

MATHS

BOOKS - KC SINHA ENGLISH

TRIGONOMTERY - JEE MAINS AND ADVANCED QUESTIONS - FOR COMPETITION

Exercise

1. Period of
$$\sin^2 heta$$
 is(A) π^2 (B) π (C) 2π (D) $\frac{\pi}{2}$

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2. The period of the function $f(x) = \sin^4 x + \cos^4 x$ is:

3.
$$\sin^2 \theta = \frac{4xy}{(x+y)^2}$$
 is true if and only if (A) $x - y \neq 0$ (B) $x = -y$
(C) $x + y \neq 0$ (D) $x \neq 0, y \neq 0$

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4. The value of
$$rac{1- an^2 \, 15^\circ}{1+ an^2 \, 15^\circ} = \,$$
 (A) 1 (B) $\sqrt{3}$ (C) $rac{\sqrt{3}}{2}$ (D) 2

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5. If
$$\tan \theta = -\frac{4}{3}$$
, then $\sin \theta$ is $-\frac{4}{5}but - \frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$
 $\frac{4}{5}but - \frac{4}{5}$ (d) none of these

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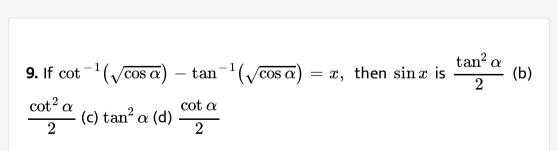
6. If $\sin(\alpha + \beta) = 1$ and $\sin(\alpha - \beta) = \frac{1}{2}$, where $0 \le \beta \le \frac{\pi}{2}$, then find the values of $\tan(\alpha + 2\beta)$ and $\tan(2\alpha + \beta)$.

7. If $y=\sin^2 heta+\cos ec^2 heta, heta
eq 0,\,$ then (A) y=0 (B) $y\leq 2$ (C) $y\geq -2$

(D) $y \geq 2$

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8. The equation $\sin x + b\cos x = c, where |c| > \sqrt{a^2 + b^2}$ has (A) a unique solution (B) infinite number of solutions (C) no solution (D) none of these



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10. Prove the following: $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

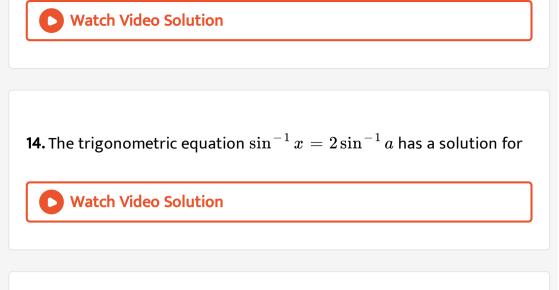
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11. In triangle
$$ABC$$
, $2ac\sin\left(\frac{1}{2}(A-B+C)\right)$ is equal to $(a)a^2 + b^2 - c^2$ (b) $c^2 + a^2 - b^2$ (c) $b^2 - c^2 - a^2$ (d) $c^2 - a^2 - b^2$

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12. In a triangle ABC, $a=4, b=3, \angle A=60^0$ then c is root of the equation $c^2-3c-7=0$ (b) $c^2+3c+7=0$ (c) $c^2-3c+7=0$ (d) $c^2+3c-7=0$

13. In a
$$\triangle ABC$$
, $\tan \frac{A}{2} = \frac{5}{6}$ and $\tan \frac{C}{2} = \frac{2}{5}$ then (A) a,c,b are in A.P. (B) a,b,c are in A.P. (C) b,a,c are in A.P. (D) a,b,c are in G.P.



15. If in a triangle
$$ABC$$
, $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides $a, b, andc$ are in A.P. b. are in G.P. c. are in H.P. d. satisfy $a + b = \cdot$

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16. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$ then the area of the triangle ABC is :

17. 10. The upper $\frac{3}{4}$ portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at the point in the horizontal plane through its foot. The tangent of the angle subtended by the pole at the same point is

18. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side 'a', is: $a \cot\left(\frac{\pi}{n}\right)$ b. $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ c. $a \cot\left(\frac{\pi}{2n}\right)$ d. $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

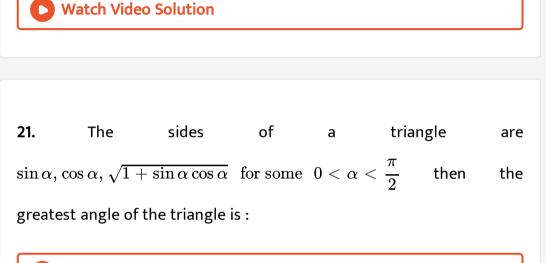
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19. Let
$$\alpha$$
 and β be such that $\pi < \alpha - \beta < 3\pi$, If $\sin \alpha + \sin \beta = -\frac{21}{65}$ and $\cos \alpha + \cos \beta = -\frac{27}{65}$, then the value of $\frac{\cos(\alpha - \beta)}{2}is$ (a) $-\frac{3}{\sqrt{130}}$ (b) $\frac{3}{\sqrt{130}}$ (c) $\frac{6}{25}$ (d) $\frac{6}{65}$

20. If $f : R \overset{\longrightarrow}{S}$, defined by $f(x) = \sin x - \sqrt{3} \cos x + 1, i son o \, , \,$ then

find the set S



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22. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60° . When he move 40 metres away from the bank, he finds the angle of elevation to be 30° . Find the height of the tree and the width of the river.

23. If
$$\cos^{-1}x - \cos^{-1}$$
. $\frac{y}{2} = lpha$, then $4x^2 - 4xy\coslpha + y^2$ is equal to

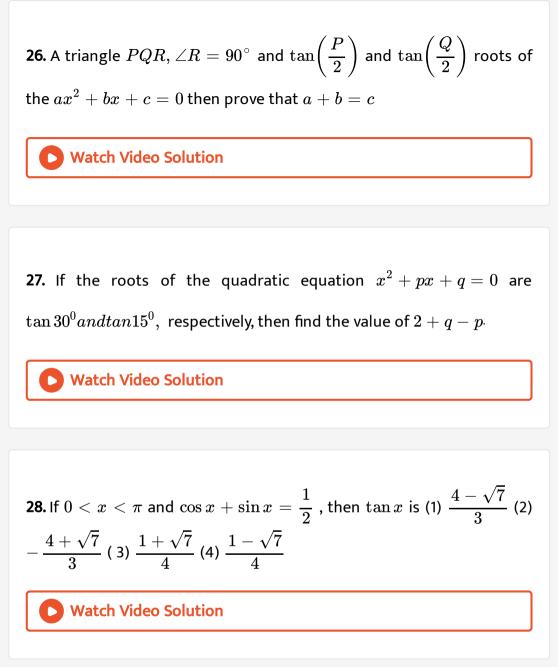
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24. In triangle ABC, let $\angle c = \frac{\pi}{2}$. If r is the inradius and R is circumradius of the triangle, then 2(r+R) is equal to a+b (b) b+cc+a (d) a+b+c

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25. If in $\triangle ABC$, the altitudes from the vertices A, B and C on opposite

sides are in HP, then sin A sin B and sin C are in



29. If
$$\sin^{-1}\left(\frac{x}{5}\right) + \cos ec^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$$
 then a value of x is: (1) 1 (2) 3 (3) 4 (4) 5

30. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB(=a) subtends an angle of 60o at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30o. The height of the tower is (1) $\frac{2a}{\sqrt{3}}$ (2) $2a\sqrt{3}$ (3) $\frac{a}{\sqrt{3}}$ (4) $a\sqrt{3}$

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31. The value of $\cot\left(\cos ec^{-1}\frac{5}{3} + \frac{\tan^{-1}2}{3}\right)$ is: (1) $\frac{6}{17}$ (2) $\frac{3}{17}$ (2) $\frac{4}{17}$ (4) $\frac{5}{17}$

32. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60o. He moves away from the pole along the line BC to a point D such that CD = 7m. From D the angle of elevation of the point A is 45o. Then the height of the pole is (1) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}m$ (2) $\frac{7\sqrt{3}}{2}\sqrt{3}+1m$ (3) $\frac{7\sqrt{3}}{2}\sqrt{3}-1m$ (4) $\frac{7\sqrt{3}}{2}\sqrt{3}+1m$

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33. Let A and B denote the statements

$$A : \cos lpha + \cos eta + \cos \gamma = 0$$

$$B : \sin lpha + \sin eta + \sin \gamma = 0$$

If
$$\cos(eta-\gamma)+\cos(\gamma-lpha)+\cos(lpha-eta)=-rac{3}{2}$$
 , then

34. Let
$$\cos(\alpha + \beta) = \frac{4}{5}$$
 and let $\sin(\alpha + \beta) = \frac{5}{13}$ where $0 \le \alpha, \beta \le \frac{\pi}{4}$, then $\tan 2\alpha =$ (1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

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35. For a regular polygon, let r and R be the radii of the inscribed and the cirumscribed circles, respectively. A false statement among the following is

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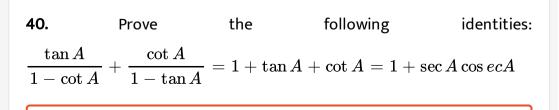
36. If
$$A = s \in {}^2 x + \cos^4 x$$
 , then for all real x : (1) $\frac{3}{4} \le A \le 1$ (2) $\frac{13}{16} \le A \le 1$ (3) $1 \le A \le 2$ (4) $\frac{3}{4} \le A \le \frac{13}{16}$

37. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **Vatch Video Solution**

38. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

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39. In a ΔPQR , if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to (1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$



41. If x, y, z are in A.P. and $tan^{-1}x$, $tan^{-1}y$ and $tan^{-1}z$ are also in A.P., then (1) 2x = 3y = 6z (2) 6x = 3y = 2z (3) 6x = 4y = 3z (4) x = y = z

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42. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, BC = p and CD = q, then AB is equal to

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43. Let
$$F_k(x)=rac{1}{k}\Big(\sin^k x+\cos^k x\Big)$$
, where $x\in R$ and $k\geq 1$, then find the value of $F_4(x)-F_6(x).$

44. A bird is sitting on the top of a vertical pole 20 m high and its elevation fron a point O on the ground is 45° , It Files off horizontally straight away from the point O.After one second , the elevation of the bird fron O is reduced to 30° Then the speed (in m/s) of the bird is

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45. Let
$$\tan^{-1} y = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$
, where $|x| < \frac{1}{\sqrt{3}}$.

Then a value of y is

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46. If the anlges of elevation of the top of a tower from three collinear points A,B and C on a line leading to the foot of the tower are 30° , 45° and 60° respectively, then the ratio AB:BC is

47. A value of heta for which $rac{2+3i\sin heta}{1-2i\sin heta}$ purely imaginary, is

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48. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30° . After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60° . Then the time taken (in minutes) by him, from B to reach the pillar is :



49. If $0 \le x \le 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is

50. If $5(an^2x - \cos^2x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is

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51. Let a vertical tower AB have its end A on the level ground. Let C be the mid point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan \beta$ is equal to : $\frac{2}{9}$ (2) $\frac{4}{9}$ (3) $\frac{6}{7}$ (4) $\frac{1}{4}$

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52. If $\pi < \alpha < \frac{3\pi}{2}$, then find the value of expression $\sqrt{4\sin^4 \alpha + \sin^2 2\alpha} + 4\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$.

53. The number of real solutions of the equation $(\sin x - x)(\cos x - x^2) = 0$ is

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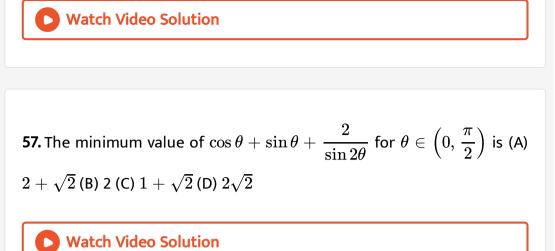
54. If
$$\sin^{-1}\left(x + \frac{x^2}{2} + \frac{x^3}{4} + \frac{x^4}{8} + ...\right) = \frac{\pi}{6}$$
, where $|x| < 2$ then the value of x is (A) $\frac{2}{5}$ (B) $\frac{3}{2}$ (C) $-\frac{2}{3}$ (D) $-\frac{3}{2}$

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55. If
$$f: \left[0, \frac{\pi}{2}\right) \to R$$
 is defined as $f(\theta) = \begin{vmatrix} 1 & \tan \theta & 1 \\ -\tan \theta & 1 & \tan \theta \\ -1 & -\tan \theta & 1 \end{vmatrix}$

Then, the range of f is

56. Prove:
$$\cot^{-1}\left(\frac{1}{2}\right) - \frac{1}{2}\cot^{-1}\left(\frac{4}{3}\right) = \frac{\pi}{4}$$



58. In a triangle ABC, $a^2\cos^2 A = b^2 + c^2$ then triangle is

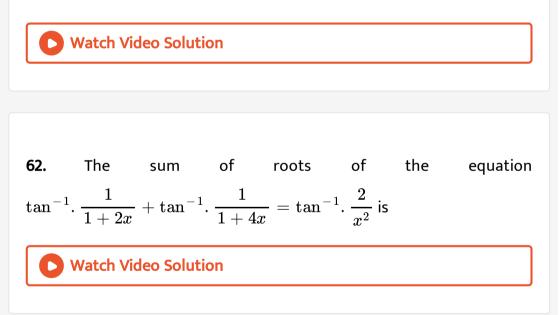
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59. The number of distinct real roots of $|\sin x \cos x \cos x \cos x \sin x \cos x \cos x \sin x| = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is 0 (b) 2 (c) 1 (d) 3

60. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$ has a solution for

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61. In a triangle, the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is



63. For $x \in (0,\pi)$ the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

64. The number of distinct solution of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is _____.

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65. In a triangle ΔXYZ , leta, bandc be the lengths of the sides opposite to the angles X, Y and Z respectively. If $2(a^2 - b^2) = c^2$ and $\lambda = \frac{\sin(X - Y)}{\sin Z}$ then possible values of n for which $\cos(n\pi\lambda) = 0$ is (are)

66. If
$$\alpha = 3\sin^{-1}\left(\frac{6}{11} \text{ and } \beta = 3\cos^{-1}\left(\frac{4}{9}\right)$$
, where the inverse trigonometric functions take only the principal values, then the correct options (s) is (are)

67. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 , are the roots of the equation $x^2 - 2x \sec \theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan \theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals:

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68. Let
$$S = \left\{x \in (-\pi,\pi) : x \neq 0, \pm \frac{\pi}{2}\right\}$$
. The sum of all distinct solutions of the equation $\sqrt{2}\sec x + \cos ecx + 2(\tan x - \cot x)$

= 0 in the set S is equal to

69. The value of
$$\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$$
 is equal to

70. In a ΔXYZ , let x,y,z be the lengths of sides opposite to the angles

X,Y,Z respectively and 2x = x + y + z. If $rac{s-x}{4} = rac{s-y}{3} = rac{s-z}{2}$ and area of incircle of the ΔXYZ is $rac{8\pi}{3}$ then

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71. If $f(x)|\cos(2x)\cos(2x)\sin(2x) - \cos x \cos x - \sin x \sin x \sin x \cos x|$, then: af'(x) = 0 at exactly three point in $(-\pi, \pi)$ bf'(x) = 0 at more than three point in $(-\pi, \pi)$ cf(x) attains its maximum at x = 0 d f(x) attains its minimum at x = 0

72. Let α, β be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true ?

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73. Let O be the origin and \overrightarrow{OX} , \overrightarrow{OY} , \overrightarrow{OZ} be three unit vector in the directions of the sides \overrightarrow{QR} , \overrightarrow{RP} , \overrightarrow{PQ} respectively, of a triangle PQR. if the triangle PQR varies , then the manimum value of $\cos(P+Q) + \cos(Q+R) + \cos(R+P)$ is