

MATHS

BOOKS - KC SINHA ENGLISH

PRINCIPLES OF MATHEMATICAL INDUCTION - FOR BOARDS

Solved Examples

1. Prove the following by using the principle of mathematical induction

for all
$$n \in N\!\!:\!\!1^3+2^3+3^3+.....$$
 $+n^3=\left(rac{n(n+1)}{2}
ight)^2$

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2. Prove the following by the principle of mathematical induction: $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{n(n+1)} = \frac{n}{n+1}$

3. Prove the following by the principle of mathematical induction: $1.\ 2+2.\ 2^2+3.\ 2^3+\ +n.2^n=(n-1)2^{n+1}+2$

4. Prove the following by using the principle of mathematical induction

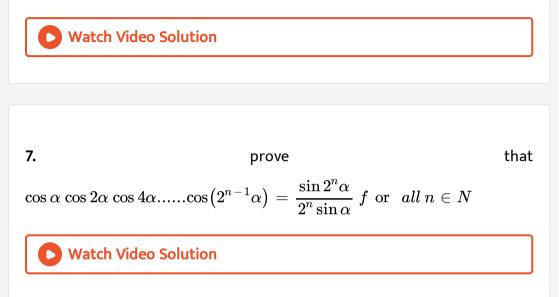
$$egin{array}{ccccccccc} {
m for} & {
m all} & n \in N & : \ 1.\ 2.\ 3+2.\ 3.\ 4+ \ \ {
m dot} \ \ \ {
m dot} \ \ \ {
m dot} + {
m n}({
m n}+1) & ({
m n}+2) = rac{n(n+1)(n+2)(n+2)(n+2)}{4} \end{array}$$

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5. Prove the following by the principle of mathematical induction: $7+77+777++777++\ddot{n}-digits7=rac{7}{81}ig(10^{n+1}-9n-10ig)$ for all $n\in NB$.

6. Prove by the principle of mathematical induction that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$

is a natural number for all $n\in N_{\cdot}$



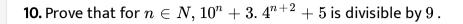
8. Show that
$$n^3 + (n+1)^3 + (n+2)^3$$
 is divisible by 9 for every natural

number n.

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9. Using the Principle of mathematical induction, show that $11^{n+2} + 12^{2n+1}$, where n is a natural number is divisible by 133.







11. Using the principle of mathematical induction. Prove that $(x^n - y^n)$ is

divisible by (x - y) for all $n \in N$.

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12. Show by using the principle of mathematical induction that for all

natural number $n > 2, 2^n > 2n + 1$

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13. Prove the following by using the principle of mathematical induction

for all
$$n \in N: 1+2+3+ + n < rac{1}{8}(2n+1)^2.$$

14. Prove the following by using the principle of mathematical induction

for all $n\in N\!:\left(2n+7
ight)<\left(n+3
ight)^{2}$.

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15. Prove that $(1+x)^n \ge (1+nx)$, for all natural number n, where

 $x \succ 1.$

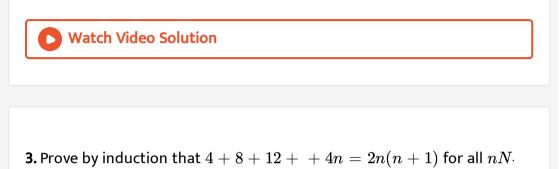
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Exercise

1. Prove that
$$:1^2 + 2^2 + 3^2 + ... + n^2 = rac{n(n+1)(2n+1)}{6}$$

2. If P(n) be the statement "10n + 3 is a prime number", then prove that

P(1) and P(2) are true but P(3) is false.



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4. Prove the following by the principle of mathematical induction: $1+2+3++n=rac{n(n+1)}{2}i\dot{e}$, the sum o the first n natural numbers is $rac{n(n+1)}{2}$.

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5. Prove by the principle of mathematical induction that for all $n\in N$: $1^2+2^2+3^2+\ +n^2=rac{1}{6}n(n+1)(2n+1)$

6. Prove the following by the principle of mathematical induction:

$$1+3+3^2+ \ +3^{n-1}={3^n-1\over 2}$$

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7. Prove the following by using the principle of mathematical induction

for all
$$n \in N$$
: $rac{1}{2} + rac{1}{4} + rac{1}{8} + +rac{1}{2^n} = 1 - rac{1}{2^n}$

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8. Prove the rule of exponents $(ab)^n = a^n b^n$ by using principle of mathematical induction for every natural number.

9. Prove the following by using the principle of mathematical induction

for all
$$n \in N$$
: $1^2 + 3^2 + 5^2 + + (2n-1)^2 = rac{n(2n-1)(2n+1)}{3}$

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10. Prove by using the principle of mathematical induction: $3.2^2 + 3.2^3 + \ldots + 3^n \cdot 2^{n+1} = \frac{12}{5}(6^n - 1)$

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11. Using the principle of mathematical induction prove that : $1.3+2.3^2+3.3^3++n.3^n=rac{(2n+1)3^{n+1}+3}{4}$ for all $n\in N$.

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12. Prove the following by using the principle of mathematical induction

for all
$$n\in N{:}a+ar+ar^2++ar^{n-1}=rac{a(r^n-1)}{r-1}$$

13. Prove the following by the principle of mathematical induction:

$$a+(a+d)+(a+2d)+ + (a+(n-1)d) = rac{n}{2}[2a+(n-1)d]$$

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14. Prove by the principle of mathematical induction that for all $nbelongs \rightarrow N$: $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$ Watch Video Solution

15. Prove the following by the principle of mathematical induction: $\frac{1}{3.7} + \frac{1}{7.11} + \frac{1}{11.15} + \dots + \frac{1}{(4n-1)(4n+3)} = \frac{n}{3(4n+3)}$

16. Prove the following by the principle of mathematical induction: $1.2+2.3+3.4++n(n+1)=rac{n(n+1)(n+2)}{3}$

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17. Prove the following by the principle of mathematical induction:

 $1.\ 3+2.\ 4+3.\ 5+\ +\ (2n-1)(2n+1)=rac{nig(4n^2+6n-1ig)}{3}$

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18. Prove the following by the principle of mathematical induction: $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$ **Watch Video Solution**

19. Prove the following by the principle of mathematical induction: $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$ 20. Prove the following by the principle of mathematical induction:

$$rac{1}{3.5} + rac{1}{5.7} + rac{1}{7.9} + rac{1}{(2n+1)(2n+3)} = rac{n}{3(2n+3)}$$

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21. Prove the following by using the principle of mathematical induction

forall
$$n \in N$$
: $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{1} + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$ \checkmark Watch Video Solution

22. Using the principle of mathematical induction prove that
$$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}++\frac{1}{1+2+3++n}=\frac{2n}{n+1}$$
 for all $n \in N$

23. Prove the following by using the principle of mathematical induction

for all
$$n \in N$$
: $\left(1+rac{1}{1}
ight) \left(1+rac{1}{2}
ight) \left(1+rac{1}{3}
ight) 1+rac{1}{n}=(n+1)$

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24. Prove, by induction, that $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ for all

positive as well as negative integral values of n

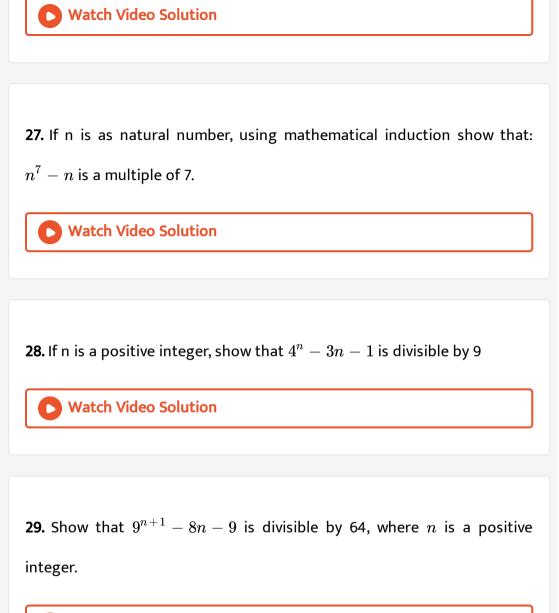
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25. Prove by the principle of mathematical induction, that

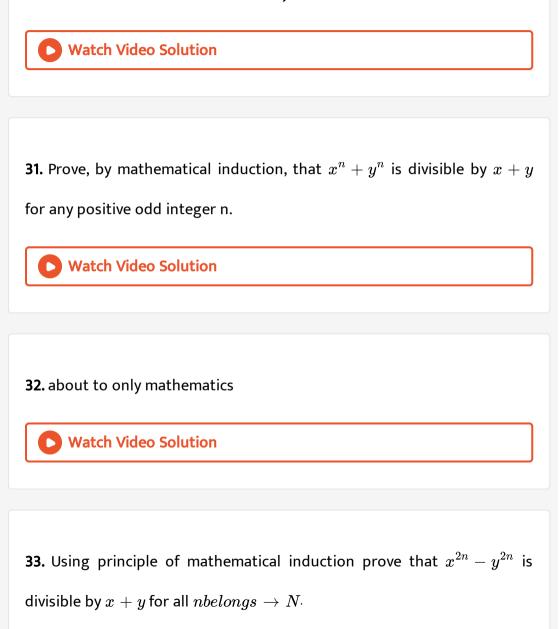
 $3 \times 6 + 6 \times 9 + 9 \times 12 + + (3n) \times (3n+3) = 3n(n+1)(n+2)$

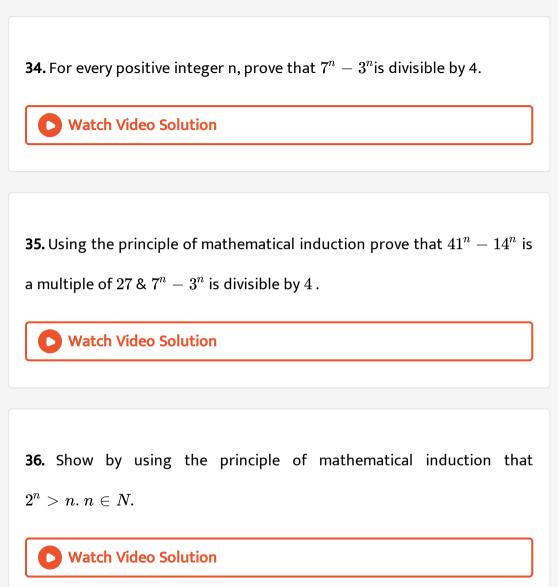
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26. n(n + 1)(n + 5) is divisible by 6.



30. Prove the following by using the principle of mathematical induction for all $n \in N$: 10^{2n-1} + 1is divisible by 11.





37. Using mathematical induction, prove the following: $1+2+3+...+n<(2n+1)^2\,orall n\in N$



