MATHS

BOOKS - KC SINHA ENGLISH

TRIGONOMETRIC EQUATIONS AND INEQUATIONS - FOR COMPETITION

Solved Examples

1. Find the solution set of inequality $\sin x > \frac{1}{2}$

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2. Find the solution set of inequation $\cos x \geq \frac{1}{2}$

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3. With the help of graph, find the solution set of inequation $\tan x > -\sqrt{3}$

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4. Find the set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$.

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5. If $A = \{\theta : 2\cos^2\theta + \sin\theta \leq 2\}$, and $B = \left\{\theta : \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}\right\}$ then the region for $(A \cap B)$ is _____

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6. In an acute angled $\triangle ABC$, the minimum value of $\tan A \tan B \tan C$ is

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7. If in a ΔABC, $\cos A + \cos B + \cos C = \frac{3}{2}$. Prove that ΔABC is an equilateral triangle.

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8. If $0 < \alpha < \beta < \frac{\pi}{2}$ show that $\alpha - \sin\alpha < \beta - \sin\beta$

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9. If $y + \log(1 + x) = 0$, then which of the following is true?

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10. Find the intervals of monotonicity of the function $y = 2x^2 - \log|x|, x \neq 0$

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11. Show that $1+x\log_e x+\sqrt{x^2+1}\geq\sqrt{1+x^2}$ for all $x>0$

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12. If $0<x<\frac{\pi}{2}$ show that $\cos x>1-\frac{x^2}{2}$

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13. Let $f(x)=x^2 and g(x)=\sin x f$ or $all x\in R$. Then the set of all x satisfying $(fogogof)(x)=(gogof)(x), where (fog)(x)=f(g(x)),$ is

(a) $\pm\sqrt{n\pi}, n\in\{0,1,2,.\}$ (b) $\pm\sqrt{n\pi}, n\in\{1,2,.\}$ (c) $\frac{\pi}{2}+2n\pi, n\in\{,-2,-1,0,1,2\}$ (d) $2n\pi, n\in\{,-2,-1,0,1,2,\}$

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14. The number of solution of the pair of equations $2\sin^2\theta - \cos 2\theta = 0 and 2\cos^2\theta = 0$ in the interval $[0, 2\pi]$ is 0 (b) 1 (c) 2 (d) 4

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15. For `0

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16. Let $S \subset (0, \pi)$ denote the set of values of x satisfying the equation $8^{1+|\cos x|+\cos^2 x+|\cos^3 x|+\ldots. \text{ to } \infty} = 4^3$ Then, S =

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17. If $4\sin^4 x + \cos^4 x = 1$, then x is equal to $(n \in Z)$

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18. If [x] dnote the greatest integer less than or equal to x then the equation $\sin x = [1 + \sin x] + [1 - \cos x][$ has no solution in

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19. If $\frac{1 - \tan x}{1 + \tan x} = \tan y$ and $x - y = \frac{\pi}{6}$, then x, y are respectively (A) $\frac{5\pi}{24}, \frac{\pi}{24}$ (B) $-\frac{7\pi}{24}, -\frac{11\pi}{24}$ (C) $-\frac{115\pi}{24}, \frac{-119\pi}{24}$ (D) none of these

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20. Solve the inequation $2^{\frac{1}{\cos^2 x}} \sqrt{y^2 - y + \frac{1}{2}} \leq 1$

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21. If $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$ then the most general values of θ equal to (A) $2n\pi + \frac{\pi}{6}$ (B) $2n\pi = \frac{5\pi}{6}$ (C) $2n\pi + \frac{7\pi}{6}$ (D) none of these

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22. If $\tan(A-B)=1, \sec(A+B)=\frac{2}{\sqrt{3}}$, then the smallest positive value of B is. $(a)\frac{25\pi}{24}$ (b) $\frac{19\pi}{24}$ (c) $\frac{13\pi}{24}$ (d) $\frac{11\pi}{24}$

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23. If $\sin x+\cos y=1$ and $\cos 2x-\cos 2y=1$ then the values of x and y are (A) $x=n\pi+(-1)^n\frac{\pi}{3}, n\in I, y=m\pi\pm\frac{\pi}{6}, m\in I$ (B) $x=n\pi+(-1)^n\frac{\pi}{6}, n\in I, y=m\pi\pm\frac{\pi}{3}, m\in I$ (C) $x=n\pi+(-1)^n\frac{\pi}{4}, n\in I, y=m\pi\pm\frac{\pi}{4}, m\in I$ (D) none of these

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24. The positive integer value of $n>3$ satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)}=\frac{1}{\sin\left(\frac{2\pi}{n}\right)}+\frac{1}{\sin\left(\frac{3\pi}{n}\right)}is$

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25. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is _____.

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26. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$(y + z)\cos 3\theta = (xyz)\sin 3\theta$

$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$

$(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$

has a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$ is _____.

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Exercise

1. If $\alpha > 0, \beta > 0, \gamma > 0$ and $\alpha + \beta + \gamma < \frac{\pi}{2}$ show that $\tan\alpha\tan\beta + \tan\beta\tan\gamma + \tan\gamma\tan\alpha < 1$

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2. If A, B, C are the angles of a right angled triangle, then $\left(\cos^2 A + \cos^2 B + \cos^2 C\right)$ equals

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3. about to only mathematics

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4. Show that : $\sin^6\alpha + \cos^6\alpha \geq \frac{1}{4}$

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5. In a triangle $ABC \cos A + \cos B + \cos C \leq k$ then $k =$

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6. In a triangle ABC $\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) = \frac{1}{8}$ prove that the triangle is equilateral

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7. Statement I $\cos 36^\circ > \sin 36^\circ$

Statement II $\cos 36^\circ > \tan 36^\circ$

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8. Show that : $\sin^4 \alpha + \cos^4 \alpha \geq \frac{1}{2}$

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9. If $0 < \alpha < \frac{\pi}{2}$, show that $\alpha - \frac{\alpha^3}{3} < \sin\alpha$

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10. For $n \in Z$, the general solution of $(\sqrt{3}-1)\sin\theta + (\sqrt{3}+1)\cos\theta = 2$ is $(n \in Z)$

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11. The general values of θ satisfying the equation $2\sin^2\theta\pi - 3\sin\theta\pi - 2 = 0$ is $(n \in Z)\cdot \quad n\pi + (-1)^n\frac{\pi}{6}$ (b) $n\pi + (-1)^n\frac{\pi}{2}$ $n\pi + (-1)^n\frac{5\pi}{6}$ (d) $n\pi + (-1)^n\frac{7\pi}{6}$

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12. Find all the solution of $4cos^2 x\sin x - 2\sin^2 x = 3\sin x$

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13. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$.

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14. The general value of x satisfying the equation satisfying the equation $\sqrt{3}s \in x + \cos x - \sqrt{3}$ is given by $x = n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{3}$, nZ b. $x = n\pi + (-1)^n\frac{\pi}{3} + \frac{\pi}{6}$, nZ c. $x = n\pi \pm \frac{\pi}{6}$, nZ d. $x = n\pi \pm \frac{\pi}{3}$, nZ

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15. The number of solution(s) of $\sin 2x + \cos 4x = 2$ in the interval $(0,\ 2\pi)$ is 0 (b) 2 (c) 3 (d) 4

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16. Let n be a positive integer such that $\sin\left(\frac{\pi}{2n}\right) + \cos\left(\frac{\pi}{2n}\right) = \frac{\sqrt{n}}{2}$ then

(A) $n = 6$ (B) $n = 1, 2, 3, \ldots .8$ (C) $n = 5$ (D) none of these

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17. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$ satisfying the equation $\left(\sqrt{3}\right)^{\sec^2 \theta} = \tan^4 \theta + 2\tan^2 \theta$ is 2 (b) 4 (c) 0 (d) 1

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18. Let $f(x) = \max \{\tan x, \cot x\}$. Then number of roots of the equation $f(x) = \frac{1}{\sqrt{3}} \in (0, 2\pi)$ is (A) 2 (B) 4 (C) 0 (D) infinite

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19. the number of solution of the equation $\tan^2 x - \sec^{10} x + 1 = 0$ in (0,10) is -

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20. The number of solutions of the equation $|\cos x - \sin x| = 2\cos x \in [0, 2\pi]$ is (A) 1 (B) 2 (C) 3 (D) 4

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21. $\sin\theta + \sqrt{3}\cos\theta = 6x - x^2 - 11, 0 \leq \theta \leq 4\pi, x \in R,$ (a) hold for no values of x and θ (b) one value of x and two values of θ (c) two values of x and two values of θ (d) two point of values of (x, θ)

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22. $2\sin^2 x + \sin^2(2x) = 2, -\pi < x < \pi\, then x equals (A) \pm\frac{\pi}{2}$ (B) $\pm\frac{\pi}{4}$ (C) $\pm\frac{3\pi}{4}$ (D) none of these

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23. Which of the following is not the general solution of $2^{\cos 2x}+1=3.2^{-\sin^2 x}$? (a) $n\pi, n\in Z$ (b) $\left(n+\frac{1}{2}\right)\pi, n\in Z$ (c) $\left(n-\frac{1}{2}\right)\pi, n\in Z$ (d) none of these

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24. If $4\cos^2\theta+\sqrt{3}=2\left(\sqrt{3}+1\right)\cos\theta,$ then θ is $2n\pi\pm\frac{\pi}{3}, n\in I$ (b) $2n\pi\pm\frac{\pi}{4}, n\in I\, 2n\pi\pm\frac{\pi}{6}, n\in I$ (d) none of these

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25. The value of θ lying between $\theta=0$ and $\theta=\frac{\pi}{2}$ and satisfying the equation

$\left|1+\sin^2\theta\cos^2\theta 4\sin 4\theta\sin^2\theta 1+\cos^2\theta 4\sin 4\theta\sin^2\theta\cos^2\theta 1+4\sin 4\theta\right|=0$

$\frac{7\pi}{24}$ (b) $\frac{5\pi}{24}$ (c) $\frac{11\pi}{24}$ (d) $\frac{\pi}{24}$

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26. The equation whose roots are n^{th} power of the roots of the equation $x^2 - 2x\cos\phi + 1 = 0$ is given by

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27. In a triangle, $a^2 + b^2 + c^2 = ca + ab\sqrt{3}$. Then the triangles is :

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28. Solve the equation $\sin^{10} x + \cos^{10} x = \frac{29}{16}\cos^4 2x$

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29. If $3\sin x + 4\cos x = 5$, then the value of $90\tan^2\left(\frac{x}{2}\right) - 60\tan\left(\frac{x}{2}\right) + 10$ is equal to

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30. If x and y are the solutions of the equatiion $12\sin x + 5\cos x = 2y^2 - 8y + 21$ the value of $144\cot\left(x\frac{y}{2}\right)$ is

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31. Four times the sum of the roots of the equation $\sin 2x + 5\sin x + 5\cos x + 1 = 0$ in the interval $[0, 50\pi] isn\pi, wheren$ is equal to

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32. If $\cos 2\theta + 9\sin 2\theta - 6\sin\theta + 54\cos\theta = 1,$ then the value of $100\tan^2\theta + 9 + 14$ is equal to

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33. Solve the equation $2(\cos x + \cos 2x) + \sin 2x(1_2\cos x) = 2s \in xf$ or $x(-\pi \leq x \leq \pi)$

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34. Find the coordinates of the points of intersection of the curves $y = \cos x, y = \sin 3x$ if $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

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35. Show that the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution.

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36. Find all the solution of $4cos^2 x \sin x - 2\sin^2 x = 3\sin x$

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37. There exists a value of theta between 0 and 2π which sastisfies the equation $\sin^4 \theta - 2\sin^2 \theta - 1 = 0$. (True/False)

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38. Find the values of $x \in (-\pi, \pi)$ which satisfy the equation $8^{1+|\cos x|+|\cos^2 x|+|\cos^3 x|+\ldots)} = 4^3$

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39. Solve the system of equations $x + y = \frac{2\pi}{3}, \cos x + \cos y = \frac{3}{2}$, where x and y are real.

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40. Consider the system linear equations in $x, y, andz$ given by $(s \in 3\theta)x - y + z = 0, (\cos 2\theta)x + 4y + 3z = 0, 2x + 7y + 7z = 0.$

Find the value of θ for which the system has a non-trivial solution.

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41. The smallest positive root of the equation $\tan x - x = 0$ lies in

$\left(0, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2}, \pi\right)$ $\left(\pi, \frac{3\pi}{2}\right)$ (d) $\left(\frac{3\pi}{2}, 2\pi\right)$

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42. Find the set of all x in the interval $[0, \pi]$ for which

$2\sin^2 x - 3\sin x + 1 \geq 0$.

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43. The value of θ lying between $\theta = 0$ and $\theta = \frac{\pi}{2}$ and satisfying the equation

$\left|1 + \sin^2\theta \cos^2\theta 4\sin 4\theta \sin^2\theta 1 + \cos^2\theta 4\sin 4\theta \sin^2\theta \cos^2\theta 1 + 4\sin 4\theta\right| = 0$

$\frac{7\pi}{24}$ (b) $\frac{5\pi}{24}$ (c) $\frac{11\pi}{24}$ (d) $\frac{\pi}{24}$

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44. The general solution of the equation $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 2x + \cos 3x$ is

$(n \in Z)\ n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (d) $2n\pi + \frac{\cos^{-1} 2}{3}$

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45. The equation $(\cos p - 1)^x \hat{} 2 + (\cos p)x + s \in p = 0$ in the variable x has real roots. The p can take any value in the interval $(0, 2\pi)$ (b) $(-\pi)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(, \pi)$

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46. In triangle ABC, angle A is greater than angle B. If the measure of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0$. Find the value of angle C (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

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47. The number of real solution of the equation. $\sin(e^x) = 5^x + 5^{-x}$ is

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48. If $\exp\left[\left(\sin^2 x + \sin^4 x + \sin^6 x + \ldots \infty\right) \ln 2\right]$ satisfies the equation $y^2 - 9y + 8 = 0$, then the value of $\frac{\cos x}{\cos x + \sin x}, 0 < x < \frac{\pi}{2}$, is

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49. The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is 0 (b) 1 (c) 2 (d) 3

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50. Determine the smallest positive value of x (in degrees) for which $\tan(x + 100^\circ) = \tan(x + 50^\circ)\tan x \tan(x - 50^\circ)$

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51. Let $2\sin^2 x + 3\sin x - 2 > 0 and x^2 - x - 2 < 0 (x$ is measured in radians). Then x lies in the interval $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (b) $\left(-1, \frac{5\pi}{6}\right)$ $(-1, 2)$ (d) $\left(\frac{\pi}{6}, 2\right)$

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52. The smallest positive value of p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has solution in $0 \leq x \leq 2\pi$ is :

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53. The general value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is_____

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54. Find all values of theta in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation $(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$

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55. Solve $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$.

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56. The number of values of x in the in interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is 0 (b) 5 (c) 6 (d) 10

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57. The number of distinct real roots of $|\sin x \cos x \cos x \cos x \sin x \cos x \cos x \cos x \sin x| = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is 0 (b) 2 (c) 1 (d) 3

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58. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is (1) 4 (2) 8 (3) 10 (4) 12

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59. The number of solutions of the pair of equations $2\sin^2\theta - \cos 2\theta = 0$ $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is

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60. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is _____.

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61. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$$(y+z)\cos 3\theta = (xyz)\sin 3\theta$$

$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$

has a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$ is _____.

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62. Let

$$P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\} \text{ and } Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$$

be two sets. Then

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63. The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)} is$$

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64. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$, is

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65. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function $\left[f(x) = 4^{-x} \hat{} 2 + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)\right]$ is defined, is (1) $[0, \pi]$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (3) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (4) $\left[0, \frac{\pi}{2}\right)$

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