MATHS

BOOKS - KC SINHA ENGLISH

INTEGRAL CALCULUS - PREVIOUS YEAR QUESTIONS - FOR COMPETITION

Solved Examples

1. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}.$ If the firm employs 25 more workers, then the new level of production of items is (1) 3000 (2) 3500 (3) 4500 (4) 2500

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2. The area (in square units) bounded by the curves $y=\sqrt{x}, 2y-x+3=0,$ x-axis, and lying in the first quadrant is

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3. If $\int f(x)dx=\Psi(x),$ then $\int x^5 f(x)^3 dx$ is equal to

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4. The area under the curve $y=|\cos x-\sin x|, 0\leq x\leq\frac{\pi}{2}$, and above x-axis is: (A) $2\sqrt{2}+2$ (B) 0 (C) $2\sqrt{2}-2$ (D) $2\sqrt{2}$

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5. If a curve passes through the point $\left(2,\frac{7}{2}\right)$ and has slope $\left(1-\frac{1}{x^2}\right)$ at any point (x,y) on it, then the ordinate of the point on the curve whose abscissa is -2 is: (A) $\frac{5}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{3}{2}$ (D) $-\frac{5}{2}$

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6. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ is

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7. The integral $\int \frac{x dx}{2 - x^2 + \sqrt{2 - x^2}}$ equals: (A) $\log\left|1 + \sqrt{2 + x^2}\right| + C$ (B) $x \log\left|1 - \sqrt{2 + x^2}\right| + C$ (C) $-\log\left|1 + \sqrt{2 - x^2}\right| + C$ (D) $x \log\left|1 - \sqrt{2 - x^2}\right| + C$

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8. The area enclosed by the curve $y = \sin x + \cos x and y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is

$(a)4\left(\sqrt{2} - 2\right)$ (b) $2\sqrt{2}$ ($\sqrt{2}$ -1) $(c)2\left(\sqrt{2}\right.$ +1) (d) $2\sqrt{2}\left(\sqrt{2} + 1\right)$

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9. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at eact point (x,y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0.$ Then, the equation of the curve is

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10. Let $f: [0, 1] \rightarrow R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1]$ Which of the following is true for $0 < x < 1$? (A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$ (C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

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11. Let $f: [0, 1] \rightarrow R$ (the set if all real numbers be a function . Suppose the function f us twice differentiable ,$f(0) = f(1) = 0$ and satisfies $f(''(x) - 2f(x) \geq e^x, x \in [0, 1]$

if the function $e^{-x}f(x)$ assume its minimum in the interval [0,1] at $x = \frac{1}{4}$, then which of the folowing is true ?

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12. Let S be the area of the region enclosed by $y - e^{-x^2}, y = 0, x = 0$ and $x = 1$. Then

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13. Let $IRI\overrightarrow{} R$ be defined as $f(x) = |x| + + x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is______

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14. If `f(x) = ∫et^2(t - 2)(t - 3)dt for t ∈ [0, x] for x ∈ (0, ∞), then (A) f has a local maximum at x = 2 (B) f is decreasing on (2, 3) (C) there exists some c∈(0, ∞) such that f"(c) = 0 (D) f has a local minimum at x = 3.

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15.

The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constan

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16. Let $f(x) = \int_{x^2}^{x^3} \frac{dt}{\ln t}$ for $x > 1$ and $g(x) = \int_1^x (2t^2 - \ln t) f(t) dt (x > 1)$, then: (a) g is increasing on $(1, \infty)$ (b) g is decreasing on $(1, \infty)$ (c) g is increasing on $(1, 2)$ and decreasing on $(2, \infty)$ (d) g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

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17. Consider the statements : P : There exists some x IR such that f(x) + 2x = 2(1+x2) Q : There exists some x IR such that 2f(x) +1 = 2x(1+x) $f(x) = (1-x)^2 \sin^2 x + x^2 \quad \forall x \in R$ Then (A) both P and Q are true (B) P is true and Q is false (C) P is false and Q is true (D) both P and Q are false.

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18. The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^2 + \ln\left(\frac{\pi + x}{\pi - x}\right)\right) \cos x dx$ is

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19. If the integral $\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln|\sin x - 2 \cos x| + k$ then a is equal to (1) -1 (2) -2 (3) 1 (4) 2

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20. The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is

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21. The area bounded between the parabolas $x^2 = \frac{y}{4} \text{and} x^2 = 9y$ and the straight line $y = 2$ is (1) $20\sqrt{2}$ (2) $\frac{10\sqrt{2}}{3}$ (3) $\frac{20\sqrt{2}}{3}$ (4) $10\sqrt{2}$

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22. If $g(x) = \int_0^x \cos^4 t dt$, then $g(x + \pi)$ equals $g(x) + g(\pi)$ (b) $g(x) - g(\pi)$ $g(x)g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

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23. $Thevalueof\int_{\sqrt{1n2}}^{\sqrt{1n3}}\frac{x\sin x^2}{\sin x^2+\sin(1n6-x^2)}dxis\ \frac{1}{4}1n\frac{3}{2}$ (b) $\frac{1}{21}n\frac{3}{2}$ $1n\frac{3}{2}$ (d) $\frac{1}{61}n\frac{3}{2}$

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24. Let the straight line x= b divide the area enclosed by $y=(1-x)^2, y=0, \text{ and } x=0$ into two parts $R_1(0\leq x\leq b)$ and $R_2(b\leq x\leq 1)$ such that $R_1-R_2=\frac{1}{4}$. Then b equals

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25. Let $f:[1,\infty]$ be a differentiable function such that $f(1)=2$. If $6\int_1^x f(t)dt=3xf(x)-x^3$ for all $x\geq 1$, then the value of $f(2)$ is

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26. Let $f : [-1, 2] \rightarrow [0, \infty)$ be a continuous function such that $f(x) = f(1-x) f$ or $all x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x f(x) dx$, and R_2 be the area of the region bounded by $y = f(x), x = -1, x = 2,$ and the x- axis . Then $R_1 = 2R_2$ (b) $R_1 = 3R_2$ (c) $2R_1 = R_2$ (d) $3R_1 = R_2$

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27. Let $y'(x) + y(x) g'(x) = g(x) g'(x), y(0) = 0, x \in R$, where $f'(x)$ denotes $\frac{df(x)}{dx}$, and $g(x)$ is a given non-constant differentiable function on R with g(0)=g(2)=0. Then the value of $y(2)$ is ....

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28. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x-axis is (1) $\frac{1}{2}$ square units (2) 1 square units (3) $\frac{3}{2}$ square units (4) $\frac{5}{2}$ square units

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29. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt}$=-k(T-t) , where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is : (1) $T^2 - \frac{1}{k}$ (2) $I - \frac{kT^2}{2}$ (3) $I - \frac{k(T-t)^2}{2}$ (4) e^{-kT}

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30. If $\frac{dy}{dx} = y + 3$ and y(0)=2, then $y(\ln 2)$ is equal to : (1) 7 (2) 5 (3) 13 (4) -2

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31. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\sin t dt$ Then f has : local maximum at π and 2π . local minimum at π and 2π local minimum at π and local maximum at 2π . local maximum at π and local minimum at 2π .

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32. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is

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