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# MATHS

## BOOKS - KC SINHA ENGLISH

## INVERSE TRIGONOMETRIC FUNCTIONS - FOR BOARDS

### Solved Examples

**1.** Evaluate : $\tan^{-1}(-1)$

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**2.** Evaluate : $\cot^{-1}(-1)$

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**3.** Evaluate : $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

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**4.** Write the value of $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$

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**5.** Find the value of $\sin\left[arc\cos\left(-\frac{1}{2}\right)\right]$

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**6.** Find the value of $\sin\left[\tan^{-1}(-\sqrt{3})+\cos^{-1}\left(\frac{\sqrt{3}}{-2}\right)\right]$

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**7.** Evaluate: (i)$\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right\}$ (ii) $\tan\left\{\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right\}$

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**8.** Find the value of $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

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**9.** Evaluate : $\sin\left[\frac{\pi}{2}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

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**10.** Write the following function in the simplest form: `tan^(-1)x/(sqrt(a^2-x^2)),|x|

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**11.** Write the following function in the simplest form: `tan^(-1)(sqrt((1-cosx)/(1+cosx))), x

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**12.** $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), a > o, \frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

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**13.** Write the following function in the simplest form: `tan^(-1)((cosx-sinx)/(cosx+sinx)), x

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**14.** Prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2.$

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**15.** Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$

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**16.** Find the value of: $\cot\left(\tan^{-1} a + \cot^{-1} a\right)$

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**17.** Find the value of : $\sin\left(\sin^{-1} x + \cos^{-1} x\right)$

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**18.** $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right], |x| < 1, y > 0 \text{ and } xy < 1$

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**19.** $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)$

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**20.** Prove that $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$

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**21.** prove that $2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\frac{\pi}{4}$

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**22.** Prove that :

$$\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$$

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**23.** Prove that: $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$

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**24.** Prove the following: $4\frac{\tan^{-1}1}{5} - \frac{\tan^{-1}1}{70} + \frac{\tan^{-1}1}{99} = \frac{\pi}{4}$

$2tan^{-1}\frac{1}{5} + sec^{-1}\frac{5\sqrt{2}}{7} + 2\frac{\tan^{-1}1}{8} = \frac{\pi}{4}$

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**25.** Prove that $\cot^{-1}9 + \cos ec^{-1}\frac{\sqrt{41}}{4} = \frac{\pi}{4}$

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**26.** Prove that $4\left(\cot^{-1}3 + cosec^{-1}\sqrt{5}\right) = \pi$

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**27.** Prove the following: $\tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

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**28.** Prove that :

$$\tan^{-1}\left(\frac{x-y}{1+xy}\right)+\tan^{-1}\left(\frac{y-z}{1+yz}\right)+\tan^{-1}\left(\frac{z-x}{1+zx}\right) = \tan^{-1}\left(\frac{x^2-y^2}{1+x^2y^2}\right)+\tan^{-1}\left(\frac{y^2-z^2}{1+y^2z^2}\right)+\tan^{-1}\left(\frac{z^2-x^2}{1+z^2x^2}\right)$$

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**29.** Prove that

$$\tan\left(\frac{\text{pi}}{4}+\frac{1}{2}\cos^{-1}\backslash\backslash\ \frac{\text{a}}{\text{b}}\right)+\tan\backslash\ \left(\frac{\text{pi}}{4}+\frac{1}{2}\cos^{-1}\backslash\backslash\ \frac{\text{a}}{\text{b}}\right)=\frac{2\text{b}}{\text{a}}.$$

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**30.** Prove that : $\sin\cot^{-1}\tan\cos^{-1}x = x$

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**31.** In any triangle ABC, if $A = \tan^{-1} 2$ and $B = \tan^{-1} 3$. Prove that $C = \frac{\pi}{4}$.

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**32.** If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

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**33.** If $\frac{\cos^{-1} x}{a} + \frac{\cos^{-1} y}{b} = \alpha$ , then $\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} = s \in^2 \alpha$

(b) $\cos^2 \alpha$ (c) $tan^2 \alpha$ (d) $\cot^2 \alpha$

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**34.** Solve the equation $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \cos ecx)$

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**35.** $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x, (x > 0)$

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**36.** Solve : $\sin^{-1}(x) + \sin^{-1}(2x) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

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**37.** Solve: $\cos^{-1}x\sqrt{3} + \cos^{-1}x = \frac{\pi}{2}$

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## Exercise

**1.** Find the value of : $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

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**2.** Find the value of : $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

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**3.** Find the principal values of each of the following: $\cot^{-1}(\sqrt{3})$

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**4.** Find the value of : $\cot^{-1}\cot\left(\frac{5\pi}{4}\right)$

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**5.** Find the value of $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

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**6.** Find the value of : $arc\sin\left(\frac{1}{2}\right) + arc\cos\left(\frac{1}{2}\right)$

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**7.** Find the value of the following: $\tan^{-1}\left(\frac{\tan(7\pi)}{6}\right)$

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**8.** Find the value of the following: $\cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right)$

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**9.** Find the value of $\sin^{-1}\left(\frac{\sin(3\pi)}{5}\right)$.

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**10.** Find the principal value of : $\tan^{-1}\sqrt{3}$

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**11.** Find the principal value of: $\sin^{-1}\left(-\frac{1}{2}\right)$

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**12.** Find the principal value of: $\tan^{-1}(-1)$

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**13.** Find the principal values of $(1)\operatorname{cosec}^{-1}(2)$ and $\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

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**14.** Find the principal value of: $\cos^{-1}\left(-\frac{1}{2}\right)$

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**15.** Find the principal value of: $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

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**16.** What is the principal value of $\cos^{-1}\left(\cos\left(\frac{2\text{pi}}{3}\right)\right)+\sin^{-1}\left(\sin\left(\frac{2\text{pi}}{3}\right)\right)$?

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**17.** Evaluate the following: $\cos\left(\tan^{-1}3/4\right)$

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**18.** Evaluate that : $\cos\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$

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**19.** Evaluate that : $2arc\sin\left(\frac{1}{2}\right)+3arc\tan(-1)+2arc\cos\left(-\frac{1}{2}\right)$

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**20.** Write the simplest form : $\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), |x|>1$

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**21.** Write the following function in the simplest form: $\frac{\tan^{-1}\left(\sqrt{1+x^2}-1\right)}{x}, x\neq 0$

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**22.** Write the simplest form : $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

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**23.** Prove that:$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

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**24.** Write the simplest form : $\cos^{-1}\left(\sqrt{\left(\frac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}\right)}\right.$

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**25.** Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-\sin x}}{\sqrt{1+x}-\sqrt{1-\sin x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}, -\frac{1}{\sqrt{2}} \leq x \leq 1$

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**26.** Prove that: $\tan^{-1}\left(\frac{2}{11}\right)+\tan^{-1}\left(\frac{7}{24}\right)=\tan^{-1}\left(\frac{1}{2}\right)$

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**27.** Prove that :

$$\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{1}{4}\right)=\frac{\pi}{4}$$

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**28.** Prove that : $\tan^{-1}\left(\frac{2a-b}{b\sqrt{3}}\right)+\tan^{-1}\left(\frac{2b-a}{a\sqrt{3}}\right)=\frac{\pi}{3}$

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**29.** $2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{4}=\tan^{-1}\frac{32}{43}$

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**30.** Prove that: $2\frac{\tan^{-1}1}{2}+\frac{\tan^{-1}1}{7}=\frac{\tan^{-1}(31)}{17}$

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**31.** Prove that :

$\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi=2\left(\tan^{-1}1+\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right.\right.$

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**32.** Prove that : $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right]=\frac{\pi}{4}$

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**33.** Find the value of $\tan\left(\frac{\sin^{-1}3}{5}+\frac{\cot^{-1}3}{2}\right)$

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**34.** Prove that : $\tan^{-1} x + \cot^{-1} y = \tan^{-1}\left(\frac{xy+1}{y-x}\right)$

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**35.** Prove that : $\tan^{-1} x + \cot^{-1}(1+x) = \tan^{-1}(1+x+x^2)$

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**36.** $\tan^{-1}\left(\frac{1}{x+y}\right) + \tan^{-1}\left(\frac{y}{x^2+xy+1}\right) = \cot^{-1} x$

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**37.** $2\cot^{-1} 5 + \cot^{-1} 7 + 2\cot^{-1} 8 = \frac{\pi}{4}$

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**38.** Prove that

$$\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{1+bc}\right)+\tan^{-1}\left(\frac{c-a}{1+ca}\right)=0,$$

$ab>(-1), bc>(-1), ca>(-1)$

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**39.** Prove that :

$$\tan^{-1}\left(\frac{a^3-b^3}{1+a^3b^3}\right)+\tan^{-1}\left(\frac{b^3-c^3}{1+b^3c^3}\right)+\tan^{-1}\left(\frac{c^3-a^3}{1+c^3a^3}\right)=0$$

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**40.** Prove that $\sin^{-1},\frac{8}{17}+\sin^{-1},\frac{3}{5}=\sin^{-1},\frac{77}{85}.$

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**41.** Prove that $\cos^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{63}{65}\right)=\frac{\pi}{2}$

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**42.** Prove that $\sin^{-1}, \frac{8}{17} + \sin^{-1}, \frac{3}{5} = \sin^{-1}, \frac{77}{85}$.

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**43.** Prove that:$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

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**44.** Prove that: $\frac{\cos^{-1} 4}{5} + \frac{\cos^{-1}(12)}{13} = \frac{\cos^{-1}(33)}{65}$

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**45.** Show that $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$.

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**46.** Prove: $\sin^{-1}\left(\frac{5}{13}\right) + \frac{\cos^{-1} 3}{5} = \frac{\tan^{-1}(63)}{16}$

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**47.** Prove that: $\frac{\sin^{-1}(12)}{13} + \frac{\cos^{-1} 4}{5} + \frac{\tan^{-1}(63)}{16} = \pi$

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**48.** Prove that:$2\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{24}{7}\right)$

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**49.** Prove that:$\frac{9\pi}{8} - \frac{9}{4}\frac{\sin^{-1} 1}{3} = \frac{9}{4}\frac{\sin^{-1}\left(2\sqrt{2}\right)}{3}$

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**50.** $\sin^{-1} x + \sin^{-1} y = \cos^{-1}\left(\sqrt{1-x^2}\sqrt{1-y^2} - xy\right)$ if $x \in [0, 1], y \in [0, 1]$

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**51.** Prove that : $4\left(\sin^{-1}\left(\frac{1}{\sqrt{10}}\right) + \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right) = \pi$

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**52.** Prove that : $\cos\left(2\sin^{-1} x\right) = 1 - 2x^2$

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**53.** Prove that : $\cos\left(\sec^{-1} x + \cos ec^{-1} x\right) = 0, |x| \geq 1$

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**54.** The value of $2\tan^{-1}\left(\cos ec\tan^{-1}x - \tan\cot^{-1}x\right)$ is equal to

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**55.** Prove that $\cos\tan^{-1}\sin\cot^{-1}x = \sqrt{\frac{x^2+1}{x^2+2}}$

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**56.** If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$ then prove that $yz + zx + xy = 1$

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**57.** If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ prove that $x + y + z = xyz$

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**58.** If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, prove that $x\sqrt{1-y^2} + y\sqrt{1-x^2} = 1$.

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**59.** If x, y, z are in A.P. and $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are also in A.P.

then show that x=y=z and y≠0

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**60.** Solution of equation $\cot^{-1} x + \sin^{-1}. \frac{1}{\sqrt{5}} = \frac{\pi}{4}$ is

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**61.** If $\sin\left(\sin^{-1}. \frac{1}{5} + \cos^{-1} x\right) = 1$, then find the value of x

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**62.** Solve : $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

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**63.** Solve : $\tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$

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**64.** Solve : $\tan^{-1}\left(\frac{1}{2}\right) = \cot^{-1} x + \tan^{-1}\left(\frac{1}{7}\right)$

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**65.** Solve : $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x$

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**66.** Solve : $\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \pi + \tan^{-1}(-7)$

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**67.** Solve : $\frac{\tan^{-1}(x-1)}{x-2}+\frac{\tan^{-1}(x+1)}{x+2}=\frac{\pi}{4}$

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**68.** Solve :

$$\sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)+\sin^{-1}\left(\frac{2\beta}{1+\beta^2}\right)=2\tan^{-1}x, |\alpha|\leq 1, |\beta|\leq 1$$

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**69.** Solve :

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=2\tan^{-1}x, |a|\leq 1, |b|\geq 0.$$

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**70.** Solve : $\tan\left(\cos^{-1}x\right)=\sin\left(\tan^{-1}2\right)$

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**71.** A value of x satisfying

$$\tan\left(\sec^{-1} x\right) = \sin\left(\cos^{-1}. \frac{1}{\sqrt{5}}\right)$$ is

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