

## MATHS

## **BOOKS - KC SINHA ENGLISH**

## SINE AND COSINE FORMULA - FOR BOARDS

## **Solved Examples**

**1.** If in any 
$$\Delta ABC$$
,  $A=30^{\circ}$  and  $B=60^{\circ}$ , then find  $a\!:\!b\!:\!c$ 



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**2.** If in any 
$$\triangle ABC$$
,  $B=60^{\circ}$  and  $b$ :  $c=\sqrt{3}$ :  $\sqrt{2}$ , then find  $A$ .



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**3.** If in  $\Delta ABC$ , a=3, b=5, c=7 find the greatest angle.



**4.** If in a  $\ \triangle\ ABC, a=rac{1}{\sqrt{6}-\sqrt{2}}, b=rac{1}{\sqrt{6}+\sqrt{2}}, C=60^\circ$  , then prove that  $c=rac{\sqrt{3}}{2}.$ 



**5.** In any  $\Delta ABC$ , prove that  $a^2+b^2+c^2=2(bc\cos A+ca\cos B+ab\cos C).$ 

**6.** In any ABC, prove that:  $a(b\cos C - \mathrm{o} sB) = b^2 - c^2$ 



7. In any  $\Delta ABC$  , prove that

$$\left(a-b
ight)^2\cos^2\!\left(rac{C}{2}
ight) + \left(a+b
ight)^2\sin^2\!\left(rac{C}{2}
ight) = c^2.$$



**8.** If in any  $\Delta ABC$ ,  $a\!:\!b\!:\!c=4\!:\!5\!:\!6$ , prove that the greatest angle is doule the smalles angle`



**9.** The sides of a tringle are 8 cm, 10 cm and 12 cm. Prove that the greatest angle is double of the smalest angle.



**10.** In any parallelogram if a and b are the length of the two non-parallel sides,  $\theta$  is the angle measure between these two sides and d is the length

that the measure of d can be given be :  $d^2 = a^2 + b^2 + 2ab\cos x$ 

of the diagonal that has a common vertex with sides a and b, then show

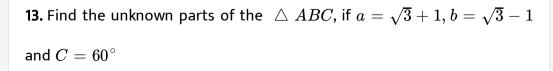


**11.** With usual notations, if in a triangle  $ABC\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$  , then prove that:  $\frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25}$ 



**12.** If in a triangle 
$$ABC, \angle C = 60^0$$
, then prove that  $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ .







In

14.

$$(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a+b+c$$

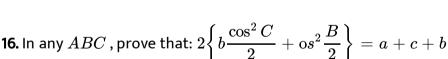
that:

any ABC , prove

**15.** In any 
$$\Delta ABC$$
, prove that :

$$rac{\cos A}{b\cos C + c\cos B} + rac{\cos B}{c\cos A + a\cos C} + rac{\cos C}{a\cos B + b\cos A} = rac{a^2 + b^2 + c}{2abc}$$

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**17.** In any ABC , prove that:  $2\Big\{arac{\sin^2C}{2}+crac{\sin^2A}{2}\Big\}=a+c-b$ 



**18.** In any triangle 
$$ABC$$
 prove that:  $\sin\!\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{a}\right)\frac{\cos A}{2}$ 



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**19.** In any  $\triangle ABC$ , prove that

$$\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2-b^2}{c^2}$$

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- **20.** In any  $\triangle ABC$ , prove that
- $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$

any triangle ABC

ln

21.

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 $a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) = 0$ 

following:

that

, prove



**22.** In any 
$$\Delta ABC$$
, prove that  $a\sin\Bigl(rac{A}{2}+B\Bigr)=(b+c)\sinrac{A}{2}$ 



**23.** In any  $\Delta ABC$ , prove that  $\frac{\sin B}{\sin C} = \frac{c - a\cos B}{b - a\cos C}$ 

**24.** In any  $\triangle ABC$ , prove that :  $\left(\frac{a-b}{c}\right)=rac{ an\left(rac{A}{2}\right)- an\left(rac{B}{2}\right)}{ an\left(rac{A}{2}\right)+ an\left(rac{B}{2}\right)}$ 



- **26.** If  $a^2, b^2, c^2$  are in A.P., prove that  $\cot A, \cot B, \cot C$  are in  $A\dot{P}$ .
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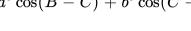
**27.** Problem on sine rule Type:-1(i) In a  $\Delta ABC$ ; If a=2; b=3 and  $\sin A=\frac{2}{3}$ ; find  $\angle B$  (ii) In a  $\Delta ABC$ ; the angle of a triangle are in AP; It is being given that  $b: c=\sqrt{3}:\sqrt{2}$ 



- **28.** In any triangle ABC, prove that:  $\frac{\sin(B-C)}{\sin(B+C)} = \frac{b^2-c^2}{a^2}$ 
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**29.** In any  $\Delta ABC$ , prove that

$$a^{3}\cos(B-C) + b^{3}\cos(C-A) + c^{3}\cos(A-B) = 3abc$$



**30.** For any triangle ABC, prove that 
$$rac{b^2-c^2}{a^2}\sin 2A+rac{c^2-a^2}{b^2}\sin 2B+rac{a^2-b^2}{c^2}\sin 2C=0$$

**31.** In any triangle ABC, prove

In any  $\Delta ABC$ , prove that

 $a^{3}\sin(B-C) + b^{3}\sin(C-A) + c^{3}\sin(A-B) = 0$ 

that:





32.

$$a\cos A + b\cos B + c\cos C = 2a\sin B\sin C$$

**33.** For any triangle ABC, prove that 
$$(b^2-c^2)\cot A+(c^2-a^2)\cot B+(a^2-b^2)\cot C=0$$



**34.** In a triangle ABC, if  $a\cos A = b\cos B$ , show that the triangle is either isosceles or right angled.



**35.** In any triangle, if  $\frac{a^2-b^2}{a^2+b^2}=\frac{\sin(A-B)}{\sin(A+B)}$ , then prove that the triangle is either right angled or isosceles



**36.** In  $\Delta ABC,\,a=3,\,b=5,\,c=6$ , find the value of :  $\sin\!\left(rac{A}{2}
ight)$ 



**37.** In  $\Delta ABC,\,a=3,\,b=5,\,c=6$ , find the value of :  $\cos\!\left(rac{A}{2}
ight)$ 

**38.** In  $\Delta ABC, \, a=3, \, b=5, \, c=6$ , find the value of :  $an\Bigl(rac{A}{2}\Bigr)$ 

**39.** In  $\Delta ABC, a=3, b=5, c=6$ , find the value of :  $\cot\left(rac{A}{2}
ight)$ 



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**40.** In any 
$$\Delta ABC$$
, prove  $(a+b-c)\cot\left(rac{B}{2}
ight)=(a-b+c)\cot\left(rac{C}{2}
ight)$ 

that



**41.** If in  $\Delta ABC$ , (a+b+c)(b+c-a)=abc, find the condition of a



- **42.** In any  $\Delta ABC$ , prove that:  $rac{a+b-c}{a+b+c}= an\Bigl(rac{A}{2}\Bigr) an\Bigl(rac{B}{2}\Bigr)$ 
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- **43.** In any ABC , prove that:  $2\Big\{arac{\sin^2C}{2}+crac{\sin^2A}{2}\Big\}=a+c-b$ 
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$$\cot\left(\frac{A}{2}\right) + \epsilon$$

44.

45.

$$\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) = \frac{a+b+c}{b+c-a}\cot\left(\frac{A}{2}\right)$$

that

any triangle

 $2a\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) = (b+c-a)\sin\left(\frac{A}{2}\right)$ 

any

 $\Delta ABC$ .

in

prove

any

that

triangle

that

that

In

$$ABC, (a+b+c)igg( anigg(rac{A}{2}igg)+ anigg(rac{B}{2}igg)igg)=2c\cotigg(rac{C}{2}igg)$$

Show

In

47.

46.

ABC, show

**47.** In any triangle ABC, 
$$2a\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)=(a+b+c)\sin\left(\frac{A}{2}\right)$$

**48.** In 
$$\Delta ABC$$
,  $\frac{1}{a}$ ,  $\frac{1}{b}$ ,  $\frac{1}{c}$  are in A.P. prove that  $\csc^2\frac{A}{2}$ ,  $\csc^2\frac{B}{2}$ ,  $\csc^2\frac{C}{2}$  are also in A.P.

**49.** In any  $\Delta ABC, \angle B=90^\circ$  , prove that  $anrac{A}{2}=\sqrt{rac{b-c}{b+c}}$ 

**50.** In  $\triangle ABC$ , if  $\angle C=90^{0}$ , then prove

that



 $\tan\left(\frac{A}{2}\right) = \sqrt{\frac{c-b}{c+b}} = \frac{a}{b+c}$ 

**52.** In a 
$$\Delta ABC$$
, if  $a=18,\,b=24,\,c=30$ , find the area of  $\Delta ABC$ 

**54.** In any  $\Delta ABC$ , prove that  $b^2\sin 2C + c^2\sin 2B = 2bc\sin A = 4\Delta$ .



**53.** In any  $\Delta ABC$ , prove that  $\Delta = \frac{b^2 + c^2 - a^2}{4\cot A}$ .

**55.** Prove that in  $\triangle ABC$ ,

# $\frac{\left(a+b+c ight)^2}{a^2+b^2+c^2} \Rightarrow \frac{\cot rac{A}{2}+\cot rac{B}{2}+\cot rac{C}{2}}{\cos A+\cot B+\cot C}$

**56.** In any  $\triangle ABC$ , If  $\cot \frac{A}{2}$ ,  $\cot \frac{B}{2}$ ,  $\cot \frac{C}{2}$  are in AP, then a,b,c are in



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 $\text{If} \qquad a\cos^2\!\left(\frac{C}{2}\right) + c\cos^2\!\left(\frac{A}{2}\right) = \frac{3b}{2}.$ To prove:  $\cot\left(\frac{A}{2}\right)$ ,  $\cot\left(\frac{B}{2}\right)$ ,  $\cot\left(\frac{C}{2}\right)$  are in A.P.



**58.** If  $P_1, P_2$  and  $P_3$  are the altitudes of a triangle from vertices A, B and C respectively and  $\Delta$  is the area of the triangle, then the value of  $\frac{1}{P_1} + \frac{1}{P_2} - \frac{1}{P_2} =$ 



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**1.** The ratio of angles in a triangle ABC is  $2\!:\!3\!:\!7$  then prove that

$$a : b : c = \sqrt{2} : 2 : (\sqrt{3} + 1)$$



**2.** If in  $\Delta ABC$ ,  $a=4,\,b=12,\,\angle B=30^{0}$  , then find  $\sin A$ .



**3.** In any  $\Delta ABC$ , prove that  $\dfrac{\sin B}{\sin(B+C)}=\dfrac{b}{a}$ 



**4.** In  $\triangle ABC$ ,

If the two angles of a triangle are  $30^\circ~{
m and}~45^\circ$  and the included side is  $(\sqrt{3}+1)$  cm, find the area of the triangle.



**5.** The angle of a triangle are in the ratio 1 : 2 : 7, prove that the ratio of the greatest side to the least side is  $\left(\sqrt{5}+1\right)$  :  $\left(\sqrt{5}-1\right)$ .



**6.** In  $\triangle ABC$ , if  $c=3.4cm,\,A=25^0,\,B=85^0$ , find a,b and angle C.



**7.** Find the greatest angle of  $\Delta ABC$  if : a=2,  $b=\sqrt{6},$   $c=\sqrt{3}+1$ 



**8.** Find the greatest angle of  $\Delta ABC$  if :  $a=m, b=n, c=\sqrt{m^2+mn+n^2}$ 

**9.** In  $\Delta ABC$ , if  $a=25,\,b=52\,$  and  $\,c=63$ , find  $\cos A$ 



**10.** In a  $\Delta ABC$ , If  $a=18,\;b=24,\;c=30$  , find the value of  $\cos A,\;\cos B\;and\;\cos C$ 



**11.** In a ABC , if  $\angle B=60^0$  , prove that (a+b+c)(a-b+c)=3ca



**12.** If the sides of a triangle are 3, 5, and 6, prove that the triangle is obtuse - angled triangle and find the obtuse angle.



**13.** In any  $\Delta ABC$ , prove that  $: rac{b^2+c^2-a^2}{c^2+a^2-b^2} = rac{ an B}{ an A}$ 



**14.** In a triangle ABC,  $a^4+b^4+c^4=2c^2ig(a^2+b^2ig)$  prove that  $C=45^\circ$  or  $135^\circ$ 



**15.** In a ABC, prove the following :  $\frac{c-b\cos A}{b-c\cos A}=\frac{\cos B}{\cos C}$ 



**16.** In  $\Delta ABC,\,a=4,\,b=6,\,c=8$ , then find the value of

 $8\cos A + 16\cos B + 4\cos C.$ 



**17.** For any triangle ABC, prove that  $(b+c)rac{\cos(B+C)}{2}=arac{\cos(B-C)}{2}$ 



**18.** For any triangle ABC, prove that  $\dfrac{a+b}{c}=\dfrac{\cos\left(\dfrac{A-B}{2}\right)}{\dfrac{\sin C}{2}}$ 



**19.** For any triangle ABC, prove that  $\frac{a-b}{c}=rac{\sin\left(rac{A-B}{2}
ight)}{\cos\left(rac{C}{2}
ight)}$ 



**20.** In any  $\Delta ABC$ , prove that : If  $b+c=2a\cos\left(\frac{B-C}{2}\right)$ , then prove that  $A=60^{0}$ .



$$h^2$$
  $a^2$ ) and

**21.** In any  $\Delta ABC$ , prove that :

$$ig(b^2-c^2ig)\cos 2A + ig(c^2-a^2ig)\cos 2B + ig(a^2-b^2ig)\cos 2C = 0$$

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## **22.** In any $\triangle ABC$ , prove that

$$\frac{1 + \cos(A - B)\cos C}{1 + \cos(A - C)\cos B} = \frac{a^2 + b^2}{a^2 + c^2}$$



 $rac{b^2 - c^2}{\cos B + \cos C} + rac{c^2 - a^2}{\cos C + \cos A} + rac{a^2 - b^2}{\cos A + \cos B} = 0$ 



- **24.** In any triangle ABC, prove that:

**23.** In any  $\Delta ABC$ , prove that

 $rac{a^2\sin(B-C)}{\sin B+s\in C}+rac{b^2\sin(C-A)}{\sin C+s\in A}+rac{c^2\sin(A-B)}{\sin A+s\in B}=0$ 

**25.** In any 
$$\Delta ABC$$
, prove that :  $\dfrac{b^2-c^2}{a^2}=\dfrac{\sin(B-C)}{\sin(B+C)}$ 

**26.** In any  $\Delta ABC$ , prove that :  $an\Bigl(rac{A}{2}+B\Bigr)=rac{c+b}{c-b} an\Bigl(rac{A}{2}\Bigr)$ 

 $a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B) = 0$ .

**28.** If A=2B, then prove that either c=b or  $a^2=b(c+b)$ 





27.

**29.** In any  $\Delta ABC$ , prove that :

 $\frac{\cos A}{a} + \frac{a}{bc} = \frac{\cos B}{b} + \frac{b}{ca} = \frac{\cos c}{c} + \frac{c}{ab}$ 



**30.** In a  $\Delta ABC$  the angles A, B, C are in A.P. show that

$$2\cosrac{A-C}{2}=rac{a+c}{\sqrt{(a^2-ac+c^2)}}$$



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**31.** In any  $\triangle ABC$ , prove that

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$$



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- **32.** In any  $\Delta ABC$ , prove that :
- $(b^2-c^2)\sin^2 A + (c^2-a^2)\sin^2 B + (a^2-b^2)\sin^2 C = 0$ 

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- **33.** In any triangle ABC , prove that following:
  - $\frac{a^2 \sin(B-C)}{\sin A} + \frac{b^2 \sin(C-A)}{\sin B} + \frac{c^2 \sin(A-B)}{\sin C} = 0$
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- is equal to

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- **35.** In a  $\triangle ABC$ , If  $\tan \frac{A}{2}$ ,  $\tan \frac{B}{2}$ ,  $\tan \frac{C}{2}$  are in A.P. then  $\cos A, \cos B, \cos C$  are in

**34.** In a  $\Delta ABC$ , if median AD is perpendicular to AB, the  $\tan A + 2 \tan B$ 

**36.** If 
$$\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$$
, prove that  $a^2, b^2, c^2$  are in A.P.

**37.** If in  $\Delta ABC$ , a=15, b=36, c=39, find  $an\left(rac{A}{2}
ight)$ 



$$(2)$$
,  $(2)$ ,  $(2)$ 

**39.** In

$$\tan\left(\frac{A}{2}\right), \tan\left(\frac{B}{2}\right), \tan\left(\frac{C}{2}\right)$$

**38.** In a  $\triangle ABC$ , if a = 18, b = 24, c = 30, find

any  $\Delta ABC$ , prove

 $\frac{b-c}{a}\cos^2\left(\frac{A}{2}\right) + \frac{c-a}{b}\cos^2\left(\frac{B}{2}\right) + \frac{a-b}{c}\cos^2\left(\frac{C}{2}\right) = 0$ 

**40.** Prove the questions :

$$1 - an. \ rac{A}{2} an. \ rac{B}{2} = rac{2c}{(a+b+c)}$$



**41.** In any 
$$ABC$$
, prove that:  $2(bosA+ca\cos B+ab\cos C)=a^2+b^2+c^2$ 

42. In any 
$$\Delta ABC$$
, prove that 
$$\frac{\cos^2\left(\frac{A}{2}\right)}{a} + \frac{\cos^2\left(\frac{B}{2}\right)}{b} + \frac{\cos^2\left(\frac{C}{2}\right)}{c} = \frac{s^2}{abc}$$

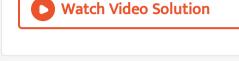
**46.** Prove that 
$$(b+c-a)igg(\cot.rac{B}{2}+\cot.rac{C}{2}igg)=2a\cot.rac{A}{2}$$

 $3\tan, \frac{A}{2}\tan, \frac{C}{2}=1$ 

If in  $\mathsf{a}\Delta ABC, \sin A, \sin B, \sin C$  are in A.P., show

that

**43.** In ABC,  $\left(\cot\left(\frac{A}{2}\right)+\cot\left(\frac{B}{2}\right)\right)\left(a\sin^2\left(\frac{B}{2}\right)+b\sin^2\left(\frac{A}{2}\right)\right)=$ 



(a)  $\cot C$  (b)  $\cot C$  (c)  $\cot \left(\frac{C}{2}\right)$  (d)  $\cot \left(\frac{C}{2}\right)$ 

**44.** In 
$$\Delta ABC$$
, prove that: 
$$\frac{\tan A}{2}.\ \frac{\tan B}{2}.\ \frac{\tan C}{2}=\sqrt{\left(1-\frac{a}{s}\right)\left(1-\frac{b}{s}\right)\left(1-\frac{c}{s}\right)}$$



**47.** Prove that 
$$(b+c-a)igg(\cot.rac{B}{2}+\cot.rac{C}{2}igg)=2a\cot.rac{A}{2}$$



**48.** In a  $\Delta ABC$ , in the sum of two sides is  $\sqrt{3}$  times their difference and the included angle is  $60^{\circ}$ , find the difference of the remaining angles.



**49.** If in  $\Delta ABC$ , the difference of two angles is  $60^0$  and the remaining angle is  $30^0$ , then find the ratio of the sides opposite to first two angles.



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**51.** The side of a triangle are in A.P. Its area is  $\frac{3}{5}$  the of an equilateral triangle of the same perimeter. Show that the sides are in the proportion 3:5:7.



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**52.** The sides of a quadrilateral are 3, 4, 5 and 6 cms. The sum of a pair of opposite angles is  $120^0$ . Show that the area of the quadrilateral is  $3\sqrt{30}$  sq.cm.



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