MATHS

BOOKS - OBJECTIVE RD SHARMA ENGLISH

CONTINUITY AND DIFFERENTIABILITY

Illustration

1. For what value of k, the function

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases},$$

is continuous at x =2.

A. 0

B. 4

C. 6

D. none of these

Answer: B

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2. The function $f: R \sim \{0\}^{\rightarrow}$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ can be made continuous at x = 0 by defining f(0) as

A. 0

B. 1

C. 2

D. -1

Answer: B

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3. If $f(x) = \frac{1 - \sin x}{(\pi - 2x)^2}, whenx \neq \frac{\pi}{2} andf\left(\frac{\pi}{2}\right) = \lambda,$ the $f(x)$ will be continuous function at $x = \frac{\pi}{2}, where\lambda =$ $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) none of these

A. $1/8$

B. $1/4$

C. $1/2$

D. none of these

Answer: A

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4. If $f(x)=\frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}$ for $x\neq\frac{\pi}{4}$, find the value which can be assigned to $f(x)$ at $x=\frac{\pi}{4}$ so that the function $f(x)$ becomes continuous every where in $\left[0, \frac{\pi}{2}\right]$.

A. 1

B. $1/2$

C. 2

D. none of these

Answer: B

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5. If $f(x) = \begin{cases} \frac{\sin(\cos x) - \cos x}{(\pi - 2x)^2} & x \neq \frac{\pi}{2} \\ k & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then k is equal to

A. 0

B. $-\frac{1}{6}$

C. $-\frac{1}{24}$

D. $-\frac{1}{48}$

Answer: D

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6. If $f(x)=\begin{cases}\frac{(4^x-1)^3}{\sin(x/4)\log(1+x^2/3)} & x\neq 0\\ k & x=0\end{cases}$ is a continous at x=0, then k=

A. $12(\log, 4)^2$

B. $96(\log, 2)^3$

C. $(\log, 4)^3$

D. none of these

Answer: B

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7. Given a real valued function f such that $f(x)=\begin{cases}\frac{\tan^2[x]}{x^2-[x]^2}, x<0 \text{ and } 1, x=0 \text{ and } \sqrt{\{x\}\cot\{x\}}, x<0\end{cases}$

where [.] represents greatest integer function then

A. $A=-3, B=-\sqrt{3}$

B. $A = 3, B = -\frac{\sqrt{3}}{2}$

C. $A = -3, B = -\frac{\sqrt{3}}{2}$

D. $A = -\frac{\sqrt{3}}{2}, B = -3$

Answer: C

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8. Prove that the greatest integer function $[x]$ is continuous at all points except at integer points.

A. N

B. Z

C. R

D. ϕ

Answer: B

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9. Let |x| be the greatest integer less than or equal to x, Then f(x)= $x\cos(\pi(x+[x])$ is continous at

A. $x = -1$

B. $x = 0$

C. $x = 2$

D. $x - 1$

Answer: B

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10. If $f(x) = \begin{cases} x^m \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is a continous at x=0, then

A. $m \in (0, \infty)$

B. $m \in (-\infty, 0)$

C. $m \in (1, \infty)$

D. $m \in (-\infty, 1)$

Answer: A

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11. $Let\ f(x) = \frac{1-\tan x}{4x-\pi}, x \neq \frac{\pi}{4}, x \in \left[0, \frac{\pi}{2}\right]$, If $f(x) is$ continuous in $\left[0, \frac{\pi}{4}\right]$, then find the value of $f\left(\frac{\pi}{4}\right)$.

A. 1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -1

Answer: C

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12. The function, $f(x) = [|x|] - |[x]|$ where [] denotes greatest integer function:

A. continous everywhere

B. continous at integer points only

C. continous at non-integer points only

D. nowhere continous

Answer: C

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13. Let f(x)= $\begin{cases} \dfrac{\tan x - \cot x}{x - \frac{\pi}{4}} & x \neq \frac{\pi}{4} \\ a & x = \frac{\pi}{4} \end{cases}$

The value of a so that f(x) is a continous at $x = \pi / 4$ is.

A. 2

B. 4

C. 3

D. 1

Answer: B

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14. $f(x) = \begin{cases} \frac{\sqrt{1+px} - \sqrt{1-px}}{x}, & -1 \le x < 0 \frac{2x+1}{x-2}, 0 \ge x \ge 1 \end{cases}$ is continuous in the interval $[-1, 1]$, then p is equal to -1 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$

(d) 1

A. -1

B. $-1/2$

C. $1/2$

D. 1

Answer: B

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15. The function f(x)= $\begin{cases} x^2/a & 0 \le x < 1 \\ a & 1 \le x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^2} & \sqrt{2} \le x < \infty \end{cases}$ and if it is continous at x=1, $\sqrt{2}, then$ a and b` is equal to

A. -2

B. -4

C. -6

D. -8

Answer: B

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16. If `f(x)={a x^2+b ,0lt=x<1 4,x=1x+3,1

A. (2,2)

B. (3,1)

C. (4,0)

D. (5,12)

Answer: D

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17. $f: R \rightarrow R$ is defined by $f(x) = \begin{cases} \dfrac{\cos 3x - \cos x}{x^2}, x \neq 0\lambda, x = 0 \end{cases}$ and f is continuous at $x = 0;$ then $\lambda =$

A. -2

B. `-4

C. -6

D. -8

Answer: B

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18. If $f(x) = \begin{cases} \frac{1-\sqrt{2}\sin x}{\pi - 4x}, & \text{if } x \neq \frac{\pi}{4} \\ a, & \text{if } x = \frac{\pi}{4} \end{cases}$ in continuous at $\frac{\pi}{4}$, then a is equal to :

A. 4

B. 2

C. 1

D. $1/4$

Answer: D

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19. Let $f(x) = \frac{\sin x}{x}, x \neq 0$. Then f(x) can be continous at x=0, if

A. $f(0) = 0$

B. $f(0) = 1$

C. $f(0) = 2$

D. $f(0) = -2$

Answer: B

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20. Let $a, b \in R, (a \in 0)$. If the funtion f defined as

$$f(x) = \begin{cases} \frac{2x^2}{a} & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^3} & \sqrt{2} < x < \infty \end{cases}$$ is a continous in $[0, \infty)$. Then, (a,b)=

A. $(\sqrt{2}, 1 - \sqrt{3})$

B. $(-\sqrt{2}, 1 - \sqrt{3})$

C. $(\sqrt{2}, -1 + \sqrt{3})$

D. $(-\sqrt{2}, 1 + \sqrt{3})$

Answer: A

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21. Let f(x)=[cosx+ sin x], $0 < x < 2\pi$, where [x] denotes the greatest integer less than or equal to x. The number of points of discontinuity of f(x) is

A. 6

B. 5

C. 4

D. 3

Answer: C

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22. If function f(x) given by

$$f(x) = \begin{cases} (\sin x)^{1/(\pi - 2x)} & x \neq \pi/2 \\ \lambda & x = \pi/2 \end{cases}$$ is continous at $x = \frac{\pi}{2}$ then λ=

A. e

B. 1

C. 0

D. none of these

Answer: B

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23. If $f(x) = \{x^2\} - (\{x\})^2$, where (x) denotes the fractional part of x, then

A. $f(x)$ is continuous at $x = 2$ but not at $x = -2$

B. $f(x)$ is continuous at $x = -2$ but not at $x = 2$

C. $f(x)$ is continuous at $x = 2 \text{ and } x = -2$

D. $f(x)$ is discontinuous at $x = 2 \text{ and } x = -2$

Answer: A

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24. If $f(x) = [x]\sin\left(\frac{\pi}{[x+1]}\right)$, where [.] denotes the greatest integer function, then the set of point of discontiuity of f in its domain is

A. Z

B. $Z - \{-1, 0\}$

C. $R - [-1, 0)$

D. none of these

Answer: B

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25. The function f(x)=(x) where (x) denotes the smallest integer $\geq x$ is

A. everywhere continuous

B. continuous at x=n, $n \in Z$

C. continuous on R-Z

D. none of these

Answer: C

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26. Let $f(x) = [x^3 - 3]$, where [.] is the greatest integer function, then the number of points in the interval (1,2) where function is discontinuous is (A) 4 (B) 5 (C) 6 (D) 7

A. 4

B. 2

C. 6

D. none of these

Answer: C

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27. Let $f(x) = \dfrac{e^{\tan x} - e^{x} + \ln(\sec x + \tan x) - x}{\tan x - x}$ be a continous function at $x = 0$. The value of $f(0)$ equals:

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 2

Answer: C

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28. Find the value of x where function ,

$f(x) = \begin{cases} x & \text{if x is rational} \\ 1 - x & \text{ if x is irrational} \end{cases}$ is continuous.

A. ∞

B. 1

C. 0

D. none of these

Answer: C

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29. It is given that f' (a) exists, then $\lim_{x \to a} \frac{xf(a) - af(x)}{x - a}$ is equal to:

A. $f(a) - af'(a)$

B. $f'(a)$

C. $-f'(a)$

D. $f(a) + af'(a)$

Answer: A

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30. If $f(2) = 4$ and $f'(2) = 1$, then find $(\lim)_{x \to 2} \frac{x f(2) - 2f(x)}{x - 2}$.

A. 2

B. 4

C. -2

D. 1

Answer: A

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31. If f(3)=6 and f'(3)=2, then $\lim_{x \to 3} \frac{xf(3) - 3f(x)}{x - 3}$ is given by

A. 6

B. 4

C. 0

D. none of these

Answer: C

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32. Let f(x) =|x| and g(x)=|x| where [.] denotes the greatest function. Then, (fog)' (-2) is

A. 0

B. 1

C. -1

D. non-existent

Answer: D

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33. If $f(x)$ is differentiable and strictly increasing function, then the value of $(\lim)_{x \to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is 1 (b) 0 (c) -1 (d) 2

A. 1

B. 0

C. -1

D. 2

Answer: C

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34. If f(x)=$\begin{cases} x-5 \text{for} & x \leq 1 \\ 4x^2-9 \text{for} & 1 < x < 2 \text{then} & f'(2+) \\ 3x+4 \text{for} & x \geq 2 \end{cases}$

A. 0

B. 2

C. 3

D. 4

Answer: C

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35. If $f : R \to R$ is defined by $f(x) = \begin{cases} \frac{x-2}{x^2-3x+2} & \text{if } x \in R-(1,2) \\ 2 & \text{if } x = 1 \\ 1 & \text{if } x = 2 \end{cases}$

them $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} =$

A. 0

B. -1

C. 1

D. $-1/2$

Answer: B

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36. If f(4)= 4, f'(4) =1 then $\lim_{x \to 4} 2\left(\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}\right)$ is equal to

A. -1

B. 1

C. 2

D. -2

Answer: B

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37. Let $f(x)$ be a twice-differentiable function and $f''(0) = 2$. Then evaluate $\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$.

A. 3a

B. 2a

C. 5a

D. 4a

Answer: A

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38. Suppose f(x) is differentibale for all x and $\lim_{h \to 0} \frac{1}{h}(1+h) = 5 \text{then f'}(1) \text{ equals}$

A. 6

B. 5

C. 4

D. 3

Answer: B

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39. If f is a real- valued differentiable function satisfying $|f(x) - f(y)| \leq (x-y)^2, x, y, \in R \text{ and } f(0) = 0$ then f(1) equals

A. 1

B. 2

C. 0

D. -1

Answer: C

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40. Let $f: R \to R$ be a function defined by $f(x) = \min\{x+1, |x|+1\}$. Then which one of the following is true?

A. $f(x) > 1$ for all $x \in R$

B. $f(x)$ is not differentiable at x=1

C. $f(x)$ is everywhere differentiable

D. $f(x)$ is not differentiable at x=0

Answer: C

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41. Let $f(x) = \begin{cases} (x-1)^2 \sin\left(\frac{1}{x-1}\right) - |x| & ; x \neq 1 \\ -1 & ; x = 1 \end{cases}$ then which one of the following is true?

A. f is differential at x=0 but not at x=1

B. f is differentiable at x=1 but not at x=0

C. f is neither differentiable at x=0 nor at x=1

D. f is differentiable at x=0 and at x=1

Answer: A

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42. Let $f: R \rightarrow R$ be a function defined by $f(x) = \max\{x, x^3\}$. The set of all points where f(x) is not differentiable, is

A. $\{-1, 1\}$

B. $\{-1, 0\}$

C. $\{0, 1\}$

D. $\{-1, 0, 1\}$

Answer: D

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43. If $f(x) = \begin{cases} x & x \leq 1 \quad \text{and } f'x(x) \\ x^2 + bx + c & x > 1 \end{cases}$ and exists finetely for all $x \in R$, then

A. $b = -1, c \in R$

B. $c = 1, b \in R$

C. $b = 1, c = -1$

D. $b = -1, c = 1$

Answer: D

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44. Let $f(x) = a + b|x| + c|x|^2$, where a,b,c are real constants. The, f'(0) exists if

A. b=0

B. c=0

C. a=0

D. b=c

Answer: A

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45. Draw a graph of the function $y = [x] + |1 - x|, \ -1 \leq x \leq 3$. Determine the points if any where this function is not differentiable.

A. $(-1, 0, 1, 2, 3)$

B. $(-1, 0, 2)$

C. $(0, 1, 2, 3)$

D. $(-1, 0, 1, 2)$

Answer: C

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46. The number of points in $(1, 3)$, where $f(x) = a\left(\left(\left[x^2\right]\right), a > 1\right.$ is not differential is

A. 0

B. 3

C. 5

D. 7

Answer: D

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47. Let $f(x) = p[x] + qe^{-[x]} + r|x|^2$, where p,q and r are real constants,

If f(x) is differential at x=0. Then,

A. $q = 0, r = 0, p \in R$

B. $p = 0, r = 0, q \in R$

C. $p = 0, q = 0, r \in R$

D. none of these

Answer: C

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48. If g is inverse of fand $f'(x) = \frac{1}{1 + x^n}$, then g'(x) equals

A. $\frac{1}{1 + (g(x)^n)}$

B. $1 + (g(x)^n)$

C. $(g(x)^n) - 1$

D. none of these

Answer: B

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49. Let f and g be differentiable functions satisfying g'(a) = 2 g(a) = b and fog = I (Identity function). Then f'(b) is equal to

A. 2

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. none of these

Answer: C

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50. If $f(x) = x + \tan x$ and f is the inverse of g, then $g'(x)$ is equal to

A. $\dfrac{1}{1+[g(x)-x]^2}$

B. $\dfrac{1}{2+[g(x)-x]^2}$

C. $\dfrac{1}{2+[g(x)-x]^2}$

D. none of these

Answer: C

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51. If g is the inverse of a function f and $f'(x) = \dfrac{1}{1+x^5}$, then g'(x) is equal to

A. $\dfrac{1}{1+(g(x))^5}$

B. $1+\{g(x)\}^5$

C. $1+x^5$

D. $5x^4$

Answer: B

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52. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{if } |x| > 2 \quad \text{then} \quad f(x) is \\ a + bx^2 & \text{if} |x| \leq 2 \end{cases}$ is differentiable at x=-2 for

A. $a = \frac{3}{4}, b = \frac{1}{6}$

B. $a = \frac{3}{4}, b = \frac{1}{16}$

C. $a = -\frac{1}{4}, b = \frac{1}{16}$

D. $a = \frac{1}{4}, b = -\frac{1}{16}$

Answer: B

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53. If the function $g(X) = \begin{cases} k\sqrt{x+1} & 0 \leq x \leq 3 \\ mx + 2 & 3 < x \leq 5 \end{cases}$ is differentiable , then the value of K+ m is

A. $\frac{10}{3}$

B. 4

C. 2

D. $\frac{16}{5}$

Answer: C

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54. Let a and b be real numbers such that the function

$g(x) = \begin{cases} -3ax^2 - 2 & x < 1 \\ bx + a^2 & x \geq 1 \end{cases}$ is differentiable for all $x \in R$

Then the possible value(s) of a is (are)

A. 1,2

B. 3,4

C. 5,6

D. 8,9

Answer: A

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55. If the function

$f(x) = \begin{cases} -x & x < 1 \\ a + \cos^{-1}(x+b) & 1 \le x \le 2 \end{cases}$ is differentiable at x=1, then $\frac{a}{b}$ is equal to

A. $\frac{-\pi - 2}{2}$

B. $-1 - \cos^{-1}$

C. $\frac{\pi}{2} + 1$

D. $\frac{\pi}{2} - 1$

Answer: C

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56. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}, 0 < x < 2$ m and n integers, $m \ne 0, n > 0$ and. If $\lim_{x \to 1+} g(x) = -1$, then

A. $n = 1, m = 1$

B. $n = 1, m = -1$

C. $n = 2, m = 2$

D. $n > 2, m = n$

Answer: C

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Section I - Solved Mcqs

1. The function $f(x) = [x]^2 - [x^2]$, where [y] is the greatest integer less than or equal to y, is discontinuous at

A. all integers

B. all integers except 0 and 1

C. all integers except 0

D. all integers except 1

Answer: D

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2. The function $f(x) = [x^2] + [-x]^2$, where [.] is GIF is

A. continuous and derivable at x=2

B. neither continuous nor derivable at x=2

C. continuous but not dervable at x=2

D. none of these

Answer: B

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3. Let $f: R \to R$ be any function. Also $g: R \to R$ is defined by $g(x) = |f(x)|$ for all x. Then g is

a. Onto if f is onto b. One-one if f is one-one c. Continuous if f is continuous d. None of these

A. onto if if is onto

B. one-one if f is one-one

C. continuous if f is continuous

D. differentiable if f is differentiable

Answer: C

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4. The left hand derivative of $f(x) = [x]\sin(\pi x)$ at $x = k, k \in Z$, is

A. $(-1)^k(k-1)\pi$

B. $(-1)^{k-1}(k-1)\pi$

C. $(-1)^k k\pi$

D. $(-1)^{k-1}k\pi$

Answer: A

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5. Which of the following functions is differentiable at $x = 0$?

A. $\cos(|x|) + |x|$

B. $\cos(|x|) - |x|$

C. $\sin(|x|) + |x|$

D. $\sin(|x|) - |x|$

Answer: D

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6. The domain of the derivative of the function

$f(x) = \{(\tan^{-1} x, \quad \text{if} \quad |x| \leq 1), \left(\frac{1}{2}(|x| - 1), \quad \text{if} \quad |x| > 1\right):\}$

A. $R - \{0\}$

B. $R - \{1\}$

C. $4 - \{-1\}$

D. $R - \{-1, 1\}$

Answer: D

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7. The set of all points where the function $f(x) = 3\sqrt{x^2|x|}$ is differentiable, is

A. $[0, \infty)$

B. $(0, \infty)$

C. $(-\infty, \infty)$

D. $(-\infty 0) \cup (0, \infty)$

Answer: D

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8. Let $f(x)=|x|+|\sin x|, x \in (-\pi/2, \pi/2)$. Then, f is

A. nowhere continuous

B. continuous and differentiable everywhere

C. nowhere differentiable

D. differentiable everywhere except at x=0

Answer: D

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9. If the function $f(x)=\left[\frac{(x-2)^3}{a}\right]\sin(x-2)+a\cos(x-2), [.]$ denotes the greatest integer function, is continuous in $[4, 6]$, then find the values of a.

A. $a \in [8, 64)$

B. $a \in [0, 8)$

C. $a \in [64, \infty)$

D. none of these

Answer: C

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10. If F(x) = $\begin{cases} \dfrac{\sin\{\cos x\}}{x - \frac{\pi}{2}}, x \neq \dfrac{\pi}{2} \text{ and } 1, x = \dfrac{\pi}{2}, \end{cases}$ where {.} represents the fractional part function, then $\lim_{x \to \pi/2} f(x)$ is

A. continuous at $x = \pi/2$

B. $\lim_{x \to \pi/2} f(x)$ but f(x) is not continuous at $x = \pi/2$

C. $\lim_{x \to \pi/2} f(x)$ does not exist

D. $\lim_{x \to \pi/2^-} f(x) = -1$

Answer: B

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11. If $\alpha, \beta(\alpha, \beta)$ are the points of discontinuity of the function $f(f(x))$, where $f(x) = \frac{1}{1-x}$, then the set of values of a foe which the points (α, β) and (a, a^2) lie on the same side of the line $x + 2y - 3 = 0$, is

A. $(-3/2, 1)$

B. $[-3/2, 1]$

C. $[1, \infty)$

D. $(-\infty, -3/2]$

Answer: A

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12. The function $f(x)$ given by $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is

A. everywhere differentiable such that $f'(x) = -\frac{2}{1+x^2}$

B. such that f'(x) $= \begin{cases} \frac{2}{1+x^2} & -1 < x < 1 \\ \frac{-2}{1+x^2} & |x| > 1 \end{cases}$

C. such that f'(x) $= \begin{cases} \frac{-2}{1+x^2} & -1 < x < 1 \\ \frac{+2}{1+x^2} & |x| > 1 \end{cases}$

D. not differentiable at infinitely many points.

Answer: B

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13. Let f(x) be the function given by $f(x) = arc\cos\left(\frac{1-x^2}{1+x^2}\right). Then$

A. f(x) is everywhere differential such that $f'(x) = \frac{2}{1+x^2}$

B. $f'(x) = \begin{cases} \frac{2}{1+x^2} & x > 0 \\ \frac{-2}{1+x^2} & x < 0 \end{cases}$

C. $f'(x) = \begin{cases} \frac{-2}{1+x^2} & x > 0 \\ \frac{2}{1+x^2} & x < 0 \end{cases}$

D. f'(x) exists at x=0

Answer: B

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14. If $f(x) = \sin^{-1}\left(2x\sqrt{1 - x^2}\right), x \in [-1, 1]$. Then

A. $f'(x) = \dfrac{2}{\sqrt{1 - x^2}}$, for all $x \in (-1, 1)$

B. $f'(x) = \begin{cases} \frac{2}{\sqrt{1-x^2}} & \text{If } |x| < \frac{1}{\sqrt{2}} \\ \frac{-2}{\sqrt{1-x^2}} & \text{If } \frac{1}{\sqrt{2}} < |x| < \frac{1}{2} \end{cases}$

C. $f'(x) = \begin{cases} \frac{-2}{\sqrt{1-x^2}} & \text{If } |x| < \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{1-x^2}} & \text{If } \frac{1}{\sqrt{2}} < |x| < 1 \end{cases}$

D. f(x) exists for all $x \in [-1, 1]$

Answer: B

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15. If $f(x) = \cos^{-1}\left(2x^2 - 1\right), x \in [-1, 1]$. Then

A. f(x) is differentiable on (-1,1) such that $f'(x) = \dfrac{-2}{\sqrt{1 - x^2}}$

B. f(x) is differentiable on $(-1, 0) \cup (0, 1)$ such that

$f'(x) = \dfrac{-2}{\sqrt{1 - x^2}}$

C. f(x) is differentiable on $(-1, 0) \cup (0, 1)$ such that

$$f'(x) = \begin{cases} \frac{-2}{\sqrt{1-x^2}} & 0 < x < 1 \\ \frac{2}{\sqrt{1-x^2}} & -1 < x < 0 \end{cases}$$

D. f(x) is differentiable on (-1,1) such that

$$f'(x) = \begin{cases} \frac{-2}{\sqrt{1-x^2}} & 0 \leq x < 1 \\ \frac{2}{\sqrt{1-x^2}} & -1 < x \leq 0 \end{cases}$$

Answer: C

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16. If $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right), x \in R$then $f'(x)$ is given by

A. $f'(x) = \frac{2}{1+x^2}$for all x $\in R(-1, 1)$

B. $f'(x) = \frac{2}{1+x^2}$for all x $\in R$

C. $F'(x) = \begin{cases} \frac{2}{1+x^2} & \text{if } |x| \leq 1 \\ \frac{-2}{1+x^2} & \text{if } |x| > 1 \end{cases}$

D. $f'(x) = \begin{cases} \frac{2}{1+x^2} & \text{if } |x| < 1 \\ \frac{-2}{1+x^2} & \text{if } |x| > 1 \end{cases}$

Answer: A

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17. If $y = \sin^{-1}(3x - 4x^3)$, then the number of points in $[-1, 1]$, where y is not differentiable is

A. $f'(x) = -\frac{3}{\sqrt{1-x^2}}$ for all $x \in (-1, 1)$

B. $f'(x) = \frac{3}{\sqrt{1-x^2}}$ for all $x \in [-1, 1]$

C. $f'(x) = \begin{cases} \frac{3}{\sqrt{1-x^2}} & \text{if } -\frac{1}{2} < x < \frac{1}{2} \\ \frac{-3}{\sqrt{1-x^2}} & \text{if } \frac{1}{2} < x < 1 \text{ or }, -1 < x < -\frac{1}{2} \end{cases}$

D. $f'(x) = \begin{cases} \frac{3}{\sqrt{1-x^2}} & \text{if } |x| < \frac{\sqrt{3}}{2} \\ \frac{-3}{\sqrt{1-x^2}} & \text{if } 1 > |x| > \frac{\sqrt{3}}{2} \end{cases}$

Answer: C

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18. If $f(x) = \cos^{-1}(4x^3 - 3x), x \in [-1, 1]$, then

A. $f'(x) = \frac{-3}{\sqrt{1-x^2}}$ for all $x \in [-1, 1]$

B. $f'(x) = \frac{-3}{\sqrt{1-x^2}}$ for all $x \in [-1, 1]$

C. $f'(x) = \begin{cases} \frac{-3}{\sqrt{1-x^2}} & \text{if } |x| < \frac{1}{2} \\ \frac{3}{\sqrt{1-x^2}} & \text{if } \frac{1}{2} < |x| < \frac{1}{2} \end{cases}$

D. $f'(x) = \begin{cases} \frac{-3}{\sqrt{1-x^2}} & \text{if } |x| < \frac{1}{2} \\ \frac{-3}{\sqrt{1-x^2}} & \text{if } < x < \frac{-1}{2}, \frac{1}{2} < x < 1 \end{cases}$

Answer: D

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19. Prove that

$$3\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & \text{if } -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & \text{if } x > \frac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & \text{if } x < -\frac{1}{\sqrt{3}} \end{cases}$$

A. $f'(3) = \frac{3}{1+x^2}$ for all x $\in R - \left\{\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}$

B. $f'(x) = \frac{3}{1+x^2}$ for all $x \in R$

C. f(x) is not differentiable at infinitely many points.

D. none of these

Answer: A

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20. The function $f(x) = \sin^{-1}(\sin x)$, is

A. continuous but not differentiable at $x = \pi$

B. continuous and differentiable at x=0

C. discontinuous at $x = -\pi$

D. none of these

Answer: B

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21. The function, $f(x) = \cos^{-1}(\cos x)$ is

A. discontinuous at infinitely many-points

B. everywhere differentiable such that f'(x)=1

C. not differentiable at $x = n\pi, n \in Z$ and $f'(x) = 1, x \neq n\pi$

D. not differentiable at $x = n\pi, n \in Z$ and $f'(x) = (-1)^n, x \in (n\pi, (n+1)\pi), n \in Z$

Answer: D

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22. The function $f(x) = \tan^{-1}(\tan x)$ is

A. everywhere continuous

B. discontinuous at $x = \frac{n\pi}{2}, n \in Z$

C. not differentiable at x

D. everywhere continuous and differentiable such that f'(x)=1 for all $x \in R$

Answer: C

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23. Number of points where the function f(x)= $\text{Maximum}\left[\text{sgn}\,(\text{x}),\ -\sqrt{9-x^2}, x^3\right]$ is continuous but not differentiable, is

A. 4

B. 2

C. 5

D. 6

Answer: C

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24. The function $f(x)=\dfrac{1}{\log|x|}$ is discontinuous at

A. {0}

B. {-1,1}

C. {-1,0,1}

D. none of these

Answer: C

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25. Let $f(x) = \frac{\sin(\pi[x - \pi])}{1 + [x^2]}$ where [] denotes the greatest integer function then f(x) is

A. continuous at integer points

B. continuous everywhere

C. differentiable once but f''(x) and f''' (x) do not exist

D. differentiable for all x

Answer: B::D

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26. If $f(x) = \begin{cases} ax^2 - b & a \leq x < 1 \\ 2 & x = 1 \\ x + 1 & 1 \leq x \leq 2 \end{cases}$ then the value of the pair (a,b) for which f(x) cannot be continuous at x=1, is

A. (2,0)

B. (1,-1)

C. (4,2)

D. (1,1)

Answer: D

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27. If $f(x) = \frac{[x]}{|x|}, x \neq 0$, where [.] denotes the greatest integer function, then f'(1) is

A. -1

B. 1

C. non-existent

D. none of these

Answer: C

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28. Let $f(x) = [|x|]$ where [.] denotes the greatest integer function, then $f'(-1)$ is

A. 0

B. 1

C. non-existent

D. none of these

Answer: C

Watch Video Solution

29. If $f(x) = [x][\sin x]$ in $(-1, 1)$ then f(x) is

A. continuous on (-1,0)

B. differentiable on (-1,1)

C. differentiable at x=0

D. none of these

Answer: A

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30. If $f(x-y), f(x)f(y)$ and $f(x+y)$ are in A.P. for all $x, y,$ and $f(0) \neq 0,$ then (a) $f(4) = f(-4)$ (b) $f(2) + f(-2) = 0$ (c) $f'(4) + f'(-4) = 0$ (d) $f'(2) = f'(-2)$

A. $f'(2) = f'(2)$

B. $f'(-3) = -f'(3)$

C. $f'(-2) + f'(2) = 0$

D. none of these

Answer: A

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31. Let $f(x) =$ Degree of $\left(u^{x^2} + u^2 + 2u + 3\right)$. Then, at $x = \sqrt{2}$, $f(x)$ is

A. continuous but not differentiable

B. differentiable

C. dicontinuous

D. none of these

Answer: A

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32. LEt $F: R \to R$is a differntiable function

$f(x+2y) = f(x) + f(2y) + 4xy$ for all $x, y \in R$

A. $f'(1) = f'(0) + 1$

B. $f'(1) = f'(0) - 1$

C. $f'(0) = f'(1) + 2$

D. $f'(0) = f'(1) - 2$

Answer: D

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33. Let $f: R \to R$ be a function given by

$f(x+y) = f(x)f(y)\text{for all} \quad x, y \in R$

If $f(x) \neq 0$ for all $x \in R$ and f'(0) exists, then f'(x) equals

A. f(x) for all $x \in R$

B. f(x) f'(0) for all $x \in R$

C. f(x)+f'(0) for all $x \in R$

D. none of these

Answer: B

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34. Let $f: R \in R$ be a function given by

$f(x+y) = f(x)f(y)$for all$x, y \in R$

If $f(x) \neq 0$, for all $x \in R$ and $f'(0) = \log 2$, then $f(x) =$

A. x^2

B. 2^x

C. $x(\log 2)$

D. e^{2x}

Answer: B

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35. Let $f: R \to R$ be a function given by

$f(x+y) = f(x)f(y)$for all $x, y \in R$

If $f(x) = 1 + xg(x)$, $\log_e 2$, where $\lim_{x \to 0} g(x) = 1. Then, f'(x) =$

A. $\log_e 2^{f(x)}$

B. $\log_e (f(x))^2$

C. $\log_e 2$

D. none of these

Answer: A

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36. Let $f: R \to R$ be a function given by $f(x+y) = f(x)f(y)$ for all x,y $\in$ R .If $f'(0) = 2$ then $f(x)$ is equal to`

A. Ae^x

B. Ae^{2x}

C. 2x

D. none of these

Answer: B

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37. If a differentiable function f defined for $x > 0$ satisfies the relation $f(x^2) = x^3, x > 0$, then what is the value of $f'(4)$?

A. 2

B. 3

C. 4

D. none of these

Answer: B

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38. If $f(x+y)=2f(x)f(y)$ for all x,y where f'(0)=3 and f(4)=2, then f'(4) is equal to

A. 6

B. 12

C. 4

D. none of these

Answer: B

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39. Let $f: R \to R$ be a function given by $f(x+y)=f(x)f(y)$ for all $x, y \in R$

If $f(x)=1+xg(x)+x^2g(x)\phi(x)$ such that $\lim_{x\to 0} g(x)=a$ and $\lim_{x\to 0} \phi$

then f'(x) is equal to

A. $(a+b)f(x)$

B. af(x)

C. bf (x)

D. abf (x)

Answer: B

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40. Let $f: R \to R$ be a function satisfying $f(x+y)=f(x)+f(y) \text{for all } x, y \in R$

If $f(x)=x^3 g(x) \text{for all } x, y \in R$, where g(x) is continuous, then f'(x) is equal to

A. g(0)

B. g'(x)

C. 0

D. none of these

Answer: C

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41. Let $f: R \to R$ be a function given by

$f(x+y) = f(x) + 2y^2 + \text{kxy for all } \; x, y \in R$

If $f(1) = 2$. Find the value of f(x)

A. $2x^2$

B. $x^2 + 3x - 2$

C. $-x^2 + 3x - 2$

D. $-x^2 + 9x - 6$

Answer: A

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42. Let $f: R \to R$ be a function satisfying $f(x+y) = f(x) + \lambda xy + 3x^2y^2$ for all $x, y \in R.$ If $f(3) = 4$ and $f(5) = 52$ then f'(x) is equal to

A. 10x

B. $-10x$

C. 20x

D. 128x

Answer: B

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43. Let f be a differential function satisfying the condition. $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all $x, y(\neq 0) \in R$ and $f(y) \neq 0$ If f'(1)=2`, then f'(x) is equal to

A. 2f(x)

B. $\frac{f(x)}{2}$

C. 2x f(x)

D. $\frac{2f(x)}{x}$

Answer: D

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44. Let f(x) be a real function not identically zero in Z, such that for all $x, y \in R \ f(x + y^{2n+1}) = f(x) = \{f(y)^{2n+1}\}, n \in Z$

If $f'(0) \geq 0$, then f'(6) is equal to

A. 0

B. 1

C. 2

D. 6

Answer: B

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45. Let $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ for all real x and y. If $f'(0)$ exits and equals -1 and $f(0) = 1$, then find $f(2)$.

A. -1

B. 1

C. 0

D. none of these

Answer: A

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46. Let $f: R \to R$ be given by $f(x+y) = f(x) - f(y) + 2xy + 1 \text{for all } x, y \in R$ If f(x) is everywhere differentiable and $f'(0) = 1$, then f'(x)=

A. 2x+1

B. 2x-1

C. x+1

D. x-1

Answer: B

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47. If $f(x) = |2 - x| + (2 + x)$, where (x)=the least integer greater than or equal to x, them

A. $\lim_{x \to 2^-} f(x) = f(2) = 2$

B. f(x) is continuous and differentiable at x=2

C. f(x) is neither continuous nor differentiable at x=2

D. f(x) is continuous and non-differentiable at x=2

Answer: C

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48. If $f(x)=\frac{[x]}{|x|}, x\neq 0$ where [.] denotes the greatest integer function, then f'(1) is

A. -1

B. 1

C. non-existent

D. ∞

Answer: C

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49. If $4x+3|y|=5y$, then y as a function of x is

A. differentiable at x=0

B. continuous at x=0

C. $\frac{dy}{dx} = 2\text{for all x}$

D. none of these

Answer: B

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50. Let $f(x) = \log_e|x-1|, x \neq 1$, then the value of $f'\left(\frac{1}{2}\right)$ is

A. -2

B. 2

C. non-existent

D. 1

Answer: A

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51. Let a function f(x) defined on [3,6] be given by $f(x) = \begin{cases} \log_e [x] & 3 \le x < 5 \\ |\log_e x| & 5 \le x < 6 \end{cases}$ then f(x) is

A. continuous and differentiable on [3,6]

B. continuous on [3,6] but not differentiable at x=4,5

C. differentiable on [3,6] but not continuous at x=4,5

D. none of these

Answer: D

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52. If $f(x) = \begin{cases} e^x & x < 2 \\ ax + b & x \ge 2 \end{cases}$ is differentiable for all $x \in R$, them

A. $a = e^2, b = -e^2$

B. $a = -e^2, b = e^2$

C. $a = b = e^2$

D. none of these

Answer: A

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53. If the function f(x) is given by $f(x) = \begin{cases} 2^{1/(x-1)} & x < 1 \\ ax^2 + bx & x \geq 1 \end{cases}$ is everywhere differentiable, then

A. a=0, b=1

B. a-0, b=0

C. a=1, b=0

D. none of these

Answer: B

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54. Let $f(x) = \sin x$, $g(x) = [x + 1]$ and $h(x) = gof(x)$ where [.] the greatest integer function. Then $h'\left(\frac{\pi}{2}\right)$ is

A. 1

B. -1

C. non-existent

D. none of these

Answer: C

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55. If $f(x) = |x - 2|$ and $g(x) = f[f(x)]$, then $g'(x) =$ .... for $x > 20$

.

A. 1

B. 2

C. -1

D. none of these

Answer: A

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56. If $f(x) = sgn(x) = \begin{cases} \frac{|x|}{x}, x \neq 0, 0, x = 0 \end{cases}$ and $g(x) = f(f(x))$, then at $x = 0, g(x)$ is

A. continuous and differentiable

B. continuous but not differentiable

C. differentiable but not continuous

D. neither continuous nor differentiable

Answer: D

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57. Let $f(x) = \cos x$ and $g(x) = [x+1], where[.]$ denotes the greatest integer function, Then $(gof)'(\pi/2)$ is

A. 0

B. 1

C. -1

D. non-existent

Answer: D

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58. $f(x) = \min\{1, \cos x, 1 - \sin x\}, -\pi \leq x \leq \pi,$ then

A. not continuous at $x = \pi/2$

B. continuous but not differentiable at x=0

C. neither continuous nor differentiable at $x = \pi/2$

D. none of these

Answer: B

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59. If [.] denotes the greatest integer function, then $f(x)=[x]+\left[x+\frac{1}{2}\right]$

A. is continuous at $x=\frac{1}{2}$

B. is discontinuous at $x=\frac{1}{2}$

C. $\lim_{x\to\left(\frac{1}{2}\right)} f(x)=2$

D. $\lim_{x\to\left(\frac{1}{2}\right)^{-}} f(x)=1$

Answer: B

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60. If $f(x)=sgn\left(x^5\right)$, then which of the following is/are false (where sgn denotes signum function)

A. continuous and differentiable

B. continuous but not differentiable

C. differentiable but not continuous

D. neither continuous nor differentiable

Answer: A

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61. If $f(x) = |x - 1|$ and $g(x) = f(f(f(x)))$, then $g'(x)$ is equal to:

A. 22

B. 20

C. 18

D. none of these

Answer: A

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62. If f(x)=$\begin{cases} \frac{1}{x} - \frac{2}{e^{2x}-1} & x \neq 0 \\ 1 & x = 0 \end{cases}$

A. f(x) is differentiable at x=0

B. f(x) is not differentiable at x=0

C. $f'(0) = \frac{1}{3}$

D. f(x) is continuous but not differenitable at x=0

Answer: A

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63. Let $f(x) = (-1)^{[x^3]}$, where [.] denotest the greatest integer function. Then,

A. f(x) is discontinuous at x $= n^{1/3}, n \in Z$

B. f(3/2)=1

C. $f'(0) = 0$for all $x \in (-1, 1)$

D. none of these

Answer: A

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64. $f(x) = \frac{1}{1-x}$ and $f^n = fofof.... of$, then the points of discontinuitym of f^(3n)(x) is/are

A. x=2

B. x=0,1

C. x=1,2

D. none of these

Answer: B

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65. Let f(x) = [n + p sin x], $x \in (0, \pi), n \in Z$, p is a prime number and [x] = the greatest integer less than or equal to x. The number of points at which f(x) is not not differentiable is :

A. p

B. p-1

C. 2p+1

D. 2p-1

Answer: D

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66. Determine the values of x for which the following functions fails to be continuous or differentiable $f(x) = \begin{cases} (1-x), & x < 1 \\ (1-x)(2-x), & 1 \leq x \leq 2 \\ (3-x), & x > 2 \end{cases}$ justify your answer.

A. x=1

B. x=2

C. x=1,2

D. none of these

Answer: B

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67. Let [x] denote the greatest integer less than or equal to x and g (x) be given by $g(x)=\begin{cases}[f(x)] & x \in (0,\pi/2) \cup (\pi/2,\pi) \\ 3 & x=\frac{\pi}{2}\end{cases}$

where, $f(x)=\dfrac{2(\sin x-\sin^n x)+|\sin x-\sin^n x|}{2(\sin x-\sin^n x)-|\sin x-\sin^n x|}, n \in R^+$ then at $x=\dfrac{\pi}{2}, g(x)$, is

A. continuous and differentiable when $n>1$

B. continuous and differentiable when $0<n<1$

C. continuous but not differentiable when $n>1$

D. continuous but not differentiable when $0<n<1$

Answer: A

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68. Let $f(x) = \begin{cases} \frac{x}{1+|x|}, & |x| \geq 1 \\ \frac{x}{1-|x|}, & |x| < 1 \end{cases}$, then domain of $f'(x)$ is:

A. discontinuous and non-differentiable at $x = -1, 1, 0$

B. discontinuous and non-differentiable at x=-1, whereas continuous and differentiable at x=0,1

C. discontinuous and non-differentiable at x=-1,1 wheras continuous and differentiable at x=0.

D. none of these

Answer: C

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69. Let $f:[0,1]\rightarrow[0,1]$be a continuous function such that $f(f(x))=1f$ or $allx\in[0,1]$then:

A. $f(x)=x$ for at least one $x\in(0,1)$

B. f(x) will be differential in [0,1]

C. f(x)+x=0 for at least one x such that $0\leq x\leq1$

D. none of these

Answer: A

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70. Let f(x) be a continuous defined for $1\leq x\leq3$. if f(x) takes rational values for all x and f(2) =10, then find the value of f(1.5)

A. 20

B. 5

C. 10

D. none of these

Answer: C

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71. Let f(x) and g(x) be two equal real function such that

$$f(x) = \frac{x}{|x|} g(x), x \neq 0$$

If g(0)=g'(0)=0 and f(x) is continuous at x=0, then f'(0) is

A. 0

B. 1

C. -1

D. non-existent

Answer: A

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72. If f(x) is periodic function with period, T, then

A. f and f' are also periodic

B. f is periodic but f' is not periodic

C. f is periodic but f' is not periodic

D. none of these

Answer: A

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73. If $f(x) = \begin{cases} \frac{e^{x[x]} - 1}{x + [x]} & x \neq 0 \\ 1 & x = 0 \end{cases}$ then

A. $\lim_{x \to 0^+} f(x) = -1$

B. $\lim_{x \to 0^-} f(x) = \frac{1}{e} - 1$

C. f(x) is continuous at x=0

D. f(x) is discontinuous at x=0

Answer: D

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74. Let $f(x)$ be defined on $[-2, 2]$ and be given by

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x-1, & 1 < x \leq 2 \end{cases} \text{ and } g(x) = f(|x|) + |f(x)|.$$

Then find $g(x)$.

A. [-2,2]`

B. $[-2, 0) \cup (0, 2]$

C. $[-2, 1) \cup (1, 2]$

D. $[-2, 0) \cup (0, 1) \cup (1, 2]$

Answer: D

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75. Check the continuity of f(x) = $\begin{cases} \frac{x^2}{2} & \text{if } 0 \le x \le 1 \\ 2x^2 - 3x + \frac{3}{2} & \text{if } 1 < x \le 2 \end{cases}$ at $x = 1$

A. f, f' and f'' are continuous in [0,2]

B. f and f' are continuous in [0,2] whereas f'' is continuous in $[0, 1] \cup (1, 2]$

C. f,f' and f'' are continuous in $[0, 1) \cup (1, 2]$

D. none of these

Answer: A

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76. If $f(x) = \begin{cases} x[x] & 0 \le x < 2 \\ (x-1)[x] & 2 \le x < 3 \end{cases}$ where [.] denotes the greatest integer function, then continutity and diffrentiability of f(x)

A. both f'(1) and f'(2) do not exist

B. f'(1) exist but f'(2) does not exist

C. f'(2) exist but f'(1) does not exist

D. both f'(1) and f'(2) exist

Answer: A

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77. If $f(x) = \begin{cases} 4 & -3 < x < -1 \\ 5 + x & -1 \leq x < 0 \\ 5 - x & 0 \leq x < 2 \\ x^2 + x - 3 & 2 < x < 3 \end{cases}$ then, f(|x|) is

A. differentiable but not continuous in (-3,3)

B. continuous but not differentiable in (-3,3)

C. continuous as well as differentiable in (-3,3)

D. neither continuous nor differentiable (-3,3)

Answer: B

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78. If $f(x) = \begin{cases} (x-a)^n \cos\left(\frac{1}{x-a}\right) & x \neq a \\ 0 & x = a \end{cases}$

then at x=a, f(x) is

A. continuous if $n > 0$ and differentiable if $n > 1$

B. continuous if $n > 1$ and differentiable if $n > 0$

C. continuous and differentiable if $n > 0$

D. none of these

Answer: A

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79. Let f(x) and g(x) be two functions given by $f(x) = -1|x-1|, -1 \leq x \leq 3$ and $g(x) = 2 - |x+1|, -2 \leq x \leq 2$

Then,

A. fog is differentiable at x=-1 and gof is differentiable at x=1

B. for is differentiable at x=-1 and gof is not differentiable at x=1

C. fog is differentiable at x=1 and gof is differentiable at x=-1

D. none of these

Answer: D

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80. Let $y = f(x)$ be definded parametrically as $y = t^2 + t|t|, x = 2t - |t|, t \in R.$ find f(x) and discuss its differentiability ,

A. continuous and differentiable in [-1,1]

B. continuous but not differentiable in [-1,1]

C. continuous in [-1,1] and differentiable in [-1,1] only

D. none of these

Answer: A

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81. Let f(x) be a function defined as

$$f(x) = \begin{cases} \int_0^x (3 + |t - 2|) & \text{if} \quad x > 4 \\ 2x + 8 & \text{if} \quad x \le 4 \end{cases}$$

Then, f(x) is

A. continuous at x=4

B. neither continuous nor differentiable at x=4

C. everywhere continuous but not differentiable at x=4

D. everywhere continuous and differentiable

Answer: C

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82. If a function y=f(x) is defined as $y = \frac{1}{t^2 - t - 6}$ and $t = \frac{1}{x - 2}$, $t \in R$. Then f(x) is discontinuous at

A. $2, \frac{2}{3}, \frac{7}{3}$

B. $2, \frac{3}{2}, \frac{7}{3}$

C. $2, \frac{2}{3}, \frac{7}{3}$

D. none of these

Answer: B

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83. Let $f(x) = x^3 - x^2 + x + 1$ and $g(x) = \begin{cases} \max f(t), & 0 \leq t \leq x \quad \text{for} \quad 0 \leq \\ 3 - x, & 1 < x \leq 2 \end{cases}$

Then, g(x) in [0, 2] is

A. continuous and differentiable on [0,2]

B. continuous but not differentiable on [0,2]

C. neither continuous nor differentiable on [0,2]

D. none of these

Answer: B

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84. If $f(x) = \sum_{r=1}^{n} a_r |x|^r$, where a_i s are real constants, then f(x) is

A. continuous at x=0 for all a_1

B. differentiable at x=0 for all $a_i \in R$

C. differentiable at x=0 for all $a_{2k+1} = 0$

D. none of these

Answer: A::C

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85. Let $f(x) = \phi(x) + \Psi(x)$ and $\phi'(a), \Psi'(a)$ are finite and definite. Then

A. f(x) is continuous at x=a

B. f(x) is differentiable on x=a

C. f'(x) is conntinuous at x=a

D. f'(x) is differentiable at x=a

Answer: A::B

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86. A function $f(x)$ is defiend in the interval $[1, 4]$ as follows:

$f(x) = \begin{cases} \log_e[x] & 1 \le x < 3 \\ |\log_e x| & 3 \le x < 4 \end{cases}$ the graph of the function of $f(x)$:

A. is broken at two points

B. is broken at exactly one point

C. does not have a definite tangent at two points

D. does not have a definite tangent at more than two points

Answer: A::C

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87. If $f(x) = \begin{cases} e^x & x < 2 \\ a + bx & x \geq 2 \end{cases}$ is differentiable for all $x \varepsilon R$ then

A. a+b=0

B. $a + 2b = e^2$

C. $b = e^2$

D. all of these

Answer: D

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88. Let f(x)= $\min\left(x^3, x^4\right)$ for all $x \in R$. Then,

A. f(x) is continuous for all x

B. f(x) is indifferentiable for all x

C. $f'(x) = 3x^2 \text{for all} \quad x > 1$

D. f(x) is not differentiable at two points

Answer: A::C

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89. Let g(x) be a polynomial of degree one and f(x) be defined by $f(x) = \begin{bmatrix} g(x) & x \leq 0 \\ \left[\frac{(1+x)}{(2+x)}\right]^{1/x} & x > 0 \end{bmatrix}$. Find the continuous function $f(x)$ satisfying $f'(1) = f(-1)$.

A. $-\frac{1}{9}(1 + 6\log_e, 3)x$

B. $\frac{1}{9}(1 + 6\log_e, 3)$

C. $-\frac{1}{9}(1 - 6\log_e, 3)x$

D. none of these

Answer: A

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90. If $f(x) = \sin(\pi(x - [x]))$, $\forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, where $[\cdot]$ denotes the greatest integer function, then

A. 4

B. 5

C. 3

D. 2

Answer: C

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91. If $f(x) = \left[\sin^2 x\right]$ ([.] denotes the greatest integer function), then

A. f is everywhere continuous

B. f is everywhere differerntiable

C. f is a constant function

D. none of these

Answer: D

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92. If $f(x) = [x^2] + \sqrt{\{x\}^2}$, where [] and {.} denote the greatest integer and fractional part functions respectively,then

A. f(x) is continuous at all integer points

B. f(x) is continuous and differentiable at x=0

C. f(x) is continuous for all $x \in Z - (1)$

D. f(x) is not differerntiable on Z

Answer: C

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93. Let f be a differentiable function satisfying $f(xy) = f(x).\, f(y).\ \forall x > 0, y > 0$ and $f(1+x) = 1 + x\{1 + g(x)\}$, where $\lim_{x \to 0} g(x) = 0$ then $\int \frac{f(x)}{f.\,(x)} dx$ is equal to

A. $\frac{x^2}{2} + C$

B. $\frac{x^3}{3} + C$

C. $\frac{x^2}{3} + C$

D. none of these

Answer: A

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94. Let $f: R \to R$ be a function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x) + f(y)}{3}$, $f(0) = 0$ and $f'(0) = 3$,then

A. a quadratic function

B. continuous but not differerntiable

C. differerntiable in R

D. bounded in R

Answer: C

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95. $f(x) = x^3 + 3x^2 - 33x - 33 \text{for} \quad x > 0$ and g be its inverse such that kg'(2)=1, then the value of k is

A. -36

B. 42

C. 12

D. none of these

Answer: D

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96. $\lim_{h \to 0} \frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$ given that $f'(2)=6$ and $f'(1)=4$ then (a) limit does not exist (b) is equal to $-\frac{3}{2}$ (c) is equal to $\frac{3}{2}$ (d) is equal to 3

A. does not exist

B. is equal to $-\frac{3}{2}$

C. is equal to $\frac{3}{2}$

D. is equal to 3

Answer: D

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97. Let $f(x)=\begin{cases} x\exp\left[\left(\frac{1}{|x|}+\frac{1}{x}\right)\right], & x \neq 0 \\ 0, & x=0 \end{cases}$ Test whether (a) f(x) is continuous at x = 0

(b) f(x) is differentiable at x = 0

A. discontinuous everywhere

B. continuous as well as differential for all x

C. continuous for all c but not differential at x=0

D. neither differential nor continuous at x=0

Answer: C

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98. Let $f(x) = \lim_{n \to \infty} \frac{(2\sin x)^{2n}}{3^n - (2\cos x)^{2n}}, n \in Z$. Then

A. at $x = n \pm \frac{\pi}{6}$, f(x) is discontinuous

B. $f\left(\frac{\pi}{3}\right) = 1$

C. f(0)=0

D. all of the above

Answer: D

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99. The function $f(x)=||x|-1|, x\in R$, is differerntiable at all $x\in R$ except at the points.

A. $1, 0, \ -1$

B. 1

C. $1, \ -1$

D. -1

Answer: A

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100. If f(x) is continuous and differentiable function $f\left(\frac{1}{n}\right)=0 \forall n \le 1$ and $n \in Z.$ then prove that

$f(0) = 0$ and $f'(0) = 0$

A. $f(x) = 0$for all $x \in N \cup (0, 1]$

B. $f(0) = 0, f'(0) = 0$

C. $f'(0) = 0, f''(0) = 0$

D. f(0) and f'(0) may or may not be zero

Answer: B

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101. The second degree polynomial f(x), satisfying f(0)=o,

$f(1) = 1, f'(x) > 0 \forall x \in (0, 1)$

A. $f(x) = \phi$

B. $f(x) = ax + (1 - a)x^2, a \in (0, \infty)$

C. $f(x) = ax + (1 - a)x^2, x \in (0, 2)$

D. non-existent

Answer: C

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102. If f''(x) =- f(x) and g(x) = f'(x) and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that F(5) =5, then F(10) is

A. 15

B. 10

C. 0

D. 15

Answer: A

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103. If f(x) $\min\left(x, x^2, x^3\right)$, then

A. f(x) is everywhere differentiable

B. $f(x) > 0 \text{for} \quad x > 1$

C. f(x) is not differentiable at three points but continuous for all $x \in R$

D. f(x) is not differerntiable for two values of x

Answer: C

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104. If $f(x) = \min\left(1, x^2, x^3\right)$, then

A. f(x) is everywhere continuous

B. f(x) is continuous and differentiable everywhere

C. f(x) is not differentiable at two points

D. f(x) is not differentiable at one points

Answer: A::D

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105. Let $f:(-1,1)\to R$ be a differentiable function with $f(0)=-1$ and $f'(0)=1$. Let $g(x)=[f(2f(x)+2)]^2$. Then $g'(0)=$ (1) -4 (2) 0 (3) 2 (4) 4

A. 0

B. -2

C. 4

D. -4

Answer: D

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106.

if $f(x)=\{\left(-x=\frac{\pi}{2}, x\le-\frac{\pi}{2}\right), \left(-\cos x, -\frac{\pi}{2}<x, \le 0\right), (x -$

A. f(x) is continuous at $x = -\frac{\pi}{2}$

B. f(x) is not differentiable at x=0

C. f(x) is differentiable at $x = 1, -\frac{3}{2}$

D. f(x) is discontinuous at x=0

Answer: D

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107. Let $f: R \to R$ be a function such that $f(x+y) = f(x) + f(y), \forall x, y \in R$. If f(x) is differentiable at x = 0, then

A. f(x) is continuous for all $x \in R$

B. f'(x) is constant for all $x \in R$

C. f(x) is differentiable for all $x \in R$

D. f(x) is differentiable only in a finite interval containing zero

Answer: D

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108. Let $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x \neq 0, x \in R \\ 0, & x = 0 \end{cases}$, then f is

A. differentiable both at x=0 and x=2

B. differentiable at x=0 but not differentiable at x=2

C. not differentiable at x=0 but differentiable at x=2

D. differentiable neither at x=0 nor at x=2

Answer: B

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109. Q. For every integer n, let a_n and b_n be real numbers. Let function $f: R \to R$ be given by a $f(x) = \{a_n + \sin \pi x, f \text{ or } x \in [2n, 2n+1],$ $b_n + \cos \pi x, f \text{ or } x \in (2n+1, 2n)$ for all integers n.

A. $a_n - b_{n+1} = -1$

B. $a_{n-1} - b_{n-1} = 0$

C. $a_n - b_n = 1$

D. $a_{n-1} - b_n = 1$

Answer: B

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110. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1), g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$ (1) $2f'(c) = g'(c)$ (2) $2f'(c) = 3g'(c)$ (3) $f'(c) = g'(c)$ (4) $f'(c) = 2g'(c)$

A. $f'(c) = g'(c)$

B. $f'(c) = 2g'(c)$

C. $2f'(c) = g'(c)$

D. $2f'(c) = 3g'(c)$

Answer: B

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111. Let $f(1): R \to R,\ f_2: [0, \infty) \to R,\ f_3: R \to R \text{ and } f_4: R \to [0, \infty)$

be a defined by

$f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x > 0 \end{cases} : f_2(x) = x^2,\ f_3(x) = \begin{cases} \sin x & \text{if x} < 0 \\ x & \text{if } x \geq 0 \end{cases}$

and $f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(f_1(x))) - 1 & \text{if } x \geq 0 \end{cases}$ Then, f_4 is

A. onto but not one-one

B. neither continuous nor one-one

C. differentiable but not one-one

D. continuous and one-one

Answer: A

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112. In Q,NO, 111, f_3 is

A. onto but not one-one

B. neither continuous nor one-one

C. differentiable but not one-one

D. continuous and one-one

Answer: C

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113. Let $f_1 : R \to R,\ f_2 : [0, \infty) \to R,\ f_3 : R \to R$ and $f_4 : R \to [0, \infty)$

be a defined by

$$f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x > 0 \end{cases} : f_2(x) = x^2,\ f_3(x) = \begin{cases} \sin x & \text{if x} < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

and $f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(f_1(x))) - 1 & \text{if } x \geq 0 \end{cases}$ then f_2 of f_1 is

A. onto but not one-one

B. neither continuous nor one-one

C. differentiable but not one-one

D. continuous and one-one

Answer: B

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114. Let $f_1 : R \to R, f_2 : [0, \infty) \to R, f_3 : R \to R$ and $f_4 : R \to [0, \infty)$

be a defined by

$f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x > 0 \end{cases} : f_2(x) = x^2, f_3(x) = \begin{cases} \sin x & \text{if x} < 0 \\ x & \text{if } x \geq 0 \end{cases}$

and $f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(f_1(x))) - 1 & \text{if } x \geq 0 \end{cases}$ then f_2 is

A. onto but not one-one

B. neither continuous nor one-one

C. differentiable but not one-one

D. continuous and one-one

Answer: D

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115. about to only mathematics

A. $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some c $\in [0, 1]$

B. $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some c $\in [0, 1]$

C. $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some c $\in [0, 1]$

D. $(f(c))^2 + (g(c))^2$ for some c $\in [0, 1]$

Answer: A::D

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116. Let $f : [a, b] \rightarrow [1, \infty)$ be a continuous function and let $g : R \rightarrow R$ be defined as

$$g(x) = \begin{cases} 0 & \text{if} \quad x < a \\ \int_a^x f(t)dt & \text{if} \quad a \leq x \leq b \\ \int_a^b f(t)dt & \text{if} \quad x > b \end{cases}$$ Then

A. g(x) is continuous but not differentiable at x=a

B. g(x) is differentiable on R

C. g(x) is continuous but not differentiable at x=b

D. g(x) is continuous and differentiable at either x=a or x=b but not both

Answer: A::C

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117. Let f:$R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1)$. Define $h: R \to R$ by

$$h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if } x \le 0 \\ \min\{f(x), g(x)\} & \text{if } x > 0 \end{cases}$$

then number of point at which h(x) is not differentiable is

A. 1

B. 2

C. 3

D. 4

Answer: C

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118. Let $g: R \rightarrow R$ be a differentiable function with $g(0) = 0, , g'(1) \neq 0$ .Let $f(x) = \begin{cases} \frac{x}{|x|} g(x), 0 \neq 0 \text{ and } 0, x = 0 \end{cases}$ and $h(x) = e^{|x|}$ for all $x \in R$. Let $(foh)(x)$ denote $f(h(x))$ and $(hof)(x)$ denote $h(f(x))$. Then which of the following is (are) true?

A. f is differentiable at x = 0

B. h is differentiable at x = 0

C. f o h is differentiable at x = 0

D. h o f is differentiable at x = 0

A. f is differentiable at $x = 0$

B. h is differentiable at $x = 0$

C. foh is differentiable at x = 0

D. hofis differentiable at $x = 0$

Answer: A::D

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119. Let $f(x) = \begin{cases} 3\sin x + a^2 - 10a + 30 & x \in Q \\ 4\cos x & x \in Q \end{cases}$ whichone of the following statements is correct?

A. f(x) is continuous for all x when a=5

B. f(x) must be continuous for all, x when a=5

C. f(x) is continuous for all x, $= 2\pi x - \tan^{-1}\left(\frac{3}{4}\right), n \in Z, \text{when a=5}$

D. f(x) is continuous for all $x = 2\pi x - \tan^{-1}\left(\frac{4}{3}\right), n \in Z$ when a=5

Answer: C

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120. If $(\lim)_{x \to 0} \frac{\{(a-n)nx - \tan x\}\sin nx}{x^2} - 0,$ where n is nonzero real number, the a is 0 (b) $\frac{n+1}{n}$ (c) n (d) $n + \frac{1}{n}$

A. 0

B. $\frac{n}{n+1}$

C. n

D. $n + \frac{1}{n}$

Answer: D

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121. The value of k for which $f(x) = \begin{cases} \frac{x^{2^{32}} - 2^{32}x + 4^{16} - 1}{(x-1)^2} & x \neq 1 \\ k & x = 1 \end{cases}$ is continuous at x=1, is

A. $2^{63} - 2^{31}$

B. $2^{65} - 2^{33}$

C. $2^{62} - 2^{31}$

D. $2^{65} - 2^{31}$

Answer: A

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122. The function $f(x) = \begin{cases} \frac{x^2}{a} & 0 \le x < 1 \\ a & 1 \le x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^2} & \sqrt{2} \le x < \infty \end{cases}$ is a continuous for $0 \le x < \infty$. Then which of the following statements is correct?

A. The number of all possible ordered pairs (a,b) is 3

B. The number of all possible ordered pairs (a,b) is 4

C. The product of all possible pairs ,b is -1

D. The product of all possible values of b is 1

Answer: A::C

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123. If $f(x) = \begin{cases} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots . + \left[\frac{n}{x}\right]\right) & x \neq 0 \\ k & x = 0 \end{cases}$ and $n \in N.$

Then the value of k for which f(x) is continuous at x=0 is

A. n

B. n+1

C. $n(n+1)$

D. `(n(n+1))/(2)

Answer: D

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124. The value of k for which $f(x) = \begin{cases} \left[1 + x\left(e^{-1/x^2}\right)\sin\left(\frac{1}{x^4}\right)\right]^{e^{1/x^2}} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at x=0, is

A. 1

B. 2

C. 3

D. 4

Answer: A

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125. Let $f(x) = \begin{cases} \sum_{r=0}^{x^2\left[\frac{1}{|x|}\right]} r & x \neq 0 \\ k & x = 0 \end{cases}$ where [.] denotes the greatest integer function. The value of k for which is continuous at x=0, is

A. 1

B. 2

C. 4

D. $\frac{1}{2}$

Answer: A

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126. If $f(x) = \begin{cases} |x| - 3, & x < 1 \\ |x - 2| + a, & x \geq 1 \end{cases}, g(x) = \begin{cases} 2 - |x|, & x < 2 \\ sgn(x) - b, & x \geq 2 \end{cases} Ifh(x) = f($

is discontinous at exactly one point, then which of the following are correct ?

A. a=3,b=0

B. a=-3,b=-1

C. a=2,b=1

D. a=0,b=3

Answer: B::C

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127. If $f: R \to R$ is a continuous function satisfying $f(0) = 1$ and $f(2x) - f(x) = x \, \forall x \varepsilon R$ and $\lim_{n \to \infty} \left(f(x) - f\left(\frac{x}{2^n}\right) \right) = P(x)$. Then $P(x)$ is

A. a constant function

B. a linear polynomial in x

C. a quadratic polynomial in x

D. a cubic polynomial in x

Answer: B

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128. Let $f: (0, \infty) \to R$ be a continuous function such that $F(x) = \int_0^{x^2} t f(t) dt$. If $F(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2) =$

A. 216

B. 219

C. 222

D. 225

Answer: B

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129. A function $f: R \to R$ is differernitable and satisfies the equation $f\left(\frac{1}{n}\right)$=0 for all integers $n \geq 1$, then

A. $f(x) = 0 \text{for all x} \in (0, 1]$

B. $f(0) = f'(0)$

C. $f(0) = 0$ but f'(0) need not be equal to 0

D. $|f(x)| \leq 1 \text{for all} x \in [0, 1]$

Answer: B

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130. Suppose $f(x)=e^{ax}+e^{bx}$, where $a\neq b$, and that f''(x) -2f'(x)-15f(x)=0 for all x. Then the product ab is

A. 25

B. 9

C. -15

D. -9

Answer: C

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131. If $f(x)=\left\{\alpha+\frac{\sin[x]}{x}, x>0 \text{ and } 2, x=0 \text{ and } \beta+\left[\frac{\sin x - x}{x^3}\right], x<0\right.$ (whlenotes the greatest integer function) if $f(x)$ is continuous at $x=0.$ then β is equal to

A. $\alpha-1$

B. $\alpha + 1$

C. $\alpha + 2$

D. $\alpha - 2$

Answer: B

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132. If a function y=f(x) is defined as

$$y = \frac{1}{t^2 - t - 6} \text{ and } t = \frac{1}{x - 2}, t \in R$$

Then, f(x) is discontinuous at

A. $2, \frac{2}{3}, \frac{7}{3}$

B. $2, \frac{3}{2}, \frac{7}{3}$

C. $2, \frac{3}{2}, \frac{5}{3}$

D. None of these

Answer: B

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133. If f(x) is continuous in [0,2] and f(0)=f(2). Then the equation f(x)=f(x+1) has

A. no real root in [0,2]

B. at least one real root in [0,1]

C. at least one real root in [0,2]

D. at least one real root in [1,2]

Answer: B::C

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134. If $\lim_{x \to a} f(x) = \lim_{x \to a} [f(x)]$ and $f(x)$ is non-constant continuous function, where [.] denotes the greatest integer function, then

A. $\lim_{x \to a}$ f(x) is an integer

B. $\lim_{x \to a}$ f(x) is not an integer

C. f(x) has a local maximum at x=a

D. f(x) has a local minimum at x=a

Answer: A::D

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135. Let $f: R \to R$ be a differentiable function at x = 0 satisfying f(0) = 0 and f'(0) = 1, then the value of $\lim_{x \to 0} \frac{1}{x} . \sum_{n=1}^{\infty} (-1)^n . f\left(\frac{x}{n}\right)$, is

A. 0

B. $-In2$

C. 1

D. e

Answer: B

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136. For $x \in R$, $f(x) = |\log_e 2 - \sin x|$ and $g(x) = f(f(x))$, then

A. g is not differerentiable at x=0

B. g'(0)=cos(log2)

C. g'(0)=-cos(log 2)

D. g is differentiable at x=0 and g'(0)=-sin (log 2)

Answer: B

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137. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ for all $x \in R$, Then, g'(2)=

A. $\frac{1}{15}$

B. $\frac{1}{5}$

C. $\frac{1}{3}$

D. 15

Answer: C

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138. Let $f: \square \rightarrow \square, g: \square \rightarrow \square$ and $h: \square \rightarrow \square$ be differentiable functions such that $f(x) = x^3 + 3x = 2, g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \varepsilon R$. Then

A. 666

B. 16

C. 66

D. 111

Answer: A

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139. If h(x)= f(f(x)) for all x $\in$ R, and f(x)=$x^3 + 3x + 2$, then h(0) equals

A. 6

B. 16

C. 2

D. 15

Answer: B

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140. In Example 138, h(0) equals

A. 66

B. 6

C. 36

D. 38

Answer: D

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141. Let $a, b \in R$ and $f: R \to R$ be defined by f(x) $= a\cos\left(\left|x^3 - x\right|\right) + b|x|\sin\left(\left|x^3 + x\right|\right)$ then f is

A. differentiable at x=0, if a=0 and b=1

B. differentiable at x=1, if a=1 and b=0

C. not differentiable at x=0, if a=1 and b=0

D. not differerntiable at x=1, if a=1 and b=1

Answer: A::B

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142. Let $f: R \to (0, \infty)$ and $g: R \to R$ be twice differentiable functions such that f'' and g'' are continuos functions of R suppose

$f'(2) = g(2) = 0, f'(2) \neq 0$ and $g'(2) \neq 0$. If

$$\lim_{x \to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1, then$$

A. f has a local maximum at x=2

B. f has a local minimum at x=2

C. $f''(2) > f(2)$

D. $f(x) - f''(x) = 0$ for at least one $x \in R$.

Answer: B::D

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143. Let $f : \left[-\frac{1}{2}, 2\right] \to R$ and $g : \left[-\frac{1}{2}, 2\right] \to R$ be functions defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|f(x) + |4x - 7|f(x)$, where [y] denotes the greatest integer less than or equal to y for $y \in R$. Then,

A. f is discontinuous exactly at three points in [-1/2,2]

B. f is discontinuous exactly at four points in [-1/2,2]

C. g is not differentiable exactly at four points in [-1/2,2]

D. g is not differentiable exactly at five points in [-1/2,2]

Answer: B::C

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Section II - Assertion Reason Type

1. if $|f(x)| \leq |x|$ for all x $\in$ R then prove that f(x) is continuous at 0.

A. 1

B. 2

C. 3

D. 4

Answer: A

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2. Let $f(x) = \begin{cases} 1+x & \text{if } \; x < 0 \\ 1+[x]+\sin x & 0 \le x < \pi/2 \\ 3 & x \ge \pi/2 \end{cases}$

Statement-1: F is a continuous on R-[1]

Statement-2: The greatest integer function is discontinuous at every integer point.

A. 1

B. 2

C. 3

D. 4

Answer: B

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3. Statement-1: The function $f(x) = [x] + x^2$ is discontinuous at all integer points.

Statement-2: The function g(x)=[x] has Z as the set of points of its discontinuous from left.

A. 1

B. 2

C. 3

D. 4

Answer: A

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4. Statement-1: If a continuous funtion on [0,1] satisfy $0 \leq f(x) \leq 1$, then there exist $c \in [0, 1]$ such that f(c)=c

Statement-2: $\lim_{x \to c} f(x) = f(c)$

A. 1

B. 2

C. 3

D. 4

Answer: B

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5. Statement-1: Let $f(x) = [3 + 4\sin x]$, where [.] denotes the greatest integer function. The number of discontinuities of f(x) in $[\pi, 2\pi]$ is 6

Statement-2: The range of f is $[-1, 0, 1, 2, 3]$

A. 1

B. 2

C. 3

D. 4

Answer: D

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6. The function $f(x) = e^{-|x|}$ is continuous everywhere but not differentiable at $x = 0$ continuous and differentiable everywhere not continuous at $x = 0$ none of these

A. 1

B. 2

C. 3

D. 4

Answer: D

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7. Statement-1: If f and g are differentiable at x=c, then min (f,g) is differentiable at x=c.

Statement-2: min (f,g) is differentiable at $x = c$ if $f(c) \neq g(c)$

A. 1

B. 2

C. 3

D. 4

Answer: D

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8. Statement-1: Let f be a differentiable function satisfying $f(x+y)=f(x)+f(y)+2xy-1$ for all $x, y \in R$ and $f'(0)=a \text{where} 0<a<1$ then, $f(x)>0$ for all x.

Statement-2: f(x) is statement-1 is of the form x^2+ax+1

A. Statement -1 is true, Statement -2 is True, Statement -2 is a correct explanation for Statement for Statement -1.

B. Statement -1 is true, Statement -2 is True, Statement -2 is not a correct explanation for Statement for Statement -1.

C. Statement -1 is true, Statement -2 is False.

D. Statement -1 is False, Statement -2 is True.

Answer: A

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9. Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is constinous, $g(0) = 0$, $g'(0) = 0, g''(0) = 0$ and $f(x) = g(x)\sin x$.

Statement I $\lim_{x \to 0} (g(x)\cot x - g(0)\cos ecx) = f''(0)$

Statement II $f'(0) = g'(0)$

A. 1

B. 2

C. 3

D. 4

Answer: B

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10. Let $f(x) = x|x|$ and $g(x) = \sin x \in x$

Statement 1 : gof is differentiable at $x = 0$ and its derivative is continuous at that point

Statement 2: gof is twice differentiable at $x = 0$

(1) Statement1 is true, Statement2 is true, Statement2 is a correct explanation for statement1

(2) Statement1 is true, Statement2 is true; Statement2 is not a correct explanation for statement1.

(3) Statement1 is true, statement2 is false.

(4) Statement1 is false, Statement2 is true

A. 1

B. 2

C. 3

D. 4

Answer: C

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11. about to only mathematics

A. 1

B. 2

C. 3

D. 4

Answer: B

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12. Define $f(x)$ as the product of two real functions $f_1(x) = x, x \varepsilon R$ and $f_2(x) = \begin{cases} \sin\frac{1}{x} & \text{if } \ x \neq 0 \\ 0 & \text{if } \ x = 0 \end{cases}$ as follows $f(x) = \begin{cases} f_1(x).\, f_2(x) & \text{if } \ x \neq 0 \\ 0 & \text{if } \ x = 0 \end{cases}$ Statement 2: $f_1(x)$ and $f_2(x)$ are continuous on IR.

A. 1

B. 2

C. 3

D. 4

Answer: C

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13. Let $f:[1,3] \to R$ be a function satisfying $\frac{x}{[x]} \le f(x) \le \sqrt{6-x}$, for all $x \ne 2$ and $f(2) = 1$, Where R is the set of all real number and [x] denotes the largest integer less than or equal to x.

Statement-1: $\lim_{x \to 2} f(x)$ exists.

Statement-2: F is continuous at x=2.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

B. Statement 1 is false, Statement 2 is true

C. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1

D. Statement 1 is true, Statement 2 is false

Answer: D

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Exercise

1. The function $f(x) = \frac{4 - x^2}{4x - x^3}$ is discontinuous at

A. discontinuous at only one point

B. discontinuous exactly at two point

C. discontinuous exactly at three point

D. None of these

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2. Let $f(x) = |x|$ and $g(x) = \left|x^3\right|$, then (a).$f(x)$ and $g(x)$ both are continuous at $x = 0$ (b) $f(x)$ and $g(x)$ both are differentiable at $x = 0$ (c) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x = 0$ (d) $f(x)$ and $g(x)$ both are not differentiable at $x = 0$

A. f(x) and g(x) btoh the continuous at x=0

B. f(x) and g(x) btoh the differentiable at x=0

C. f(x) is differentiable but g(x) is not differentiable at x=0

D. f(x) and g(x) both are not differentiable at x=0.

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3. The function $f(x) = \sin^{-1}(\cos x)$ is discontinuous at $x = 0$ (b) continuous at $x = 0$ (c) differentiable at $x = 0$ (d) none of these

A. discontinuous at x=0

B. continuous at x=0

C. differentiable at x=0

D. None of these

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4. The set of points where the function $f(x) = x|x|$ is differentiable is $(-\infty, \infty)$ (b) $(-\infty, 0) \cup (0, \infty)$ (c) $(0, \infty)$ (d) $[0, \infty]$

A. $(-\infty, \infty)$

B. $(-\infty, 0) \cup (0, \infty)$

C. $(0, \infty)$

D. $[0, \infty]$

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5. On the interval $I = [-2, 2]$, if the function $f(x) = \begin{cases} (x+1)e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then which of the following hold good ?

A. is continuous for all $x \in I - [0]$

B. assumes all intermediate values from $f(-2) \rightarrow f(2)$

C. has a maximum value equal to 3/e.

D. all of the above

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6. If $f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)} & x \neq -2 \\ 2 & x = -2 \end{cases}$, then f(x) is

A. continuous at x=-2

B. not continuous at x=-2

C. differentiable at x=-2

D. continous but not derivable at x=-2

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7. Let $f(x) = (x + |x|)|x|$. Then, for all x f is continuous

A. f and f' are continuous

B. f is differentiale for some x

C. f' is not continuous

D. f'' is continuous

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8. The set of all points where the function $f(x)=\sqrt{1-e^{-x^2}}$ is differentiable is

A. $(-\infty, \infty)$

B. $(-\infty, 0) \cup (0, \infty)$

C. $(-1, \infty)$

D. None of these

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9. The function $f(x)=e^{-|x|}$ is continuous everywhere but not differentiable at $x=0$ (b) continuous and differentiable everywhere (c) not continuous at $x=0$ (d) none of these

A. continuous everywhere but not differentiable at x=0

B. continuous and differentiable everywhere

C. not continuous at x=0

D. None of these

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10. The function $f(x) = [\cos x]$ is

A. everywhere continuous and differentiable

B. everywhere continuous but not differentiable at $(2n+1)\pi/2, n \in Z$

C. neither continuous nor differentiable at $(2n+1)\pi/2, n \in Z$

D. None of these

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11. If $f(x)=\sqrt{1-\sqrt{1-x^2}}$, then $f(x)$ is (a) continuous on [-1, 1] and differentiable on (-1, 1) (b) continuous on [-1, 1] and differentiable on (− 1 , 0) ∪ (0 , 1)` (c) continuous and differentiable on [-1, 1] (d) none of these

A. continuous of [-1,1] and differentiable on (-1,1)

B. continuous on [-1,1] and differentiable aon $(-1,0) \in (0,1)$

C. continuous and differentiable on [-1,1]

D. None of these

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12. If $f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ then f (x) is differentiable in the interval :

A. [-1,1]

B. $R-[-1,1]$

C. $R-[-1,1]$

D. None of these

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13. about to only mathematics

A. a=b=c=0

B. a=0,b=0,$c \in R$

C. $b = c = 0, a \in R$

D. $c = 0, a = 0, b \in R$

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14. If $f(x) = |x - a|\varphi(x)$, where $\varphi(x)$ is continuous function, then $f'(a^+) = \varphi(a)$ (b) $f'(a^-) = -\varphi(a)$ $f'(a^+) = f'(a^-)$ (d) none of these

A. $F'(a^+) = \phi(a)$

B. $f'(a^-) = \phi(a)$

C. $f'(a^+) = f'(a^-)$

D. None of these

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15. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots + \frac{x^2}{(1+x^2)^n} +$, then at $x = 0$, $f(x)$ (a)has no limit (b) is discontinuous (c)is continuous but not differentiable (d) is differentiable

A. has no limit

B. is discontinuous

C. is continuous but not differentiable

D. is differentiable

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16. If $f(x) = |\log_{10} x|$ then at $x = 1$.

A. f(x) is continuous and $f'(1^+) = \log_{10} e, f'(1^-) = -\log_{10} e$

B. f(x) is continuous and $f'(1^+) = \log_{10} e, f'(1^-) = \log_{10} e$

C. f(x) is continuous and $f'(1^-) = \log_{10} e, f'(1^+) = -\log_{10} e$

D. None of these

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17. If $f(x) = |\log_e x|, then$

A. $f'(1^+) = 1, f'(1^-) = -1$

B. $f'(1^-) = -1, f'(1^+) = 0$

C. $f'(1) = 1, f'(1^-) = 0$

D. None of these

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18. If $f(x) = |\log_e |x||, \quad \text{then} \quad f'(x)$ equals

A. f(x) is continuous and differentiable for all x in its domain

B. f(x) is continuous for all x in its domain but not differentiable at $x = \pm 1$

C. f(x) is neither continuous nor differentiable at $x = \pm 1$

D. None of these

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19. Let $f(x) = \begin{cases} \frac{1}{|x|} & f \text{ or } |x| \geq 1ax^2 + b & f \text{ or } |x| < 1 \end{cases}$. If $f(x)$ is continuous and differentiable at any point, then $a = \frac{1}{2}, \quad b = -\frac{3}{2}$ (b) $a = -\frac{1}{2}, \quad b = \frac{3}{2}$ (c) $a = 1, \quad b = -1$ (d) none of these

A. $a = \frac{1}{2}, b = -\frac{3}{2}$

B. $a = -\frac{1}{2}, b = \frac{3}{2}$

C. a=1,b=-1

D. None of these

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20. Let $h(x) = \min \{x, x^2\}$ for every real number of x. Then, which one of the following is true?

(a) h is not continuous for all x

(b) h is differentiable for all x

(c) $h'(x) = 1$, for all x

(d) h is not differentiable at two values of x.

A. h is continuous for all x

B. h is differentiable for all x

C. $h'(x) = 1\text{for all} \quad x > 1$

D. h is not differentiable at two values of x

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21. If $f(x) = \begin{cases} \dfrac{36^x - 9^x - 4^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}}, x \neq 0k, x = 0 \end{cases}$ is continuous at $x = 0$, then k equal $16\sqrt{2}\log 2 \log 3$ (b) $16\sqrt{2} \in 6$ $16\sqrt{2} \in 2In3$ (d) none of these

A. $16\sqrt{2}\log 2 \log 3$

B. $16\sqrt{2} \quad \text{In} \quad 6$

C. $16\sqrt{2} \quad \text{In} \quad 2 \quad \text{In} \quad 3$

D. None of these

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22. $f(x) = \{|x-4|f \text{ or } x \le 1\frac{x^3}{2} - x^2 + 3x + \frac{1}{2}f \text{ or } x < 1$, then 1) f(x) is continuous at x=1 and x=4 2) f(x) is differentiable at x=4 3) f(x) is continuous and differentiable at x=1 4) f(x) is only continuous at x=1

A. f(x) is continuous at x=1 and x=4

B. f(x) is differentiable at x=4

C. f(x) is continuous and differentiable at x=1

D. f(x) is not continuous at x=1

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23. Let $f(x)=\begin{cases}\sin 2x & \text{if } 0\le x\le \frac{\pi}{6}\\ ax+b & \text{if } \frac{\pi}{6}<x<1\end{cases}$ If $f(x)$ and $f'(x)$ are continuous then a & b are (A) $a=1, b=\frac{1}{\sqrt{2}}+\frac{\pi}{6}$ (B) $a=\frac{1}{\sqrt{2}}, b=\frac{1}{\sqrt{2}}$ (C) $a=1, b=\frac{\sqrt{3}}{2}-\frac{\pi}{6}$ (D) None of these

A. $a=1, b=\frac{1}{\sqrt{2}}+\frac{\pi}{6}$

B. $a=\frac{1}{\sqrt{2}}, b=\frac{1}{\sqrt{2}}$

C. $a=1, b=\frac{\sqrt{3}}{2}-\frac{\pi}{6}$

D. None of these

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24. Let $f(x)=\begin{cases}\int_0^x \{5+|1-t|\}dt & \text{if} x<2\\ 5x+1 & \text{if} x\ge 2\end{cases}$ then:

A. f(x) is continuous at x=2

B. f(x) is continuous but not differentiable at x=2

C. f(x) is everywhere differentiable

D. the right derivative of f(x) at x=2 does not exist

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25. The function f defined by $f(x) = \begin{cases} \frac{\sin x^2}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ is

A. continuous and derivative at x=0

B. neither continuous nor derivative at x=0

C. continuous but not derivable at x=0

D. None of these

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26. If f(x) is continuous at x=0 and f(0)=2, then $\lim_{x \to 0} \frac{\int_0^x f(u)\,du}{x} \; is$

A. 0

B. 2

C. f(2)

D. None of these

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27. If $f(x)$ defined by `f(x)={(|x^2-x|)/(x^2-x1,x=0),x!=0,1-1,x=1 then (A)f(x) is continuous for all x (B) for all x except at x=0 (C) for all x except at x=1 (D)for all x except at x=0 and x=1

A. x

B. x except at x=0

C. x except at x=1

D. x except at x=0 and x=1

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28. If

$$f(x) = \begin{cases} \frac{1-\sin x}{(\pi - 2x)^2} \frac{\log \sin x}{(\log(1+\pi^2 - 4\pi x + 4x^2))}, x \ne \frac{\pi}{2}, k\ at x = \frac{\pi}{2} \end{cases}$$

is continuous at $x = \frac{\pi}{2}, then k = \ -\frac{1}{16}$ (b) $-\frac{1}{32}$ (c) $-\frac{1}{64}$ (d) $-\frac{1}{28}$

A. $-\frac{1}{16}$

B. $-\frac{1}{32}$

C. $-(1)(64)$

D. $-\frac{1}{28}$

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29. The set of points of differentiable of the function

$$f(x) = \begin{cases} \frac{\sqrt{x+1}-1}{\sqrt{x}} & \text{for} \quad x \ne 0 \\ 0 & f \text{ or } \ x = 0 \end{cases}$$

A. R

B. $[0, \infty)$

C. $(-\infty, 0)$

D. $R-(0)$

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30. The set of points where the function $f(x)=|x-1|e^x$ is differentiable, is

A. R

B. $R-[1]$

C. $R-[-1]$

D. $R-(0)$

Answer: B

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31. If $f(x)=(x+1)^{\cot x}$ be continuous at $x=0$, the $f(0)$ is equal to

A. 0

B. 1/e

C. e

D. None of these

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32. If $f(x)=\begin{cases}\frac{\sqrt{x+1}-1}{\sqrt{x}} & \text{for } x\neq 0\\ 0 & f \text{ or } x=0\end{cases}$ and f(x) is continuous at x=0, then the value of k is

A. a-b

B. a+b

C. loga+log b

D. None of these

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33. The function $f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, x \neq 0 0, x = 0 \end{cases}$ is continuous at $x = 0$ is not continuous at $x = 0$ is not continuous at $x = 0,$ but can be made continuous at $x = 0$ (d) none of these

A. is continuous at x=0

B. is not continuous at x=0

C. is not continuous at x-0, but can be made continuous at x=0

D. None of these

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34. $Let f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & x < 4 \\ a + b & x = 4 \\ \frac{x-4}{|x-4|} + b & x > 4 \end{cases}$

then f(x) is continuous at x=4 when

A. a=0, b=0

B. a=1,b=1

C. a=-1,b=1

D. a=1,b=-1

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35. If the function $f(x) = \{(\cos x)^{\frac{1}{x}}, x \neq 0k, x = 0$ is continuous at $x = 0$, then the value of k is (a)0 (b) 1 (c) -1 (d) None of these

A. 0

B. 1

C. -1

D. e

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36. If the function $f(x) = |x| + |x-1|$, then

A. f(x) is continuous at x=0 as well as at x=1

B. f(x) is continuous at x=0, but not at x=1

C. f(x) is continuous at x=1, but not at x=0

D. None of these

Answer: A

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37. Let

$f(x) = \begin{cases} \frac{x^4 - 5x^2 + 4}{|(x-1)(x-2)|}, & x \neq 1,\ 16 \\ , & x = 1, \\ 12, & x = 2 \end{cases}$

. Then, $f(x)$ is continuous on the set R (b) $R - \{1\}$ (c) $R - \{2\}$ (d) $R - \{1,\ 2\}$

A. R

B. $R - [1]$

C. $R - [2]$

D. $R - [1, 2]$

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38. If the function f as defined below is continuous at x=0find the values of a,b and c

$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, x < 0 \text{ and } c, x = 0, \text{ and } \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}\end{cases}$

A. $a = -\frac{3}{2}, b = 0, c = \frac{1}{2}$

B. $a = -\frac{3}{2}, b = 1, c = -\frac{1}{2}$

C. $a = -\frac{3}{2}, b \in R - [0], c = \frac{1}{2}$

D. None of these

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39. If $f(x) = \begin{bmatrix} mx + 1 & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n & \text{if } x > \frac{\pi}{2} \end{bmatrix}$ is continuous at $x = \frac{\pi}{2}$, then find the relation between m and n.

A. m=1,n=0

B. $m = \frac{n\pi}{2} + 1$

C. $n = \frac{m\pi}{2}$

D. $m = n = \frac{\pi}{2}$

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40. The value of $f(0)$, so that $f(x)=\frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$

becomes continuous for all, x is given by

A. $a^{3/2}$

B. $a^{1/2}$

C. $-a^{1/2}$

D. $-a^{3/2}$

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41. (a) Draw the graph of

$$f(x)==\begin{cases}1, & |x|\geq 1\\ \frac{1}{n^2}, & \frac{1}{n}<|x|<\frac{1}{n-1}, n=2,3,\ldots\\ 0, & x=0\end{cases}$$

(b) Sketch the region $y\leq -1$.

(c) Sketch the region $|x|<3$.

A. is discontinuous at finitely many points

B. is continuous everywhere

C. is discontinuous only at $x = \pm\frac{1}{n}, n \in Z - (0)$ and $x = 0$

D. None of these

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42. The value of $f(0)$, so that the function $f(x) = \frac{(27-2x)^2 - 3}{9 - 3(243+5x)^{1/5} - 2}(x \neq 0)$ is continuous, is given $\frac{2}{3}$ (b) 6 (c) 2 (d) 4

A. $\frac{2}{3}$

B. 6

C. 2

D. 4

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43. The value of $f(0)$ so that the function $f(x)=\frac{2-(256-7x)^{\frac{1}{8}}}{(5x+32)^{1/5}-2}, x\neq 0$ is continuous everywhere, is given by

A. -1

B. 1

C. 26

D. None of these

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44. The following functions are continuous on $(0, \pi)$

(a) $\tan x$

(b) $\int_0^x t\sin\frac{1}{t}dt$

(c) $\begin{cases} -1 & 0 < x \leq \frac{3\pi}{4} \\ 2\sin\left(\frac{2}{9}x\right) & \frac{3\pi}{4} < x < \pi \end{cases}$

(d) $\begin{cases} x\sin x & 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2}\sin(\pi + x) & \frac{\pi}{2} < x < \pi \end{cases}$

A. tan x

B. $\int_0^x t\sin\frac{1}{t}dt$

C. $\begin{cases} -1 & 0 < x \leq \frac{3\pi}{4} \\ 2\sin\left(\frac{2}{9}x\right) & \frac{3\pi}{4} < x < \pi \end{cases}$

D. $\begin{cases} x\sin x & 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2}\sin(\pi + x) & \frac{\pi}{2} < x < \pi \end{cases}$

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45. If $f(x) = x\frac{\sin 1}{x},\ x \neq 0$, then the value of the function at $x = 0$, so that the function is continuous at $x = 0$, is (a) 0 (b) -1 (c) 1 (d) indeterminate

A. 1

B. -1

C. 0

D. intermediate

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46. Let $f(x) = [x]$ and $g(x) = \begin{cases} 0, & x \in Z \\ x^2, & x \in R - Z \end{cases}$, then (where $[\cdot]$ denotes greatest integer function)

A. $_{x \to 1}$ exists, but g(x) is not continuous at x=1

B. $\lim_{x \to 1}$ does not exist and f(x) is not continuous at $x = 1$

C. gof is continuous for all x

D. fog is continuous for all x

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47. Let $f(x) = \lim_{n \to \infty} m(\sin x)^{2n}$ then which of the following is not true?

A. continuous at $x = \pi / 2$

B. discontinuous at $x = \pi / 2$

C. discontinuous at $x = -\pi / 2$

D. discontinuous at infinite number of points

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48. Let f(x) be a function differentiable at x=c. Then $\lim_{x \to c} f(x)$ equals

A. $f'(c)$

B. $f''(c)$

C. $\frac{1}{f(c)}$

D. None of these

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49. If $\left(\lim\right)_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists finitely, write the value of $\left(\lim\right)_{x \to c} f(x)$.

A. $\lim_{x \to c} f(x) = f(c)$

B. $\lim_{x \to c} f'(x) = f'(c)$

C. $\lim_{x \to c}$ f(x) does not eixst

D. $\lim_{x \to c}$ f(x) may or may not exist

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50. if $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1 + x^2)} & x \neq 0 \\ 0 & x = 0 \end{cases}$

A. f(x) is not continuous at x=0

B. f(x) is continuous and differentiable at x=0

C. f(x) is continuous at x=0 but not differentiable at x=0

D. None of these

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51. The function f(x)=|x|+|x-1| is

A. continuous at x=1, but not differentiable

B. both continous and differentiable at x=1

C. not continuous at x=1

D. None of these

Answer: A

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52. For the function f(x)= $\begin{cases} |x-3| & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & x < 1 \end{cases}$ which one of the folllowing is incorrect

A. continuous at x=1,

B. derivable at x=1

C. continuous at x=3

D. derivable at x=-3

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53. Let $f(x) = \begin{cases} x^n \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ Then f(x) is continuous but not differentiable at $x = 0$. If

A. $n \in (0, 1)$

B. $n \in [1, \infty)$

C. $n \in (-\infty, 0)$

D. $n = 0$

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54. If $4x + 3|y| = 5y$, then y as a function of x is

A. continuous at x=0

B. derivable at x=0

C. $\frac{dy}{dx} = \frac{1}{2}$ for all x

D. none of these

Answer: A

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55. If $f(x) = x^3 sgn(x)$, then

A. f is derivable at x=0

B. f is continuous but not derivable at x=0

C. LHD at x=0 is 1

D. RHD at x=0 is 1

Answer: A

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56. For a real number y, Let [y] denotes the geatest integer less than or equal to y Let $f(x)=\frac{\tan(\pi[x-\pi])}{1+[x]^2}$. then

A. discontinuous at some x

B. continuous at all,, x but f'(x) does not exist for some x

C. f'(x) exists for all x, but f''(x) does not exist

D. f'(x) exists for all x

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57. If $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right) & x\neq 0 \\ 0 & x=0 \end{cases}$, then

A. f and f' are continuous at x=0

B. f is derivable at x=0 and f' is continuous at x=0

C. f is derivable at x=0 and f' is not continuous at x=0

D. f is derivable at x=0

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58. The following functions are differentiable on (-1,2)

A. $\int_x^{2x} (\log t)^2 dt$

B. $\int_x^{2x} \frac{\sin t}{t} dt$

C. $\int_{x}^{2x} \frac{1-t+t^2}{1+t+t^2} dt$

D. None of these

Answer: C

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59. If $f(x) = \sqrt{x + 2\sqrt{2x-4}} + \sqrt{x - 2\sqrt{2x-4}}$ then the value of 10 $f'(102^{+})$, is

A. $(-\infty, \infty)$

B. $(2, \infty) - [4]$

C. $[2, \infty)$

D. None of these

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60. The derivative of $f(x) = |x|^3 at x = 0,$ is

A. -1

B. 0

C. does not exist

D. None of these

Answer: B

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61. If $f(x) = x\left(\sqrt{x} + \sqrt{(x+1)}\right)$, then

A. f is continuous but not differentiable at x=0

B. f is differentiable at x=0

C. f is differentiable but not continuous at x=0

D. f is not differentiable at x=0

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62. Write the value of the derivative of $f(x) = |x-1| + |x-3|\ at\ x=3.$

A. -2

B. 0

C. 2

D. does not exist

Answer: B

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63. If $f(x) = [x \sin \pi x]$, then which of the following, is incorrect,

A. f (x) is continuous at x=0

B. f(x) is continuous at (-1,0)

C. f(x) is differentiable at x=1

D. f(x) is differentiable in (-1,1)

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64. The function $f(x) = 1 + |\sin x|$, is

A. continuous no where

B. continuous everywhere and not differentiable at infinetly many points

C. differentiable no where

D. differentiable at x=0

Answer: B

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65. If $f(x) = \begin{cases} 1 & x < 0 \\ 1 + \sin x & 0 \le x < \frac{\pi}{2} \end{cases}$ then derivative of f(x) x=0

A. is equal to 1

B. is equal to 0

C. is equal to -1

D. does not exist

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66. Let [x] denotes the greatest integer less than or equal to x and $f(x) = \left[\tan^2 x\right]$. Then

A. $\lim_{x \to 0} f(x)$ does not exist

B. f(x) is continuous at x=0

C. f(x) is not continuous at x=0

D. f'(0)=1

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67. A function $f: R \rightarrow R$ satisfies the equation $f(x+y) = f(x)f(y), \forall x, y$ in R and $f(x) \neq 0$ for any x in R . Let the function be differentiable at x = 0 and f'(0) = 2. Show that $f'(x) = 2f(x), \forall x$ in R. Hence, determine f(x)

A. f(x)

B. $-f(x)$

C. 2f(x)

D. None of these

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68. Let f(x) be defined on R such that $f(1) = 2, f(2) = 8$ and $f(u+v) = f(u) + kuv - 2v^2$ for all $u, v \in R$ (k is a fixed constant). Then,

A. $f'(x) = 8x$

B. $f(x) = 8x$

C. $f'(x) = x$

D. None of these

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69. Let f(x) be a function satisfying $f(x+y) = f(x) + f(y)$ and $f(x) = xg(x)f$ or $all x . y \in$ R. which g(x) is continuous then prove that $f'(x) = g(0)$

A. $f'(x) = g'(x)$

B. $f'(x) = g(x)$

C. $f'(x) = g(0)$

D. None of these

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70. If $f(x) = \begin{cases} ax^2 - b & |x| < 1 \\ \frac{1}{|x|} & |x| \geq 1 \end{cases}$ is differentiable at x=1, then

A. $a = \frac{1}{2}, b = -\frac{1}{2}$

B. $a = -\frac{1}{2}, b = -\frac{3}{2}$

C. $a = b = \frac{1}{2}$

D. $a = b = -\frac{1}{2}$

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71. If $f(x) = (x - x_0)\phi(x)$ and $\phi(x)$ is continuous at x=x_0. Then $f'(x_0)$ is equal to

A. $\phi'(x_0)$

B. $\phi(x_0)$

C. $x_0\phi(x_0)$

D. None of these

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72. Let $f(x + y) = f(x)f(y)$ for all x and y, and $f(5) = 2, f'(0) = 3,$ then $f'(5)$ is equal to:

A. 6

B. 3

C. 5

D. None of these

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73. If f be a function satisfying $f(x+y)=f(x)+f(y), \forall x, y \in R$. If f(1) = k, then f(n), $n \in N$ is equal to

A. 4

B. 1

C. $1/2$

D. 8

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74. Let $f(x+y)=f(x)f(y)$ for all $x, y, \in R$, suppose that $f(3)=3$ and $f'(0)=2$ then $f'(3)$ is equal to-

A. 22

B. 44

C. 28

D. None of these

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75. Let $f(x+y)=f(x)+f(y)$ and $f(x)=x^2g(x)\,\forall x, y \in R$ where $g(x)$ is continuous then $f'(x)$ is

A. g'(x)

B. g(0)

C. g(0)+g'(x)

D. 0

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76. Let $f(x+y) = f(x)f(y)$ for all $x, y \varepsilon R$ and $f(x) = 1 + x\phi(x)ln2$ where $\lim_{x \to 0} \phi(x) = 1$ then $f, (x)$ is

A. g'(x)

B. g(x)

C. f(x)

D. None of these

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77. Let $f(x+y) = f(x)f(y)$ and $f(x) = 1 + (\sin 2x)g(x)$ where g(x) is continuous. Then, f'(x) equals

A. 1+ab

B. ab

C. a/b

D. None of these

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78. Let $f(x+y)=f(x)f(y) \text{ and } f(x)=1+(\sin 2x)g(x)$ where g(x) is continuous. Then, f'(x) equals

A. f(x)g(0)

B. 2f(x)g(0)

C. 2g(0)

D. None of these

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79. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x=c$. Then $g'(f(x))$ equal. $f'(c)$ (b) $\frac{1}{f'(c)}$ (c) $f(c)$ (d) none of these

A. f'(c)

B. $\frac{1}{f'(c)}$

C. f(c)

D. None of these

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80. Let g(x) be the inverse of f(x) and $f'(x) = \frac{1}{1+x^3}$.Find g'(x) in terms of g(x).

A. $\frac{1}{1+(g(x))^3}$

B. $\frac{1}{1+(f(x))^3}$

C. $1+(g(x))^3$

D. $1+(f(x))^3$

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81. Let $f(x)=\begin{cases} x^n \sin\frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$ Then f(x) is continuous but not differentiable at $x=0$. If

A. $n\in(0,1]$

B. $n\in[1,\infty)$

C. $n\in(1,\infty)$

D. $n\in(-\infty,0)$

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82. If for a continuous function $f, f(0)=f(1)=0, f'(1)=2$ and $y(x)=f(e^x)e^{f(x)}$, then y'(0) is equal to

A. 1

B. 2

C. 0

D. None of these

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83. Let f(x) be a function such that $f(x+y) = f(x) + f(y)$ and $f(x) = \sin x g(x)$ for all $x, y \in R$. If g(x) is a continuous functions such that g(0)=k, then f'(x) is equal to

A. k

B. kx

C. kg(x)

D. None of these

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84. Let $f(0, \pi) \to R$ be defined as $f(x) = \begin{cases} \frac{1-\sin x}{(\pi - 2x)^2} \cdot \frac{In \sin x}{(In(1+\pi^2 - 4\pi x + 4x^2))} & x \neq \frac{\pi}{2} \\ k & x = \frac{\pi}{2} \end{cases}$ If a continuous at $x = \frac{\pi}{2}$, then the value of $8\sqrt{|k|}, is$

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85. If $f(x) = \frac{e^{2x} - (1+4x)^{1/2}}{\ln(1-x^2)}$ for $x \neq 0$, then f has

A. an irremovable discontinuity at x=0

B. a removable discontinuity at x=0 and f(0)=-4

C. a removable discontinuity at x=0 and $f(0) = -\frac{1}{4}$

D. a removable discontinuity at x=0 and $f(0) = 4$

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86. Let $f(x) = \begin{cases} \dfrac{ex^2 - \frac{2}{\pi}\sin^{-1}\sqrt{1-x}}{In\left(1+\sqrt{x}\right)} & x \in (0, 1) \\ k & x \leq 0 \end{cases}$ be a continuous at x=0, then the value of k, is

A. $1 + \dfrac{2}{\pi}$

B. $1 - \dfrac{2}{\pi}$

C. $\dfrac{2}{\pi}$

D. $-\dfrac{2}{\pi}$

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87. Let $f(x) = \begin{cases} x^3 & x < 1 \\ ax^2 + bx + c & : x \geq 1 \end{cases}$. If f''(1) exists, then the value of $\left(a^2 + b^2 + c^2\right)$ is

A. 20

B. 21

C. 19

D. 17

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