MATHS

BOOKS - KC SINHA ENGLISH

PRINCIPLES OF MATHEMATICAL INDUCTION - FOR BOARDS

Solved Examples

1. Prove the following by using the principle of mathematical induction for all $n \in N$: $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots. + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

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2. Prove the following by the principle of mathematical induction: $\frac{1}{1.\,2} + \frac{1}{2.\,3} + \frac{1}{3.\,4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

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3. Prove the following by the principle of mathematical induction:

$1.\ 2+2.\ 2^2+3.\ 2^3+\ +n.2^n=(n-1)2^{n+1}+2$

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4. Prove the following by using the principle of mathematical induction for all $n \in N$:

$1.\ 2.\ 3+2.\ 3.\ 4+\ \text{dot}\ \ \text{dot}\ \ \text{dot}+\text{n}(\text{n}+1)\ \ (\text{n}+2)=\frac{n(n+1)(n+2)(}{4}$

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5. Prove the following by the principle of mathematical induction: $7+77+777+\ +777+\ +\ddot{n}-digits7=\frac{7}{81}\left(10^{n+1}-9n-10\right)$ for all $n \in NB\cdot$

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6. Prove by the principle of mathematical induction that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number for all $n \in N$.

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7. prove that

$\cos\alpha\cos 2\alpha\cos 4\alpha......\cos\left(2^{n-1}\alpha\right) = \frac{\sin 2^n\alpha}{2^n\sin\alpha}\ f\ \text{or}\ \ all\ n \in N$

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8. Show that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9 for every natural number n.

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9. Using the Principle of mathematical induction, show that $11^{n+2} + 12^{2n+1}$, where n is a natural number is divisible by 133.

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10. Prove that for $n \in N$, $10^n + 3. 4^{n+2} + 5$ is divisible by 9 .

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11. Using the principle of mathematical induction. Prove that $(x^n - y^n)$ is divisible by $(x - y)$ for all $n \in N$.

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12. Show by using the principle of mathematical induction that for all natural number $n > 2$, $2^n > 2n + 1$

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13. Prove the following by using the principle of mathematical induction for all $n \in N$:$1 + 2 + 3 + \vdots + n < \frac{1}{8}(2n + 1)^2$.

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14. Prove the following by using the principle of mathematical induction for all $n \in N : (2n+7) < (n+3)^2$.

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15. Prove that $(1+x)^n \geq (1+nx)$, for all natural number n, where $x \succ 1.$

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Exercise

1. Prove that :$1^2 + 2^2 + 3^2 + \quad + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

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2. If $P(n)$ be the statement "$10n + 3$ is a prime number", then prove that P(1) and P(2) are true but P(3) is false.

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3. Prove by induction that $4 + 8 + 12 + \quad + 4n = 2n(n+1)$ for all nN.

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4. Prove the following by the principle of mathematical induction: $1 + 2 + 3 + \quad + n = \frac{n(n+1)}{2}i\dot{e}$, the sum o the first n natural numbers is $\frac{n(n+1)}{2}$.

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5. Prove by the principle of mathematical induction that for all $n \in N$: $1^2 + 2^2 + 3^2 + \quad + n^2 = \frac{1}{6}n(n+1)(2n+1)$

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6. Prove the following by the principle of mathematical induction:

$$1+3+3^2+\quad+3^{n-1}=\frac{3^n-1}{2}$$

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7. Prove the following by using the principle of mathematical induction for all $n \in N$: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\vdots+\frac{1}{2^n}=1-\frac{1}{2^n}$

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8. Prove the rule of exponents $(ab)^n=a^nb^n$ by using principle of mathematical induction for every natural number.

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9. Prove the following by using the principle of mathematical induction for all $n \in N$:$1^2 + 3^2 + 5^2 + \vdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

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10. Prove by using the principle of mathemtical induction: $3.2^2 + 3.\ 2^3 + \ldots + 3^n.2^{n+1} = \frac{12}{5}(6^n - 1)$

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11. Using the principle of mathematical induction prove that : $1.\ 3 + 2.\ 3^2 + 3.\ 3^3 + \ + n.3^n = \frac{(2n+1)3^{n+1} + 3}{4}$ for all $n \in N$.

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12. Prove the following by using the principle of mathematical induction for all $n \in N$:$a + ar + ar^2 + \vdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$

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13. Prove the following by the principle of mathematical induction:

$$a+(a+d)+(a+2d)+\quad+(a+(n-1)d)=\frac{n}{2}[2a+(n-1)d]$$

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14. Prove by the principle of mathematical induction that for all $nbelongs \rightarrow N$:

$$\frac{1}{1.\,3}+\frac{1}{3.5}+\frac{1}{5.7}+\quad+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$$

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15. Prove the following by the principle of mathematical induction:

$$\frac{1}{3.\,7}+\frac{1}{7.\,11}+\frac{1}{11.\,15}+\ldots.\,+\frac{1}{(4n-1)(4n+3)}=\frac{n}{3(4n+3)}$$

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16. Prove the following by the principle of mathematical induction:

$$1.2 + 2.3 + 3.4 + \quad + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

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17. Prove the following by the principle of mathematical induction:

$$1.3 + 2.4 + 3.5 + \quad + (2n-1)(2n+1) = \frac{n\left(4n^2 + 6n - 1\right)}{3}$$

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18. Prove the following by the principle of mathematical induction:

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots. + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

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19. Prove the following by the principle of mathematical induction:

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \quad + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$

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20. Prove the following by the principle of mathematical induction:

$$\frac{1}{3.\ 5} + \frac{1}{5.\ 7} + \frac{1}{7.\ 9} + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

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21. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{1.\ 2.\ 3} + \frac{1}{2.\ 3.\ 4} + \frac{1}{3.\ 4.\ 5} + + \vdots \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

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22. Using the principle of mathematical induction prove that

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + + \frac{1}{1+2+3+ \ + n} = \frac{2n}{n+}$$

for all $n \in N$

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23. Prove the following by using the principle of mathematical induction for all $n \in N$: $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right) \vdots 1+\frac{1}{n}=(n+1)$

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24. Prove, by induction, that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for all positive as well as negative integral values of n

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25. Prove by the principle of mathematical induction, that

$3 \times 6 + 6 \times 9 + 9 \times 12 + ... + (3n) \times (3n+3) = 3n(n+1)(n+2)$

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26. $n(n+1)(n+5)$ is divisible by 6.

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27. If n is as natural number, using mathematical induction show that: $n^7 - n$ is a multiple of 7.

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28. If n is a positive integer, show that $4^n - 3n - 1$ is divisible by 9

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29. Show that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a positive integer.

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30. Prove the following by using the principle of mathematical induction for all $n \in N$:$10^{2n-1} + 1$is divisible by 11.

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31. Prove, by mathematical induction, that $x^n + y^n$ is divisible by $x + y$ for any positive odd integer n.

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33. Using principle of mathematical induction prove that $x^{2n} - y^{2n}$ is divisible by $x + y$ for all $nbelongs \rightarrow N$.

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34. For every positive integer n, prove that $7^n - 3^n$is divisible by 4.

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35. Using the principle of mathematical induction prove that $41^n - 14^n$ is a multiple of 27 & $7^n - 3^n$ is divisible by 4 .

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36. Show by using the principle of mathematical induction that $2^n > n. n \in N.$

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37. Using mathematical induction, prove the following: $1 + 2 + 3 + \dots + n < (2n+1)^2 \forall n \in N$

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