MATHS

BOOKS - KC SINHA ENGLISH

TRIGONOMTERY - JEE MAINS AND ADVANCED QUESTIONS - FOR COMPETITION

Exercise

1. Period of $\sin^2\theta$ is(A) π^2 (B) π (C) 2π (D) $\frac{\pi}{2}$

Watch Video Solution

2. The period of the function $f(x)=\sin^4 x+\cos^4 x$ is:

Watch Video Solution

3. $\sin^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if (A) $x - y \neq 0$ (B) $x = -y$ (C) $x + y \neq 0$ (D) $x \neq 0, y \neq 0$

Watch Video Solution

4. The value of $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ} =$ (A) 1 (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) 2

Watch Video Solution

5. If $\tan\theta = -\frac{4}{3}, then \sin\theta$ is $-\frac{4}{5} but \neg \frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$ $\frac{4}{5} but \neg -\frac{4}{5}$ (d) none of these

Watch Video Solution

6. If $\sin(\alpha+\beta) = 1 \, and \, \sin(\alpha-\beta) = \frac{1}{2}, \, where \, 0 \leq , \beta \leq \frac{\pi}{2}$, then find the values of $\tan(\alpha+2\beta) and \tan(2\alpha+\beta)$.

Watch Video Solution

7. If $y = \sin^2\theta + \cos ec^2\theta, \theta \neq 0,$ then (A) $y = 0$ (B) $y \leq 2$ (C) $y \geq -2$ (D) $y \geq 2$

Watch Video Solution

8. The equation $\sin x + b\cos x = c, where|c| > \sqrt{a^2 + b^2}$ has (A) a unique solution (B) infinite number of solutions (C) no solution (D) none of these

Watch Video Solution

9. If $\cot^{-1}\left(\sqrt{\cos\alpha}\right) - \tan^{-1}\left(\sqrt{\cos\alpha}\right) = x,$ then $\sin x$ is $\frac{\tan^2\alpha}{2}$ (b) $\frac{\cot^2\alpha}{2}$ (c) $\tan^2\alpha$ (d) $\frac{\cot\alpha}{2}$

Watch Video Solution

10. Prove the following: $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

Watch Video Solution

11. In triangle $ABC, 2ac\sin\left(\frac{1}{2}(A - B + C)\right)$ is equal to $(a)a^2 + b^2 - c^2$ (b) $c^2 + a^2 - b^2$ (c) $b^2 - c^2 - a^2$ (d) $c^2 - a^2 - b^2$

Watch Video Solution

12. In a triangle $ABC, a = 4, b = 3, \angle A = 60^0$ then c is root of the equation $c^2 - 3c - 7 = 0$ (b) $c^2 + 3c + 7 = 0$ (c) $c^2 - 3c + 7 = 0$ (d) $c^2 + 3c - 7 = 0$

Watch Video Solution

13. In a $\triangle ABC, \tan\frac{A}{2} = \frac{5}{6}$ and $\tan\frac{C}{2} = \frac{2}{5}$ then (A) a,c,b are in A.P. (B) a,b,c are in A.P. (C) b,a,c are in A.P. (D) a,b,c are in G.P.

Watch Video Solution

14. The trigonometric equation $\sin^{-1} x = 2\sin^{-1} a$ has a solution for

Watch Video Solution

15. If in a triangle $ABC, a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides $a, b, andc$ are in A.P. b. are in G.P. c. are in H.P. d. satisfy $a + b = \cdot$

Watch Video Solution

16. In a triangle ABC, medians AD and BE are drawn. If $AD = 4, \angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$ then the area of the triangle ABC is :

Watch Video Solution

17. 10. The upper $\frac{3}{4}$ portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at the point in the horizontal plane through its foot. The tangent of the angle subtended by the pole at the same point is

Watch Video Solution

18. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side $'a'$, is: $a\cot\left(\frac{\pi}{n}\right)$ b. $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ c. $a\cot\left(\frac{\pi}{2n}\right)$ d. $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

Watch Video Solution

19. Let α and β be such that $\pi<\alpha-\beta<3\pi$, If $\sin\alpha+\sin\beta=-\frac{21}{65}$ and $\cos\alpha+\cos\beta=-\frac{27}{65}$, then the value of $\frac{\cos(\alpha-\beta)}{2} is$ (a) $-\frac{3}{\sqrt{130}}$ (b) $\frac{3}{\sqrt{130}}$ (c) $\frac{6}{25}$ (d) $\frac{6}{65}$

Watch Video Solution

20. If $f: \overrightarrow{RS}$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1, ison \rightarrow$, then find the set S.

Watch Video Solution

21. The sides of a triangle are $\sin\alpha, \cos\alpha, \sqrt{1 + \sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$ then the greatest angle of the triangle is :

Watch Video Solution

22. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60^0. When he move 40 metres away from the bank, he finds the angle of elevation to be 30^0. Find the height of the tree and the width of the river.

Watch Video Solution

23. If $\cos^{-1}x - \cos^{-1}.\ \frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

Watch Video Solution

24. In triangle ABC, let $\angle c = \frac{\pi}{2}$. If r is the inradius and R is circumradius of the triangle, then $2(r+R)$ is equal to $a+b$ (b) $b+c$ $c+a$ (d) $a+b+c$

Watch Video Solution

25. If in ΔABC, the altitudes from the vertices A, B and C on opposite sides are in HP, then sin A sin B and sin C are in

Watch Video Solution

26. A triangle PQR, $\angle R = 90^\circ$ and $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ roots of the $ax^2 + bx + c = 0$ then prove that $a + b = c$

Watch Video Solution

27. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^0 and tan 15^0$, respectively, then find the value of $2 + q - p$.

Watch Video Solution

28. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is (1) $\frac{4 - \sqrt{7}}{3}$ (2) $-\frac{4 + \sqrt{7}}{3}$ (3) $\frac{1 + \sqrt{7}}{4}$ (4) $\frac{1 - \sqrt{7}}{4}$

Watch Video Solution

29. If $\sin^{-1}\left(\frac{x}{5}\right)+\cos ec^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$ then a value of x is: (1) 1 (2) 3 (3) 4 (4) 5

Watch Video Solution

30. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that $AB(=a)$ subtends an angle of $60o$ at the foot of the tower, and the angle of elevation of the top of the tower from A or B is $30o$. The height of the tower is (1) $\frac{2a}{\sqrt{3}}$ (2) $2a\sqrt{3}$ (3) $\frac{a}{\sqrt{3}}$ (4) $a\sqrt{3}$

Watch Video Solution

31. The value of $\cot\left(\cos ec^{-1}\frac{5}{3}+\frac{\tan^{-1}2}{3}\right)$ is: (1) $\frac{6}{17}$ (2) $\frac{3}{17}$ (2) $\frac{4}{17}$ (4) $\frac{5}{17}$

Watch Video Solution

32. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is $60o$. He moves away from the pole along the line BC to a point D such that $CD = 7m$. From D the angle of elevation of the point A is $45o$. Then the height of the pole is (1) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}m$ (2) $\frac{7\sqrt{3}}{2}\sqrt{3}+1m$ (3) $\frac{7\sqrt{3}}{2}\sqrt{3}-1m$ (4) $\frac{7\sqrt{3}}{2}\sqrt{3}+1m$

Watch Video Solution

33. Let A and B denote the statements

$A: \cos\alpha + \cos\beta + \cos\gamma = 0$

$B: \sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then

Watch Video Solution

34. Let $\cos(\alpha+\beta)=\frac{4}{5}$ and let $\sin(\alpha+\beta)=\frac{5}{13}$ where $0\leq\alpha,\beta\leq\frac{\pi}{4}$, then $tan2\alpha=$ (1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

Watch Video Solution

35. For a regular polygon, let r and R be the radii of the inscribed and the cirumscribed circles, respectively. A false statement among the following is

Watch Video Solution

36. If $A=s\in^2 x+\cos^4 x$, then for all real x : (1) $\frac{3}{4}\leq A\leq 1$ (2) $\frac{13}{16}\leq A\leq 1$ (3) $1\leq A\leq 2$ (4) $\frac{3}{4}\leq A\leq\frac{13}{16}$

Watch Video Solution

37. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are

Watch Video Solution

38. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

Watch Video Solution

39. In a ΔPQR, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to (1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

Watch Video Solution

40. Prove the following identities: $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = 1 + \tan A + \cot A = 1 + \sec A \cos ecA$

Watch Video Solution

Watch Video Solution

41. If x, y, z are in A.P. and $tan^{-1}x$, $tan^{-1}y$ and $tan^{-1}z$ are also in A.P., then (1) $2x = 3y = 6z$ (2) $6x = 3y = 2z$ (3) $6x = 4y = 3z$ (4) $x = y = z$

Watch Video Solution

42. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to

Watch Video Solution

43. Let $F_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$, where $x \in R$ and $k \geq 1$, then find the value of $F_4(x) - F_6(x)$.

Watch Video Solution

44. A bird is sitting on the top of a vertical pole 20 m high and its elevation fron a point O on the ground is 45°,It Files off horizontally straight away from the point O.After one second , the elevation of the bird fron O is reduced to 30° Then the speed (in m/s) of the bird is

Watch Video Solution

45. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is

Watch Video Solution

46. If the anlges of elevation of the top of a tower from three collinear points A,B and C on a line leading to the foot of the tower are $30^\circ, 45^\circ$ and 60° respectively , then the ratio AB:BC is

Watch Video Solution

47. A value of θ for which $\dfrac{2+3i\sin\theta}{1-2i\sin\theta}$ purely imaginary, is

Watch Video Solution

48. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60° . Then the time taken (in minutes) by him, from B to reach the pillar is :

Watch Video Solution

49. If $0 \leq x \leq 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is

Watch Video Solution

50. If $5\left(\tan^2 x - \cos^2 x\right) = 2\cos 2x + 9$, then the value of $\cos 4x$ is

Watch Video Solution

51. Let a vertical tower AB have its end A on the level ground. Let C be the mid point of AB and P be a point on the ground such that $AP = 2AB$. If $\angle BPC = \beta$, then $\tan \beta$ is equal to : $\frac{2}{9}$ (2) $\frac{4}{9}$ (3) $\frac{6}{7}$ (4) $\frac{1}{4}$

Watch Video Solution

52. If $\pi < \alpha < \frac{3\pi}{2}$, then find the value of expression $\sqrt{4\sin^4 \alpha + \sin^2 2\alpha} + 4\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$.

Watch Video Solution

53. The number of real solutions of the equation $(\sin x - x)(\cos x - x^2) = 0$ is

Watch Video Solution

54. If $\sin^{-1}\left(x + \frac{x^2}{2} + \frac{x^3}{4} + \frac{x^4}{8} + \dots\right) = \frac{\pi}{6}$, where $|x| < 2$ then the value of x is (A) $\frac{2}{5}$ (B) $\frac{3}{2}$ (C) $-\frac{2}{3}$ (D) $-\frac{3}{2}$

Watch Video Solution

55. If $f: \left[0, \frac{\pi}{2}\right) \rightarrow R$ is defined as $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$

Then, the range of f is

Watch Video Solution

56. Prove: $\cot^{-1}\left(\frac{1}{2}\right) - \frac{1}{2}\cot^{-1}\left(\frac{4}{3}\right) = \frac{\pi}{4}$

Watch Video Solution

57. The minimum value of $\cos\theta + \sin\theta + \dfrac{2}{\sin 2\theta}$ for $\theta \in \left(0, \dfrac{\pi}{2}\right)$ is (A) $2 + \sqrt{2}$ (B) 2 (C) $1 + \sqrt{2}$ (D) $2\sqrt{2}$

Watch Video Solution

58. In a triangle ABC, $a^2\cos^2 A = b^2 + c^2$ then triangle is

Watch Video Solution

59. The number of distinct real roots of $|\sin x \cos x \cos x \cos x \sin x \cos x \cos x \cos x \sin x| = 0$ in the interval $-\dfrac{\pi}{4} \le x \le \dfrac{\pi}{4}$ is 0 (b) 2 (c) 1 (d) 3

Watch Video Solution

60. The trigonometric equation $\sin^{-1} x = 2\sin^{-1} a$ has a solution for

Watch Video Solution

61. In a triangle, the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is

Watch Video Solution

62. The sum of roots of the equation $\tan^{-1}.\frac{1}{1+2x} + \tan^{-1}.\frac{1}{1+4x} = \tan^{-1}.\frac{2}{x^2}$ is

Watch Video Solution

63. For $x \in (0, \pi)$ the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

Watch Video Solution

Watch Video Solution

64. The number of distinct solution of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is ______.

Watch Video Solution

65. In a triangle ΔXYZ, leta, bandc be the lengths of the sides opposite to the angles X, Y and Z respectively.If $2(a^2 - b^2) = c^2$ and $\lambda = \frac{\sin(X - Y)}{\sin Z}$ then possible values of n for which $\cos(n\pi\lambda) = 0$ is (are)

Watch Video Solution

66. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct options (s) is (are)

Watch Video Solution

67. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1, are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals:

Watch Video Solution

68. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{2}\sec x + \cos ecx + 2(\tan x - \cot x)$ $= 0$ in the set S is equal to

Watch Video Solution

69. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to

Watch Video Solution

70. In a $\Delta XYZ,$ let x,y,z be the lengths of sides opposite to the angles X, Y, Z respectively and $2x = x + y + z.$ If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the ΔXYZ is $\frac{8\pi}{3}$ then

Watch Video Solution

71. If $f(x)|\cos(2x)\cos(2x)\sin(2x) - \cos x \cos x - \sin x \sin x \sin x \cos x|, then:$ a$f'(x) = 0$ at exactly three point in $(-\pi, \pi)$ b$f'(x) = 0$ at more than three point in $(-\pi, \pi)$ c$f(x)$ attains its maximum at $x = 0$ d $f(x)$ attains its minimum at $x = 0$

Watch Video Solution

72. Let α, β be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true ?

Watch Video Solution

73. Let O be the origin and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vector in the directions of the sides $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$ respectively , of a triangle PQR.

if the triangle PQR varies , then the manimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is

Watch Video Solution