MATHS

BOOKS - KC SINHA ENGLISH

TRIGONOMETRIC FUNCTIONS - MULTIPLE AND SUBMULTIPLE OF ANGLES - FOR BOARDS

Solved Examples

1. If $\sin A = \frac{3}{5}$ and $0^\circ < A < 90^\circ$, find the value of $\sin 2A, \cos 2A, \tan 2A,$ and $\sin 4A.$

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2. If $\tan\theta = \frac{b}{a}$, then find the value of $a\cos 2\theta + b\sin 2\theta.$

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3. If $\tan\alpha = \frac{1}{7}, \sin\beta = \frac{1}{\sqrt{10}},$ prove that$\alpha + 2\beta = \frac{\pi}{4}$, where `0

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4. Prove that: $\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 2\sec 2\theta$.

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6. Prove that

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cos 8\alpha = \cot\alpha$

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7. Prove that $\cos\theta + \cos(120^\circ + \theta) + \cos(120^\circ - \theta) = 0$.

Hence deduce that

$\cos^3\theta + \cos^3(120^\circ + \theta) + \cos^3(120^\circ - \theta) = \frac{3}{4}\cos 3\theta$.

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8. Prove that: $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$

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9. Prove that : $2\sin^2\theta + 4\cos(\theta + \alpha)\sin\alpha\sin\theta + \cos 2(\alpha + \theta)$ is independent of θ.

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10. Prove that : $\cos^3 A\cos 3A + \sin^3 A\sin 3A = \cos^3 2A$

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11. $\tan\alpha.\ \tan(60^\circ - \alpha).\ \tan(60^\circ + \alpha) = \tan 3\alpha$

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12. Prove that:

$$\sin^3 A + \sin^3\left(\frac{2\pi}{3} + A\right) + \sin^3\left(\frac{4\pi}{3} + A\right) = -\frac{3}{4}\sin 3A.$$

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13. Prove that $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

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14. If $2\ tan\alpha = 3tan\beta,$ prove that $\tan(\alpha - \beta) = \dfrac{s \in 2\beta}{5 - \cos 2\beta}$

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15. If $\sin\alpha + \sin\beta = a \; and \; \cos\alpha + \cos\beta = b,$ show that : $\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$

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16. If $(2^n+1)\theta = \pi$ then $2^n \cos\theta \cos 2\theta \cos 2^2\theta \ldots\ldots \cos 2^{n-1}\theta =$

A. -1

B. 1

C. $\frac{1}{2}$

D. none of these

Answer: null

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17. Prove that : $\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \tan\left(\frac{\theta}{2}\right)$

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18. Show that

$$\frac{1+\tan.\frac{\theta}{2}}{1-\tan.\frac{\theta}{2}}=\frac{1+\sin\theta}{\cos\theta}=\tan\left(\frac{\pi}{2}+\frac{\theta}{2}\right)$$

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19. Prove that $(\cos A+\cos B)^2+(\sin A+\sin B)^2=4\cos^2\left(\frac{A-B}{2}\right)$.

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20. Find the value of $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}$.

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21. Find the value of $\tan\frac{\pi}{8}$.

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22. If $\tan x = -\frac{4}{3}, \frac{\pi}{2} < x < \pi$, then find the value of $\sin. \frac{x}{2}, \cos. \frac{x}{2}$ and $\tan. \frac{x}{2}$.

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23. Find the value of $\sin. \frac{23\pi}{24}$.

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24. If $\alpha = 112^\circ 30'$, find the value of $\sin\alpha$ and $\cos\alpha$

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25. Prove that: $\sin^2 24^0 - \sin^2 6^0 = \frac{\sqrt{5}-1}{8}$

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26. If $A = \tan 6^\circ \tan 42^\circ$ and $B = \cot 66^\circ \cot 78^\circ$, then

A. $A = 2B$

B. $A = \frac{1}{3}B$

C. $A = B$

D. $3A = 2B$

Answer: null

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27. Show that $\sin 47^\circ + \sin 61^\circ - \sin 11^\circ - \sin 25^\circ = \cos 7^\circ$

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28. Prove that: $s \in 12^0 s \in 48^0 s \in 54^0 = \frac{1}{8}$.

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29. Prove that $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\frac{\tan\theta}{2}\right)=\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$

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30. If α and β are distinct roots of $a\cos\theta+b\sin\theta=c$, prove that $\sin(\alpha+\beta)=\frac{2ab}{a^2+b^2}$

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31. If $\sin\alpha+\sin\beta=a$ and $\cos\alpha+\cos\beta=b$ then $\tan\left(\frac{\alpha-\beta}{2}\right)$ is equal to

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32. If $\cos\theta=\cos\alpha\cos\beta$ then $\tan\left(\frac{\theta+\alpha}{2}\right)\tan\left(\frac{\theta-\alpha}{2}\right)=$

(i)$\tan^2\left(\frac{\alpha}{2}\right)$ (ii)$\tan^2\left(\frac{\beta}{2}\right)$ (iii)$\tan^2\left(\frac{\theta}{2}\right)$ (iv) $\cot^2\left(\frac{\beta}{2}\right)$

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33. If $\cos\theta = \dfrac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta}$, prove that one value of $(\tan)\dfrac{\theta}{2} = \dfrac{\frac{\tan\alpha}{2} - \frac{\tan\beta}{2}}{1 - tan\frac{\alpha}{2}\frac{\tan\beta}{2}}$.

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Exercise

1. If $\tan\theta = \dfrac{a}{b}$, where $0 < \theta < \dfrac{\pi}{4}$ and $b > a > 0$, find the value of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$.

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2. If $\tan\theta = -\dfrac{3}{4}$ and $\dfrac{\pi}{2} < \theta < \pi$, find the values of $\sin\theta$, $\cos\theta$ and $\cot\theta$.

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3. Let $\frac{3\pi}{4} < \theta < \pi$ and $\sqrt{2\cot\theta + \frac{1}{\sin^2\theta}} = k - \cot\theta$ then $k =$

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4. Prove that $\cot\theta - \tan\theta = 2\cot 2\theta.$

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5. Prove that $\frac{1 - \tan^2\left(\frac{\pi}{4} - A\right)}{1 + \tan^2\left(\frac{\pi}{4} - A\right)} = \sin 2A.$

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6. Prove that $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \sec 2\theta - \tan 2\theta.$

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7. Prove that $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)=2\tan 2\theta.$

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8. If $\cos\theta=\frac{1}{2}\left(a+\frac{1}{a}\right)$, show that $\cos 2\theta=\frac{1}{2}\cdot\left(a^2+\frac{1}{a^2}\right)$

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9. Prove that : $\cos^2\theta+\sin^2\theta\cos 2\beta=\cos^2\beta+\sin^2\beta\cos 2\theta$

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10. Prove that $1+\tan 2\theta\tan\theta=\sec 2\theta.$

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11. Prove that : $\frac{1+\sin 2A-\cos 2A}{1+\sin 2A+\cos 2A}=\tan A$

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12. Prove that $\frac{1+\sin 2\theta}{1-\sin 2\theta} = \left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2$

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13. Show that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4.$

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14. Prove that $\cos ecA - 2\cot 2A\cos A = 2\sin A.$

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15. Prove that: $\cot^2 A - \tan^2 A = 4\cos 2A\cos ec2A$

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16. Prove that : $\frac{1+\sin 2A}{\cos 2A} = \frac{\cos A + \sin A}{\cos A - \sin A} = \tan\left(\frac{\pi}{4} + A\right)$

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18. Prove that: $\cos^2 A + \cos^2\left(A + \frac{\pi}{3}\right) + \cos^2\left(A - \frac{\pi}{3}\right) = \frac{3}{2}$

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19. Prove that

$\frac{2\cos 2^n\theta + 1}{2\cos\theta + 1} = (2\cos\theta - 1)(2\cos 2\theta 1) \times \left(\cos 2^2\theta - 1\right).....\left(2\cos 2^{n-1}\theta -\right.$

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20. prove that :

$$\frac{\tan 2^n\theta}{\tan\theta} = (1+\sec 2\theta)\left(1+\sec 2^2\theta\right)\left(1+\sec 2^3\theta\right)\ldots(1+\sec 2^n\theta)$$

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21. Prove that : $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x.$

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22. Prove that $\cos\theta\cos 2\theta\ldots\cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n\sin\theta}, \forall n \in N.$

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23. Show that :

$$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4\left(\sin^6 x + \cos^6 x\right) = 13.$$

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24. Show that : $2\left(\sin^6 x + \cos^6 x\right) - 3\left(\sin^4 x + \cos^4 x\right) + 1 = 0.$

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25. Show that $\cos^2\theta + \cos^2\theta(\alpha+\theta) - 2\cos\alpha\cos\theta\cos(\alpha+\theta)$ is independent of θ.

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26. Prove that $4\left(\cos^3 10^\circ + \sin^3 20^\circ\right) = 3(\cos 10^\circ + \sin 20^\circ).$

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27. Prove that : $\sin\theta\cos^3\theta - \cos\theta\sin^3\theta = \frac{1}{4}\sin 4\theta.$

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28. Prove that $\cos^3\theta\sin 3\theta + \sin^3\theta\cos 3\theta = \frac{3}{4}\sin 4\theta.$

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29. If $\tan\theta + \tan(60^\circ + \theta) + \tan(120^\circ + \theta) = 3$ then $\theta =$

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31. $\cot A + \cot(60^\circ + A) + \cot(120^\circ + A) = 3\cot 3A.$

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32. Prove that:$\cos 4x = 1 - 8\sin^2 x\cos^2 x$

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34. Prove that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1.$

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35. Prove that

$\cos 4\theta - \cos 4\alpha = 8(\cos\theta - \cos\alpha)(\cos\theta + \cos\alpha)(\cos\theta - \sin\alpha)(\cos\theta + \sin\alpha$

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36. Prove that :$\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}.$

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37. If $\tan x = \frac{b}{a}$, prove that $a\cos 2x + b\sin 2x = a$

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38. If $\tan^2\theta = 1 + 2\tan^2\phi$, prove that $\cos 2\phi = 1 + 2\cos 2\theta$.

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39. If α and β be acute angle and $\cos 2\pi \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$, than $\tan\alpha$ is

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40. If $\tan\beta = 3\tan\alpha$, prove that $\tan(\alpha + \beta) = \frac{2\sin 2\beta}{1 + \cos 2\beta}$

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41. If $x \sin \alpha = y \cos \alpha$, prove that : $\frac{x}{\sec 2\alpha} + \frac{y}{\cos ec 2\alpha} = x$

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42. If $\tan 2\theta = \sec 2\alpha$, prove that $\sin 2\theta = \frac{1 - \tan^4 \alpha}{1 + \tan^4 \alpha}$

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43. If $\alpha = \frac{\pi}{3}$, prove that $\cos \alpha \cdot \cos 2\alpha \cdot \cos 3\alpha \cdot \cos 4\alpha \cdot \cos 5\alpha \cdot \cos 6\alpha = -\frac{1}{16}$.

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44. Prove that $\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right) = \frac{1}{16}$

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45. If $\tan\theta\tan\phi = \sqrt{\frac{a-b}{a+b}}$, prove that

$a - b\cos 2\theta)(a - b\cos 2\phi)$ is independent of θ and ϕ.

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46. Prove that $\frac{2\sin\theta - \sin 2\theta}{2\sin\theta + \sin 2\theta} = \tan^2\frac{\theta}{2}$

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47. Prove that $\cot\left(\frac{\theta}{2}\right) - \tan\left(\frac{\theta}{2}\right) = 2\cot\theta$

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48. Prove that : $\frac{1+\sin\theta}{1-\sin\theta} = \tan^2\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

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49. Prove that $\sec\theta + \tan\theta = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

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50. $\frac{\sin\alpha + \sin\beta - \sin(\alpha+\beta)}{\sin\alpha + \sin\beta + \sin(\alpha+\beta)} = \tan\left(\frac{\alpha}{2}\right)\tan\left(\frac{\beta}{2}\right)$

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51. Prove that :$\tan\left(\frac{\pi}{4} - \frac{A}{2}\right) = \sec A - \tan A = \sqrt{\frac{1-\sin A}{1+\sin A}}$

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52. Prove that $\cos ec\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\cos ec\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = 2\sec\theta$

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53. Prove that $(\cos A - \cos B)^2 + (\sin A - \sin B)^2 = 4\sin^2\left(\frac{A-B}{2}\right)$.

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54. Prove that : $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2\frac{x+y}{2}$

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55. $\cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\frac{5\pi}{8}\right) + \cos^2\left(\frac{7\pi}{8}\right) = 2$

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56. Prove that: $s \in^4 \frac{\pi}{8} + s \in^4 \frac{3\pi}{8} + s \in^4 \frac{5\pi}{8} + s \in^4 \frac{7\pi}{8} = \frac{3}{2}$

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57. If $\tan x = \frac{3}{4}$, `pi

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58. If $\sin x = \frac{1}{4}, \frac{\pi}{2} < x < \pi$, find the values of $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$.

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59. If $\cos x = -\frac{1}{3}$, x lies in III quardrnat, then find the values of $\sin\left(\frac{x}{2}\right), \cos\left(\frac{x}{2}\right)$ and $\tan\left(\frac{x}{2}\right)$.

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60. Prove that:

$$\left(1+\cos.\frac{\pi}{8}\right)\left(1+\cos.\frac{3\pi}{8}\right)\left(1+\cos.\frac{5\pi}{8}\right)\left(1+\cos.\frac{7\pi}{8}\right)=\frac{1}{8}$$

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61. Show that $\cot 142\frac{1^\circ}{2} = \sqrt{2}+\sqrt{3}-2-\sqrt{6}$.

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62. Prove that $\sin^2 48^\circ - \cos^2 12^\circ = -\frac{\sqrt{5}+1}{8}$

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63. $4\left(\sin 24^0 + \cos 6^0\right) = \sqrt{3} + \sqrt{15}$

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64. $\cot 6^0 \cot 42^0 \cot 66^0 \cot 78^0 = 1$

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65. Prove that $\tan 12^\circ \tan 24^\circ \tan 48^\circ \tan 84^\circ = 1.$

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66. Prove that: $\sin 6^0 \cos 42^0 \cos 60^0 s \in 78^0 = \frac{1}{16}$

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67. Prove that: $\sin\left(\frac{\pi}{5}\right)\sin\left(\frac{2\pi}{5}\right)\sin\left(3\frac{\pi}{5}\right)\sin\left(4\frac{\pi}{5}\right) = \frac{5}{16}$

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68. Prove that $\cos 36^\circ \cos 72^\circ \cos 108^\circ \cos 144^\circ = 1/16.$

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69. Prove:

$\cos\left(\frac{\pi}{15}\right)\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{3\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{5\pi}{15}\right)\cos\left(\frac{6\pi}{15}\right)\cos\left(\frac{7\pi}{15}\right) = \frac{1}{2^7}$

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71. If $tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}}\frac{\tan\alpha}{2}, then \cos\alpha = \quad 1 - e\cos(\cos\theta + e)$ (b) $\frac{1+e\cos\theta}{\cos\theta - e}$ (c) $\frac{1-e\cos\theta}{\cos\theta - e}$ (d) $\frac{\cos\theta - e}{1-e\cos\theta}$

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72. If $\sin\alpha + \sin\beta = a \; and \; \cos\alpha + \cos\beta = b,$ show that : $\sin(\alpha+\beta) = \frac{2a\,b}{a^2+b^2}$

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73. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b,$ prove that : $\cos(\alpha-\beta) = \frac{1}{2}(a^2+b^2-2)$

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74. If α and β are the two different roots of equations $a\cos\theta + b\sin\theta = c$, prove that

(a) $\tan(\alpha+\beta) = \dfrac{2ab}{a^2-b^2}$ (b) $\cos(\alpha+\beta) = \dfrac{a^2-b^2}{a^2+b^2}$

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75. If α and β are the solution of the equation $a\cos\theta + b\sin\theta = c$, then show that : $\cos(\alpha+\beta) = \dfrac{a^2-b^2}{a^2+b^2}$

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76. If $\cos\alpha + \cos\beta = \dfrac{1}{3}$ and $\sin\alpha + \sin\beta = \dfrac{1}{4}$ prove that $\cos\left[\dfrac{\alpha-\beta}{2}\right] = \pm\left(\dfrac{5}{24}\right)$

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77. If $2\tan\left(\frac{\alpha}{2}\right) = \tan\left(\frac{\beta}{2}\right)$, prove that $\cos\alpha = \frac{3+5\cos\beta}{5+3\cos\beta}$.

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78. If $\sin\alpha = \frac{4}{5}$ and $\cos\beta = \frac{5}{13}$, prove that $\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{8}{\sqrt{65}}$.

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79. If $\sec(\phi+\alpha)+\sec(\phi-\alpha) = 2\sec\phi$, prove that $\cos\phi = \pm\sqrt{2}\cos\frac{\alpha}{2}, \left(\phi \neq \frac{\pi}{2}\right)$.

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80. If $\cos\theta = \frac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta}$, prove that one value of $\tan\left(\frac{\theta}{2}\right) = \frac{\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)}{1-\tan\left(\frac{\alpha}{2}\right)\tan\left(\frac{\beta}{2}\right)}$.

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81. If $\cos\theta = \dfrac{\cos\alpha - \cos\beta}{1 - \cos\alpha\cos\beta}$, then prove that one of the values of $\tan.\dfrac{\theta}{2}$ is $\tan.\dfrac{\alpha}{2}\cot.\dfrac{\beta}{2}$

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