MATHS

BOOKS - KC SINHA ENGLISH

QUADRATIC EQUATIONS - FOR COMPETITION

Solved Examples

1. If the roots of equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ be equal prove that a, b, c are in H.P.

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2. If a, b, c are nonzero real numbers and $az^2+bz+c+i=0$ has purely imaginary roots, then prove that $a=b^2c$

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3. If $a+b+c=0(a,b,c$ are real), then prove that equation $(b-x)^2-4(a-x)(c-x)=0$ has real roots and the roots will not be equal unless $a=b=c$.

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4. If $P(x)=ax^2+bx+c$ and $Q(x)=-ax^2+dx+c$ where $ac\neq 0$ then $P(x)Q(x)=0$ has atleast

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5. Prove that the roots of equation $bx^2+(b-c)x+b-c-a=0$ are real if those of equatiion $ax^2+2bx+b=0$ are imaginary and vice versa where $a, b, c\varepsilon R$.

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6. The number of integral values of 'm' less than 50, so that the roots of the quadratic equation $mx^2 + (2m-1)x + (m-2) = 0$ are rational, are

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7. Statement (1) : If a and b are integers and roots of $x^2 + ax + b = 0$ are rational then they must be integers. Statement (2): If the coefficient of x^2 in a quadratic equation is unity then its roots must be integers

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8. If r is the ratio of the roots of the equation $ax^2 + bx + c = 0$, show that $\frac{(r+1)^2}{r} = \frac{b^2}{ac}$

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9. If one root of the equation $(l-m)x^2 + lm + 1 = 0$ is double the other and if l is real, then the great value of m is

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10. If one root of the equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other, then $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b$ is equal to

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11. If $\alpha and \beta$ are the roots of $x^2 - p(x+1) - c = 0 and S_n = \alpha^n + \beta,$ then $aS_{n+1} + bS_n + cS_{n-1} = 0$ and hence find S_5.

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12. Let x_1, x_2 be the roots of the equation $x^2 - 3x + A = 0$ and x_3, x_4 be those of equation $x^2 - 12x + B = 0$ and x_1, x_2, x_3, x_4 form an increasing G.P. find A and B.

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13. Let $p and q$ be the roots of the equation $x^2 - 2x + A = 0$ and let $r and s$ be the roots of the equation $x^2 - 18x + B = 0$. If $p < q < r < s$ are in arithmetic progression, then $A =$ and $B = \ldots$

(1997, 2M)

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14. If $x^2 - ax + b = 0$ and $x^2 - px + q = 0$ have a root in common then the second equation has equal roots show that $b + q = \frac{ap}{2}$

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15. If $ax^2 + 2bx + c = 0$ and $x^2 + 2b_1x + c_1 = 0$ have a common root and $\frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1}$ are in show that a_1, b_1, c_1 are in G.P.

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16. If a, b, c, a_1, b_1, c_1 are rational and equations $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ have one and only one root in common, prove that $b^2 - ac$ and $b_1^2 - a_1c_1$ must be perfect squares.

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17. Find the vaues of p if the equations $3x^2 - 2x + p = 0$ and $6x^2 - 17x + 12 = 0$ have a common root.

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18. If the quadratic equations $x^2 + bx + ca = 0 \& x^2 + cx + ab = 0$ (where $a \neq 0$) have a common root. prove that the equation containing their other root is $x^2 + ax + bc = 0$

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19. If p,q,r,s are real and $pr > 4(q+s)$ then show that at least one of the equations $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ has real roots.

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20. If the roots of $ax^2 + 2bx + c = 0$ be possible and different then show that the roots of $(a+c)\left(ax^2 + 2bx + 2c\right) = 2\left(ac - b^2\right)\left(x^2 + 1\right)$ will be impossible and vice versa

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21. If α, β are the roots of $x^2 + px + q = 0 and x^{2n} + p^n x^n + q^n = 0 and if (\alpha / \beta), (\beta / \alpha)$ are the roots of $x^n + 1 + (x+1)^n = 0, the \cap (\in N)$ a. must be an odd integer b. may be any integer c. must be an even integer d. cannot say anything

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22. Approach to solve greatest integer function of x and fractional part of x ; (i) Let [x] and {x} represent the greatest integer and fractional part of x ; respectively Solve $4\{x\} = x + [x]$

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23. If $b > a$ then show that the equation $(x - a)(x - b) - 1 = 0$ has one root less than a and other root greater than b.

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24. Let $-1 \leq p \leq 1$, show that the equation $4x^3 - 3x - p = 0$ has a unique root in the interval $\left[\frac{1}{2}, 1\right]$ and identify it.

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25. If α is a real root of the quadratic equation $ax^2 + bx + c = 0 and \beta$ ils a real root of $-ax^2 + bx + c = 0,$ then show that there is a root γ of equation $(a/2)x^2 + bx + c = 0$ whilch lies between $a and \beta\cdot$

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26. If $2a + 3b + 6c = 0,$ then prove that at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval (0,1).

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27. Thus $f(0) = f(1)$ and hence equation $f'(x) = 0$ has at least one root between 0 and 1. Show that equation $(x-1)^5 + (2x+1)^9 + (x+1)^{21} = 0$ has exactly one real root.

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28. Find the positive solutions of the system of equations $x^{x+y} = y^n$ and $y^{x+y} = x^{2n}.\ y^n$, where $n > 0$

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29. For $a \leq 0,$ determine all roots of the equaton $x^2 - 2a|x - a| - 3a^2 = 0.$

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30. Find all integers x for which $(5x - 1) < (x + 1)^2 < (7x - 3)$.

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31. Show that the expression $\dfrac{x^2 - 3x + 4}{x^2 + 3x + 4}$ lies between $\dfrac{1}{7}$ and 7 for real values of x.

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32. Find the range of $f(x) = \frac{x^2 + 34x - 71}{x^2 + 2x - 7}$

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33. If x is real, show that the expression $\frac{4x^2 + 36x + 9}{12x^2 + 8x + 1}$ can have any real value .

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34. Prove that if x is real, the expression $\frac{(x-a)(x-c)}{x-b}$ is capable of assuming all values if $a > b > c$ or $a < b < c$.

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35. Prove that $\left|\frac{12x}{4x^2 + 9}\right| \leq 1$ for all real values of x the equality being satisfied only if $|x| = \frac{3}{2}$

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36. Prove that if the equation $x^2+9y^2-4x+3=0$ is satisfied for real values of $x and y, then x$ must lie between 1 and 3 and y must lie between-1/3 and 1/3.

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37. For what real values of a, will the expression $x^2-ax+1-2a^2$, for the real x, be always positive ?

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38. For what real values of k both the roots of equation $x^2+2(k-3)x+9=\,-0$ lie between -6 and 1.

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39. Find all the values of the parameter a for which the inequality $a9^x + 4(a-1)3^x + a > 1$ is satisfied for all real values of x.

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40. If the equation $x^2 + px + q = 0,$ the coefficient of x was incorrectly written as 17 instead of 13. Thetn roots were found to be $-2 \text{ and } -15.$ Then correct roots are :

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41. If the roots of the quadratic equation $ax^2 + cx + c = 0$ are in the ratio $p : q$ show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{c}{a}} = 0,$ where a, c are real numbers, such that $a > 0$

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42. Find the number of quadratic equations, which are unchanged by squaring their roots.

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43. a, b, c are positive real numbers forming a G.P. ILf $ax^2 + 2bx + c = 0 and dx^2 + 2ex + f = 0$ have a common root, then prove that $d/a, e/b, f/c$ are in A.P.

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44. the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has.

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45. The roots of the equation $(q-r)x^2 + (r-p)x + p - q = 0$ are (A) $\frac{r-p}{q-r}, 1$ (B) $\frac{p-q}{q-r}, 1$ (C) $\frac{q-r}{p-q}, 1$ (D) $\frac{r-p}{p-q}, 1$

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46. If α and β are the roots equation $ax^2 - 2bx + c = 0, then \alpha^3\beta^3 + \alpha^2\beta^3 + \alpha^3\beta^2 =$ (A) $\frac{c^2}{a^3}(c + 2b)$ (B) $\frac{c^2}{c^3}(c - 2b)$ (C) $b\frac{c^2}{a^3}$ (D) none of these

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47. If c, d are the roots of the equation $(x - a)(x - b) - k = 0$, prove that a, b are roots of the equation $(x - c)(x - d) + k = 0.$

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48. If $a, b, c \in R$ and the equation $ax^2 + bx + c = 0$ and $x^2 + x + 1 = 0$ have a common root, then

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49. If $ax^2+2bx+c=0$ and $a_1x^2+2b_1x+c_1=0$ have a common root and $\frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1}$ are in AP then a_1, b_1, c_1 are in (A) A.P. (B) G.P. (C) H.P. (D) none of these

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50. The sum of all real values of k for which the expression $x^2+2xy+ky^2+2x+k=0$ can be resolved into linear factors is :

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51. Equation $(a+5)x^2-(2a+1)x+(a-1)=0$ will have roots equal in magnitude but opposite in sign if $a=$ (A) 1 (B) -1 (C) 2 (D) $-\frac{1}{2}$

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52. Let f(x) be defined by $f(x)=x-[x], 0\neq x\in R$, where [x] is the greatest integer less than or equal to x then the number of solutions of

$f(x) + f\left(\frac{1}{x}\right) = 1$

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53. If $0 < x < 1000$ and $\left[\frac{x}{2}\right] + \left[\frac{x}{3}\right] + \left[\frac{x}{5}\right] = \frac{31}{30}x$, where $[x]$ is the greatest integer less than or equal to x the number of possible values of x is

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54. If the equations $ax + by = 1$ and $cx^2 + dy^2 = 1$ have only one solution, prove that $\frac{a^2}{c} + \frac{b^2}{d} = 1$ and $x = \frac{a}{c}, y = \frac{b}{d}$

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55. If α, β are the roots of the equations $x^2 + px + q = 0$ then one of the roots of the equation $qx^2 - (p^2 - 2q)x + q = 0$ is (A) 0 (B) 1 (C) $\frac{\alpha}{\beta}$ (D) $\alpha\beta$

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56. Let α and β be the roots of the equation $x^2+x+1=0$. The equation whose roots are α^{29}, β^{17} is (A) $x^2-x+1=0$ (B) $x^2+x+1=0$ (C) $x^2-x-1=0$ (D) $x^2+x-1=0$

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57. If $x \varepsilon R$, then the number of real solutions of the equation $3^x+3^{-x}=\log_{10} 99$ is (A) 0 (B) 1 (C) 2 (D) more than 2

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58. Number of real roots of the equation $2^x+2^{x-1}+2^{x-2}=7^x+7^{x-1}+7^{x-2}$ is

(A) 4

(B) 2

(C) 1

(D) 0

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59. Roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are real and equal, then (A) $a+b+c \neq 0$ (B) a, b, c are in H.P. (C) a, b, c are in A.P. (D) a, b, c are in G.P.

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60. Let $f(x) = ax^2 + bx + c, a, b, c\varepsilon Ra \neq 0$ such that $f(x) > 0 \forall x\varepsilon R$ also let $g(x) = f(x) + f'(x) + f''(x)$. Then (A) $g(x) < 0 \forall x\varepsilon R$ (B) $g(x) > 0 \forall x\varepsilon R$ (C) $g(x) = 0$ has real roots (D) $g(x) = 0$ has non real complex roots

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61. If $\alpha and \beta$ are the roots of $x^2 + px + q = 0 and \alpha^4, \beta^4$ are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always. one positive and one negative root two positive roots two negative roots cannot say anything

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62. If $P(x) = x^2 + ax + b$ and $Q(x) = x^2 + a_1x + b_1, a, b, a_1, b_1 \varepsilon R$ and equation $P(x). Q(x) = 0$ has at most one real root, then

(A) $(1 + a + b)(1 + a_1 + b_1) > 0$ (B) $(1 + a + b)(1 + a_1 + b_1) < 0$

(C) $\dfrac{1 + a + b}{1 + a_1 \_ b_1} > 0$ (D) $1 + a + b > 0$

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63. Find product of all real values of x satisfying $(5 + 2\sqrt{6})^{x^2 - 3} + (5 - 2\sqrt{6})^{x^2 - 3} = 10.$

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64. The set of values of a for which the inequation $x^2 + ax + a^2 + 6a < 0$ is satisfied for all $x \varepsilon (1, 2)$ lies in the interval

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65. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to sum of the squares of their reciprocals, then bc^2, ca^2, ab^2 are in

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66. If the equation $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0, a, b, c \varepsilon R$ have a common root, then $a : b : c$ is

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67. If a, b, and c are odd integers, then prove that roots of $ax^2 + bx + c = 0$ cancont be rational.

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68. If the equation $f(x) = ax^2 + bx + c = 0$ has no real root, then $(a + b + c)c$ is (A) $= 0$ (B) > 0 (C) < 0 (D) not real

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69. If $2a + 3b + 6c = 0,$ then prove that at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval (0,1).

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70. If $f(x) = x$ has non real roots, then the equation $f(f(x)) = x$ (A) has all real and unequal roots (B) has some real and non real roots (C) has all real and equal roots (D) has all non real roots

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71. Consider the quadratic equation$x^2 - mx + 1 = 0$ with two roots α and β such that α+β=m and $\alpha\beta$=1 The value of m for which both the roots of the equation are less than unity are (A) $]-\infty, -2]$ (B) $[-2, 2]$ (C) $[2, \infty]$ (D) $]-\infty, -2] \cup [2, \infty]$

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72. Consider the quadratic equation$x^2 - mx + 1 = 0$ with two roots α and β such that α+β=m and $\alpha\beta$=1 The value of m for which both the roots of the equation are greater then unity re (A) $[2, \infty]$ (B) $]- -\infty, 2]$ (C) $[-2, 2]$ (D) none of these

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73. Consider the quadratic equation$x^2 - mx + 1 = 0$ with two roots α and β such that α+β=m and $\alpha\beta$=1 The values of m for which $\alpha < 1$ and $\beta > 1$ are (A) $[-2, \infty[$ (B) $[-2, 2]$ (C) $[2, \infty]$ (D) $]-\infty, -2]$

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74. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$ such that $\alpha, \beta, \gamma, \delta$ are in G.P. and $p \geq 2. Ifa, b, c\varepsilon\{1, 2, 3, 4, 5\}$, let the number of equation of the form $ax^2 + bx + c = 0$ which have real roots be r, then the minium value of p q r =

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75. Let α, β and γ be the roots of equation $f(x) = 0$, where $f(x) = x^3 + x^2 - 5x - 1 = 0.$ then the value of $|[\alpha] + [\beta] + [\gamma]|, where[.]$ denotes the integer function, is equal to

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Exercise

1. If the roots of the equation $ax^2+bx+c=0$ be in the ratio m:n, prove that $\sqrt{\frac{m}{n}}+\sqrt{\frac{n}{m}}+\frac{b}{\sqrt{ac}}=0$

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2. If α, β are the roots of the equation $x^2-px+q=0$, find the quadratic equation the roots of the which are $(\alpha^2-\beta^2)(\alpha^3-\beta^3)$ and $\alpha^3\beta^2+\alpha^2\beta^3$.

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3. If n and r are positive integers such that$0 < r < n$ then show that the roots of the quadratic equation$nC_rx^2+2.^nC_{r+1}x+^nC_{r+2}=0$ are real.

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4. If a, b, c, are nonzero, unequal rational numbers, then prove that the roots of the equation $(abc)^2x^2+3x^2cx+b^2cx-6a^2-ab+2b^2=0$ are rational .

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5. If α_1, α_2 be the roots of equation $x^2+px+q=0$ and β_1, β be those of equation $x^2+rx+s=0$ and the system of equations $\alpha_1 y+\alpha_2 z=0$ and $\beta_1 y+\beta_2 z=0$ has non trivial solution, show that $\frac{p^2}{r^2}=\frac{q}{s}$

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6. If a,b,c are the roots of the equation $x^3+px^2+qx+r=0$ such that $c^2=-ab$ show that $\left(2q-p^2\right)^3.r=(pq-4r)^3$.

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7. Let $\alpha + i\beta; \alpha, \beta \in R,$ be a root of the equation $x^3 + qx + r = 0; q, r \in R$. A real cubic equation, independent of $\alpha \& \beta$, whose one root is 2α is (a)$x^3 + qx - r = 0$ (b) $x^3 - qx + 4 = 0$ (c) $x^3 + 2qx + r = 0$ (d) None of these

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8. Find the values of k for which $5x^2 - 4x + 2 + k(4x^2 - 2x - 1) = 0$ has real and equal roots.

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9. Find the value of m for which the product of the roots of the equation $5x^2 - 4x + 2 + m(4x^2 - 2x - 1) = 0$ is 2

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10. Find the value of m for which the sum of the roots of the equation $5x^2 - 4x + 2 + m(4x^2 - 2x - 1) = 0$ is 6.

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11. If the sum of the rotsof the equation $px^2 + qx + r = 0$ be equal to the sum of their squares, show that $2pr = pq + q^2$

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12. In copying a quadratic equation of the form $x^2 + px + q = 0$, the coefficient of x was wrongly written as -10 in place of -11 and the roots were found to be 4 and 6 . find the roots of the correct equation.

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13. Solve for x: $\sqrt{11x - 6} + \sqrt{x - 1} = \sqrt{4x + 5}$

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14. If x and y satisfy the equation $y = 2[x] + 3$ and $y = 3[x - 2]$ simultaneously, where [.] denotes the greatest integer function, then $[x + y]$ is equal to

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15. $|x + 1| - |x| + 3|x - 1| - 2|x - 2| = x + 2$. Solve for x

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16. Solve $\left|x^2 + 4x + 3\right| + 2x + 5 = 0$.

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17. Show that the equation $(x-1)^5+(x+2)^7+(7x-5)^9=10$ has exactly one root.

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18. Solve $\frac{1}{[x]}+\frac{1}{[2x]}=\{x\}+\frac{1}{3}$, where [ ] denotes the greatest integer function and { } denotes fractional part of x.

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19. Solve for $x: 4^x 3^{x-1/2}=3^{x+1/2}-2^{2x-1}$.

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20. Find the values of x,satisfying the equation $\log_{10}\left(98+\sqrt{x^3-x^2-12x+36}\right)=2$ is

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21. Solve:

$(\log)_{(2x+3)}\left(6x^2+23+21\right)+(\log)_{(3x+7)}\left(4x^2+12x+9\right)=4$

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22. If S is the set of all real x such that $\frac{2x-1}{2x^3+3x^2+x}$ is $\left(-\infty,-\frac{3}{2}\right)$ b. $\left(-\frac{3}{2},\frac{1}{4}\right)$ c. $\left(-\frac{1}{4},\frac{1}{2}\right)$ d. $\left(\frac{1}{2},3\right)$ e. None of these

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23. Find the value of x such that $\log_{10}\left(x^2-2x-2\right)\leq 0$

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24. For real x, the function $(x-a)(x-b)/(x-c)$ will assume all real values provided $a>b>c$ b. `a c > bd. a

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25. If x, a, b are real prove that

$4(a-x)\left(x-a+\sqrt{a^2+b^2}\right) \not> a^2+b^2$

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26. Prove that for real values of x, $\left(ax^2+3x-4\right)/\left(3x-4^2+a\right)$ may have any value provided a lies between 1 and 7.

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27. if α, β, γ are roots of $2x^3+x^2-7=0$ then find the value of $\sum_{\alpha,\beta,\gamma}\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$

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28. The equation $x^3 + px^2 + qx + r = 0$ and $x^3 + p'x^2 + q'x + r' = 0$ have two common roots, find the quadratic whose roots are these two common roots.

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29. FIND that condition that the roots of equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in G.P.

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30. Show that one of the roots of equation $ax^2 + bx + c = 0$ may be reciprocal of one of the roots of $a_1x^2 + b_1x + c_1 = 0$ if $(aa_1 - cc_1)^2 = (bc_1 - ab_1)(b_1c - a_1b)$

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31. If every pair from among the equations $x^2 + px + qr = 0$, and $x^2 + rx + pq = 0$ have a common root, then $\left(\frac{\text{sum of all distinct roots}}{\text{Product of all distinct roots}}\right)$ is

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32. If $a < b < c < d$, then for any real non-zero λ, the quadratic equation $(x-a)(x-c) + \lambda(x-b)(x-d) = 0$,has real roots for

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33. Show that the following equation can have at most one real root $3x^5 - 5x^3 + 21x + 3\sin x + 4\cos x + 5 = 0$

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34. If $e^{(\cos^2 x + \cos^4 + \cos^x \ldots\ldots)\log 3}$ satisfies the equation $t^2 - 8t - 9 = 0$ then the value of $\tan x$, $\left(0 < x < \frac{\pi}{2}\right)$ is

(A) $\sqrt{3}$

(B) $\sqrt{2}$

(C) 1

(D) $\frac{1}{\sqrt{2}}$

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35. Let $a = \cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)$, $A = a + a^2 + a^4$ and $B = a^3 + a^5 + a^6$, then A and B are the roots of the equation (A) $x^2 - x + 2 = 0$ (B) $x^2 - x - 2 = 0$ (C) $x^2 + x + 2 = 0$ (D) none of these

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36. The number of real solution of $\sin(e^x) = 5^x + 5^{-x} \in [0, 1]$ is

(A) 0

(B) 1

(C) 2

(D) 4

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37. If $\left(x^2 - 3x + 2\right)$ is a factor of $x^4 - px^2 + q = 0$, then the values of p and q are

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38. Equation $\frac{a}{x-1} + \frac{b}{x-2} + \frac{c}{x-3} = 0(a, b, c > 0)$ has (A) two imaginary roots (B) one real roots in (1,2) and other in (2,3) (C) no real root in [1,4] (D) two real roots in (1,2)

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39. If α is a root of the equation $4x^2 + 3x - 1 = 0$ and $f(x) = 4x^2 - 3x + 1$, then $2(f(\alpha) + (\alpha))$ is equal to

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40. The number of solution of equation $|x-1| = e^x$ is

(A) 0

(B) 1

(C) 2

(D) none of these

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41. If $p, q, r, s \in R$, then the equation $\left(x^2+px+3q\right)\left(-x^2+rx+q\right)\left(-x^2+sx-2q\right)=0$ has

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42. If $\alpha and \beta$ are the roots of $x^2+px+q=0 and \alpha^4, \beta^4$ are the roots of $x^2-rx+s=0$, then the equation $x^2-4qx+2q^2-r=0$ has always. one positive and one negative root two positive roots two negative roots cannot say anything

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43. If $a+b+c>\frac{9c}{4}$and the equation $ax^2+2bx-5c=0$ has non-real complex roots, then

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44. If $a, b, c\varepsilon R(a\neq 0)$ and $a+2b+4c=0$ then equatio $ax^2+bx+c=0$ has

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45. If p, q be non zero real numbes and $f(x)\neq 0, x\in[0, 2]$ also $f(x)>0$ and $\int_0^1 f(x).\left(x^2+px+q\right)dx=\int_1^2 f(x).\left(x^2+px+q\right)dx=0$ then

equation $x^2 + px + q = 0$ has (A) two imginary roots (B) no root in $(0, 2)$ (C) one root in $(0, 1)$ and other in $(1, 2)$ (D) one root in $(-\infty, 0)$ and other in $(2, \infty)$

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46. The number of real roots of $x^8 - x^5 + x^2 - x + 1 = 0$ is

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47. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 + bx + c = 0$, then (A) $(a-c)^2 = b^2 + c^2$ (B) $(a+c)^2 = b^2 - c^2$ (C) $a^2 = b^2 - 2ac$ (D) $a^2 + b^2 - 2ac = 0$

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48. If $x^2 + ax + b$ is an integer for every integer x , then :

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49. If $x^2 + ax + b$ is an integer for every integer x , then :

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50. If $0 < \alpha < \frac{\pi}{4}$ equation $(x - \sin\alpha)(x - \cos\alpha) - 2 = 0$ has (A) both roots in $(\sin\alpha, \cos\alpha)$ (B) both roots in $(\cos\alpha, \sin\alpha)$ (C) one root in $(-\infty, \cos\alpha)$ and other in $(\sin\alpha, \infty)$ (D) one root in $(-\infty, \sin\alpha)$ and other in $(\cos\alpha, \infty)$

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51. Number of roots of the equation $\sin x + \cos x = x^2 - 2x + \sqrt{6}$ is

(A) 0

(B) 2

(C) 4

(D) an odd number

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52. Let $f(x) = x^3 - 6x^2 + 3(1+\pi)x + 7, p > q > r, then \frac{\{x - f(p)\}(x - f(r)\}}{x - f(q)}$ has no value in (A) (p,q) (B) (q,r) (C) (r, ∞) (D) none of these

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53. If expression $x^2 - 4cx + b^2 > 0f \text{ or } all x \varepsilon R$ and $a^2 + c^2 < ab$ then range of the function $\frac{x+a}{x^2+bx+c^2}$ is (A) $(0, \infty)$ (B) $(0, \infty)$ (C) $(-\infty, \infty)$ (D) none of these

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54. If the equation $(\lambda - 1)x^2 + (\lambda + 1)x + \lambda - 1 = 0$ has real roots then $\lambda = \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos 3\theta}{\sin 3\theta}$ for

(A) only one value of θ

(B) for infinitely many values of θ

(C) for no value of θ

(D) of only two values of θ

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55. If α and β are roots of equation $x^2 + px + q = 0$ and $f(n) = \alpha^n + \beta^n$, then (i) $f(n+1) + pf(n) - qf(n-1) = 0$ (ii) $f(n+1) - pf(n) + qf(n-1) = 0$ (iii) $f(n+1) + pf(n) + qf(n-1) = 0$ (iv) $f(n+1) - pf(n) - qf(n-1) = 0$

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56. If t_n denotes the nth term of an A.P. and $t_p = \frac{1}{q}, t_q = \frac{1}{p}$ then which one of the following is necessarily a root of the equation $(p + 2q - 3r)x^2 + (q + 2r - 3p)x + (r + 2p - 3q) = 0$ (A) t_p (B) t_q (C) t_{pq} (D) t_{p+q}

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57. If α and β $(\alpha' < '\beta')arethe\sqrt[s]{o}ftheequation$x^2+b x+c=0,$wherec$<0

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58. α **and** β **are the roots of the equation** $x^2 + px + p^3 = 0, (p \neq 0)$**. If the point** (α, β) **lie on the curve** $x = y^2$ **then the roots of the given equation are (A) 4,-2 (B) 4,2 (C) 1,-1 (D) 1,1**

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59. If $\alpha \text{ and } \beta$ **are the roots of the equation** $x^2 - ax + b = 0 \text{ and } A_n = \alpha^n + \beta^n,$ **then which of the following is true ?**

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60. If the difference between the roots of $x^2 + ax + b = 0$ is same as that of $x^2 + bx + a = 0$ $a \neq b$, then:

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61. If x satisfies $|x - 1| + x - 2| + |x - 3| \geq 6,$ then (a) $0 \leq x \leq 4$ (b) $x \leq 4$ (c) $x \leq 0 \text{ or } x \geq 4$ (d) none of these

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62. Let a, b, c be nonzero real numbers such that

$$\int_0^1 (1 + \cos^8 x)(ax^2 + bx + c)dx = \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c)dx = 0$$

Then show that the equation $ax^2 + bx + c = 0$ will have one root between 0 and 1 and other root between 1 and 2.

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63. If α and β are the roots of a quadratic equation such that $\alpha + \beta = 2, \alpha^4 + \beta^4 = 272$, then the quadratic equation is

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64. The minimum $valueof$|x-3|+|x-2|+|x-5|` is (A) 3 (B) 7 (C) 5 (D) 9

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65. Let $[x]$ denote the integral part of a real number x and $\{x\} = x - [x]$ then solution of $4\{x\} = x + [x]$ are (A) $\pm\frac{2}{3}, 0$ (B) $\pm\frac{4}{3}, 0$ (C) $0, \frac{5}{3}$ (D) $\pm 2, 0$

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66. The equation $\left|x^2 - x - 6\right| = x + 2$ has :

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67. If equation $x^2-(2+m)x+1(m^2-4m+4)=0$ has coincident roots then (A) $m=0, m=1$ (B) $m=0, m=2$ (C) $m=\frac{2}{3}, m=6$ (D) $m=\frac{2}{3}, m=1$

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68. If $f(x)=2x^3+mx^2-13x+n$ and 2 and 3 are 2 roots of the equations f(x)=0, then values of m and n are

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69. If $y=\frac{x^2-3x+1}{2x^2-3x+2}$, where x is real, the value of y lies between (A) $-1 \leq y \leq \frac{5}{7}$ (B) $-\frac{1}{2} \leq y \leq \frac{5}{7}$ (C) $\frac{5}{7} < y < 1$ (D) none of these

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70. If one of the values of x of the equation $2x^2 - 6x + k = 0$ be $\frac{1}{2}(a + 5i)$, find the values of a and k.

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71. If $f(x)$ is a continuous function and attains only rational values and $f(0) = 3$, then roots of equation $f(1)x^2 + f(3)x + f(5) = 0$ as

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72. If a,b,c,d are unequal positive numbes, then the roots of equation $\frac{x}{x-a} + \frac{x}{x-b} + \frac{x}{x-c} + x + d = 0$ are necessarily (A) all real (B) all imaginary (C) two real and two imaginary roots (D) at least two real

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73. The number of solutions of te equation $\left|2x^2 - 5x + 3\right| + x - 1 = 0$ is (A) 1 (B) 2 (C) 0 (D) infinite

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74. The set of value of a for which both the roots of the equation $x^2 - (2a - 1)x + a = 0$ are positie is

(A) $\left\{\frac{2 - \sqrt{3}}{2}\right\}$

(B) $\left\{\frac{2 - \sqrt{3}}{2}, \frac{2 + \sqrt{3}}{2}\right\}$

(C) $\left[\left(2 + \frac{\sqrt{3}}{2}, \infty\right)\right.$

(D) none of these

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75. If the root of the equation $(a - 1)\left(x^2 + x + 1\right)^2 = (a + 1)\left(x^4 + x^2 + 1\right)$ are real and distinct,

then the value of $a \in \ (-\infty, 3]$ b. $(-\infty, \ -2) \cup (2, \infty)$ c. $[-2, 2]$ d. $[-3, \infty)$

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76. If the product of the roots of the equation $2x^2 + ax + 4\sin a = 0$ is 1, then roots will be imaginary, if

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77. The quadratic equation whose roots are x and y intercepts of the line passing through (1,1) and making a triangle of area A with the axes is

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78. If α and β are solution of $\sin^2 x + a\sin x + b = 0$ as well as that of $\cos^2 x + cocsx + d = 0$, such that $\sin\alpha \neq \sin\beta$ and $\cos\alpha \neq \cos\beta$ then $\sin(\alpha + \beta)$ is equal to

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79. The roots α and β of the quadratic equation $ax^2 + bx + c = 0$ are and of opposite sing. The roots of the equation $\alpha(x-\beta)^2 + \beta(x-\alpha)^2 = 0$ are

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80. If $a, b, c\varepsilon\{1, 2, 3, 4, 5\}$, the number of equations of the form $ax^2 + bx + c = 0$ which have real roots is (A) 25 (B) 26 (C) 27 (D) 24

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81. The number of real solutions of the equation $-x^2 + x - 1 = \sin^4 x$ is

(A) 1

(B) 2

(C) 0

(D) 4

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82. Solve the equation $(6-x)^4+(8-x)^4=16$

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83. If $x, a_1, a_2, a_3, \ldots . a_n \varepsilon R$ and $(x-a_1+a_2)^2+(x-a_2+a_3)^2+\ldots\ldots+(x-a_{n-1}+a_n)^2\leq 0$, then $a_1, a_2, a_3 \ldots\ldots\ldots . a_n$ are in (A) AP (B) GP (C) HP (D) none of these

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84. The expression $ax^2+2bx+b$ has same sign as that of b for every real x, then the roots of equation $bx^2+(b-c)x+b-c-a=0$ are (A) real and equal (B) real and unequal (C) imaginary (D) none of these

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85. Let $\alpha + i\beta; \alpha, \beta \in R,$ be a root of the equation $x^3 + qx + r = 0; q, r \in R.$ A real cubic equation, independent of $\alpha \& \beta,$ whose one root is 2α is (a)$x^3 + qx - r = 0$ (b) $x^3 - qx + 4 = 0$ (c) $x^3 + 2qx + r = 0$ (d) None of these

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86. The equation $\sin x = x^2 + x + 1$ has (A) `one real solution (B) n real solution (C) more thanone real solution (D) two positive solutons

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87. If $p, q, r \in R$ and the quadratic equation $px^2 + qx + r = 0$ has no real roots, then (A) $p(p + q + r) > 0$ (B) $(p + q + r) > 0$ (C) $q(p + q + r) > 0$ (D) $p + q + r > 0$

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88. If $p, q, r \varepsilon R$ and are distinct the equation $(x-p)^5 + (x-q)^5 + (x-r)^5 = 0$ has

(A) four imaginary and one real root

(B) two imaginary and three real roots

(C) all the roots real

(D) none of these

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89. Let S denotes the set of real values of 'a' for which the roots of the equation $x^2 - ax - a^2 = 0$ exceeds 'a', then S equals to

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90. Let $f(x) = x^2 + bx + c$ and $g(x) = af(x) + bf'(x) + cf''(x). Iff(x) > 0 \forall$

then the sufficient condition of $g(x)$ to be $> 0 \forall x \varepsilon R$ is (A) $c > 0$ (B) $b > 0$ (C) $b < 0$ (D) $c < 0$

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91. Find the set of values of k for which $x^2 - kx + \sin^{-1}(\sin 4) > 0$ for all real x .

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92. Let a, b, c be three distinct positive real numbers then number of real roots of $ax^2 + 2b|x| + c = 0$ is (A) 0 (B) 1 (C) 2 (D) 4

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93. The constant term of the quadratic expression $\sum_{k=2}^{n}\left(x - \frac{1}{k-1}\right)\left(x - \frac{1}{k}\right)$, as $n \to \infty$ is

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94. If $x^2 + ax + b$ is an integer for every integer x , then :

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95. If a, b are roots of $x^2 + px + q = 0$ and c, d are the roots $x^2 - px + r = 0 then a^2 + b^2 + c^2 + d^2$ equals **(A)** $p^2 - q - r$ **(B)** $p^2 + q + r$ **(C)** $p^2 + q^2 - r^2$ **(D)** $2(p^2 - q + r)$

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96. If two roots of the equation

$(p-1)(x^2+x+1)^2 - (p+1)(x^4+x^2+1) = 0$ are real and distinct and $f(x) = \frac{1-x}{1+x}$, then $f(f(x)) + f\left(f\left(\frac{1}{x}\right)\right)$ is equal to

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97. Te least value of $|a|$ for which $\tan\theta$ and $\cot\theta$ are the roots of the equation $x^2 + ax + b = 0$ is (A) 2 (B) 1 (C) $\frac{1}{2}$ (D) 0

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98. If $(y^2 - 5y + 3)(x62 + x + 1) < 2x$ for all $x \in R,$ then fin the interval in which y m lies.

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99. If $P(x)$ be a polynomial satisfying the identity $P(x^2) + 2x^2 + 10x = 2xP(x+1) + 3$, then $P(x)$ is

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100. Let a,b,c be positive real parameter and $ax^2 + \frac{b}{x^2} \geq c, \forall x \varepsilon R$ then

(A) $c^2 \geq 4ab$

(B) $4c \geq b^2$

(C) $4bc \geq c^2$

(D) $4ac < b^2$

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101. The quadratic equatin $(2x - a)(2x - c) + \lambda(x - 2b)(x - 2d) = 0$, (where $0 < 4a < 4b < c < 4d$) has (A) a root between 2 b and 2d for all λ (B) as root between b nd d for all $-ve\lambda$ (C) a root between b and d for all $+ve\lambda$ (D) none of these

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102. The set of values of c for which $x^3 - 6x^2 + 9x - c$ is of the form $(x - a)^2(x - b)$ (a, b is real) is given by

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103. The number of real roots (s) of the equation $x^2 \tan x = 1$ lying between 0 and 2π is /are

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104. If 1 lies between the roots of the quadratic equation $3x^2 - (3\sin\theta)x - 2\cos^2\theta = 0,$ then : (A) $-\frac{\pi}{3} < \theta < \frac{5\pi}{3}$ (B) $n\pi < \theta < 2n\pi$ (C) $2n\pi + \frac{\pi}{6} < \theta < 2n\pi + \frac{5\pi}{6}$ (D) none of these

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105. Let α and β be the real and distinct roots of the equation $ax^2 + bx + c = |c|, (a > 0)$ and p, q be the real and distinct roots of the equation $ax^2 + bx + c = 0.$ Then which of the following is true? (A) p and q lie between α and β (B) p and q lies outside (α, β) (C) only p lies between α and β (D) only q lies between $(\alpha$ and $\beta)$

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106. The roots of the equation $ax^2 + bx + c = 0, a \in R^+,$ are two consecutive odd positive

(A) $|b| \leq 4a$

(B) $|b| \geq 4a$

(C) $|b| \geq 2a$

(D) none of these

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107. If equation $x^5 + 10x^2 + x + 5 = 0$ has one roots as alpha then (A) $[\alpha] = -3$(where [.] denotes the greatest integer function) (B) number of roots between -2 and -1 is three (C) number of real roots is 3 (D) equation has at least one positive root

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108. The equation

$\frac{A}{x - a_1} + \frac{A_2}{x - a_2} + \frac{A_3}{x - a_3} = 0, where A_1, A_2, A_3 > 0$ and $a_1 < a_2 < a$

has two real roots lying in the invervals.

(A) (a_1, a_2) and (a_2, a_3) **(B)** $(-\infty, a_1)$ and (a_3, ∞) **(C)** (A_1, A_3) and (A_2, A_3) **(D)** none of these

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109. If both roots of the equation $x^2 - 2ax + a^2 - 1 = 0$ lie between -3 and 4 ,then [a] is/are , where [ ] represents the greatest ineger function

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110. If α be the number of solutons of equation $[\sin x] = |x|, where[x]$ denotes the integral part of x and m be the greatest value of $\cos\left(x^2 + xe^x - [x]\right)$ in the interval $[-1, 1]$, then **(A)** $\alpha = m$ **(B)** $\alpha < m$ **(C)** $\alpha > m$ **(D)** $\alpha \neq m$

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111. If m be the number of integral solutions of equation $2x^2 - 3xy - 9y^2 - 11 = 0$ and n be the roots of $x^3 - [x] - 3 = 0$, then m

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112. If the roots of equation $ax^2 + bx + 10 = 0$ are not real and distinct where $a, b \varepsilon R, \text{ and } m \text{ and } n$ are values of a and b respectively for which $5a + b$ is minimum then the family of lines $m(4x + 2y + 3) + n(x - y - 10 = 0$ are concurrent at (A) $(1, -1)$ (B) $\left(-\frac{1}{6}, -\frac{7}{6}\right)$ (C) $(1, 1)$ (D) none of these

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113. If $[x]$ denotes the integral part of x and $k = \sin^{-1}\left(\frac{1+t^2}{2t}\right) > 0$ then integral valueof α for which the equation

$(x-[k])(x+\alpha)-1=0$ **has integral roots is (A)** $1(B)2(C)4$ **(D) none of these**

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114. If $[x]$ **denotes the integral part of x and** $m=\left[\frac{|x|}{1+x^2}\right], n=$ **integral values of** $\frac{1}{2-\sin 3x}$ **then (A)** $m \neq n$ **(B)** $m>n$ **(C)** $m+n=0$ **(D)** $n^m=0$

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115. If 1 lies between the roots of the equation $y^2-my+1=0$ **and [x] denotes the greatest integer less than or equal to x, then the values of** $\left[\left(\frac{4|x|}{|x|^2+16}\right)^m\right]$**, is**

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116. If for $x > 0 f(x) = (a - x^n)^{\frac{1}{n}}, g(x) = x^2 + px + q, p, q \varepsilon R$ and equation $g(x) - x = 0$ has imaginary roots, then number of real roots of equation $g(g(x)) - f(f(x)) = 0$ is

(A) 0

(B) 2

(C) 4

(D) none of these

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117. Let $f(x) = x^3 + x^2 + 10x + 7\sin x$, then the equation $\frac{1}{y - f(1)} + \frac{2}{y - f(2)} + \frac{3}{y - f(3)} = 0$ has (A) no real root (B) one real roots (C) two real roots (D) more than two real roots

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118. If $0 < \alpha < \beta < \gamma < \frac{\pi}{2}$ then the equation $\frac{1}{x - \sin\alpha} + \frac{1}{x - \sin\beta} + \frac{1}{x - \sin\gamma} = 0$ has

(A) imaginary roots

(B) real and equal roots

(C) real and unequal roots

(D) rational roots

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119. IF $a = \frac{x^2 - 2x + 4}{x^2 + 2x + 4}$ and equation of lines AB and CD be $3y = x \text{ and } y = 3x$ respectively, then for all real x, point $P(a, a^2)$ (A) lies in the acute angle between lines AB and CD (B) lies in the obtuse angle between lines AB and CD (C) cannot be in the acute angle between lines AB and CD (D) cannot lie in the obtuse angle between lines AB and CD

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120. If $f(x) = 3^x + 4^x + 5^x - 6^x, then f(x) < f(3)$ for (A) only one value of x (B) no value of x (C) only two value of x (D) infinitely many value

of x

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121. If α_1, α_2 are the roots of equation $x62 - px + 1 = 0 and \beta_1, \beta_2$ are those of equation $x^2 = qx + 1 = 0$ and vector $\alpha_1 \hat{i} + \beta_1 \hat{j}$ is parallel to $\alpha_2 \hat{i} + \beta_2 \hat{j}$, then $p = \pm q$ b. $p = \pm 2q$ c. $p = 2q$ d. none of these

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122. If α_1, α_2 be the roots of the equation $x^2 - px + 1 = 0$ and β_1, β_2 be those of equatiion $x^2 - qx + 1 = 0$ and $\vec{u} = \alpha_1 \hat{i} + \alpha_2 \hat{j}$, and $\vec{v} = \beta_1 \hat{i} + \beta_2 \hat{j}$ is parallel.

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123. If $a, b, c, d\varepsilon R$ and $f(x) = ax^3 + bx^2 - cx + d$ has local extrema at two points of opposite signs and $ab > 0$ then roots of equation $ax^2 + bx + c = 0$ (A) are necessarily negative (B) have necessarily negative real parts (C) have necessarily positive real parts (D) are necessarily positive

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124. Let $f(x) = Ax^2 + Bx + c, where A, B, C$ are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers $2A, A + B, and C$ are all integer. Conversely, prove that if the number $2A, A + B, and C$ are all integers, then $f(x)$ is an integer whenever x is integer.

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125. Let $f(x) = Ax^2 + Bx + c, where A, B, C$ are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers

$2A, A+B, andC$ are all integer. Conversely, prove that if the number $2A, A+B, andC$ are all integers, then $f(x)$ is an integer whenever x is integer.

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126. If $a(p+q)^2+2bpq+c=0$ and $a(p+r)^2+2bpr+c=0(a\neq 0)$, then which one is correct? a) $qr=p^2$ b) $qr=p^2+\frac{c}{a}$ c) none of these d) either a) or b)

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127. If α and $\beta(\alpha<\beta)$ are the roots of the equation $x^2+bx+c=0$ where $c<0<b$, then

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128. Let $\alpha and \beta$ be the roots of $x^2 - x + p = 0 and \gamma and \delta$ be the root of $x^2 - 4x + q = 0$. If $\alpha, \beta, and \gamma, \delta$ are in G.P., then the integral values of $p and q$, respectively, are $-2, -32$ b. $-2, 3$ c. $-6, 3$ d. $-6, -32$

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129. If $2a + 3b + 6c = 0$, then prove that at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval (0,1).

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130. if α, β be roots of $x^2 - 3x + a = 0$ and γ, δ are roots of $x^2 - 12x + b = 0$ and $\alpha, \beta, \gamma, \delta$(in order) form a increasing GP then find the value of $a \& b$

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131. If the difference of the roots of the equation $x^2 + kx + 7 = 0$ is 6, then possible values of k are (A) 4 (B)-4 (C) 8 D)-8

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132. If x real and $y = \frac{x^2 - x + 3}{x + 2}$, then (A) $y \geq 1$ (B) $y \geq 11$ (C) $y \leq -11$ (D) $-11 < y < 1$

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133. Let $f(x) = \frac{3}{x-2} + \frac{4}{x-3} + \frac{5}{x-4}$. Then $f(x) = 0$ has (A) exactly one real root in (2,3) (B) exactly one real root in (3,4) (C) at least one real root in (2,3) (D) none of these

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134. Let $f(x) = ax^3 + bx^2 + cx + 1$ has exterma at $x = \alpha, \beta$ such that $\alpha\beta < 0$ and $f(\alpha)f(\beta) < 0$ f . Then the equation $f(x) = 0$ has (a)three equal real roots (b)one negative root if $f(\alpha)$ < 0 and $f(\beta)$ >0 (c)one positive root if $f(\alpha)$ < 0 and $f(\beta)$ >0 (d) none of these

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135. If every pair from among the equations $x^2 + px + qr = 0$, and $x^2 + rx + pq = 0$ have a common root, then $\left(\frac{\text{sum of all distinct roots}}{\text{Product of all distinct roots}}\right)$ is

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136. If every pair from among the equations $x^2 + px + qr = 0$, and $x^2 + rx + pq = 0$ have a common root, then $\left(\frac{\text{sum of all distinct roots}}{\text{Product of all distinct roots}}\right)$ is

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137. If $a+b+2c=0, c \ne 0,$ then equation $ax^2+bx+c=0$ has (A) at least one root in (0,1) (B) at least one root in (0,2) (C) at least on root in (-1,1) (D) none of these

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138. If all the roots of $z^3+az^2+bz+c=0$ are of unit modulus, then (A) $|a| \le 3$ (B) $|b| \le 3$ (C) $|c|=1$ (D) none of these

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139. If the product of the roots of the equatiin $2x^2+ax+4\sin a=0$ is 1, then the roots will be imaginary if

(A) $a\varepsilon R$

(B) $a\varepsilon\left\{\frac{-7\pi}{6}, \frac{\pi}{6}\right\}$

(C) $a\varepsilon\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

(D) none of these

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140. If p and q are odd integers, then the equation $x^2 + 2px + 2q = 0$ (A) has no integral root (B) has no rational root (C) has no irrational root (D) has no imaginary root

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141. Suppose that $f(x)$ isa quadratic expresson positive for all real x. If $g(x) = f(x) + f'(x) + f''(x)$, then for any real `x

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142. Let $f(x)$ be a quadratic expression which is positive for all real x and $g(x) = f(x) + f'(x) + f''(x)$.A quadratic expression $f(x)$ has same sign as that coefficient of x^2 for all real$x$ if and only if the roots of the corresponding equation $f(x) = 0$ are imaginary.Which of the following

holds true? (A) $g(0)g(1) < 0$ (B) $g(0)g(-1) < 0$ (C) $g(0)f(1)f(2) > 0$

(D) $f(0)f(1)f(2) < 0$

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143. let $f(x)$ be a polynomial function of degree 2 and $f(x) > 0$ for all $x \in R$. if $g(x) = f(x) + f'(x) + f''(x)$, then for any x show that $g(x) > 0$

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144. Let $\alpha + \iota\beta$,$\alpha, \beta \varepsilon R$ be a root of $x^3 + qx + r = 0$If γ be a real root of equation $x^3 + qx + r = 0$ then γ

(A) -2α

(B) α

(C) 2α

(D) $-\alpha$

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145. Let $\alpha + i\beta; \alpha, \beta \in R,$ be a root of the equation $x^3 + qx + r = 0; q, r \in R$. A real cubic equation, independent of $\alpha \& \beta$, whose one root is 2α is (a)$x^3 + qx - r = 0$ (b) $x^3 - qx + 4 = 0$ (c) $x^3 + 2qx + r = 0$ (d) None of these

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146. The number of solution of equation $|x - 1| = e^x$ is (A) 0 (B) 1 (C) 2 (D) none of these

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147. If α is root of equation $f(x) = 0$ then the value of

$$\left(\alpha + \frac{1}{\alpha}\right)^2 + \left(\alpha^2 + \frac{1}{\alpha^2}\right)^2 + \left(\alpha^3 + \frac{1}{\alpha^3}\right) + \dots\dots\dots + \left(\alpha^6 + \frac{1}{\alpha^6}\right)^2$$

is (A) 18 (B) 54 (C) 6 (D) 12

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148. Find the Domain and Range of $f(x) = \dfrac{x-3}{4-x}$

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149. The set of all value of a for which one root of equation $x^2 - ax + 1 = 0$ is less than unity and other greater than unity

(A) $(-\infty, 2)$

(B) $(2, \infty)$

(C) $(1, \infty)$

(D) none of these

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150. The set of all values of a for which both roots of equation $x^2 - 2ax + a^2 - 1 = 0$ lies between -2 and 4 is (A) $(-1, 2)$ (B) $(1, 3)$ (C) $(-1, 3)$ (D) none of these

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151. If $a, b, c(abc^2)x^2 + 3a^2cx + b^2cx - 6a^2 - ab + 2b^2 = 0$ ares rational.

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152. If n and r are positive of the equation $x^2 - bx + c = 0$ then show that the roots of the quadratic equatin $\hat{} cC_r x^2 + 2.^n C_{r+1} x +^n C_{r+2} = 0$ are real.

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153. If $ax^3 + bx^2 + cx + d$ has local extremum at two points of opposite signs then roots of equation $ax^2 + bx + c = 0$ are necessarily (A) rational (B) real and unequal (C) real and equal (D) imaginary

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154. If α and β are the roots of the equation $ax^2 + bx + c = 0$ then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$.Also if a quadratic equation $f(x) = 0$ has both roots between m and n then $f(m)$ and $f(n)$ must have same sign. It is given that all the quadratic equations are of form $ax^2 - bx + c = 0$ $a, b, c \varepsilon N$ have two distict real roots between 0 and 1 .The least value of a for which such a quadratic equation exists is (A) 3 (B) 4 (C) 5 (D) 6

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155. If α and β are the roots of the equation $ax^2 + bx + c = 0$ then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$.Also if a quadratic equation $f(x) = 0$ has both roots between m and n then $f(m)$ and $f(n)$ must have same sign. It is given that all the quadratic equations are of form $ax^2 - bx + c = 0$ $a, b, c \varepsilon N$ have two distict real roots between 0 and 1 .The least value of b for which such a quadratic equation exists is (A) 3 (B) 4 (C) 5 (D) 6

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156. If α and β are the roots of the equation $ax^2+bx+c=0$ then $ax^2+bx+c=a(x-\alpha)(x-\beta)$.Also if a quadratic equation $f(x)=0$ has both roots between m and n then $f(m)$ and $f(n)$ must have same sign. It is given that all the quadratic equations are of form $ax^2-bx+c=0$ $a, b, c\varepsilon N$ have two distict real roots between 0 and 1. The least value of c for which such a quadratic equation exists is (A) 1 (B) 2 (C) 3 (D) 4

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157. The number of real root (s) of the equation $x^2 \tan x=1$ lying between 0 and 2π is /are.

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158. Find the number of quadratic equations, which are unchanged by squaring their roots.

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159. If x and y satisfy the equation $y = 2[x] + 3$ and $y = 3[x - 2]$ simultaneously, where [.] denotes the greatest integer function, then $[x + y]$ is equal to

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160. Given that α, γ are roots of the equation $Ax^2 - 4x + 1 = 0$, and $\beta, \delta 1$ the equation of $Bx^2 - 6x + 1 = 0$, such that α, β, γ and δ are in H.P., then

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161. Let α be the root of the equation $ax^2 + bx + c = 0$ and β be the root of the equation $ax^2 - bx - c = 0$ where$\alpha < \beta$ Assertion (A): Equation $ax^2 + 2bx + 2c = 0$ has exactly one root between α and β.,

Reason(R): A continuous function $f(x)$ vanishes odd number of times between a and b if $f(a)$ and $f(b)$ have opposite signs.

A. Both A and R are true and R is the correct explanation of A

B. Both A and R are true and R is not the correct explanation of A

C. A is true but R is false.

D. A is false but R is true.

Answer: null

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162. Let $f(x) = ax^3 + bx^2 + cx + d = 0$ have extremum of two different points of opposite signsAssertion (A): Equation $ax^2 + bx + c = 0$ has distinct real roots. , Reason (R): A differentiable function $f(x)$ has extremum only at points where $f'(x) = 0$.

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163. Assertion (A): Equation $(x-p)(x-q)-r=0$ where $p, q, r \varepsilon R$ and $0 < p < q < r$ has roots in (p, q), Reason(R): A polynomial equation $f(x)=0$ has odd number of roots between a and $b(a < b)$ if $f(a)$ and $f(b)$ have opposite signs

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164. Assertion (A): Equation $(x-a)(x-b)-2=0, a < b$ has one root less than a and other root greater than b. , Reason (R): A polynomial equation $f'(x)=0$ has even number of roots between a and b if $f(a)$ and $f(b)$ have opposite signs. .

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165. Assertion (A): For $0 < a < b < c$ equation $(x-a)(x-b)-c=0$ has no roots in (a, b)

Reason (R):For a continuous function $f(x)$equation $f'(x)=0$ has at least one root between a and b if $f(a)$ and $f(b)$ are equal.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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166. Assertion (A): For $\alpha < \beta$ equation $(x - \cos\alpha)(x - \cos\beta) - 2 = 0$ has one root less than $\cos\beta$ and other greater than $\cos\alpha$., Reason (R): Quadratic expression$ax^2 + bx + c$ has sign opposite to that of a between the roots α and β of equation $ax^2 + bx + c = 0$ if $\alpha < \beta$.

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167. LET the equation $ax^2 + bx + c = 0$ has no real roots Assertion (A): $c(a + b + c) > 0$, Reason (R): A quadratic expression $ax^2 + bx + c$ has signs same as that of al for all real x if the roots of the corresponding equation $ax^2 + bx + c = 0$ are imaginary. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te

correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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168. Assertion (A): Quadratic equation $f(x) = 0$ has real and distinct roots. Reason (R): quadratic equation f(x)=0 has even number of roots between p and $q(p < q)$ if $f(p) \text{ and } f(q)$ have same sign. (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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169. Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots $\alpha and \beta, where \alpha \langle -1 and \beta \rangle 1$, then show that $1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$

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170. The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + bx + \gamma = 0$ are in A.P. Find the intervals in which β and γ lie.

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171. The sum of all real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is

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172. The equation $\sqrt{(x+1)} - \sqrt{(x-1)} = \sqrt{(4x-1)}$ has

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173. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c and d denote the lengths of sides of the quadrilateral, prove that $2 \leq a_2 + b_2 + c_2 + d_2 \leq 4$

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174. Let $f(x)=Ax^2+Bx+c, whereA, B, C$ are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers $2A, A+B, andC$ are all integer. Conversely, prove that if the number $2A, A+B, andC$ are all integers, then $f(x)$ is an integer whenever x is integer.

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175. In a triangle $PQR, \angle R=\frac{\pi}{2}$.If $\tan\left(\frac{P}{2}\right)\&\tan\left(\frac{Q}{2}\right)$, are the roots of the equation $ax^2+bx+c=(a\neq0)$ then

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176. If the roots of the equation $x^2-2ax+a^2-a-3=0$ are real and less than 3, then (a)$a<2$ b. $2< \ -a\leq3$ c. `34

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177. If α and β $(\alpha' < '\beta')arethe\sqrt[s]{o}ftheequation$x^2+b x+c=0,$where$c<0

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178. If $b > a,$ then the equation $(x-a)(x-b)-1=0$ has both roots in (a, b) both roots in $(-\infty, a)$ both roots in $(b, +\infty)$ one root in $(-\infty, a)$ and the other in $(b, +\infty)$

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179. For the equation $3x^2+px+3=0, p>0,$ if one of the root is square of the other, then p is equal to $1/3$ b. 1 c. 3 d. $2/3$

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180. If α, β are the roots of $ax^2 + bx + c = 0, (a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0, (A \neq 0)$ for some constant δ then prove that $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$

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181. Let $\alpha and \beta$ be the roots of $x^2 - x + p = 0 and \gamma and \delta$ be the root of $x^2 - 4x + q = 0$. If $\alpha, \beta, and \gamma, \delta$ are in G.P., then the integral values of $pandq$, respectively, are $-2, \ -32$ b. $-2, 3$ c. $-6, 3$ d. $-6, \ -32$

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182. Let $-1 \leq p \leq 1$. Show that the equation $4x^3 - 3x - p = 0$ has a unique root in the interval [1/2,1] and identify it.

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183. Let a, b, c be real numbers with $a \neq 0$ and α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β.

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184. The number of solution of $\log_4(x-1) = \log_2(x-3)$ is :

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185. Let $f(x) = (1+b^2)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is (a) $[0, \}$ b. $\left(0, \frac{1}{2}\right)$ c. $\frac{1}{2}, 1$ d. $(0, 1]$

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186. The set of all real numbers x for which $x^2 - |x+2| + x > 0$ is $(-\infty, -2)$ b. $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ c. $(-\infty, -1) \cup (1, \infty)$ d. $(\sqrt{2}, \infty)$

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187. If $x^2 + (a-b)x + (1-a-b) = 0. where a, b \in R,$ then find the values of a for which equation has unequal real roots for all values of b.

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188. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min $f(x) > max g(x)$, then the relation between b and c is

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189. For all 'x' , $x^2 + 2ax + (10 - 3a) > 0$, then the interval in which 'a' lies is :

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190. If one root is square of the other root of the equation $x^2 + px + q = 0,$ then the relation between $p and q$ is

$p^3 - q(3p - 1) + q^2 = 0$ $p^3 - q(3p + 1) + q^2 = 0$

$p^3 + q(3p - 1) + q^2 = 0$ $p^3 + q(3p + 1) + q^2 = 0$

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191. If α, β are the roots of $ax^2 + bx + c = 0, (a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0, (A \neq 0)$ for some constant δ then prove that $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$

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192. Let a,b,c be the sides of a triangle. Now two of them are equal to $\lambda \varepsilon R$. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ are real then

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193. Let a and b are the roots of the equation $x^2 - 10xc - 11d = 0$ and those of $x^2 - 10ax - 11b = 0, \cdot$ are c and d then find the value of `a+b+c+d

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194. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$, then the value of r is (1) $\frac{2}{9}(p-q)(2q-p)$ (2) $\frac{2}{9}(q-p)(2p-q)$ (3) $\frac{2}{9}(q-2p)(2q-p)$ (4) $\frac{2}{9}(2p-q)(2q-p)$

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195. The smallest value of k for which both roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real distinct and have value at least 4, is

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