doubtnut

India's Number 1 Education App

MATHS

BOOKS - KC SINHA ENGLISH

PROPERTIES OF TRIANGLE - FOR COMPETITION

Solved Examples

1. If in a triangle ABC, $\frac{\tan A}{1}=\frac{\tan B}{2}=\frac{\tan C}{3}$ then prove that $6\sqrt{2a}=3\sqrt{5b}=2\sqrt{10c}$

Watch Video Solution

2. The sides of a triangle are $x^2+x+1, 2x+1, \text{ and } x^2-1$. Prove that the greatest angle is 120°

Watch Video Solution

3. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smalles one. Determine the sides of the triangle.

Watch Video Solution

4. If in triangle $ABC, \cos A \cos B + \sin A \sin B \sin C = 1.$ Show that $a:b:c = 1:1:\sqrt{2}$

Watch Video Solution

5. In $ABC,$ if $\sin^3\theta = \sin(A-\theta)\sin(B-\theta)\sin(C-\theta)$, then prove that $\cot\theta = \cot A + \cot B + \cot C.$

Watch Video Solution

6. If in a triangle of base 'a', the ratio of the other two sides is r (<1).Show that the altitude of the triangle is less than or equal to $\frac{ar}{1-r^2}$

Watch Video Solution

7. Given the base of a triangle, the opposite angle A, and the product k^2 of the other two sides, show that it is not possible for a to be less than $2k\frac{\sin A}{2}$

Watch Video Solution

8. In a triangle ABC, the vertices A,B,C are at distances of p,q,r fom the orthocentre respectively. Show that $aqr + brp + cpq = abc$

Watch Video Solution

9. Prove that a triangle ABC is equilateral if and only if $\tan A + \tan B + \tan C = 3\sqrt{3}$.

Watch Video Solution

10. If the sides of triangle ABC are in G.P with common ratio $r(r < 1)$, show that $r < \frac{1}{2}(\sqrt{5} + 1)$

Watch Video Solution

11. If in a triangle $r_1 = r_2 + r_3 + r$, prove that the triangle is right angled.

Watch Video Solution

12. If $A + B + C = \pi$, prove that

(a) $\tan 3A + \tan 3B + \tan 3C = \tan 3A \tan 3B \tan 3C$

(b) $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

Watch Video Solution

13. about to only mathematics

Watch Video Solution

14. Prove that the sum of the radii of the radii of the circles, which are, respectively, inscribed and circumscribed about a polygon of n sides, whose side length is a, is $\frac{1}{2}a\frac{\cot\pi}{2n}$.

Watch Video Solution

15. The sides of a quadrilateral are 3, 4, 5 and 6 cms. The sum of a pair of opposite angles is $120^0. Showt\hat{t} heareaofthe \quad rilateralis$3sqrt(30)` sq.cm.

Watch Video Solution

16. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the area of the quadrilateral is $4\sqrt{3}$, find the remaining two sides

Watch Video Solution

17. A cyclic quadrilateral $ABCD$ of areal $\frac{3\sqrt{3}}{4}$ is inscribed in unit circle. If one of its side $AB = 1,$ and the diagonal $BD = \sqrt{3},$ find the lengths of the other sides.

Watch Video Solution

18. In a cyclic quadrilateral ABCD, prove that $\tan^2\frac{B}{2} = \frac{(s-a)(s-b)}{(s-c)(s-d)}, a, b, c,$ and d being the lengths of sides ABC, CD and DA respectively and s is semi-perimeter of quadrilateral.

Watch Video Solution

19. In triangle ABC, prove that $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \leq \frac{1}{8}$ and hence, prove that $co\sec\frac{A}{2} + co\sec\frac{B}{2} + co\sec\frac{C}{2} \geq 6$.

Watch Video Solution

20. The side of a triangle inscribed in a given circle subtends angles $\alpha, \beta and \gamma$ at the centre. The minimum value of the arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right), \cos\left(\beta + \frac{\pi}{2}\right)$, and $\cos\left(\gamma + \frac{\pi}{2}\right)$ is equal to ____

Watch Video Solution

21. If A +B+C=π, prove that $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} \geq 1$

Watch Video Solution

22. Let $1 < m < 3. \ln a\Delta ABC, \quad \text{if} \quad 2b = (m+1)a \text{ and } \cos A = \frac{1}{2}\sqrt{\frac{(m-1)(r}{m}}$ prove that there are two values to the third side, one of which is m times the other.

Watch Video Solution

23. Let A,B,C, be three angles such that $A = \frac{\pi}{4}$ and $\tan B, \tan C = p$. Find all possible values of p such that A, B, C are the angles of a triangle.

Watch Video Solution

24. Two sides of triangle are of lengths $\sqrt{6}$ and 4 and the angle opposite to smaller side is 30°, then how many such triangles are possible ?

Watch Video Solution

25. If the angle $A, BandC$ of a triangle are in an arithmetic propression and if $a, bandc$ denote the lengths of the sides opposite to $A, BandC$ respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$

Watch Video Solution

26. Let $ABCD$ be a quadrilateral with area 18 , side AB parallel to the side $CD, andAB = 2CD$. Let AD be perpendicular to $ABandCD$. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is $a = 3$ (b) 2 (c) $\frac{3}{2}$ (d) 1

Watch Video Solution

27. One angle of an isosceles triangle is 120^0 and the radius of its incricel is $\sqrt{3}$. Then the area of the triangle in sq. units is $7 + 12\sqrt{3}$ (b) $12 - 7\sqrt{3}$ $12 + 7\sqrt{3}$ (d) 4π

Watch Video Solution

28. A triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\frac{\sin^2 A}{2}$. If $a, b and c,$ denote the length of the sides of the triangle opposite to the angles $A, B, and C$, respectively, then (a) $b + c = 4a$ (b) $b + c = 2a$ (c)the locus of point A is an ellipse (d)the locus of point A is a pair of straight lines

Watch Video Solution

29. Internal bisector of $\angle A$ of ΔABC meets side BC to D. A line drawn through D perpendicular to AD intersects the side AC at E and side AB at. F. If a,b,c represent sides of $\Delta ABC,$ then

Watch Video Solution

30. If in a $\triangle ABC, \cos A. \cos B + \sin A. \sin B. \sin C = 1, then$ (A) $A = B$ (B) $C = \frac{\pi}{2}$ (C) $AC = BC$ (D) $AB = \sqrt{2}AC$

Watch Video Solution

31. In a ΔABC, if $r = r_2 + r_3 - r_1$ and $A > \frac{\pi}{3}$ then range of $\frac{s}{a}$ contains

(A) $\left(\frac{1}{2}, 2\right)$

(B) $[1, 2)$

(C) $\left(\frac{1}{2}, 3\right)$

(D) $(3, \infty)$

Watch Video Solution

32. Let us consider a triangle ABC having BC=5 cm, CA=4cm, AB=3cm, D,E are points on BC such BD = DE= EC, $\angle CAE = \theta$, then:

AE^2 is equal to

Watch Video Solution

33. In triangle ABC, $R(b+c)=a\sqrt{bc}$, where R is the circumradius of the triangle. Then the triangle is

Watch Video Solution

34. In acute angled triangle ABC, AD is the altitude. Circle drawn with AD as its diameter cuts $ABandACatPandQ$, respectively. Length of PQ is equal to $/(2R)$ (b) $\frac{abc}{4R^2}$ $2R\sin A\sin B\sin C$ (d) Δ/R

Watch Video Solution

35. Statement 1. If A is the area and $2s$ is the perimeter of a $\triangle ABC$, then $A\leq\frac{s^2}{3\sqrt{3}}$,

Statement 2. $A.M\geq G.M.$

(A) Both Statements are false

(B) Both Statement 1 and Statement 2 are true

(C) Statement 1 is true but Statement 2 is false.

(D) Statement 1 is flse but Stastement 2 is true

Watch Video Solution

36. Radius of circumcircle of $\triangle DEF$ is

$(A) R$

$(B)\ \frac{R}{2}$

$(C)\ \frac{R}{4}$

(D) none of these

Watch Video Solution

37. If $\cot A + \cot B + \cot C = k\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)$ then the value of k is

$(A)\ R^2$

$(B)\ 2R$

$(C)\ \triangle$ `(D)$a^2 + b^2 + c^2$

Watch Video Solution

38. Let $ABCandABC'$ be two non-congruent triangles with sides $AB=4, AC=AC'=2\sqrt{2}$ and angle $B=30^0$. The absolute value of the difference between the areas of these triangles is

Watch Video Solution

39. ABC is a triangle. Its area is 12 sq. cm. and base is 6 cm. the difference of base angle is 60^0. If A be the angle opposite to the base, then the value of $8\sin A-6\cos A$ is......

Watch Video Solution

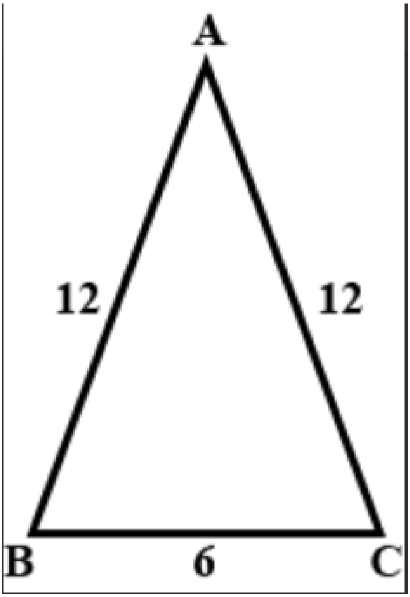
40. perpendiculars are drawn from the angles $A, BandC$ of an acute-angled triangle on the opposite sides, and produced to meet the circumscribing circle. If these produced parts are α, β, γ, respectively, then show that $\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=2(\tan A+\tan B+tanC)\cdot$

Watch Video Solution

41. The sides of a triangle are in AP. If the angles A and C are the greatest and smallest angle respectively, then $4(1 - \cos A)(1 - \cos C)$ is equal to

Watch Video Solution

42. The radius of the circle passing through the vertices of the triangle ABC, is



Watch Video Solution

43. Three circles touch one another externally. The tangents at their points of contact meet at a point whose distance from a point of contact is 4. Find the ratio of the product of the radii to the sum of the radii of the circles.

Watch Video Solution

44. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90o - \frac{1}{2}A$,$90o - \frac{1}{2}B$ and $90o - \frac{1}{2}C$

Watch Video Solution

Exercise

1. A ring,10 cm in diameter, is suspended from a point 12cm above its centre by 6 equal strings attached to its circumference at equal intervals. Find the cosine of the angle between consecutive strings.

Watch Video Solution

2. The angle of a triangle are in the ratio 1 : 2 : 7, prove that the ratio of the greatest side to the least side is $(\sqrt{5}+1):(\sqrt{5}-1)$.

Watch Video Solution

3. If the base angles of triangle are $\frac{22}{12} and 112\frac{1}{2^0}$, then prove that the altitude of the triangle is equal to $\frac{1}{2}$ of its base.

Watch Video Solution

4. If f,g,h are internal bisectoirs of the angles of a triangle ABC, show that $\frac{1}{f}\cos,\frac{A}{2}+\frac{1}{g}\cos,\frac{B}{2}+\frac{1}{h}\cos,\frac{C}{2}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

Watch Video Solution

5. In triangle ABC, medians AD and CE are drawn $AD = 5, \angle DAC = \pi/8$, and $\angle ACE = \pi/4$, then the area of the triangle ABC is equal to

Watch Video Solution

6. In ΔABC,

if the sides are $7, 4\sqrt{3}$ and $\sqrt{13}$ cm, prove that the smallest angle is 30°.

Watch Video Solution

7. In an isosceles right angled triangle , a straight line drwan from the mid - point of one of equal sides to the opposite angle . It divides the angle into two parts , θ and $(\pi/4 - \theta)$. Then $\tan\theta$ and $\tan[(\pi/4) - \theta]$ are equal to

Watch Video Solution

8. If the roots of the equation $x^3 - px^2 + qx - r = 0$ are in A.P., then prove that, $2p^3 - 9pq + 27r = 0$

Watch Video Solution

9. In any ΔABC, $\prod\left(\frac{\sin^2 A + \sin A + 1}{\sin A}\right)$ is always greater than

Watch Video Solution

10. In a $\triangle ABC$, Prove that

$$\sin^4 A + \sin^4 B + \sin^4 C = \frac{3}{2} + 2\cos A\cos B\cos C + \frac{1}{2}\cos 2A\cos 2B\cos 2$$

Watch Video Solution

11. Prove that,

$$\frac{a\sin(B-C)}{b^2-c^2} = \frac{b\sin(C-A)}{c^2-a^2} = \frac{c\sin(A-B)}{a^2-b^2}$$

Watch Video Solution

12. In any triangle ABC prove that: $\sin\left(\frac{B-C}{2}\right)=\left(\frac{b-c}{a}\right)\frac{\cos A}{2}$

Watch Video Solution

13. If in a $\Delta ABC, \sin^3 A+\sin^3 B+\sin^3 C$

$=3\sin A.\sin B.\sin C,$ then find the valueof determinant

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$$

Watch Video Solution

14. In a triangle ABC, Prove that: $\sin^3 A+\sin^3 B+\sin^3 C$= $3\frac{\cos A}{2}\frac{\cos B}{2}\frac{\cos C}{2}+\frac{\cos(3A)}{2}\frac{\cos(3B)}{2}\frac{\cos(3C)}{2}$

Watch Video Solution

15. Prove that $\left(\frac{\cot A}{2}+\frac{\cot B}{2}\right)\left(a\frac{\sin^2 B}{2}+b\frac{\sin^2 A}{2}\right)=\text{ot}\frac{C}{2}$

Watch Video Solution

16. If $pandq$ are perpendicular from the angular points A and B of ABC drawn to any line through the vertex C, then prove that $a^2b^2\sin^2 C = a^2p^2 + b^2q^2 - 2abpq\cos C$.

Watch Video Solution

17. Let O be a point inside a triangle ABC such that $\angle OAB = \angle OBC = \angle OCA = \omega$, then show that:

$\cot\omega = \cot A + \cot B + \cot C$

$\cos ec^2\omega = \cos ec^2 A + \cos ec^2 B + \cos ec^2 C$

Watch Video Solution

18. If x,y,z are perpendicular from circum centre of the sides of the ΔABC respectively. Prove that $\frac{a}{z} + \frac{b}{y} + \frac{c}{z} = \frac{abc}{4xyz}$

Watch Video Solution

19. Prove that a triangle ABC is equilateral if and only if $\tan A + \tan B + \tan C = 3\sqrt{3}$.

Watch Video Solution

20. In a triange ABC, if $\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) = \frac{1}{8}$ prove that the triangle is equilateral.

Watch Video Solution

21. If in a triangle $ABC, \cos A + 2\cos B + \cos C = 2$ prove that the sides of the triangle are in AP

Watch Video Solution

22. If in ΔABC, $(a-b)(s-c)=(b-c)(s-a)$, prove that r_1, r_2, r_3 are in A.P.

Watch Video Solution

23. . In a $\Delta ABC, If \tan\left(\frac{A}{2}\right), \tan\left(\frac{B}{2}\right), \tan\left(\frac{C}{2}\right)$, are in H.P.,then a,b,c are in

Watch Video Solution

24. If the sides of triangle in A.P. and $\angle C = 90 + \angle A$ then prove that sides will be in ratio $\sqrt{7}+1 : \sqrt{7} : \sqrt{7}-1$

Watch Video Solution

25. If the sides a,b,c of a triangle are in Arithmetic progressioni then find the value of $\tan\left(\frac{A}{2}\right)+\tan\left(\frac{C}{2}\right)$ in terms of $\cot\left(\frac{B}{2}\right)$

Watch Video Solution

26. Prove that $r_1 + r_2 + r_3 - r = 4R$

Watch Video Solution

27. prove that : triangle ABC, $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$

Watch Video Solution

28. To show that $\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \frac{1}{r^2} = \frac{\sum a^2}{S^2}$

Watch Video Solution

29. If A, A_1, A_2, A_3 are the areas of the inscribed and escribed of a ΔABC, then

Watch Video Solution

30. Prove that : $\frac{r_1}{bc}+\frac{r_2}{ca}+\frac{r_3}{ab}=\frac{1}{r}-\frac{1}{2R}$

Watch Video Solution

31. ABC is an isosceles triangle inscribed in a circle of radius $r\cdot$ If $AB=AC$ and h is the altitude from A to $BC,$ then triangle ABC has perimeter $P=2\left(\sqrt{2hr-h^2}+\sqrt{2hr}\right)$ and area A= ___________ and = _________ and also $(\lim)_{x\overrightarrow{0}}\frac{A}{P^3}=_{-}\ \ -_{-}\ -$

Watch Video Solution

32. If p_1, p_2, p_3 re the altitudes of the triangle ABC from the vertices A, B and C respectivel. Prove that $\frac{\cos A}{p_1}+\frac{\cos B}{p^2}+\frac{\cos C}{p_3}=\frac{1}{R}$

Watch Video Solution

33. Three circles whose radii are a,b and c and c touch one other externally and the tangents at their points of contact meet in a point. Prove that the distance of this point from either of their points of contact is $\left(\frac{abc}{a+b+c}\right)^{\frac{1}{2}}$.

Watch Video Solution

34. In a triangle ABC prove that $r_1 r_2 r_3 = r^3 \cot^2\left(\frac{A}{2}\right).\cot^2\left(\frac{B}{2}\right).\cot^2\left(\frac{C}{2}\right)$

Watch Video Solution

35. Prove that : $(r_1 + r_2)\frac{\tan(C)}{2} = (r_3 - r)\frac{\cot(C)}{2} = c$

Watch Video Solution

36. Prove that : $4R \sin A \sin B \sin\mathbb{C} = a \cos A + b \cos B + c \cos C$

Watch Video Solution

37. Prove that $(r_1 - r)(r_2 - r)(r_3 - r) = 4Rr^2$

Watch Video Solution

38. In a triangle ABC, prove that $r^2 + r_1^2 + r_2^2 + r_3^2 = 16R^2 - a^2 - b^2 - c^2$.

Watch Video Solution

39. If I is the incentre and I_1, I_2, I_3 are the centre of escribed circles of the $\triangle ABC$. Prove that

$II_1 . II_2 . II_3 = 16R^2 r$.

Watch Video Solution

40. If 1 is the incentre and $1_1, 1_2, 1_3$ are the centre of escribed circles of the $\triangle ABC$. Prove that

$II_1, II_2, III_3 = 16R^2r.$

Watch Video Solution

41. $\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab} =$

Watch Video Solution

42. Prove that :

$$\frac{r_1}{(s-b)(s-c)} + \frac{r_2}{(s-c)(s-a)} + \frac{r_3}{(s-a)(s-b)} = \frac{3}{r}$$

Watch Video Solution

43. If the distances of the vertices of a triangle =ABC from the points of contacts of the incercle with sides are $\alpha, \beta and \gamma$ then prove that

$$r^2 = \frac{\alpha\beta\gamma}{\alpha+\beta+\gamma}$$

Watch Video Solution

44. If in a triangle $\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right)=2$ then the triangle is right angled (b) isosceles equilateral (d) none of these

Watch Video Solution

45. In a triangle ABC, prove that the ratio of the area of the incircle to that of the triangle is $\pi : \cot\left(\frac{A}{2}\right)\cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)$

Watch Video Solution

46. For a regular polygon, let r and R be the radii of the inscribed and the cirumscribed circles, respectively. A false statement among the following is

Watch Video Solution

47. A square whose side is 2 cm, has its corners cut away so as to form a regular octagon, find its area.

Watch Video Solution

48. An equilateral triangle and a regular hexagon has same perimeter. Find the ratio of their areas.

Watch Video Solution

49. The ratio of the area of a regular polygon of n sides inscribed in a circle to that of the polygon of same number of sides circumscribing the same is $3:4$. Then the value of n is

Watch Video Solution

50. A cyclic quadrilateral $ABCD$ of areal $\frac{3\sqrt{3}}{4}$ is inscribed in unit circle. If one of its side $AB = 1,$ and the diagonal $BD = \sqrt{3},$ find the lengths of the other sides.

Watch Video Solution

51. about to only mathematics

Watch Video Solution

52. In an acute-angled triangle ABC, $\tan A + \tan B + \tan C$

Watch Video Solution

53. If in a triamgle ABC, θ is the angle determined by $\cos\theta = (a - b)/c,$ then

Watch Video Solution

54. If R be the circum radius and r the in radius of a triangle ABC, show that $R \geq 2r$

Watch Video Solution

55. If $A+B+C=\pi$, prove that: $\cot^2 A+\cot^2 B+\cot^2 C \geq 1$

Watch Video Solution

56. In acute angled $\triangle ABC$ prove that $\tan^2 A+\tan^2 B+\tan^2 C \geq 9.$

Watch Video Solution

57. In ΔABC, prove that $\operatorname{cosec}\frac{A}{2}+\operatorname{cosec}\frac{B}{2}+\operatorname{cosec}\frac{C}{2} \geq 6$.

Watch Video Solution

58. Prove that in $\triangle ABC, 2\cos A\cos B\cos C \leq \frac{1}{4}$.

Watch Video Solution

59. Three equal circles each of radius r touch one another. The radius of the circle touching all the three given circles internally is $(2+\sqrt{3})r$ (b) $\frac{(2+\sqrt{3})}{\sqrt{3}}r$ $\frac{(2-\sqrt{3})}{\sqrt{3}}r$ (d) $(2-\sqrt{3})r$

Watch Video Solution

60. In aΔABC, prove that

$$\sum_{r=0}^{n} .^n C_r a^r b^{n-r}\cos(rB-(n-r)A)=c^n.$$

Watch Video Solution

61. If $\triangle$ is the area and 2s is the perimeter of $\triangle ABC$, then prove that

$$\triangle \leq \frac{s^2}{3\sqrt{3}}$$

Watch Video Solution

62. The sides of a triangle are $3x + 4y$, $4x + 3y$ and $5x + 5y$ units, where $x > 0$, $y > 0$. The triangle is

Watch Video Solution

63. In a $\triangle ABC$, $\cos ecA[\sin B.\cos C + \cos B.\sin C] =$

(A) $\frac{c}{a}$

(B) $\frac{a}{c}$

(C) 1

(D) none of these

Watch Video Solution

64. If the data given to construct a triangle ABC are a = 5, b= 7, sin $A = 3/4$, then is it possible to construct?

Watch Video Solution

Watch Video Solution

65. If in a triangle the angles are in the ratio as $1:2:3$, prove that the corresponding sides are $1:\sqrt{3}:2$.

Watch Video Solution

66. If three sides a,b,c of a triangle ABC are in arithmetic progression, then the value of $\cot\left(\frac{A}{2}\right), \cot\left(\frac{B}{2}\right), \cot\left(\frac{C}{2}\right)$ are in

Watch Video Solution

67. If $b=3, c=4, and B=\frac{\pi}{3},$ then find the number of triangles that can be constructed.

Watch Video Solution

68. In a triangle $ABC, a=4, b=3, \angle A=60^0$ then c is root of the equation $c^2-3c-7=0$ (b) $c^2+3c+7=0$ (c) $c^2-3c+7=0$ (d) $c^2+3c-7=0$

Watch Video Solution

69. If in a triangle $ABC, 3\sin A=6\sin B=2\sqrt{3}\sin C$, then the angle A is

Watch Video Solution

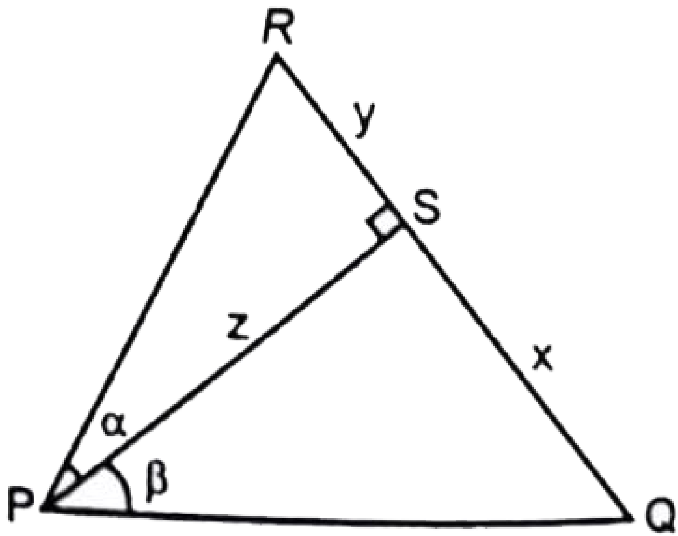
70. The number of triangles ABC that can be formed with $\sin A=\frac{5}{13}, a=3$ and $b=8$ is

Watch Video Solution

71. The sides of a triangle are $\alpha - \beta, \alpha + \beta$ and $\sqrt{3\alpha^2 + \beta^2}, (\alpha > \beta > 0)$. Its largest angle is

Watch Video Solution

72. In a ΔPQR (as shown in figure) if $x : y : z = 2 : 3 : 6$, then the value of $\angle QPR$ is :



Watch Video Solution

73. If in a $\Delta ABC, \angle C = 90^\circ$, then find the maximum value of sin A sin B.

Watch Video Solution

74. In an isosceles right angled triangle ABC, $\angle B = 90^0$, AD is the median then $\frac{\sin\angle BAD}{\sin\angle CAD}$ is (A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 1 (D) none of these

Watch Video Solution

75. If in a ΔABC, c = 3b and C - B = 90°, then tanB=

Watch Video Solution

76. If the lenghts of the sides of a triangle are 3,5 and 7, then the largest angle of the triangle is

Watch Video Solution

77. In a ΔABC if a = 7 , b = 8 and c = 9 , then the length of the line joining B to the mid-points of AC is

Watch Video Solution

78. If in ΔABC, the distance of the vertices from the orthocenter are x,y, and z then prove that $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{abc}{xyz}$

Watch Video Solution

79. If the sides of a triangle are in the ratio $3:7:8$, then find $R:r$

Watch Video Solution

80. If the sides of a triangle are in GP and its largest angle is twice tha smallset then the common ratio r satisfies the inequality

Watch Video Solution

Watch Video Solution

81. If in a $\triangle ABC, a^2 \cos^2 A = b^2 + c^2,$ then angle A is

(A) less than 45^0 (B) more than 45^0 and less than 90^0

(C) right angled (D) obtuse angle

Watch Video Solution

82. The perimeter of a triangle ABC is six times the arithmetic mean of the sines of its angles. If the side a is 1, then find angle A

Watch Video Solution

83. In a tiangle ABC if angle C is obtuse, prove that $\tan A \tan B < 1$

Watch Video Solution

84. In an equilateral triangle, the inradius, circumradius, and one of the exradii are in the ratio

Watch Video Solution

85. The ratio of the area of triangle inscribed in ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to that of triangle formed by the corresponding points on the auxiliary circle is 0.5. Then, find the eccentricity of the ellipse.

Watch Video Solution

86. If in a triangle ABC, the altitude AM be the bisector of $\angle BAD$, where D is the mid point of side BC, then prove that $\left(b^2 - c^2\right) = \frac{a^2}{2}$.

Watch Video Solution

87. In a `/_\ABC, tan, A/2 = 5/6 and tan, C/2 = 2/5 then (A) a,c,b are in A.P. (B) a,b,c are in A.P. (C) b,a,c are in A.P. (D) a,b,c are in G.P.

Watch Video Solution

88. The sides of a triangle are $3x + 4y$, $4x + 3y$ and $5x + 5y$ units, where $x > 0, y > 0$. The triangle is

Watch Video Solution

89. In triangle ABC, AD is the altitude from A. If $b > c, \angle C = 23^0, and AD = \frac{abc}{b^2} - c^2$, then $\angle B =$_ _ _

Watch Video Solution

90. A circle is inscribed in an equilateral triangle of side a. Find the area of any square inscribed in this circle.

Watch Video Solution

91. about to only mathematics

Watch Video Solution

92. Let $f(x+y)=f(x).\,f(y)$ for all x and y $f(1)=2$ If in a triangle

`ABC, a =f (3),b=f(1)+f (3), c=f (2)+f (3), then 2A is equal to

Watch Video Solution

93. In triangle ABC, angle A is greater than angle B. If the measure of angles A and B satisfy the equation $3\sin x-4\sin^3 x-k=0$. Find the value of angle C (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

Watch Video Solution

94. In a $\Delta ABC, \angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$ let D divide BC internally in the ratio $1:3$, then $\frac{\sin(\angle BAD)}{\sin(\angle CAD)}$ is equal to :

Watch Video Solution

95. If a, b,c be the sides of a triangle ABC and if roots of equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 90$ are equal then $\frac{\sin^2 A}{2}, \frac{\sin^2 B}{2}, \frac{\sin^2 C}{2}$ are in

Watch Video Solution

96. In a $\triangle ABC, b^2 + c^2 = 1999a^2$, then $\frac{\cot B + \cot C}{\cot A} =$ (A) 1/1999 (B) 1/999 (C) 999 (D) 1999

Watch Video Solution

97. If $(1+ax)^n = 1 + 8x + 24x^2 + \ldots$, then a =... and n= ....

Watch Video Solution

98. If equations $ax^2 + bx + c = 0$ and $4x^2 + 5x + 6 = 0$ have a comon root, where a,b,c are the sides of $\triangle ABC$ opposite to angles A,B,C respectively, then 2a= (A) c (B) 2c (C) 3c (D) 4c

Watch Video Solution

99. In a ΔABC, if $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$, prove that $\angle A = 90^\circ$.

Watch Video Solution

100. In triangle ABC, $a : b : c = 4 : 5 : 6$. Then find the ratio of the radius of the circumcircle to that of the incircle

Watch Video Solution

101. In triangle ABC,$\frac{\sin A + \sin B + \sin C}{\sin A + \sin B - \sin C}$ is equal to

Watch Video Solution

102. If cosA + cosB $= 4\sin^2\left(\frac{C}{2}\right)$, then

Watch Video Solution

103. If twice the square of the diameter of the circle is equal to half the sum of the squares of the sides of incribed triangle ABC,then $\sin^2 A + \sin^2 B + \sin^2 C$ is equal to

Watch Video Solution

104. If the base angles of triangle are $\frac{22}{12} and 112 \frac{1}{2^0}$, then prove that the altitude of the triangle is equal to $\frac{1}{2}$ of its base.

Watch Video Solution

105. Let ABC be an isosceles triangle with base BC. If r is the radius of the circle inscribsed in ΔABC and r_1 is the radius of the circle ecribed opposite to the angle A, then the product $r_1 r$ can be equal to (where R is the radius of the circumcircle of ΔABC)

Watch Video Solution

106. If Δ represents the area of acute angled triangle ABC, then $\sqrt{a^2b^2-4\Delta^2}+\sqrt{b^2c^2-4\Delta^2}+\sqrt{c^2a^2-4\Delta^2}=$ (a) $a^2+b^2+c^2$ (b) $\frac{a^2+b^2+c^2}{2}$ (c) $ab\cos C+bc\cos A+ca\cos B$ (d) $ab\sin C+bc\sin A+ca\sin B$

Watch Video Solution

107. If in a $\triangle ABC, a=6, b=3$ and $\cos(A-B)=\frac{4}{5}$ then (A) $C=\frac{\pi}{4}$ (B) $A=\frac{\sin^{-1}2}{\sqrt{5}}$ (C) $ar(\triangle ABC)=9$ (D) none of these

Watch Video Solution

108. In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P, then the length of the third side can be (a) $5-\sqrt{6}$ (b) $3\sqrt{3}$ (c) 5 (d) $5+\sqrt{6}$

Watch Video Solution

109. In a triangle ABC, points D and E are taken on side BC such that BD= DE= EC. If angle ADE = angle AED = θ, then: (A) tan θ= 3 tan B (B) 3 tan θ = tan C (C) (6tan θ)/($\tan^2$ θ-9)= tanA (D) $\angle B$=$\angle C$

Watch Video Solution

110. For any triangle ABC, prove that $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^2+b^2+c^2}{2abc}$

Watch Video Solution

111. about to only mathematics

Watch Video Solution

112. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x} dx$, is

Watch Video Solution

113. If a and b be the length of the sides and c the length of hypotenuse of a right anlged triangle then (A) $a+b>c$ (B) $a^2+b^2=c^2$ (C) $a^3+b^3<c^3$ (D) $a^n+b^n<c^n$ for$n \geq 3$,$n=Z$

Watch Video Solution

114. If in ΔABC, $\angle A = 90^\circ$ and c, sin B cos B are rational numbers, then show a and b are rational .

Watch Video Solution

115. In triangle ABC, the value of $\begin{vmatrix} e^{-i2A} & e^{iC} & e^{iB} \\ e^{iC} & e^{-i2B} & e^{iA} \\ e^{iB} & e^{iA} & e^{-i2C} \end{vmatrix}$

Watch Video Solution

116. If a, b, c, d and p are different real numbers such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\leq 0$, then show that a, b, c and d are in G.P.

Watch Video Solution

117. In a triangle, $a^2+b^2+c^2=ca+ab\sqrt{3}$. Then the triangles is :

Watch Video Solution

118. If all the vertices of a triangle have integral coordinates, then the triangle may be (a) right-angle (b) equilateral (c) isosceles (d) none of these

Watch Video Solution

119. In a triangle, the lengths of the two larger sides are 10 and 9 respectively. If the angles are in A.P.,

Watch Video Solution

120. If the tangents of the angles A, B of a ΔABC ...satisfy the equation $abx^2 - c^2x + ab = 0,$ then the triangle is:

Watch Video Solution

121. In a triangle ABC, points D and E are taken on side BC such that BD= DE= EC. If angle ADE = angle AED = θ, then: (A) tan θ= 3 tan B (B) 3 tan θ = tan C (C) (6tan θ)/($\tan^2$ θ-9)= tanA (D) $\angle B$=$\angle C$

Watch Video Solution

122. If in a ΔABC, if $a^4+b^4+c^4=2c^2(a^2+b^2)$, prove that $C=45^0$ or 135^0.

Watch Video Solution

123. Statement-1: If the measures of two angles of a triangle are 45 ° and 60 °, then the ratio of the smallest and the greatest sides are $(\sqrt{3}-1):1$

Statement-2: The greatest side of a triangle is opposite to its greatest angle.

Watch Video Solution

124. Statement 1. In any triangle ABC, if $a:b:c=4:5:6, then R:r=16:7$, Statement 2. In any triangle $\frac{R}{r}=\frac{abc}{4s}$ (A) Both Statement 1 and Statement 2 are true and Statement 2 is the correct explanation of Statement 1 (B) Both Statement 1 and Statement 2 are true and Statement 2 is not the correct explanatioin of Statement 1 (C) Statement 1 is true but Statement 2 is false. (D) Statement 1 is false but Stastement 2 is true

Watch Video Solution

125. $\sin\left\{2\cos^{-1}\left(-\frac{3}{5}\right)\right\}$ is equal to (a) $6/25$ (b) $24/25$ (c) $4/5$ (d) $-24/25$

Watch Video Solution

126. Three circles touch one another externally. The tangents at their points of contact meet at a point whose distance from a point of contact is 4. Find the ratio of the product of the radii to the sum of the radii of the circles.

Watch Video Solution

127. If $A+B+C=\pi$, prove that

(a) $\tan 3A+\tan 3B+\tan 3C=\tan 3A\tan 3B\tan 3C$

(b) $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

Watch Video Solution

128. Given the base of a triangle, the opposite angle A, and the product k^2 of the other two sides, show that it is not possible for a to be less than $2k\frac{\sin A}{2}$

Watch Video Solution

129. In a triangle ABC the sides b and c are the roots of the equation $x^2 - 61x + 820 = 0$ and $A = \tan^{-1}\left(\frac{4}{3}\right) thena^2 + 3$ is equal to

Watch Video Solution

130. If in a triangle $ABC, Rr(\sin A + \sin B + \sin C) = 96$ then the square of the area of the triangle ABC is.......

Watch Video Solution

131. The sides of a quadrilateral are 3, 4, 5 and 6 cms. The sum of a pair of opposite angles is $120^0. Showt\hat{t} heareaofthe\ rilateralis$3sqrt(30)` sq.cm.

Watch Video Solution

132. Three circles touch one another externally. The tangents at their points of contact meet at a point whose distance from a point of contact is 4. Find the ratio of the product of the radii to the sum of the radii of the circles.

Watch Video Solution

133. In triangle ABC, $a:b:c=4:5:6$. Then find the ratio of the radius of the circumcircle to that of the incircle

Watch Video Solution

134. If p_1, p_2, p_3, be the altitudes of a triangle ABC from the vertices A, B, C respectively and Δ be the area of the triangle ABC, prove that :

$$\frac{p}{p_1}+\frac{1}{p_2}-\frac{1}{p_3}=\frac{2ab\cos^2, \frac{C}{2}}{\Delta(a+b+c)}$$

Watch Video Solution

135. If the sides of a quadrilateral touch a circle, prove that the sum of a pair of opposite sides is equal to the sum of the other pair.

Watch Video Solution

136. If in triangle ABC, $a = (1+\sqrt{3})cm$, $b = 2cm$, and $\angle C = 6$ then find the other two angles and the third side

Watch Video Solution

137. If a circle is inscribed in right angled triangle ABC with right angle at B, show that the diameter of the circle is equal to $AB + BC - AC$.

Watch Video Solution

138. If a triangle is inscribed in a circle, then prove that the product of any two sides of the triangle is equal to the product of the diameter and the perpendicular distance of the thrid side from the opposite vertex.

Watch Video Solution

139. ABC is a triangle and D is the middle point of BC. If AD is perpendicular to AC, then prove that $\cos A . \cos C = \frac{2(c^2 - a^2)}{3ac}$

Watch Video Solution

140. If angles A,B, and C of a triangle ABC are in A.P. and if $\frac{b}{c} = \frac{\sqrt{3}}{\sqrt{2}}$, then find angle A

Watch Video Solution

141. The exradii $r_1, r_2,$ and r_3 of ΔABC are in H.P. show that its sides a, b, and c are in A.P.

Watch Video Solution

142. If in a ΔABC, $\cos A + \cos B + \cos C = \frac{3}{2}$. Prove that ΔABC is an equilateral triangle.

Watch Video Solution

143. With usual notion, if in triangle ABC, $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}, then prove that \frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25}$

Watch Video Solution

144. AB is a diameter of a circle and C is any point on the circumference of the circle. Then a) the area of ABC is maximum when it is isosceles b) the area of ABC is minimum when it is isosceles c) the perimeter of ABC is minimum when it is isosceles d) none of these

Watch Video Solution

145. about to only mathematics

Watch Video Solution

146. If in triangle ABC, $\cos A \cos B + \sin A \sin B \sin C = 1$. Show that $a : b : c = 1 : 1 : \sqrt{2}$

Watch Video Solution

147. Prove that If any A B C are distinct positive numbers , then the expression (b+c-a)(c+a-b)(a+b-c)-abc is negative

Watch Video Solution

148. In a triangle, the lengths of the two larger sides are 10 and 9 respectively. If the angles are in A.P.,

Watch Video Solution

149. If the angles of a triangle are $30^0 and 45^0$ and the included side is $(\sqrt{3}+1)cm$ then the area of the triangle is_____.

Watch Video Solution

150. about to only mathematics

Watch Video Solution

151. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smalles one. Determine the sides of the triangle.

Watch Video Solution

152. $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c}=\frac{a}{ac}+\frac{b}{ca}$, then the values of the angle A is

Watch Video Solution

Watch Video Solution

153. A circle is inscribed in an equilateral triangle of side $a \cdot$ The area of any square inscribed in this circle is _____.

Watch Video Solution

154. about to only mathematics

Watch Video Solution

155. Consider the following statements concerning a ΔABC

(i) The sides a,b,c and area of triangle are rational.

(ii) $a, \tan\frac{B}{2}, \tan\frac{C}{2}$

(iii) $a, \sin A \sin B, \sin C$ are rational .

Prove that $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (i)$

Watch Video Solution

156. IF the lengths of the side of triangle are $3, 5 \text{ and } 7,$ then the largest angle of the triangle is $\frac{\pi}{2}$ (b) $\frac{5\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{4}$

Watch Video Solution

157. In triangle ABC, $a:b:c = 4:5:6$. Then find the ratio of the radius of the circumcircle to that of the incircle

Watch Video Solution

158. Let A,B,C, be three angles such that $A = \frac{\pi}{4}$ and $\tan B, \tan C = p$. Find all possible values of p such that A, B, C are the angles of a triangle.

Watch Video Solution

159. If in a triangle $PQR; \sin P, \sin Q, \sin R$ are in A.P; then (A)the altitudes are in AP (B)the altitudes are in HP (C)the altitudes are in GP (D)the medians are in AP

Watch Video Solution

160. Prove that a triangle ABC is equilateral if and only if $\tan A + \tan B + \tan C = 3\sqrt{3}$.

Watch Video Solution

161. Let ABC be a triangle having O and I as its circumcentre and incentre, respectively. If R and r are the circumradius and the inradius respectively, then prove that (IO) 2 =R 2 −2Rr. Further show that the triangle BIO is right angled triangle if and only if b is the arithmetic mean of a and c.

Watch Video Solution

162. In triangle ABC, $2ac\sin\left(\frac{1}{2}(A-B+C)\right)$ is equal to

Watch Video Solution

163. In triangle ABC, let $\angle C=\pi/2$. If r is the inradius and R is circumradius of the triangle, then $2(r+R)$ is equal to

Watch Video Solution

164. In any triangle ABC prove that $\cot\left(\frac{A}{2}\right)+\cot\left(\frac{B}{2}\right)+\cot\left(\frac{C}{2}\right)=\cot\left(\frac{A}{2}\right)\cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)$

Watch Video Solution

165. Let $PQandRS$ be tangent at the extremities of the diameter PR of a circle of radius r. If $PSandRQ$ intersect at a point X on the circumference of the circle, then prove that $2r=\sqrt{PQxRS}$.

Watch Video Solution

Watch Video Solution

166. If Δ is the area of a triangle with side lengths a, b, and c, then show that $\Delta \leq \frac{1}{2}\sqrt{(a+b+c)abc}$. Also show that equality occurs in the above inequality if and only if $a=b=c$

Watch Video Solution

167. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle $ABC(R$ being the radius of the circumcircle)?

(a) $a, \sin A, \sin B$ (b) $a, b, c(c)$a ,sinB ,R(d)a ,sinA ,R`

Watch Video Solution

168. If the angles of a triangle are in the ratio $4:1:1,$ then the ratio of the longest side to the perimeter is

Watch Video Solution

169. If a, b, c are the sides of a triangle such that $a:b:c = 1:\sqrt{3}:2$, then ratio $A:B:C$ is equal to 3:2:1 b. 3:1:2 c. 1:2:3 d. 1:3:2

Watch Video Solution

170. In an equilateral triangle, three coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. The area of the triangle is 2sqrt(3)(b)6+4sqrt(3)12+(7sqrt(3))/4(d)3+(7sqrt(3))/4`

Watch Video Solution

171. One angle of an isosceles triangle is 120^0 and the radius of its incricel is $\sqrt{3}$. Then the area of the triangle in sq. units is $7+12\sqrt{3}$ (b) $12-7\sqrt{3}$ $12+7\sqrt{3}$ (d) 4π

Watch Video Solution

172. Let a,b,c be the sides of a triangle. Now two of them are equal to $\lambda \varepsilon R$. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ are real then

Watch Video Solution

173. Internal bisector of $\angle A$ of ΔABC meets side BC to D. A line drawn through D perpendicular to AD intersects the side AC at E and side AB at. F. If a,b,c represent sides of ΔABC, then

Watch Video Solution

174. A triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\frac{\sin^2 A}{2}$. If $a, bandc,$ denote the length of the sides of the triangle opposite to the angles $A, B, andC$, respectively, then (a) $b+c=4a$ (b) $b+c=2a$ (c)the locus of point A is an ellipse (d)the locus of point A is a pair of straight lines

Watch Video Solution

Watch Video Solution

175. Consider a triangle ABC and let $a, bandc$ denote the lengths of the sides opposite to vertices $A, B, andC$, respectively. Suppose $a = 6, b = 10,$ and the area of triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then the value of r^2 is

Watch Video Solution

176. If the angle $A, BandC$ of a triangle are in an arithmetic propression and if $a, bandc$ denote the lengths of the sides opposite to $A, BandC$ respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$

Watch Video Solution

177. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a , b and c denote the lengths of the side opposite to A, B ,and C respectively. The value(s) of x for which $a = x^2 + x + 1, b = x^2 - 1,$ and $c = 2x + 1$ is(are) $-(2 + \sqrt{3})$ (b) $1 + \sqrt{3}$ (c)$2 + \sqrt{3}$ (d) $4\sqrt{3}$

Watch Video Solution

178. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side $'a'$, is: $a\cot\left(\frac{\pi}{n}\right)$ b. $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ c. $a\cot\left(\frac{\pi}{2n}\right)$ d. $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

Watch Video Solution

179. In a triangle ABC, medians AD and BE are drawn. If $AD = 4, \angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$ then the area of the triangle ABC is :

Watch Video Solution

180. If in a triangle ABC, $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$, then the sides $a, b, and c$ are in A.P. b. are in G.P. c. are in H.P. d. satisfy $a+b=\cdot$

Watch Video Solution

181. The sides of a triangle are $\sin\alpha, \cos\alpha, \sqrt{1+\sin\alpha\cos\alpha}$ for some $0<\alpha<\frac{\pi}{2}$ then the greatest angle of the triangle is :

Watch Video Solution

182. In triangle ABC, let $\angle c=\frac{\pi}{2}$. If r is the inradius and R is circumradius of the triangle, then $2(r+R)$ is equal to $a+b$ (b) $b+c$ $c+a$ (d) $a+b+c$

Watch Video Solution

183. If in ΔABC, the altitudes from the vertices A, B and C on opposite sides are in HP, then sin A sin B and sin C are in

Watch Video Solution