively such that $BP: AP = 2:1$. Given that $f(1) = 1$. Answer the question:Equation of curve is (A) $y = \frac{1}{x}$ (B) $y = \frac{1}{x^2}$ (C) $y = \frac{1}{x^3}$ (D) none of these

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149. A tangent drawn to the curve $y = f(x)$ at $P(x, y)$ cuts the x-axis and y-axis at A and B respectively such that $BP: AP = 2:1$. Given that $f(1) = 1$. Answer the question:The curve passes through the point (A) $\left(2, \frac{1}{4}\right)$ (B) $\left(2, \frac{1}{2}\right)$ (C) $\left(2, \frac{1}{8}\right)$ (D) none of these

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150. A tangent drawn to the curve $y = f(x)$ at $P(x, y)$ cuts the x-axis and y-axis at A and B respectively such that $BP : AP = 2 : 1$. Given that $f(1) = 1$. Answer the question:Equation of normal to curve at $(1, 1)$ is (A) $x - 4y + 3 = 0$ (B) $x - 3y + 2 = 0$ (C) $x - 2y + 1 = 0$ (D) none of these

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151. A pair of curves $y = f_1(x)$ and $y = f_2(x)$ are such that following conditions are satisfied.(i) The tangents drawn at points with equal abscissae intersect on y-axis.(ii) The normals drawn at points with equal abscissae intersect on x-axis. Answer the question:Which of the following is true (A) $f'_1(x) + f'_2(x) = c$ (B) $f'_1(x) - f'_2(x) = c$ (C) $f'^2_1(x) - f'^2_1(x) = c$ (D) $f'^2_1(x) + f'^2_2(x) = c$

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152. Curves $y = f(x)$ passing through the point $(0, 1)$ and $y = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{n}\right)$ are such that the tangents drawn to them at the point with equal abscissae intersect on x-axis. Answer the question:The equation of curve $y = f(x)$

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153. Curves $y = f(x)$ passing through the point $(0, 1)$ and $y = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{n}\right)$ are such that the tangents drawn to them at the point with equal abscissae intersect on x axis. find Curve $y = f(x)$

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154. Curves $y = f(x)$ passing through the point $(0, 1)$ and $y = \int_{-\infty}^{x} f(t)dt$ passing through the point $\left(0, \frac{1}{n}\right)$ are such that the tangents drawn to them at the point with equal abscissae intersect on x axis. find Curve $y = f(x)$

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155. A differential equation of the form $\frac{dy}{dx} + Py = Q$ is said to be a linear differential equation. Integrating factor of this differential equation is $e^{\int P dx}$ and its solution is given by $y.\, e^{\int P dx} = \int \left(Q e^{\int P dx}\right) dx + c.$ Answer the question:Solution of differential equation $(1 + y^2) dx + \left(x - e^{-\tan^{-1} y} dy = 0\right.$ is (A) $y = \tan^{-1} x + c$ (B) $y e^{\tan^{-1} x} = \tan^{-1} x + c$ (C) $x e^{\tan^{-1} y} = \tan^{-1} y + c$

(D) none of these

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156. Let the $f(x)$ be differentiabe function on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \left(\frac{t^2 f(x) - x^2 f(t)}{t^2 - x^2}\right) = \frac{1}{2} \forall x > 0,$ then $f(x)$ is:

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157. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis and the y-axis in point A AND $B,$ respectively, such that $\frac{1}{OA} + \frac{1}{OB}$ =1, where O is the origin. Find the equation of such a curve passing through $(5. 4)$

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158. For $x \in R, x \neq 0,$ if $y(x)$ differential function such that $x \int_1^x y(t)dt = (x+1)\int_1^x ty(t)dt,$ then $y(x)$ equals: (where C is a constant.)

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159. A curve $y = f(x)$ satisfies $\frac{d^2y}{dx^2} = 6x - 4$ and $f(x)$ has local minimum value 5 at $x = 1$. If a and b be the global maximum and global minimum values of $f(x)$ in interval $[0, 2]$, then ab is equal to...

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160. A line is drawn from a point p(x,y) on curve y=f(x), making an angle with the x-axis which is supplementaty to the one made by the tangent to the curve at p(x,y). The line meets the x-axis at A. another line perpendicular to the first, if drawn from p(x,y) meeting the y-axis at B. If OA=OB, where O is origin, find all curve which passes through (1,1)

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