MATHS

BOOKS - OBJECTIVE RD SHARMA ENGLISH

DEFINITE INTEGRALS

Illustration

1. The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}dx$ is :

A. $\frac{\pi}{2}+1$

B. $\frac{\pi}{2}-1$

C. -1

D. 1

Answer: B

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2. The value of $I=\int_0^{\pi/2}\frac{(\sin x+\cos)^2}{\sqrt{1+\sin 2x}}dx$ is

A. 3

B. 1

C. 2

D. 0

Answer: C

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3. If $I(m,n)=\int_0^1 t^m(1+t)^n.\,dt$, then the expression for $I(m,n)$ in terms of $I(m+1,n-1)$ is:

A. $\frac{2^n}{m+1}-\frac{n}{m+1}I(m+1,n-1)$

B. $\frac{n}{m+1}I(m+1,n-1)$

C. $\dfrac{2^n}{m+1} - \dfrac{n}{m+1} I(m+1, n-1)$

D. $\dfrac{m}{n+1} I(m+1, n-1)$

Answer: B

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4. If a$f(x)$+b$f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} - 5, x \neq 0, a \neq b$, then $\displaystyle\int_1^2 f(x)dx$ equals

A. $\dfrac{(\text{In}2 - 5)a + \frac{13}{2}b}{a^2 - b^2}$

B. $\dfrac{(\text{In}2 - 5)a + \frac{7b}{2}}{a^2 - b^2}$

C. $\dfrac{(5 - \text{In}2)a + \frac{7b}{2}}{a^2 - b^2}$

D. none of these

Answer: B

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5. $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$ is equal to

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6. If $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8}$, then $a = \ldots\ldots$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. 1

D. $\frac{1}{2}$

Answer: D

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7. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$ is π (b) $\frac{\sqrt{3}}{2}$ (c) $2\sqrt{2}$ (d) none of these

A. $\frac{\sqrt{3}}{2}$

B. $2\sqrt{2}$

C. 2

D. $-\sqrt{2}$

Answer: D

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8. For real number u, $\frac{-\pi}{2} < \tan^{-1} u < \frac{\pi}{2}$ and $0 < \cot^{-1} u < \pi$, then the value of $\frac{\int_0^1 \cot^{-1}\left(l - x + x^2\right) dx}{\int_0^1 \tan^{-1} x dx}$, is :

A. log 2

B. $\frac{\pi}{2} + \log 2$

C. log 4

D. $\frac{\pi}{2} - \log 4$

Answer: A

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9. Let $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}.\ x < 0.$ If $\int_1^4 \frac{2e^{\sin x^2}}{x}dx = F(k) - F(1)$ then find the possible value of k.

A. 4

B. 8

C. 16

D. 32

Answer: C

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10. Evaluate the following integral: $\int_0^{\pi/4} \frac{s \in x + \cos x}{3 + s \in 2x}dx$

A. log 2

B. log 3

C. $\frac{1}{4}\log 3$

D. $\frac{1}{8}\log 3$

Answer: C

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11. If $I_1 = \int_e^{e^2} \frac{dx}{\ln x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx_1$ then

A. $I_1 = I_2$

B. $2I_1 = I_2$

C. $I_1 = 2I_2$

D. none of these

Answer: A

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12. The value of integral $\int_0^{\log 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} dx$ is

A. $3 + 2\pi$

B. $4 - \pi$

C. $2 + \pi$

D. none of these

Answer: B

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13. Given $\int_1^2 e^{x^2} dx = a$, the value of $\int_a^{e^4} \sqrt{\ln(x)} dx$ is :

A. $e^4 - e$

B. $e^4 - a$

C. $2e^4 - a$

D. $2e^4 - e - a$

Answer: D

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14. If $\int_0^1 \frac{\sin t}{1+t} dt = \alpha,$ then the value of the integral $\int_{4\pi-2}^{4\pi} \frac{\sin\left(\frac{t}{2}\right)}{4\pi+2-t} dt$ is (1)2α (2) -2α (3) α (4) $-\alpha$

A. 2α

B. -2α

C. α

D. $-\alpha$

Answer: D

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15. Evaluate the following definite integral: $\int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2}+\frac{\pi}{4}\right) dx$

A. 2π

B. e^{π}

C. 0

D. $2\sqrt{2}$

Answer: C

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16. If $I_n = \int_0^{\pi/4} \tan^n \theta d\theta$, then $I_8 + I_6$ equals

A. $\frac{1}{4}$

B. $\frac{1}{5}$

C. $\frac{1}{6}$

D. $\frac{1}{7}$

Answer: D

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17. $\int_0^2 \frac{x^3}{(x^2+1)^{3/2}} dx$ is equal to

A. $(\sqrt{2}-1)^2$

B. $\frac{(\sqrt{2}-1)}{\sqrt{2}}$

C. $\frac{\sqrt{2}-1}{\sqrt{2}}$

D. none of these

Answer: D

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18. The value of the integral $I = \int_0^a \frac{x^4}{(a^2+x^2)^4} dx$ is

A. $\frac{1}{16a^3}\left(\frac{\pi}{4}-\frac{1}{3}\right)$

B. $\frac{1}{16a^3}\left(\frac{\pi}{4}+\frac{1}{3}\right)$

C. $\frac{a^3}{16}\left(\frac{\pi}{4}-\frac{1}{3}\right)$

D. $\frac{a^3}{16}\left(\frac{\pi}{4}+\frac{1}{3}\right)$

Answer: A

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19. Let $f(x)=7\tan^8 x+7\tan^6 x-3\tan^4 x-3\tan^2 x$ for all $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then the correct expression (s) is (are) (a) $\int_0^{\frac{\pi}{4}} x f(x)dx=\frac{1}{12}$ (b) $\int_0^{\frac{\pi}{4}} f(x)dx=0$ (c) $\int_0^{\frac{\pi}{4}} x f(x)=\frac{1}{6}$ (d) $\int_0^{\frac{\pi}{4}} f(x)dx=\frac{1}{12}$

A. $\int_0^{\pi/4} x f(x)dx=\frac{1}{12}$

B. $\int_0^{\pi/4} x f(x)dx=0$

C. $\int_0^{\pi/4} x f(x)dx=\frac{1}{6}$

D. $\int_0^{\pi/4} f(x)dx=1$

Answer: A,B

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20. If $\alpha = \int_0^1 \left(e^{9x+3\tan^{-1}x}\right)\left(\frac{12+9x^2}{1+x^2}\right)dx$ where $\tan^{-1}x$ takes only principal values, then, find the value of $\left((\log)_e|1+\alpha| - \frac{3\pi}{4}\right)$

A. 9

B. 8

C. 7

D. 6

Answer: A

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21. The value of $\int_0^{\pi} |\cos x| dx$, is

A. 1

B. 2

C. 0

D. 4

Answer: B

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22. $\int_0^2 \left|x^2+2x-3\right| dx$ is equal to

A. 4

B. 6

C. 3

D. 2

Answer: A

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23. $\int_0^3 [x]dx$ is equal to

A. 2

B. 4

C. 3

D. 1

Answer: C

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24. $\int_0^{1.5} [x^2]dx$ is equal to

A. 2

B. $2-\sqrt{2}$

C. $2+\sqrt{2}$

D. $\sqrt{2}$

Answer: B

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25. $\int_0^2 [x^2]\,dx$ is equal to

A. $5-\sqrt{2}$

B. $5-\sqrt{3}-\sqrt{2}$

C. $5-\sqrt{3}$

D. none of these

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26. Let $f(x)=x-[x]$, for every real number x (where, [x] is integral part of x). Then , the value of $\int_{-1}^{1} f(x)dx$ is equal to

A. 1

B. 2

C. 0

D. $1/2$

Answer: A

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27. If {x} denotes the fractional part of x, then $\int_0^2 \{x\} dx$ is equal to

A. 1

B. 2

C. $\frac{1}{2}$

D. 4

Answer: A

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28. $\int_0^4 \{\sqrt{x}\}$ is equal to, where {x} denotes the fraction part of x.

A. $\frac{2}{3}$

B. $\frac{16}{3}$

C. $\frac{5}{3}$

D. $\frac{7}{3}$

Answer: D

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29. For any real number x, the value of $\int_0^x [x]dx$, is

A. x[x]

B. x[x]-[x]([x]+1)

C. $x[x] - \frac{1}{2}[x]([x]+1)$

D. none of these

Answer: C

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30. If {x} denotes the fractional part of x, then $\int_0^x \left(\{x\} - \frac{1}{2}\right)dx$ is equal to

A. $\frac{1}{2}\{x\}(\{x\}+1)$

B. $\frac{1}{2}\{x\}(\{x\}-1)$

C. $\{x\}(\{x\}-1)$

D. $\{x\}(\{x\}+1)$

Answer: B

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31. The value of : $\int_{\pi/6}^{5\pi/6}\sqrt{4-4\sin^2 t}dt$, is

A. 0

B. 2

C. 1

D. none of these

Answer: B

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32. If for a real number y, $[y]$ is the greatest integral function less, then or equal to y, then the value of the integral $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}[2\sin x]dx$ is $-\pi$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

A. $-\pi$

B. 0

C. $-\pi/2$

D. $\pi/2$

Answer: C

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33. If [.] stands for the greatest integer function, then $\int_1^2 [3x]dx$ is equal to

A. 3

B. 4

C. 5

D. 6

Answer: B

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34. The value of the integral $\int_0^{0.9}[x-2[x]]dx$, where [.] denotes the greatest integer function, is

A. 0.9

B. 0

C. 1.8

D. -0.9

Answer: B

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35. Let $[\cdot]$ denotes the greatest integer function then the value of $\int_0^{15} x\left[x^2\right]$ dx is :

A. $\frac{3}{2}$

B. $\frac{3}{4}$

C. $\frac{5}{4}$

D. 0

Answer: B

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36. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}dx$ is equals to (a)$\pi-4$ (b) $\frac{2\pi}{3}-4-\sqrt{3}$ (c)$\frac{2\pi}{3}-4-\sqrt{3}$ (d)$4\sqrt{3}-4-\frac{\pi}{3}$

A. $4\sqrt{3}-4$

B. $4\sqrt{3}-4-\frac{\pi}{3}$

C. $\pi-4$

D. $\frac{2\pi}{3}-4-4\sqrt{3}$

Answer: B

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37. The value of $\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2} \left(1 - x^2\right)^5 \right\} dx$ is

A. 1

B. 2

C. 8

D. 4

Answer: B

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38. Let $f: R \overrightarrow{R}$ be a function defined by $f(x) = \{[x], (x \leq 2) \ (0, x > 2)$

where $[x]$ is the greatest integer less than or equal to $x\cdot$ If

$$I = \int_{-1}^{2} \frac{x f\left(x^2\right)}{2 + f(x+1)} dx, then\ the\ value\ of\ (4I - 1) is$$

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 8

D. $-\frac{1}{4}$

Answer: A

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39. The value of $\int_1^a [x]f'(x)dx$,where $a > 1$, and $[x]$ denotes the greatest integer not exceeding x, is

(A) $af(a) - \{f(1)f(2) + \ldots . + f([a])\}$

(B) $[a]f(a) - \{f(1) + f(2) + \ldots\ldots + f([a])\}$

(C) $[a]f(a) - \{f(1) + f(2) + \ldots\ldots. + fA\}$

(D) $af([a]) - \{f(1) + f(2) + \ldots\ldots + fA\}$

A. $af(a) - \{f(1) + f(2) + \ldots\ldots + f([a])\}$

B. $[a]f(a) - \{f(1) + f(2) + \ldots\ldots + f([a])\}$

C. $[a]f([a]) - \{f(1) + f(2) + \ldots\ldots + f(a)\}$

D. $af([a]) - \{f(1) + f(2) + \ldots\ldots + f(a)\}$

Answer: B

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40. If m,$n \in N$, then

$$\int_0^{\pi/2} \frac{(\sin^m x)^{\frac{1}{n}}}{(\sin^m x)^{\frac{1}{n}} + (\cos^m x)^{\frac{1}{n}}} dx$$ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2n}$

D. $\frac{\pi}{4n}$

Answer: B

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41. Evaluate : $\int_0^{\frac{\pi}{2}} \log(\tan x) dx$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 0

D. none of these

Answer: C

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42. The value of $\int_0^{\infty} \frac{\log x}{1+x^2} dx$, is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 0

D. none of these

Answer: C

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43. Evaluate $\int_0^{\pi/4} \log(1+\tan\theta)d\theta$

A. $\frac{\pi}{8}\log_e 2$

B. $\frac{\pi}{4}\log_e 2$

C. $-\frac{\pi}{8}\log_e 2$

D. $-\frac{\pi}{4}\log_e 2$

Answer: A

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44. The value of integral $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$, is

A. $\frac{\pi}{8}\log_e 2$

B. $\frac{\pi}{4}\log_e 2$

C. $-\frac{\pi}{8}\log_e 2$

D. $-\frac{\pi}{4}\log_e 2$

Answer: A

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45. The value of the integral $\int_0^a \frac{1}{x+\sqrt{a^2-x^2}} dx$, is

A. π

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. none of these

Answer: C

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46. The value of $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x) dx$, is

A. $\frac{\pi}{2}\log 2$

B. $-\frac{\pi}{2}\log 2$

C. $\pi \log 2$

D. $-\pi \log 2$

Answer: B

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47. The value of $\int_0^{\pi/2n} \frac{1}{1+\cot nx} dx$, is

A. 0

B. $\frac{\pi}{4n}$

C. $\frac{\pi}{2n}$

D. $\frac{\pi}{2}$

Answer: B

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48. $\int_0^{\pi/2} \sin 2x \log \tan x dx$ is equal to

A. π

B. $\frac{\pi}{2}$

C. 0

D. 1

Answer: C

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49. The value of the integral $\int_0^{\pi} \frac{1}{e^{\cos x} + 1} dx$, is

A. π

B. 0

C. 2π

D. $\frac{\pi}{2}$

Answer: D

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50. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

A. $\frac{\pi^2}{2}$

B. $\frac{\pi^2}{4}$

C. $\frac{\pi^2}{8}$

D. $\frac{\pi^2}{16}$

Answer: B

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51. The integral $\int_0^{\pi} x f(\sin x) dx$ is equal to

A. $\frac{\pi}{2}\int_0^{\pi} f(\sin x) dx$

B. $\frac{\pi}{4}\int_0^{\pi}(\sin x)dx$

C. $\frac{\pi}{2}\int_0^{\pi/2}(\sin x)dx$

D. $\frac{\pi}{2}\int_0^{\pi/2}f(\cos x)dx$

Answer: A

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52. $\int_0^{\pi}[\cot x]dx$, where [.] denotes the greatest integer function, is equal to

A. $\frac{\pi}{2}$

B. 1

C. -1

D. $-\frac{\pi}{2}$

Answer: B

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53. The value of the integral

$$\int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right) dx, \text{ is}$$

A. 0

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. none of these

Answer: A

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54. The value of $\int_{-1}^{1} \left\{\sqrt{1+x+x^2} - \sqrt{1-x+x^2}\right\} dx$, is

A. 0

B. 1

C. -1

D. none of these

Answer: A

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55. The value of $\dfrac{\left\{\int_{-1/2}^{1/2} \cos 2x . \log\left(\frac{1+x}{1-x}\right) dx\right\}}{\left\{\int_{0}^{1/2} \cos 2x . \log\left(\frac{1+x}{1-x}\right) dx\right\}}$ is

A. 0

B. 4

C. 2

D. 8

Answer: A

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56. The value of $\int_{-2}^{2}\left(ax^3+bx+c\right)dx$ depends on

A. B

B. C

C. A

D. none of these

Answer: B

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57. The value of the integral

$\int_{-1}^{1}\log\left(x+\sqrt{x^2+1}\right)dx$ is

A. 0

B. log2

C. $\log 1/2$

D. none of these

Answer: A

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58. Evaluate: $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}}\left[(x+\pi)^3+\cos^2(x+3\pi)\right]dx$

A. $\frac{\pi^4}{32}$

B. $\frac{\pi^4}{32}+\frac{\pi}{2}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}-1$

Answer: C

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59. $\int_{-2}^{0}\{x^3+3x^2+3x+3+(x+1)\cos(x+1)\}dx$ is equal to

A. 0

B. 3

C. 4

D. 1

Answer: C

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60. $\int_{-\pi/2}^{\pi/2} \frac{1}{e^{\sin x}+1} dx$ is equal to

A. 0

B. 1

C. $-\frac{\pi}{2}$

D. $\frac{\pi}{2}$

Answer: D

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61. The value of $\int_{-\pi/4}^{\pi/4} \frac{e^x \sec^2 x}{e^{2x} - 1} dx$, is

A. 0

B. 2

C. 4

D. none of these

Answer: A

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62. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1 + e^x} dx$

A. 0

B. -1

C. 1

D. none of these

Answer: C

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63. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ is

A. $\frac{\pi^2}{4} - 2$

B. $\frac{\pi^2}{4} + 2$

C. $\pi^2 + e^{\pi/2}$

D. 0

Answer: A

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64. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$, is

A. π

B. $a\pi$

C. $\pi / 2$

D. 2π

Answer: C

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65. $\int_{-\pi}^{\pi} \frac{e^{\sin x}}{e^{\sin x}+e^{-\sin x}} dx$ is equal to

A. $\frac{\pi}{2}$

B. 2π

C. π

D. $\frac{\pi}{4}$

Answer: C

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66. The value of $\int_{-2}^{2} \frac{3x^2}{1+e^x} dx$, is

A. 8

B. 2

C. 4

D. 0

Answer: A

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67. The value of $\int_0^{2\pi} \cos^5 x dx$, is

A. 0

B. π

C. 2π

D. $\pi / 2$

Answer: A

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68. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is

(A) $\frac{\pi}{2}$

(B) π

(C) 0

(D) 2π

A. 2π

B. π

C. $\pi / 4$

D. 0

Answer: B

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69. The value of $\int_0^{\pi} \cos^{11} x dx$, is

A. π

B. 11π

C. 0

D. -11π

Answer: C

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70. $I = \int_0^{2\pi} \frac{1}{1 + e^{\sin x}} dx$ is equal to

A. π

B. 2π

C. $\frac{\pi}{2}$

D. none of these

Answer: A

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71. Prove that: $\int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} + \cos^{2n} x} dx$

A. π

B. 2π

C. π^2

D. $\frac{1}{2}\pi^2$

Answer: C

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72. If [x] stands for the greatest integar function, then $\int_4^{10} \frac{[x^2] dx}{[x^2 - 28x + 196] + [x^2]}$ is

A. 0

B. 1

C. 3

D. none of these

Answer: C

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73. The valued of $\int_{\sqrt{In2}}^{\sqrt{In3}} \frac{x\sin x^2}{\sin x^2 + \sin(In6 - x^2)} dx$ is

A. $\frac{1}{4}\text{In}\frac{3}{2}$

B. $\frac{1}{2}\text{In}\frac{3}{2}$

C. $\text{In}\frac{3}{2}$

D. $\frac{1}{6}\text{In}\frac{3}{2}$

Answer: A

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74. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$ is equal to

A. 1

B. 6

C. 2

D. 4

Answer: A

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75. $\int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x}$ is equal to

A. 4

B. -1

C. -2

D. 2

Answer: D

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76. If $f(x)=\frac{e^x}{1+e^x}, I_1=\int_{f(-a)}^{f(a)} xg(x(1-x))dx,$ and $I_2=\int_{f(-a)}^{f(a)} g(x(1-x))dx,$ then the value of $\frac{I_2}{I_1}$ is

A. -1

B. $1/2$

C. 2

D. 1

Answer: C

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77. If f(x)=f(a+b-x), then $\int_a^b x f(x) dx$ is equal to

A. $(a+b)\int_a^b f(x)dx$

B. $\frac{1}{2}(a+b)\int_a^b f(x)dx$

C. $(b-a)\int_a^b f(x)dx$

D. $\frac{1}{2}(b-a)\int_a^{\frac{b-a}{b}} f(x)dx$

Answer: B

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78. Evaluate the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{12}$

D. 0

Answer: C

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79. $I=\int_{\pi/5}^{3\pi/10}\frac{\sin x}{\sin x+\cos x}dx$ is equal to

A. (π)

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. none of these

Answer: D

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80. Find the value of integral $\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx.$

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. 2

D. 1

Answer: B

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81. Evaluate $\int_{-4}^{-5} e^{(x+5)^2} dx + 3\int_{1/3}^{2/3} e^{9\left(x-\frac{2}{3}\right)^2} dx.$

A. 1

B. 0

C. -1

D. none of these

Answer: B

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82. The value of the integral

$$\int_0^{400\pi}\sqrt{1-\cos 2x}\,dx,$$ is

A. $200\sqrt{2}$

B. $400\sqrt{2}$

C. $800\sqrt{2}$

D. none of these

Answer: C

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83. If $n \in N$, then $\int_0^n (x-[x])dx$ is equal to

A. n

B. $n/2$

C. 2n

D. none of these

Answer: B

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84. The value of $\int_0^{100} e^{x-[x]}\,dx$, is

A. 100e

B. 100(e-1)

C. 100(e+1)

D. none of these

Answer: B

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85. The value of $\int_0^{32\pi/3}\sqrt{1+\cos 2x}dx$ is

A. $\dfrac{44+\sqrt{3}}{\sqrt{2}}$

B. $\dfrac{44-\sqrt{3}}{\sqrt{2}}$

C. $\dfrac{22-\sqrt{3}}{\sqrt{2}}$

D. $\dfrac{22+\sqrt{3}}{\sqrt{2}}$

Answer: B

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86. The value of $\int_0^{[x]}(x-[x])dx$,where [x] is the greatest integer of x is equal to

A. [x]

B. 2[x]

C. $(1/2)[x]$

D. none of these

Answer: C

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87. $\frac{d}{dx}\left(\int_{x^2}^{(x^3)}\frac{1}{\log t}dt\right.$ is equal to

A. $x^2 - x$

B. $(x^2 - x)\log x$

C. $\frac{x^2 - x}{\log x}$

D. $\frac{x - 1^2}{\log x}$

Answer: C

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88. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)dt, then:$

(a) $g'\left(\frac{\pi}{2}\right) = -2\pi$ (b) $g'\left(-\frac{\pi}{2}\right) = -2\pi$ (c) $g'\left(-\frac{\pi}{2}\right) = 2\pi$ (d) $g'\left(\frac{\pi}{2}\right) = 2\pi$

A. $g'\left(\frac{\pi}{2}\right) = -2\pi$

B. $g'\left(-\frac{\pi}{2}\right) = 2\pi$

C. $g'\left(\frac{\pi}{2}\right) = 0$

D. $g'\left(-\frac{\pi}{2}\right) = 0$

Answer: C::D

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89. If ϕ (x) $= \int_{1/x}^{\sqrt{x}} \sin(t^2)dt$ then ϕ' (1)is equal to

A. sin 1

B. 2 sin 1

C. $(3/2)\sin 1$

D. none of these

Answer: C

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90. $\lim_{x \to \infty} \dfrac{\int_0^{2x} te^{t^2}\,dt}{e^{4x^2}}$ equals

A. 0

B. 2

C. $\dfrac{1}{2}$

D. ∞

Answer: C

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91. Let $f : \overrightarrow{RR}$ be a differentiable function having $f(2) = 6, f'(2) = \frac{1}{48}$.

Then evaluate $\lim_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$

A. 18

B. 12

C. 36

D. 24

Answer: A

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92. $\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t) dt}{x^2 - \frac{\pi^2}{16}}$ is equal to

A. $\frac{8}{\pi} f(2)$

B. $\frac{2}{\pi} f(2)$

C. $\frac{2}{\pi} f\left(\frac{1}{2}\right)$

D. 4f(2)

Answer: A

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93. $If \int_{\sin x}^{1} t^2 f(t) dt = 1 - \sin x, where. x \in \left(0, \frac{\pi}{2}\right),$ then find the value of $f\left(\frac{1}{\sqrt{3}}\right).$

A. 3

B. $\sqrt{3}$

C. $\frac{1}{3}$

D. none of these

Answer: A

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94. $Ify = \int_0^x f(t)\sin\{k(x-t)\}dt, thenprovethat \frac{d^2y}{dx^2} + k^2y = kf(x)\cdot$

A. f(x)

B. k f(x)

C. $k^2 f(x)$

D. none of these

Answer: B

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95. Find out the area bounded by the curve $y = \int_{1/8}^{\sin^2 x} (\sin^{-1}\sqrt{t})\,dt + \int_{1/8}^{\cos^2 x} (\cos^{-1}\sqrt{t})\,dt (0 \le x \le \pi/2)$ and the curve satisfying the differential equation $y(x+y^3)\,dx = x(y^3 - x)\,dy$ passing through $(4, -2)$.

A. π

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. none of these

Answer: C

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96. Let $f:(0, \infty) \in R$ be given

$f(x)=\int_{1/x}^{x} e^{t+\frac{1}{t}} \frac{1}{t} dt$, then

A. f(x) is monotonically increasing on $[1, \infty]$

B. f(x) is monotonically increasing on (0,1)

C. f(x) is monotonocally decreasing on (0,1)

D. $f(2^x)$is an odd function of x on R

Answer: A::C::D

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97. Let $I = \int\limits_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $f = \int\limits_o^1 \frac{\cos x}{\sqrt{x}} dx$

Then , which one of the following is true ?

A. $I > \frac{2}{3}$ and $J > 2$

B. $I < \frac{2}{3}$ and $J < 2$

C. $I < \frac{2}{3}$ and $J > 2$

D. $I > \frac{2}{3}$ and $J < 2$

Answer: B

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98. The value of the integral $\int_0^1 e^{x^2} dx$ lies in the integral

A. (0,1)

B. (-1,0)

C. (1, e)

D. none of these

Answer: C

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99. The smallest interval [a,b] such that

$$\int_0^1 \frac{1}{\sqrt{1+x^4}}dx \in [a, b], \text{ is}$$

A. $[1/\sqrt{2}, 1]$

B. [0,1]

C. $[1/2, 1]$

D. [3//4,1]`

Answer: A

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100. If $I = \int_0^1 \frac{1}{1 + x^{\pi/2}} dx$then

A. $\text{In}2 < 1 < \frac{\pi}{4}$

B. $I < \text{In} \quad 2$

C. $\text{In} \quad > \frac{\pi}{4}$

D. none of these

Answer: A

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101. Let $f(a) > 0$, and let f (x) be non-decreasing continuous function in [a, b], Then $\frac{1}{b-a} \int_a^b f(x) dx$ has the :

A. maximum value f(b) and minimum value f(a)

B. maximum value b f(b) and minimum value a f(a)

C. maximum value f(a) and minimum value f(b)

D. none of these

Answer: A

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102. If $I\sum_{k=1}^{98}\int_{k}^{k+1}\frac{k+1}{x(x+1)}dx, then:$ (a) $I<\frac{49}{50}$ (b) $I>(\log)_e 99$ (c) $I>\frac{49}{50}$ (d) $I<(\log)_e 99$

A. $I>\log_e 99$

B. $I<\log_e 99$

C. $I<\frac{49}{50}$

D. $I>\frac{49}{50}$

Answer: B::D

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103. Let $f:\left[\frac{1}{2},1\right]\to R$ (the set of all real numbers) be a positive, non-constant, and differentiable function such that $f'(x)<2f(x)\Big)andf\left(\frac{1}{2}\right)=1$. Then the value of $\int_{\frac{1}{2}}^{1}f(x)dx$ lies in the interval (a) $(2e-1,2e)$ (b) $(3-1,2e-1)$ $\left(\frac{e-1}{2},e-1\right)$ (d) $\left(0,\frac{e-1}{2}\right)$

A. (2e-1,2e)

B. e-1,2e-1)

C. $\left(\frac{e-1}{2},e-1\right)$

D. $\left(0,\frac{e-1}{2}\right)$

Answer: D

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104. Let $f'(x)=\frac{192x^3}{2+\sin^4\pi x}$ for all $x\in R$ with $f\left(\frac{1}{2}\right)=0$. If $m\leq\int_{\frac{1}{2}}^{1}f(x)dx\leq M$, then the possible values of m and M are-

A. m=13,M=24

B. $m = \frac{1}{4}, M = \frac{1}{2}$

C. m=-11, M=0

D. m=1,M=12

Answer: D

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105. Let $f: R \rightarrow R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of $f(1n5)$ is_____

A. 5

B. 0

C. 1

D. -5

Answer: B

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106. The least value of the function $\phi(x)=\int_{5\pi/4}^{x}(3\sin t+4\cos t)dt$ on the intervel $\left(\left[\frac{5\pi}{4},\frac{4\pi}{3}\right)\right]\Big)$ is

A. $\sqrt{3}+\frac{3}{2}$

B. $-2\sqrt{3}+\frac{3}{2}+\frac{1}{\sqrt{2}}$

C. $\frac{3}{2}+\frac{1}{\sqrt{2}}$

D. none of these

Answer: B

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107. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{r/n}$ is

A. e+1

B. e-1

C. 1-e

D. e

Answer: B

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108. $\underset{n \to \infty}{\mathrm{Lt}} \sum_{2n+1}^{3n} \frac{n}{r^2 - n^2}$ is equal to :

A. $\ln \sqrt{\frac{2}{3}}$

B. ln $\sqrt{\frac{3}{2}}$

C. $\ln \frac{2}{3}$

D. $\ln \frac{3}{2}$

Answer: B

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109. The value of $\lim_{n \to \infty} \frac{1}{n} . \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ is equal to

A. $1 + \sqrt{5}$

B. $-1 + \sqrt{5}$

C. $-1 + \sqrt{2}$

D. $1\sqrt{2}$

Answer: B

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110. $\lim_{n \to \infty} \left\{ \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{n+n} \right\}$ is, equal to

A. $\log_e 3$

B. 0

C. $\log_e 2$

D. 1

Answer: C

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111. $\lim_{n \to \infty} \left\{ \frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + ... + \frac{n}{n^2+n^2} \right\}$ is equal to

A. 1

B. 0

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

Answer: C

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112. $\lim_{n \to \infty} \left\{ \frac{1^m + 2^m + 3^m + ... + n^m}{n^{m+1}} \right\}$ equals

A. $\frac{1}{m+1}$

B. $\frac{1}{m+2}$

C. $\frac{1}{m}$

D. none of these

Answer: A

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113. $\lim_{n \to \infty} \frac{(n!)^{1/n}}{n}$ equals

A. e

B. e^{-1}

C. 1

D. none of these

Answer: B

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114. $\lim_{n \to \infty} \left\{ \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{3n} \right\} =$

A. log2

B. log3

C. log5

D. 0

Answer: B

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115. Evaluate the following limit:

$$\lim_{n \to \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \ldots\ldots\ldots\ldots + \frac{1}{n} \sec^2 1 \right]$$

A. $\frac{1}{2} \tan 1$

B. tan 1

C. $\frac{1}{2} \text{cosec} 1$

D. $\frac{1}{2}\sec 1$

Answer: A

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116. $\lim_{n \to \infty} \left(\frac{(n+1)(n+2)(n+3).......2n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to

A. $\frac{18}{e^4}$

B. $\frac{27}{e^2}$

C. $\frac{9}{e^2}$

D. $3\log 3 - 2$

Answer: C

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117. For $a \varepsilon R$ (the set of all real numbers) $a \neq -1$, $\lim_{n \to \infty} \frac{(1^a + 2^a + \ldots\ldots\ldots + n^a)}{(n+1)^{a-1}[(na+1)+(n+2)+\ldots\ldots\ldots\ldots+(na+n)}$

Then $a =$

A. 5,7

B. $7, \frac{17}{2}$

C. $-\frac{15}{2}, -\frac{17}{2}$

D. $7, \frac{15}{2}$

Answer: B

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118. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$ for $n = 1, 2, 3, \ldots\ldots\ldots$ Then

A. $S_n < \frac{\pi}{3\sqrt{3}}, T_n > \frac{\pi}{3\sqrt{3}}$

B. $S_n \geq \frac{\pi}{3\sqrt{3}}, T_n < \frac{\pi}{3\sqrt{3}}$

C. $S_n > \frac{\pi}{3\sqrt{3}}, T_n > \frac{\pi}{3\sqrt{3}}$

D. $S_n < \frac{\pi}{3\sqrt{3}}, T_n < \frac{\pi}{3\sqrt{3}}$

Answer: A

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119. The value of $\int_{-\pi}^{\pi} \sin^3 x \cos^2 x dx$ is.....

A. $\frac{3\pi}{216}$

B. $\frac{3\pi}{512}$

C. $\frac{\pi}{512}$

D. none of these

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120. The value of $\int_{-1}^{1} (x|x|)dx$ is equal to

A. 0

B. 1

C. -1

D. $1/2$

Answer: B

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121. The value of $\int_0^x d[t]$, is

A. x

B. $x^2/2$

C. $-x$

D. $[x]$

Answer: D

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122. The value of $\int_0^2 x d([x]-x)$, is

A. $1/2$

B. 1

C. -1

D. 0

Answer: B

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123. The value of the integral $\int_0^3 (x^2+1) d[x]$ is, where[*] is the greatest integer function

A. 12

B. 15

C. 17

D. 19

Answer: C

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Section I - Solved Mcqs

1. $\int_0^{10} |x(x-1)(x-2)| dx$ is equal to

A. 160.05

B. 1600.5

C. 16.005

D. none of these

Answer: B

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2. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt,$ then the value of $f(1)$, is

A. $1/2$

B. 0

C. 1

D. $-1/2$

Answer: A

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3. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} xf[x(1-x)]dx$, $I_2 = \int_{1-k}^{k} f[x(1-x)]dx$, where $2k-1>0$. Then $\frac{I_1}{I_2}$ is

A. 2

B. k

C. $1/2$

D. 1

Answer: C

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4. If $g(x) = \int_0^x \cos^4 t dt$, then $g(x+\pi)$ equals to (a) $\frac{g(x)}{g(\pi)}$ (b) $g(x)+g(\pi)$ (c)$g(x)-g(\pi)$ (d)$g(x).\, g(\pi)$

A. $g(x)+g(\pi)$

B. $g(x)-g(\pi)$

C. $g(x)g(\pi)$

D. $g(x)/g(\pi)$

Answer: A

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5. If $l_n = \int_0^{\pi/4} \tan^n x dx, n \in N \text{then} I_{n+2} + I_n$ equals

A. $\frac{1}{n}$

B. $\frac{1}{n-1}$

C. $\frac{1}{n+1}$

D. $\frac{1}{n+2}$

Answer: C

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6. If $\Rightarrow I_n = \int_0^{\pi/4} \tan^n x dx$, then for any positive integer, n, the value of $n(I_{n+1} + I_{n-1})$ is,

A. 1

B. 2

C. $\pi/4$

D. π

Answer: A

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7. Find the value of $\int_{-1}^{1}\frac{d}{dx}\left(\tan^{-1}\left(\frac{1}{x}\right)\right)dx$

A. $\pi/2$

B. $-\pi/4$

C. $-\pi/2$

D. none of these

Answer: A

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8. The value of the integral

$\int_{-1}^{3}\left(\tan^{1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$ is equal to

A. π

B. 2π

C. 4π

D. none of these

Answer: B

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9. If = $\int_1^e (\log x)^n dx, \text{then} I_n + nI_{n-1}$ is equal to

A. $1/e$

B. e

C. $e - 1$

D. none of these

Answer: B

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10. If = $\int_0^1 x^n e^{-x} dx \text{for} n \in N \text{then} I_n - nI_{n-1} =$

A. e

B. $1/e$

C. $-1/e$

D. none of these

Answer: C

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11. The value of $\int_{1/n}^{(an-1)/n} \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}} dx$, is

A. $\frac{a}{2}$

B. $\frac{1}{2n}(na+2)$

C. $\frac{na-2}{2n}$

D. none of these

Answer: C

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12. The value of the integral $\int_0^{\pi/2} \log|\tan x \cot x| dx$ is

A. $\pi \log 2$

B. $-\pi \log 2$

C. $\frac{\pi}{2} \log 2$

D. none of these

Answer: D

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13. If $I_1 = \int_x^1 \frac{1}{1+t^2} dt \text{ and } I_2 = \int_1^{1/x} \frac{1}{1+t^2} dt \text{for} x > 0$ then,

A. $I_1 = I_2$

B. $I_1 > I_2$

C. $I_2 = I_1$

D. none of these

Answer: A

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14. The value of $\int_{\frac{1}{e} \to \tan x} \frac{tdt}{1+t^2} + \int_{\frac{1}{e} \to \cot x} \frac{dt}{t \cdot (1+t^2)} =$

A. 0

B. 1

C. e

D. none of these

Answer: B

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15. The absolute value of $\int_{10}^{19} \frac{\cos x}{1+x^8} dx$, is

A. less than 10^{-7}

B. more than 10^{-7}

C. less than 10^{-8}

D. none of these

Answer: A

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16. If f(x) is an odd pefiodc function defined on the interval [T/2,T/2], where T is rthe perod of f(x). Then $\phi(x) = \int_a^x f(t) dt,$ is

A. periodic with period T

B. non-periodic

C. periodic with period 2T

D. perodic with period aT

Answer: A

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17. If $\int_{\pi/2}^{\theta} \sin x dx = \sin 2\theta$ then the of θ satisfying $0 < \theta < \pi$, is

A. $3\pi/2$

B. $\pi/6$

C. $5\pi/6$

D. $\pi/2$

Answer: D

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18. If f(x) is periodic function with period, T, then

A. $\int_a^b f(x)dx = \int_a^{b+T} f(x)dx$

B. $\int_a^b f(x)dx = \int_{a+T}^{b} f(x)dx$

C. $\int_a^b f(x)dx = \int_{a+T}^{b+T} f(x)dx$

D. $\int_a^b f(x)dx = \int_{a+T}^{b+2T} f(x)dx$

Answer: C

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19. The value of $\lim_{n \to \infty} \frac{1}{n}\{(n+1)(n+2)(n+3)\dots(n+n)\}^{1/n}$ is equal to

A. e

B. $1/e$

C. $2/e$

D. $4/e$

Answer: D

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20. The points of extremum of $\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)\,dt$ are

A. $x = 1, \; -1$

B. $x = \; -1, 2$

C. $x = 2, 1$

D. $x = \; -2, 1$

Answer: A

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21. $\int_{-2}^{2} \min(x - [x], -x - [x])dx$ equals, where [x] represents greates integer less than or equal to x.

A. 2

B. 1

C. 4

D. 0

Answer: B

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22. The integral $\int_0^a \frac{g(x)}{f(x) + f(a-x)}dx$ vanishes, if

A. g(x) is odd

B. f(x) =f(a-x)

C. g(x)=-g(a-x)

D. f(a-x)g=g(x)

Answer: C

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23. If $\frac{1}{\sqrt{a}}\int_1^a\left(\frac{3}{2}\sqrt{x}+1-\frac{1}{\sqrt{x}}\right)dx<4$ then 'a' may take values :

A. 0

B. 4

C. 9

D. none of these

Answer: D

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24. Evaluate $\frac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$ (where $[x]$ and $\{x\}$ are integral and fractional parts of x respectively and $n\varepsilon N$).

A. $\frac{1}{n-1}$

B. $\frac{1}{n}$

C. n

D. $n-1$

Answer: D

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25. If $f(x) = \min\{|x-1|, |x|, |x+1|,$ then the value of $\int_{-1}^{1} f(x)dx$ is equal to

A. 1

B. 2

C. 0

D. none of these

Answer: D

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26. The value of $\int_0^{100}\left[\tan^{-1}x\right]dx$ is equal to (where $[.]$ denotes the greatest integer function)

A. 100

B. $100-\tan^{-1}1$

C. 100- tan 1

D. none of these

Answer: C

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27. The value of $\int_{-1}^{10} sgn(x-[x])dx$ is equal to (where, [.] denotes the greatest integer function)

A. 10

B. 11

C. 9

D. $11/2$

Answer: B

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28. If $n \in N$, then $\int_{-n}^{n}(-1)^{[x]}dx$ equals

A. 2n

B. n

C. n^2

D. none of these

Answer: D

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29. The value of $\therefore \int_0^{[x]} \frac{2^x}{2^{[x]}} dx$ is

A. log 2

B. $\frac{[x]}{\log 2}$

C. $\frac{1}{2} \frac{[x]}{\log 2}$

D. none of these

Answer: B

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30. If f(x) is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all non zero x then, $\int_{\sin\theta}^{\text{cosec}\theta} f(x) dx$ equals

A. $\sin\theta + \text{cosec}\theta$

B. $\sin^2\theta$

C. $\text{cosec}^2\theta$

D. none of these

Answer: D

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31. If $f(x) = \begin{cases} e^{\cos x}\sin x & |x| \le 2 \\ 2 & otherwise \end{cases}$. Then $\int_{-2}^{3} f(x)dx =$ ______

A. 0

B. 1

C. 2

D. 3

Answer: C

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32. The value of the integral $\int_{e^{-1}}^{e^2}\left|\frac{\ln x}{x}\right|dx$ is:

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. 3

D. 5

Answer: B

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33. about to only mathematics

A. $-\frac{3}{2} \leq g(2) < \frac{1}{2}$

B. $0 \leq g(2) \leq 2$

C. $\frac{3}{2} \leq g(2) \leq \frac{5}{2}$

D. $2 < g(2) < 4$

Answer: B

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34. Let $f: (0, \infty) \to R$ and $F(x) = \int_0^x f(t)dt. IfF(x^2) = x^2(1+x)$,

then f (4) is equal to

A. $5/4$

B. 7

C. 4

D. 2

Answer: C

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35. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left([x]+1n\left(\frac{1+x}{1-x}\right)\right)dx$ is equal to (where [.] represents the greatest integer function)

A. $-\frac{1}{2}$

B. 0

C. 1

D. $2\ln\left(\frac{1}{2}\right)$

Answer: A

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36. Let $T>0$ be a fixed real number. Suppose f is continuous function such that for all $x\in R$, $f(x+T)=f(x)\cdot$ If $I=\int_0^T f(x)dx,$ then the value of $\int_3^{3+3T} f(2x)dx$ is $\frac{3}{2}I$ (b) $2I$ (c) $3I$ (d) $6I$

A. $\frac{3}{2}I$

B. 2I

C. 3I

D. 6I

Answer: C

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37. Let $f(x) = \int_{1}^{x} \sqrt{2-t^2}dt\cdot$ Then the real roots of the equation $x^2 - f'(x) = 0$ are ± 1 (b) $\pm \frac{1}{\sqrt{2}}$ $\pm \frac{1}{2}$ (d) 0 and 1

A. ± 1

B. $\pm \frac{1}{\sqrt{2}}$

C. $\pm \frac{1}{2}$

D. o and 1

Answer: A

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38. If $f\left(\frac{1}{x}\right) + x^2 f(x) = 0, x > 0 \text{ and } I = \int_{1/x}^{x} f(t)dt, \frac{1}{2} \leq x \leq 2x,$

then I is equal to

A. $f(2) - f\left(\frac{1}{2}\right)$

B. $f\left(\frac{1}{2}\right) - f(2)$

C. 0

D. none of these

Answer: C

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39. If $\left|\int_a^b f(x)dx\right| = \int_a^b |f(x)|dx, a < b, \text{then } \ f(x) = 0$ has

A. exactly one root in (a,b)

B. at least one root in (a,b)

C. no root in (a,b)

D. none of these

Answer: C

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40. Let f(x) be an odd continous function which is periodic with period 2.

If $g(x) = \int_0^x f(t)dt$, then

A. g(x) is an odd function

B. $g(n) = 0 \text{for all} n \in N$

C. $g(2n) = 0 \text{for all} n \in N$

D. g(x) is non periodic

Answer: C

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41. All the value of d for which

$$\int_1^2 \{a^2 + (4 - 4a)x + 4x^3\}dx \leq 12$$ are given by

A. a=3

B. $a \leq 4$

C. $0 \leq a \leq 3$

D. none of these

Answer: A

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42. Let $f(x)$ be a function defined by $f(x) = \int_1^x t(t^2 - 3t + 2)dt, 1 < x < 3$ then the maximum value of $f(x)$ is

A. [0,2]

B. [-1/4,4]

C. [-1/4,2]

D. none of these

Answer: C

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43. If $\int_0^x \{t\}dt = \int_0^{\{x\}} tdt(\text{where}x > 0 \ne Z \text{ and } \text{ and } \{\cdot\}$ represents fractional part function), then

A. $x \in (0, 1)$

B. $[x] = 1$

C. $x \in (1, 6) - I$

D. none of these

Answer: A

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44. Let $f(x) = \max.\{x+|x|, x-[x]\}$, where [x] denotes the greatest integer less than or equal to x, then $\int_{-2}^{2} f(x)$ is equal to

A. 3

B. 5

C. 1

D. none of these

Answer: B

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45. $\Rightarrow \int_0^{\infty}\left[\frac{2}{e^x}\right]dx$ (where [*] denotes the greatest integer function) equals

A. $\log_e 2$

B. e^2

C. 0

D. $2/e$

Answer: A

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46. $If \int_0^1 \frac{e^t dt}{t+1} = a, the \ne valuate \int_{b-1}^{b} \frac{e^{-t} dt}{t-b-1}$

A. ae^{-b}

B. $-ae^{-b}$

C. be^b

D. none of these

Answer: B

Watch Video Solution

47. Evaluate: $\int_{-1}^{4} f(x)dx = 4 and \int_{2}^{4} (3 - f(x))dx = 7$, then find the value of $\int_{-1}^{2} f(x)dx$.

A. 2

B. -3

C. 5

D. none of these

Answer: D

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48. For $x \varepsilon R$, and a continuous function f let $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} x f\{x(2-x)\}dx$ and $I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f\{x(2-x)\}dx$.

Then $\frac{I_1}{I_2}$ is

A. 0

B. 1

C. 2

D. 3

Answer: B

Watch Video Solution

49. $\int_1^4 \log_e [x] dx$ equals

A. $\log_e 6$

B. $\log_e 3$

C. $\log_e 2$

D. none of these

Answer: A

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50. If [.] denotes greatest integer function, then the value of $\int_{-\pi/2}^{\pi/2}\left(\left[\frac{\pi}{2}\right]+\right)dx$ is

A. π

B. $\pi/2$

C. 0

D. $-\pi/2$

Answer: C

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51. $\int_0^{100\pi}\left(\sum_{r=1}^{10}\tan rx\right)dx$ is equal to

A. 100π

B. -100π

C. 1

D. none of these

Answer: D

Watch Video Solution

52. If

$$I_1 = \int_0^{\pi/2} \cos(\sin x)dx,\ I_2 = \int_0^{\pi/2} \sin(\cos x)dx \text{ and } I_3 = \int_0^{\pi/2} \cos x dx$$

then

A. $I_1 > I_3 > I_2$

B. $I_3 > I_1 > I_2$

C. $I_1 > I_2 > I_3$

D. $I_3 > I_2 > I_1$

Answer: A

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53. For any $n \in N$, the value of the intergral $\int_0^{\pi} \frac{\sin 2nx}{\sin x} dx$ is,

A. π

B. 2π

C. $-\pi$

D. none of these

Answer: D

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54. For any $n \in N$, $\int_0^{\pi} \frac{\sin^2 nx}{\sin^2 x} dx$ is equal to

A. π

B. $n\pi$

C. 0

D. none of these

Answer: B

Watch Video Solution

55. For any $n \in N, \int_0^{\pi} \frac{\sin(2n+1)x}{\sin x} dx$ is equal to

A. π

B. 0

C. $n\pi$

D. $(2n+1) = \pi$

Answer: A

Watch Video Solution

56. If $\int_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots\ldots\ldots$ then

A. $I_n = I_{n+2}$

B. $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

C. $\sum_{m=1}^{10} I_{2m} = 0$

D. $I_n = I_{n+1}$

Answer: D

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57. If $I_n = \int_0^{\pi/4} \tan^n x dx$, then $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$, ... from\

A. am A.P.

B. a G.P.

C. a H.P.

D. none of these

Answer: A

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58. Let f(x) be a function defined on R satisfyin $f(x) = f(1-x)$ for all $x \in R$. Then $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x dx$ equals

A. $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x dx$

B. $2\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x dx$

C. $f(x)\sin dx dx$

D. none of these

Answer: D

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59. Evaluate: $5050\frac{\int 01\left(1-x^{50}\right)^{100} dx}{\int 01\left(1-x^{50}\right)^{101} dx}$

A. 5049

B. 5051

C. 5050

D. none of these

Answer: B

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60. If f and g are continuous functions on $[0, \pi]$ satisfying $f(x) + f(\pi - x) = 1 = g(x) + g(\pi - x)$ then $\int_0^{\pi} [f(x) + g(x)]dx$ is equal to

A. π

B. 2π

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{2}$

Answer: A

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61. If f(x) and g(x) are two continuous functions defined on [-a,a] then the the value of $\int_{-a}^{a}\{f(x)f+(-x)\}\{g(x)-g(-x)\}dx$ is,

A. 2a

B. f(a) g(a)

C. a

D. none of these

Answer: D

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62. Let f (x) be a conitnuous function defined on [0,a] such that $f(a-x)=f(x)\text{for all}x\in[0,a]$. If $\int_{0}^{a/2}f(x)dx=\alpha$, then $\int_{0}^{a}f(x)dx$ is equal to

A. α

B. 2α

C. 0

D. none of these

Answer: B

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63. The value of the integral $\int_0^{\pi/2} \sin 2nx \cot x dx$, where n is a positive integer, is

A. $\frac{\pi}{2}$

B. $-\pi$

C. π

D. none of these

Answer: A

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64. Evaluate $\int_1^{e^6}\left[\frac{\log x}{3}\right]dx,$ where [.] denotes the greatest integer function.

A. 0

B. e^{6-e^3}

C. e^6+e^3

D. e^3-e^6

Answer: B

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65. For any natural number n, theb value of $\Rightarrow\int_0^{n^2}\left[\sqrt{x}\right]dx,$ is

A. $\frac{n(n+1)(4n+1)}{6}$

B. $\dfrac{n(n-1)(4n+1)}{6}$

C. $\dfrac{n(n-1)(4n-1)}{6}$

D. none of these

Answer: B

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66. The value of the integral $\int_a^{a+\pi/2}(|\sin x|+|\cos x|)dx$ is

A. $a\pi$

B. $2a\pi$

C. $\dfrac{a\pi}{2}$

D. indenpedent of a

Answer: D

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67. If $\Rightarrow I_n = \int_a^{a+\pi/2} \frac{\cos^2 nx}{\sin x} dx, \text{ then} I_2 - I_1, I_3 - I_2, I_4 - I_3$ are in

A. G.P.

B. A.P.

C. H.P.

D. none of these

Answer: C

Watch Video Solution

68. Let f(x) be a polynomial of degree 2 satisfying $f(0) = 1, f(0) = -2 \text{ and } f''(0) = 6$, then $\int_{-1}^{2} f(x)$ is equal to

A. 6

B. 2

C. 9

D. none of these

Answer: C

Watch Video Solution

69. The value of $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$ where [.] denotes greatest integer function , is

A. 1

B. 2

C. $4 - \sin 4$

D. none of these

Answer: B

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70. $f(x)=\int_0^x f(t)dt=x+\int_x^1 tf(t)dt$, then the value of $f(1)$ is

A. $1/2$

B. 0

C. 1

D. $-1/2$

Answer: A

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71. If $f(x)=\int_0^{\sin x}\cos^{-1}t\,dt+\int_0^{\cos x}\sin^{-1}t\,dt, 0<x<\frac{\pi}{2}$ then $f(\pi/4)$ is equal to

A. $\frac{\pi}{\sqrt{2}}$

B. $1+\frac{\pi}{2\sqrt{2}}$

C. 1

D. none of these

Answer: D

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72. Let $f(x)$ be a continous function such that $\int_m^{n+1} f(x)dx = n^3, n \in Z$ Then , the value of $\int_{-30^3 f(x) dx}$ is

A. 9

B. -27

C. -9

D. none of these

Answer: B

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73. Let $f(x) = \frac{e^x + 1}{e^x - 1}$ and $\int_0^1 x^3 . \frac{e^x + 1}{e^x - 1} dx = \alpha \text{Then, } \int_{-1}^1 t^3 f(t) dt$ is equal to

A. 0

B. α

C. 2α

D. none of these

Answer: C

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74. If $\int_0^1 x e^{x^2} dx = \alpha \int_0^1 e^{x^2} dx$, then

A. $\alpha \in (0, 1)$

B. $(1, 2)$

C. $(-\infty, 0)$

D. $(0, \infty)$

Answer: A

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75. If $I = \int_0^1 \left(1 + e^{-x^2}\right) dx$ then, s

A. $I = \ \in (1, 2)$

B. $I \in (0, 1)$

C. $I \in \left(1 + \frac{1}{e}, 2\right]$

D. none of these

Answer: C

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76. If $I = \int_0^1 \frac{x}{8 + x^3} dx$ then the smallest interval is which I less is

A. (0,1/8)

B. (0,1/9)

C. (0,1/10)

D. (0,1/7)

Answer: B

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77. Let $f: R \to R$ be a continous function given by $f(x+y) = f(x)f(y)\text{for all}x, y \in R$. if $\int_0^2 f(x)dx = \alpha$ then $\int_{-2}^2 f(x)$ is equal to

A. 2α

B. α

C. 0

D. none of these

Answer: C

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78. Let f beintegrable over $[0, a]$ for any real value of a.

If $I_1 = \int_0^{\pi/2} \cos\theta f(\sin\theta + \cos^2\theta)\,d\theta$ and $I_2 = \int_0^{\pi/2} \sin 2\theta f(\sin\theta + \cos^2\theta)\,d\theta$, then

A. $I_1 = I_2$

B. $I_1 = -I_2$

C. $I_1 = 2I_2$

D. $I_1 = 2I_2$

Answer: A

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79. The value of $\lim_{x \to 0} \frac{2\int_0^{\cos x} \cos^{-1}(t)}{2x - \sin 2x} dx$ is

A. 0

B. $1/2$

C. $-1/2$

D. $2/3$

Answer: C

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80. If $I_1=\int_1^{\sin\theta}\frac{x}{1+x^2}dx$ and $I_2\int_1^{\operatorname{cosec}\theta}\frac{1}{x(x^{2+1})}dx$ then the value of $\begin{vmatrix} I_1 & I_1^2 & I_2 \\ e^{I_1+I_2} & I_2^2 & -1 \\ 1 & I_1^2+I_2^2 & -1 \end{vmatrix}$, is

A. $\sin\theta$

B. $\operatorname{cosec}\theta$

C. 0

D. 1

Answer: C

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81. If $f(x) = \int_1^x \frac{\log t}{1+t} dt \text{then} f(x) + f\left(\frac{1}{x}\right)$ is equal to

A. $(\log_e x)^2$

B. $\frac{2}{3} \log_e x$

C. $\frac{1}{2} \log_e x$

D. $\frac{1}{2} (\log_e x)^2$

Answer: D

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82. Let $F(x) = f(x) + f\left(\frac{1}{x}\right), \text{where} f(x) = \int_1^x \frac{\log t}{1+t} dt$ Then F (e) equals

A. 1

B. 2

C. $1/2$

D. 0

Answer: C

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83. $\int_0^x \frac{bt\cos 4t - a\sin 4t}{t^2}dt = \frac{a\sin 4x}{x}$ foe all $x \neq 0$, then a and b are given by

A. $a = \frac{1}{4}, b = 1$

B. $a = 2, b = 2$

C. $a = -1, b = 4$

D. $a = 2, b = 4$

Answer: A

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84. Let $f: R \to R$, $f(x) = \begin{cases} |x-[x]| & [x] \text{is odd} \\ |x-[x+1]| & [x] \text{is even} \end{cases}$ where [.] denotes greatest integer function, then

A. $\frac{5}{2}$

B. $\frac{3}{2}$

C. 5

D. 3

Answer: D

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85. If $f(x) = \sin x + \cos x$ and $g(x) = \begin{cases} \frac{|x|}{x} & ,x \neq 0 \\ 2 & ,x = 0 \end{cases}$ then the value of $\int_{-\pi/4}^{2\pi} gof(x) dx$ is equal to (a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ (c) π (d) None of these

A. $3\pi/4$

B. $\pi/4$

C. π

D. none of these

Answer: B

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86. If $x \in \left[(4n+1)\frac{\pi}{2}, (4n+3)\frac{\pi}{2}\right]$ and $n \in N$, then the value of $\int_0^x [\cos t]dt$, is

A. $(2n-1)\frac{\pi}{2} - x$

B. $(2n-1)\frac{\pi}{\pi}/(2) - x$

C. $(2n+1)\frac{\pi}{2} - x$

D. $(2n+1)\frac{\pi}{2} + x$

Answer: C

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87. If $f : R \in R$ is continuous and differentiable function such that

$$\int_{-1}^{x} f(t)dt + f'''(3)\int_{x}^{0} dt = \int_{1}^{x} t^3 dt - f'(1)\int_{0}^{x} t^2 dt + f'(2)\int_{x}^{3} r dt,$$

then the value of f'(4), is

A. $48 - 8f'(1) + f'(2)$

B. $48 - 8f'(1) - f''(2)$

C. $48 + 8f'(1) - f'(2)$

D. none of these

Answer: B

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88. Let $l_1 = \int_0^1 \frac{e^x}{1+x} dx$ and $l_2 = \int_0^1 \frac{x^2}{e^{x^3(2-x^3)}} dx$. Then $\frac{l_1}{l_2}$ is equal to

A. $\frac{3}{e}$

B. $\frac{e}{3}$

C. 3e

D. $\frac{1}{3e}$

Answer: C

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89. Let $f(x) = \begin{cases} 1-|x|, |x| \leq 1 \\ 0, |x| > 1 \end{cases}$ and $g(x) = f(x-1)$for all$x \in R$ The value of $\int_{-3}^{3} g(x)dx$ is

A. 2

B. 3

C. 4

D. 5

Answer: A

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90. If f(x) $= \frac{x-1}{x+1}, f^2(x) = f(f(x)), \dots\dots, \dots\dots f^{k+1}(x) = f\left(f^k(x)\right)$,k=1,2,3,......and $g(x) = f^{1998}(x)$ then $\int_{1/e}^{1} g(x)dx$ is equal to

A. 0

B. 1

C. -1

D. e

Answer: C

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91. If $f: R \in R$ be such that

f(x) $= \sqrt{\sin(\cos x)} + \text{In}\left(-2\cos^2 x + 3\cos x - 1\right),$ then

$\int_{x1}^{x2} \left[\cos x - \frac{1}{2}\right] dx$ is equal to, where $x1. x2 \in D$ and [.] denotes the greatest integer function,

A. 0

B. $\frac{1}{2}(x_2 - x_1)$

C. $x_1 - x_2$

D. $\frac{1}{2}(x_1 - x_2)$

Answer: A

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92. If $\int_e^x tf(t)dt = \sin x - x\cos x - \frac{x^2}{2}$ for all $x \in R - \{0\}$, then the value of $f\left(\frac{\pi}{6}\right)$ will be equal to

A. 0

B. 1

C. $-\frac{1}{2}$

D. none of these

Answer: C

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93. If $f(x) = \int_0^x \{f(t)\}^{-1} dt$ and $\int_0^1 \{f(t)\}^{-1} = \sqrt{2}$

A. $\sqrt{2x}$

B. $\sqrt{2 \log_e x}$

C. $\sqrt{3x - 1}$

D. none of these

Answer: A

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94. If $f(x)$ is differentiable and $\int_0^{t^2} x f(x) dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals (a)$\frac{2}{5}$ (b) $-\frac{5}{2}$ (c)1 (d) $\frac{5}{2}$

A. $2/5$

B. $-5/2$

C. 1

D. $5/2$

Answer: A

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95. The value of $\int_{-2}^{2} |1 - x^2| dx$ is______

A. $\frac{1}{3}$

B. $\frac{14}{3}$

C. $\frac{7}{3}$

D. (28)/(3)`

Answer: D

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96. The integral $\int_0^{\pi} x f(\sin x)\,dx$ is equal to

A. $\pi \int_0^{\pi} f(\cos x)\,dx$

B. $\pi \int_0^{\pi} f(\sin x)\,dx$

C. $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x)\,dx$

D. $\pi \int_0^{\pi/2} f(\cos x)\,dx$

Answer: D

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97. If $f(x)=\frac{e^x}{1+e^x}, I_1=\int_{f(-a)}^{f(a)} xg(x(1-x))dx,$ and $I_2=\int_{f(-a)}^{f(a)} g(x(1-x))dx,$ then the value of $\frac{I_2}{I_1}$ is

A. 1

B. -3

C. -1

D. 2

Answer: D

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98. The value of $\int_{-2}^{2} |[x]|dx$ is equal to

A. 1

B. 2

C. 3

D. 4

Answer: D

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99. The value

$$\int^{2}_{-(-2)}\left\{p \ \text{In}\left(\frac{1+x}{1-x}\right)+q \ \text{In} \ \left(\frac{1-x}{1+x}\right)-2+r\right\}dx$$ depends on the value of

A. p

B. q

C. r

D. p and q

Answer: C

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100. $7\left(\int_0^1 \frac{x^4(1-x)^4 dx}{1+x^2}+\pi\right)$ is equal to

A. 22

B. 23

C. 20

D. 21

Answer: A

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101. The value of $\lim_{x \to 0} \frac{1}{x^3}\int_0^x \frac{t\ln(1+t)}{t^4+4}dt$

A. 0

B. $1/12$

C. $1/24$

D. $1/24$

Answer: B

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102. Let f be the function defined on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) = \frac{\sin\left(\frac{9x}{2}\right)}{\sin\left(\frac{x}{2}\right)}$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is (asked as Match the following question)

A. 0

B. 2

C. 4

D. 6

Answer: C

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103. Let f be a real valued functional defined on the interval $(-1, 1)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}dt$, for all $x\varepsilon(-1, 1)$ and let f^{-1} be the inverse function of f. Then $f^{-1})'(2)$ is equal to

A. 1

B. $1/3$

C. $1/2$

D. $1/e$

Answer: B

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104. For any real number x, let $[x]$ denote the largest integer less than or equal to x, Let f be a real-valued function defined on the interval $[-10, 10]$ be $f(x) = \{x - [x], \quad \text{if} \quad [x]$ is odd, $1 + [x] - x, \quad \text{if} \quad [x]$ is even Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x dx$ is____

A. 3

B. 4

C. 6

D. 9

Answer: B

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105. about to only mathematics

A. f''(x)exits for all $(x \in (0, \infty))$

B. f'(x) exits for all $x \in (0, \infty))$ and f'(x) is continous on $(0, \infty)$ but not differentiable on $(0, \infty)$

C. there exits $\alpha > 1$ such that $|f'(x)| < f(x)$ for all $x \in (\alpha, \infty))$

D. there exits $\beta > 0$ such that $|f'(x)| + f'(x) \leq \beta$ for all $x \in (0, \infty)$

Answer: C

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106. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$ for all $x \varepsilon [0, 1]$, $p(0) = 1$ and $p(1) = 41$.

Then $\int_0^1 p(x)dx$ is equals to (a)42 (b)$\sqrt{41}$ (c)21 (d)41

A. 41

B. 42

C. $\sqrt{41}$

D. 21

Answer: D

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107. If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$ then the value of $f\left(\frac{\pi}{6}\right)$ is ___

A. $\frac{\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{2}$

Answer: A

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108. The value of $\frac{\pi^2}{1n3}\int_{\frac{7}{6}}^{\frac{5}{6}} \sec(\pi x)dxis\_\_$

A. $\frac{2\pi}{3}$

B. $\frac{\pi}{2}$

C. π

D. $\frac{4\pi}{3}$

Answer: C

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109. The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2+\log\frac{\pi-x}{\pi+x}\right)\cos x dx$

A. 0

B. $\frac{\pi^2}{2}-4$

C. $\frac{\pi^2}{2}-4$

D. $\frac{\pi^2}{2}$

Answer: B

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110. Let $f:(0,1)\rightarrow(0,1)$ be a differentiable functino such that $f'(x)\neq 0$ for all $x\in(0,1)$ and $f\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$. Suppose for all $x\lim_{t\rightarrow x}\left(\frac{\int_0^1\sqrt{1-(f(s))^2}ds-\int_0^x\sqrt{1-(f(s))^2}ds}{f(t)-f(x)}\right)=f(x)$. Then the value of $f\left(\frac{1}{4}\right)$ belongs to:

A. $\left\{\left(\sqrt{7}, \sqrt{15}\right)\right\}$

B. $\left\{\dfrac{\sqrt{7}}{2}, \dfrac{\sqrt{15}}{2}\right\}$

C. $\left\{\dfrac{\sqrt{7}}{3}, \dfrac{\sqrt{15}}{3}\right\}$

D. $\left\{\dfrac{\sqrt{7}}{4}, \dfrac{\sqrt{15}}{4}\right\}$

Answer: D

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111. The value of the integral $\int_0^2 \dfrac{\log\left(x^2+2\right)}{(x+2)^2}$, dx is

A. $\dfrac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}+\dfrac{5}{12}\log 2-\dfrac{1}{4}\log 3$

B. $\dfrac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}-\dfrac{5}{12}\log 2-\dfrac{1}{12}\log 3$

C. $\dfrac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}+\dfrac{5}{12}\log 2+\dfrac{1}{4}\log 3$

D. $\dfrac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}-\dfrac{5}{12}\log 2+\dfrac{1}{12}\log 3$

Answer: D

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112. The following integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\cos ecx)^{17} dx$ is equal to

$(a) \int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$ $(b) \int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{17} du$

$(c) \int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{17} du$ $(d) \int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16} du$

A. $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$

B. $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17} du$

C. $\int_0^{\log(1+\sqrt{2})} (e^u - e^{-u})^{17} du$

D. $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16} du$

Answer: A

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113. Let $f:[0,2]\overrightarrow{R}$ be a function which is continuous on [0,2] and is differentiable on (0,2) with $f(0)=1$

$Let: F(x) = \int_0^{x^2} f(\sqrt{t})\,dt f$ or $x \in [0,2] \dot{I} f F'(x) = f'(x)$. for all $x \in (0,2)$, then $F(2)$ equals $e^2 - 1$ (b) $e^4 - 1$ $e - 1$ (d) e^4

A. $e^2 - 1$

B. $e^4 - 1$

C. $e - 1$

D. e^4

Answer: B

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114. Given that for each $a \varepsilon (0,1)$, $\lim\ (h \to 0^+) \int_h^{1-h} t^{-a}(1-t)^{a-1}dt$ exists. Let this limit be $g(a)$. In addition it is given the function $g(a)$ is differentiable on $(0,1)$.

The value of $g\left(\frac{1}{2}\right)$ is a. $\frac{\pi}{2}$ b. π c. $-\frac{\pi}{2}$ d. 0

A. π

B. 2π

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: A

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115. Given that for each $a\varepsilon(0, 1)$, $\lim\left(h \to 0^+\right)\int_h^{1-h} t^{-a}(1-t)^{a-1}dt$ exists. Let this limit be $g(a)$. In addition it is given the function $g(a)$ is differentiable on $(0, 1)$.

The value of $g\left(\frac{1}{2}\right)$ is a. $\frac{\pi}{2}$ b. π c. $-\frac{\pi}{2}$ d. 0

A. $\pi/2$

B. π

C. $-\pi/2$

D. 0

Answer: D

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116. The option(s) with the values of $a and L$ that satisfy the following equation is (are) $\frac{\int 04\pi e^t\left(s\in^6 at+\cos^4 at\right)dt}{\int 0\pi e^t\left(s\in^6 at+\cos^4 at\right)dt}=L$ $a=2, L=\frac{e^{4\pi}-1}{e^{\pi}-1}$ (b) $a=2, L=\frac{e^{4\pi}+1}{e^{\pi}+1}$ $a=4, L=\frac{e^{4\pi}-1}{e^{\pi}-1}$ (d) $a=4, L=\frac{e^{4\pi}+1}{e^{\pi}+1}$

A. $a=2, L=\frac{e^{4\pi}-1}{e^{\pi}-1}$

B. $a=2, L=\frac{e^{4\pi}+1}{e^{\pi}+1}$

C. $a=4, L=\frac{e^{4\pi}-1}{e^{\pi}-1}$

D. $a=4, L=\frac{e^{4\pi}+1}{e^{\pi}+1}$

Answer: A::C

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117. Let $f: R \to R$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t)dt$ for all $x \in [-1, 2]$ and $G(x) = \int_{-1}^{x} t|f(f(t))|dt$ for all $x \in [-1, 2]$. If $\lim_{x \to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, Then the value of $f\left(\frac{1}{2}\right)$ is

A. 7

B. 8

C. 9

D. 6

Answer: A

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118. Let F : R $\to$ R be a thrice differentiable function . Suppose that $F(1) = 0, F(3) = -4$ and $F(x) < 0$ for all $x \in (1, 3)$. $f(x) = xF(x)$ for all $x \in R$.

If $\int_1^3 x^2 F'(x)dx = -12$ and $\int_1^3 x^3 F''(x)dx = 40$, then the correct expression (s) is //are

A. $9f'(3) + f'(1) - 32 = 0$

B. $\int_1^3 f(x)dx = 12$

C. $9f'(3) - f'(1) + 32 = 0$

D. $\int_1^3 f(x)dx = -12$

Answer: C::D

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119. Let $f:(0, \infty) \to R$ be a continuous function such that $f(x) = \int_0^x tf(t)dt$. If $f(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$, is equal to

A. 216

B. 219

C. 222

D. 225

Answer: B

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120. Let $f:[0,1]\rightarrow R$ be a differentiable functions with non-increasing derivative such that $f(0)=0,\ f'(1)>0$, then

A. $\int_0^1 \frac{1}{f^2(x)+1}dx > \frac{f(1)}{f'(1)}$

B. $\int_0^1 \frac{1}{f^2(x)+1}dx < \frac{f(1)}{f'(1)}$

C. $\int_0^1 \frac{1}{f^2(x)+1}dx < \frac{\tan^{-1}(f(1))}{f'(1)}$

D. $\int_0^1 \frac{1}{f^2(x)+1}dx = \frac{f(1)}{f'(1)}$

Answer: C

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121. Let $f: R \to R$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t)dt.$ $f(x)$ increases for

A. $\frac{-2}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $-\frac{1}{3}$

Answer: B

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122. If f(x) is a continuous function such that $f(x) > 0$ for all $x > 0$ and $(f(x))^{2020} = 1 + \int_0^x f(t)dt,$ then the value of $\{f(2020)\}^{2019}$ is equal to

A. 2019

B. 2020

C. 2021

D. 2018

Answer: B

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123. If a function y=f(x) such that f'(x) is continuous function and satisfies $(f(x))^2 = k + \int_0^x \left[\{f(t)\}^2 + \{f'(t)\}^2\right] dt, k \in R^+$, then

A. f(x) is an increasing function for all $x \in R$

B. f(x) is a bounded function

C. f(x) is neither even nor odd function

D. If k=100, then f(0)=10.

Answer: A::C

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124. The maximum value of f(x) $= \int_0^1 t\sin(x+\pi t)dt$ is

A. $\frac{1}{\pi}\sqrt{\pi^2+4}$

B. $\frac{1}{\pi^2}\sqrt{\pi^2+4}$

C. $\sqrt{\pi^2+4}$

D. $\frac{1}{2\pi^2}\sqrt{\pi^2+4}$

Answer: B

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125. If $I_n = \int_0^\pi e^x \sin^n x \quad \mathrm{dx\ then} \quad \frac{I_3}{I_1}$ is equal to

A. $\frac{3}{5}$

B. $\frac{1}{5}$

C. 1

D. $\frac{2}{5}$

Answer: A

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126. If $\lambda = \int_0^1 \frac{e^t}{1+t}$, then find the value of $\int_0^1 e^t \log_e(1+t)dt$

A. k

B. 2k

C. e In 2-k

D. none of these

Answer: C

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127. If $k \in N$ and $I_k = \int_{-2kp}^{2k\pi} |\sin x|[\sin x]dx$, where [.] denotes the greatest integer function, then $\int_{k=1}^{100} I_k$ equal to

A. -10100

B. -40400

C. -20200

D. none of these

Answer: C

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128. The value of

$$\int_{-1}^{1} \frac{\log\left(x+\sqrt{1+x^2}\right)}{x+\log\left(x+\sqrt{1+x^2}\right)} f(x)dx - \int_{-1}^{1} \frac{\log\left(x+\sqrt{1+x^2}\right)}{x+\log\left(x+\sqrt{1+x^2}\right)} f(-x)$$

,

A. 0

B. $2\int_{0}^{1} \frac{\log\left(x+\sqrt{1+x^2}\right)}{x+\log\left(x+\sqrt{1+x^2}\right)} \{f(x)-f(-x)\}dx$

C. 2f(x)

D. none of these

Answer: A

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129. If $\int_0^1 \alpha e^{\beta x^2} \sin(x+k)dx = 0$ for some $\alpha, \beta \in R, \alpha \neq 0$, then the value of k can belong to the interval

A. $\left[\frac{\pi}{3}, \frac{5\pi}{12}\right]$

B. $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

C. $\left[\frac{3\pi}{4}, \frac{5\pi}{6}\right]$

D. $\left[-\frac{\pi}{2}, -\frac{\pi}{3}\right]$

Answer: C

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130. $\int_0^{[x]/3} \frac{8^x}{2^{[3x]}} dx$ where [.] denotes the greatest integer function, is equal to

A. $\frac{[x]}{\text{In}2}$

B. $\frac{[x]}{\text{In}2}$

C. $\frac{2[x]}{\text{In}2}$

D. $\frac{[x]}{\text{In}8}$

Answer: D

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131. Let f(x)=In $\left[\cos|x| + \frac{1}{2}\right]$ where [.] denotes the greatest integer function, then $\int_{x_1}^{x_2} \lim_{n \to \infty} \left(\frac{\{f(x)\}^n}{x^2 + \tan^2 x}\right) dx$ is equal to, where $x_1, x_2 \in \left[\frac{-\pi}{6}, \frac{\pi}{6}\right]$

A. 1

B. 2

C. -1

D. 0

Answer: D

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132. $\lim_{x \to 0} \left(\frac{\int_0^x x e^{t^2} dt}{1 + x - e^x} \right)$ is equal to

A. 1

B. -1

C. 2

D. -2

Answer: D

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133. If $\int_{2x^2}^{x^3} (Inx) f(t) dt = x^2 - 2x + 5$, then f(8)=

A. $\frac{1}{\text{In}2}$

B. $\frac{1}{2\text{In}2}$

C. $\frac{1}{3\text{In}2}$

D. $\frac{1}{4\text{In}2}$

Answer: B

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134. $\lim_{x \to 0} \frac{\int_{-x}^{x} f(t) dt}{\int_{0}^{2x} f(t+4) dt}$ is equal to

A. f(0)

B. 0

C. $\frac{f(4)}{f(0)}$

D. $\frac{f(0)}{f(4)}$

Answer: D

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135. IF $f(x+f(y))=f(x)+y \forall x, y \in R$ and $f(0)=1,$ then $\int_0^{10} f(10-x)dx$ is equal to

A. 1

B. 10

C. $\int_0^1 f(x)dx$

D. $10\int_0^1 f(x)dx$

Answer: D

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136. If $\alpha, \beta(\beta > \alpha)$, are the roots of $g(x) - ax^2 + bx + c = 0$ and $f(x)$ is an even function, then $\int_{\alpha}^{\beta} \frac{e}{e^{f\left(\frac{g(x)}{x-\alpha}\right)} + e^{f\left(\frac{g(x)}{x-\beta}\right)}} = \left|\frac{b}{2a}\right|$ (b) $\frac{\sqrt{b^2 - 4ac}}{|2a|}$ $\left|\frac{b}{a}\right|$ (d) none of these

A. $\left|\frac{b}{a}\right|$

B. $\left|\frac{b}{2a}\right|$

C. $\frac{\sqrt{b^2 - 4ac}}{|2a|}$

D. none of these

Answer: C

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137. The value of the constant $a > 0$ such that $\int_0^a [\tan^{-1}\sqrt{x}]dx = \int_0^a [\cot^{-1}\sqrt{x}]dx$, where[.] denotes the greatest integer function, is

A. $\frac{2(3 + \cos 4)}{1 - \cos 4}$

B. $\dfrac{2(3-\cos 4)}{1+\cos 4}$

C. $\dfrac{2(3-\cos 4)}{1-\cos 4}$

D. $\dfrac{2(3+\cos 4)}{1+\cos 4}$

Answer: A

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138. If f(x) is a continuous function in $[0, \pi]$ such that f(0)=f(x)=0, then the value of

$\int_0^{\pi/2} \{f(2x) - f''(2x)\}\sin x \cos x dx$ is equal to

A. π

B. 2π

C. 3π

D. 0

Answer: D

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139. Let $f: R \to R$ be a continuous function and $f(x) = f(2x)$ is true $\forall x \in R$. If $f(1) = 3$, then the value of $\int_{-1}^{1} f(f(x))dx$ is equal to

A. 3f(0)

B. 0

C. 3f(3)

D. 6

Answer: D

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140. The value of

$$\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x)\, d\left(x - \frac{[x]}{1!} + \frac{[x]^2}{2!} - \frac{[x]^3}{3!} + \dots\right)$$

where [x] is greatest function, is

A. $\frac{1}{n}$

B. $\frac{1}{n+2}$

C. $\frac{1}{n-1}$

D. $\frac{1}{n-2}$

Answer: C

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141. The value of the definite integral

$\int_{t+2\pi}^{t+5\pi/2}\left\{\sin^{-1}(\cos x)+\cos^{-1}(\cos x)\right\}$dx is equal to

A. $\frac{\pi^2}{2}$

B. $\frac{\pi^2}{8}$

C. $\frac{\pi^2}{4}$

D. none of these

Answer: C

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142. If f(x) is an integrable function on $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and

$$I_1 = \int_{\pi/6}^{\pi/3} \sec^2\theta f(2\sin 2\theta)d\theta \text{ and } I_2 = \int_{\pi/6}^{\pi/3} \text{cosec}^2\theta f(2\sin 2\theta)d\theta,$$

then

A. $I_1 = 2I_2$

B. $I_1 = 3I_2$

C. $2I_1 = I_2$

D. none of these

Answer: D

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143. Let

$$f(x) = \lim_{n \to \infty} \left\{ \frac{n^n(x+n)\left(x+\frac{n}{2}\right)....\left(x+\frac{n}{2}\right)}{n!(x^2+n^2)\left(x^2+\frac{n^2}{4}\right)...\left(x^2+\frac{n^2}{n^2}\right)} \right\}^{x/n} \text{ for all } x > 0$$

. Then,

A. $f\left(\frac{1}{2}\right) \geq f(1)$

B. $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$

C. $f'(2) \leq 0$

D. $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$

Answer: B::C

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144. The total number for distinct $x\varepsilon[0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4}dt = 2x - 1$ is ________.

A. 1

B. 2

C. 3

D. infinitely many

Answer: A

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145. For $x \in R, x \neq 0,$ if $y(x)$ differential function such that $x\int_1^x y(t)dt = (x+1)\int_1^x ty(t)dt,$ then $y(x)$ equals: (where C is a constant.)

A. $Cx^3e^{1/x}$

B. $\frac{C}{x^2}e^{-1/x}$

C. $\frac{C}{x^2}e^{-1/x}$

D. $\frac{C}{x^3}e^{-1/x}$

Answer: D

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Section II - Assertion Reason Type

1. Statement-1: $\int_0^{\pi/2} x \cot x dx = \frac{\pi}{2}\log 2$

Statement-2: $\int_0^{\pi/2} \log \sin x dx = -\frac{\pi}{2}\log 2$

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: A

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2. Statement-1: $\int_0^{n\pi+v} |\sin x| dx = 2n + 1 - \cos v$ where $n \in N$ and $0 \leq v < \pi$.

Stetement-2: If f(x) is a periodic function with period T, then

(i) $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$, where $n \in N$

and (ii) $\int_{nT}^{nT+a} f(x)dx = \int_0^a f(x)dx$, where $n \in N$

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: A

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3. Let $I_n = \int_0^{\pi/4} \tan^n x dx.$

Statement-1: $\frac{1}{n+1} < 2I_n < \frac{1}{n-1}$ for all n=2,3,4,.....

Statement-2: $I_n + I_{n-2} = \frac{1}{n-1}$,n=3,4,5,......

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: A

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4. Statement-1: If $f(x) = \int_1^x \frac{\log_e t}{1+t+t^2} dt$, then $f(x) = f\left(\frac{1}{x}\right)$ for all $x gr 0$.

Statement-2:If f(x) $= \int_1^x \frac{\log_e t}{1+t} dt$, then $f(x) + f\left(\frac{1}{x}\right) = \frac{(\log_e x)^2}{2}$

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: B

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5. Let $n \in N$ such that $n > 1$.

Statement-1: $\int_{\infty}^{0} \frac{1}{1+x^n} dx = \int_{0}^{1} \frac{1}{(1-x^n)^{1/n}} dx$

Statement-2: $\int_{a}^{b} f(x) dx = \int_{a}^{b} f(a+b-x) dx$

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: B

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6. Statement-1:For any $n \in N$, we have

$$\int_0^{n\pi} \left|\frac{\sin x}{x}\right| dx \geq \frac{2}{\pi}\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots. + \frac{1}{n}\right)$$

Statement-2: $\frac{\sin x}{x} \geq \frac{2}{\pi} \text{on} \left(0, \frac{\pi}{2}\right)$

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: B

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7. Statement-1: $\int_0^1 \frac{\cos x}{1+x^2} dx > \frac{\pi}{4}\cos 1$

Statement-2: If f(x) and g(x) are continuous on [a,b], then

$\int_a^b f(x)g(x)dx = f(c)\int_a^b g(x)$ for some $c \in (a, b)$.

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: A

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8. Statement-1:

$$\int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt = \frac{\pi}{4}$$ for all x.

Statement-2: $\frac{d}{dx} \int_{\theta(x)}^{\psi(x)} f(t)\, dt = \psi'(x) f(\psi(x)) - \theta'(x) f(\theta(x))$

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: A

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9. Statement-1: $\int_0^{\pi/2} \frac{1}{1 + \tan^3 x} dx = \frac{\pi}{4}$

Statement-2: $\int_0^a f(x) dx = \int_0^a f(a + x) dx$

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: C

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10. Let F(x) $= \int_a^{x^2} \cos\sqrt{t}dt$

Statement-1: F'(x)=cos x

Statement-2: If f(x) $= \int_a^x \phi(t)dt$, then f'(x)=ϕ(x).

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: D

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11. $I_n = \int_0^{\pi/4} \tan^n x dx$, where $n \in N$

Statement-1: $\int_0^{\pi/4} \tan^4 x dx = \frac{3\pi - 8}{12}$

Statement-2: $I_n + I_{n-2} = \frac{1}{n-1}$

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: A

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12. Statement-1: The value of the integral

$\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt{\tan x}}dx$ is equal to $\frac{\pi}{6}$

Statement-2: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

A. Statement-1 is true, Statement-2 is True,Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is True, Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is True, Statement-2 is False.

D. Statement-1 is False, Statement-2 is True.

Answer: D

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Exercise

1. If $I=\int_{-2}^{2} dx$, then I equals

A. 6

B. 8

C. 4

D. 21

Answer: C

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2. The value of the integral $\int_{\alpha}^{\beta} \frac{1}{\sqrt{(x-\alpha)(\beta-x)}} dx \quad \text{for} \quad \beta > \alpha$, is

A. $\sin^{-1}(\alpha / \beta)$

B. $\pi / 2$

C. $\sin^{-1}(\beta / 2\alpha)$

D. π

Answer: D

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3. $\int_{1}^{\frac{4\sqrt{3}}{3}-1} \frac{x+2}{\sqrt{x^2+2x-3}} dx$ equal to

A. $\frac{2}{\sqrt{3}} - \frac{1}{2}\log 3$

B. $\frac{2}{\sqrt{3}} + \log(2\sqrt{3}-1)$

C. $\frac{2\sqrt{3}}{3} - \frac{1}{2}\log(\sqrt{3}+2)$

D. $\frac{2\sqrt{3}}{3} + \frac{1}{2}\log(\sqrt{3}+2)$

Answer: B

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4. $\int_0^{\pi^2/4} \sin\sqrt{x}dx$ equals to

A. 0

B. 1

C. 2

D. 4

Answer: C

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5. $\int_{1/2}^{2} |\log_{10} x| dx$ equals to

A. $\log 10(8/e)$

B. $\frac{1}{2}\log 10(8/e)$

C. $\log 10(2/e)$

D. none of these

Answer: B

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6. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)d\theta,\ a>0$

A. 0

B. 1

C. 2

D. none of these

Answer: A

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7. The value of the integral $\int_{-\pi/3}^{\pi/3} \frac{x \sin x}{\cos^2 x} dx$, is

A. $(\pi/3 - \log \tan 3\pi/2)$

B. $2(2\pi/3 - \log \tan 5\pi/12)$

C. $3(\pi/2 - \log \sin \pi/12)$

D. none of these

Answer: B

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8. The value of $\int_1^{7\sqrt{2}} \frac{1}{x(2x^7+1)dx}$ is

A. $\log(6/5)$

B. $6\log(6/5)$

C. $(1/7)\log(6/5)$

D. $(1/12)\log(6/5)$

Answer: C

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9. The value of $\int_{-1}^{3}\{|x-2|+[x]\}dx$, where [.] denotes the greatest integer function, is equal to

A. 7

B. 5

C. 4

D. 3

Answer: A

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10. If f(x)= $= \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$ then the value of $\int_0^{\pi/2} f(x)dx$, is

A. 3

B. $2/3$

C. $1/3$

D. 0

Answer: C

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11. Evaluate $\lim_{x \to \infty} \frac{\left(\int_0^x e^{x^2} dx\right)^2}{\int_0^x e^{2x^2} dx}$.

A. 1

B. 2

C. 3

D. 0

Answer: D

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12. The value of $\int_1^4 e^{\sqrt{x}} dx$, is

A. e^2

B. $2e^2$

C. $4e^2$

D. $3e^2$

Answer: B

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13. The value of $\int_0^{1000} e^{x-[x]}dx$, is ([.] denotes the greatest integer function) :

A. $\frac{e^{1000}-1}{1000}$

B. $\frac{e^{1000}-1}{e-1}$

C. 1000(e-1)

D. $\frac{e-1}{1000}$

Answer: C

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14. The value of the integral $\int_0^{100} \sin(x-[x])\pi dx$, is

A. $100/\pi$

B. $200/\pi$

C. 100π

D. 200π

Answer: B

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15. The difference between the greatest and least values of the function $\phi(x) = \int_0^x (t+1)$dt on [2,3], is\

A. 3

B. 2

C. $7/2$

D. $11/2$

Answer: C

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16. The value of $\int_0^1 \frac{2^{2x+1} - 5^{2x-1}}{10^x} dx$ is

A. $\frac{3}{5}\left\{\frac{2}{\log e\left(\frac{2}{5}\right)} + \frac{1}{2\log e\left(\frac{5}{2}\right)}\right\}$

B. $-\frac{3}{5}\left\{\frac{2}{\log e\left(\frac{2}{5}\right)} + \frac{1}{2\log e\left(\frac{5}{2}\right)}\right\}$

C. $\frac{3}{5}\left\{\frac{2}{\log e^{\left(\frac{2}{5}\right)}} - \frac{1}{2\log e^{\left(\frac{5}{2}\right)}}\right\}$

D. none of these

Answer: B

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17. The value of $\int_0^{\pi/2} \frac{\cos 3x + 1}{2\cos x - 1} dx$ is

A. 2

B. 1

C. $1/2$

D. 0

Answer: B

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18. The value of $\int_0^{16\pi/3} |\sin x| dx$ is

A. 21

B. $21/2$

C. 10

D. 11

Answer: B

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19. If $\int_0^{n\pi} f(\cos^2 x) dx = k \int_0^{\pi} f(\cos^2 x) dx$, then the value of k, is

A. 1

B. n

C. $n/2$

D. none of these

Answer: B

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20. The value of $\int_{-\pi}^{\pi} \sin x f(\cos x) dx$ is

A. π

B. 2π

C. 2f(1)

D. none of these

Answer: D

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21. If $a < \int_0^{2\pi} \frac{1}{10+3\cos x} dx < b$. Then the ordered pair (a,b) is

A. $\left(\frac{2\pi}{7}, \frac{2\pi}{3}\right)$

B. $\left(\frac{2\pi}{13}, \frac{2\pi}{7}\right)$

C. $\left(\frac{\pi}{10}, \frac{2\pi}{13}\right)$

D. none of these

Answer: B

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22. The value of the integral $\int_0^{\infty} \frac{x \log x}{(1+x^2)^2} dx, is$ (a)0 (b) log 7 (c) 5 log 13 (d) none of these

A. 1

B. 0

C. 2

D. none of these

Answer: B

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23. The value of the integral $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^2 x}\,dx$ is

A. 0

B. $2/3$

C. $4/3$

D. none of these

Answer: C

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24. The value of the integral $\int_{-\frac{\pi}{2}}^{\pi/2} \sqrt{\frac{1+\cos 2x}{2}} dx$ is

A. -2

B. 2

C. 0

D. -3

Answer: B

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25. Let $I_1 = \int_1^2 \frac{x}{\sqrt{1+x^2}} dx$ and $I_2 = \int_1^2 \frac{1}{x} dx$.Then

A. $I_1 > I_2$

B. $I_2 > I_1$

C. $I_1 = I_2$

D. $I_2 > 2I_2$

Answer: A

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26. Evaluate the following integral: $\int_0^{\pi/4} \frac{s \in x + \cos x}{3 + s \in 2x} dx$

A. $-\frac{1}{4}\log 3$

B. $\frac{1}{4}\log 3$

C. $-\frac{1}{3}\log 4$

D. none of these

Answer: B

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27. The value of the integral $\int_0^{\pi/4} \frac{\sin\theta + \cos\theta}{9 + 16\sin 2\theta} d\theta$, is

A. log 3

B. log 2

C. $\frac{1}{20}\log 3$

D. $\frac{1}{20}\log 2$

Answer: C

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28. Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right), x > 0.$ If $\int_1^4 \frac{3}{x}e^{\sin x} \, \hat{} \, 3dx = F(k) - F(1)$, then one of the possible values of k, is: 15 (b) 16 (c) 63 (d) 64

A. 64

B. 15

C. 16

D. 63

Answer: A

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29. If $I = \int_{-1}^{1}\left(\left[x^2\right] + \log\left(\frac{2+x}{2-x}\right)\right)dx$ where [x] denotes the greatest integer $\leq x$, then I equals :

A. -2

B. -1

C. 0

D. 1

Answer: C

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30. The value of $\int_{-\pi/2}^{\pi/2}(x^2 + x\cos x + \tan^5 x + 1)dx$ is equal to

A. 0

B. 2

C. π

D. none of these

Answer: D

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31. Evaluate: $\int_{-1}^{4} f(x)dx = 4 and \int_{2}^{4} (3 - f(x))dx = 7,$ then find the value of $\int_{-1}^{2} f(x)dx \cdot$

A. 2

B. -3

C. -5

D. none of these

Answer: C

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32. The value of $I=\int_0^{\pi/2}\frac{1}{1+\cos x}dx$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 1

D. π

Answer: C

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33. about to only mathematics

A. $\frac{\pi}{2}$

B. $\sqrt{2}\log(\sqrt{2}+1)$

C. $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$

D. none of these

Answer: C

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34. The value of integral $\int_a^b \frac{|x|}{x} dx, a < b$ is :

A. |a|-|b|

B. |b|-|a|

C. |a|-|b|

D. |b|-|a|

Answer: B

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35. The value of the integral

$\int_0^{2\pi} \frac{\sin 2\theta}{a - b\cos\theta} d\theta$ when $a > b > 0$, is

A. 1

B. π

C. $\pi/2$

D. 0

Answer: D

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36. The value of the integral $I=\int_0^1 x(1-x)^n dx$ is equal to

A. $\dfrac{1}{n+1}+\dfrac{1}{n+2}$

B. $\dfrac{1}{(n+1)(n+2)}$

C. $\dfrac{1}{n+2}-\dfrac{1}{n+1}$

D. $2\left(\dfrac{1}{n+1}-\dfrac{1}{n+2}\right)$

Answer: B

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37. The value of the integral

$$\int_0^{3\alpha} \cos ec(x-\alpha)\cos ec(x-2\alpha)dx$$ is

A. $2\sec\alpha\log\left(\frac{1}{2}\operatorname{cosec}\alpha\right)$

B. $2\sec\alpha\log\left(\frac{1}{2}\sec\ \ \alpha\right)$

C. $2\operatorname{cosec}\alpha\log(\sec\alpha)$

D. $2\ \ \operatorname{cosec}\ \ \alpha\log\left(\frac{1}{2}\sec\alpha\right)$

Answer: D

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38. The value of the integral $\int_0^{\pi}\frac{\sin 2kx}{\sin x}dx$, where $k\in I$, is :

A. π

B. $\frac{\pi}{2}$

C. $\frac{k\pi}{2}$

D. 0

Answer: D

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39. The value of the integral $\int_0^1 \frac{dx}{x^2 + 2x\cos\alpha + 1}$ is equal to $\sin\alpha$ (b) $\alpha\sin\alpha$ $\frac{\alpha}{2\sin\alpha}$ (d) $\frac{\alpha}{2}\sin\alpha$

A. $\sin\alpha$

B. $\alpha\sin\alpha$

C. $\frac{\alpha}{2\sin\alpha}$

D. $\frac{\alpha}{2}\sin\alpha$

Answer: C

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40. The greater value of $F(x) = \int_1^x |t| dt$ on the interval $[-1/2, 1/2]$, is

A. $\frac{3}{8}$

B. $\frac{1}{2}$

C. $-\frac{3}{8}$

D. $-\frac{1}{2}$

Answer: C

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41. The value of the integral $\int_0^{\pi/2} |\sin x - \cos x| dx$, is

A. 0

B. $2(\sqrt{2} - 1)$

C. $2\sqrt{2}$

D. $2(\sqrt{2} + 1)$

Answer: A

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42. The value of the integral $\int_{-\pi/4}^{\pi/4} \sin^{-4} x dx$, is

A. $-\frac{8}{3}$

B. $\frac{3}{2}$

C. $\frac{8}{3}$

D. none of these

Answer: A

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43. The value of the integral $I = \int_1^{\infty} \frac{x^2 - 2}{x^3\sqrt{x^2 - 1}} dx$, is

A. 0

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. none of these

Answer: A

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44. $\int_0^1 |\sin 2\pi x| dx$ id equal to

A. 0

B. $-\frac{1}{\pi}$

C. $\frac{1}{\pi}$

D. $\frac{2}{\pi}$

Answer: D

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45. Evaluate: $\int_{-\pi}^{\pi}(\cos ax + s \in bx)^2 dx$

A. $-\pi$

B. 0

C. π

D. 2π

Answer: D

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46. The value of the definite integral $\int_0^1 \left(1 + e^{-x} \ \hat{} \ 2\right) dx$ (b) $2 \ 1 + e^{-1}$

(d) none of these

A. -1

B. 2

C. $1 + e^{-1}$

D. none of these

Answer: D

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47. If $I=\int_{-\pi}^{\pi}\frac{\sin^2}{1+a^x}dx, a>0$, then I equals

A. π

B. $\frac{\pi}{2}$

C. $a\pi$

D. $a\frac{\pi}{2}$

Answer: B

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48. If n is an odd natural number, then

$$\int_{-\pi/6}^{\pi/6} \frac{\pi + 4x^n}{-\sin\left(x + \frac{\pi}{6}\right)dx =}$$

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49. If $I_1 = \int_0^x e^{\text{zx}}\ e^{-z^2}dz$ and $I_2 = \int_0^x e^{-z^2/4}dz$, them

A. $I_1 = e^x I_2$

B. $I_1 = e^{x^2} I_2$

C. $I_1 = e^{x^2/2} I_2$

D. none of these

Answer: D

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50. $\int_0^{1/2} |\sin \pi x| dx$ is equal to

A. 0

B. π

C. $-\pi$

D. $1/\pi$

Answer: D

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51. If $f(x)=\int_{0}^{x}\log\left(\frac{1-t}{1+t}\right)dt$, then discuss whether even or odd?

A. an even function

B. an odd function

C. a periodic function

D. none of these

Answer: A

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52. $\int_{1/3}^{3} \frac{1}{x} \sin\left(\frac{1}{x} - x\right) dx$ is equal to

A. $\frac{\sqrt{3}}{2}$

B. $\frac{\sqrt{3}\pi}{2}$

C. 0

D. none of these

Answer: C

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53. If F(x) $= \int_{x^2}^{x^3} \log t dt (x > 0)$, then F'(x) equals

A. $(9x^2 - 4x)\log x$

B. $(4x - 9x^2)\log x$

C. $(9x^2 + 4x)\log x$

D. none of these

Answer: A

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54. If $I = \int_0^1 \frac{dx}{\sqrt{1+x^4}} dx$ then

A. $I > 2$

B. $I \neq \frac{\sqrt{5}}{2}$

C. $I > \frac{\sqrt{7}}{2}$

D. none of these

Answer: B

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55. If $I = \int_0^1 \frac{dx}{1+x^4}$, then

A. $I > 0.78$

B. $I < 0.78$

C. $I > 1$

D. none of these

Answer: D

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56. The value of $\int_{-1}^{1} (x|x|)dx$ is equal to

A. 2

B. 1

C. 0

D. none of these

Answer: C

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57. If $\int_0^{\pi/2} \cos^n x \sin^n x dx = \lambda \int_0^{\pi/2} \sin^n x \; \mathrm{dx}$, then $\lambda =$

A. $\frac{1}{2^{n-1}}$

B. $\frac{1}{2^{n+1}}$

C. $\frac{1}{2^n}$

D. $\frac{1}{2}$

Answer: C

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58. The value of $\int_{1/e}^{e} \frac{|\log x|}{x^2} dx$, is

A. 2

B. $\frac{2}{e}$

C. $2\left(1 - \frac{1}{e}\right)$

D. 0

Answer: C

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59. $\int_{ac}^{bc} f(x)dx$, where $c \neq 0$, is also equal to :

A. $\frac{1}{c}\int_a^b f(x)dx$

B. $\int_a^b f(x)dx$

C. $c\int_a^b f(x)dx$

D. $c\int_{ac^2}^{bc^2} f(x)dx$

Answer: B

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60. $\frac{d}{dx}\left(\int_{f(x)}^{g(x)} \phi(t)dt\right)$ is equal to

A. $\phi(g(x)) - \phi(f(x))$

B. $\frac{1}{2}[\phi(g(x))]^2 - \frac{1}{2}[\phi(f(x))]^2$

C. $g'(x)\phi(g(x)) - f'(x)\phi(f(x))$

D. $\phi(g(x))g'(x)0 - \phi'(f(x))f'(x)$

Answer: C

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61. If f(x) $= ae^{2x} + be^{x} + cx$, satisfies the conditions f(0)=-1, f'(log 2)=31, $\int_0^{\log 4} (f(x) - cx)dx = \frac{39}{2}$, then

A. $a = 5, b = -6, c = 3$

B. $a = 5, b = 6, c = 7$

C. $a = -5, b = 6, c = 0$

D. none of these

Answer: A

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62. The value of $\int_0^2 \left|\cos\frac{\pi}{2}t\right| dt$ is equal to

A. 2π

B. $\pi/2$

C. $3/4\pi$

D. $4/\pi$

Answer: D

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63. If $\int_0^1 \cot^{-1}(1-x+x^2)dx = k\int_0^1 \tan^{-1}x dx$, then k=`

A. 1

B. 2

C. π

D. 2π

Answer: B

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64. If $0 < a < 1$, then $\int_{-1}^{1} \frac{1}{\sqrt{1 - 2ax + a^2}} dx$ is equal to

A. 1

B. 2π

C. $\pi/2$

D. $\frac{\log\left(\frac{a+1}{a-1}\right)}{a}$

Answer: D

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65. The value of $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x} dx$, is

A. π

B. 2π

C. $\pi/2$

D. $3\pi/2$

Answer: C

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66. If a is a fixed real number such that f(a-x)+f(a+x)=0, then $\int_0^{2a} f(x)$ dx=

A. $\frac{a}{2}$

B. 0

C. $-\frac{a}{2}$

D. 2a

Answer: B

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67. The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$, is

A. 2

B. $3/4$

C. 0

D. 1

Answer: C

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68. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is

A. 1

B. 0

C. -1

D. 2

Answer: B

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69. The value of $\int_0^{2\pi} |\cos x - \sin x| dx$ is

A. $\frac{4}{\sqrt{2}}$

B. $2\sqrt{2}$

C. $\frac{2}{\sqrt{2}}$

D. $4\sqrt{2}$

Answer: D

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70. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then

A. $I_1 > I_2$ and $I_4 > I_3$

B. $I_2 > I_1$ and $I_3 > I_4$

C. $I_1 > I_2$ and $I_3 > I_4$

D. none of these

Answer: A

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71. Consider the integrals

$I_1 = \int_0^1 e^{-x} \cos^2 x dx$, $I_2 = \int_0^1 e^{-x^2} \cos^2 x dx$, $I_3 = \int_0^1 e^{-x^2} dx$ and $I_4 = \int_0^1 e^{-(1/2)x^2} dx$. The greatest of these integrals, is

A. I_1

B. I_2

C. I_3

D. I_4

Answer: D

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72. If f(x)=f(a+b-x) for all $x \in [a, b]$ and $\int_a^b x f(x) dx = k \int_a^b f(x) dx$, then the value of k, is

A. $\frac{a+b}{2}$

B. $\frac{a-b}{2}$

C. $\frac{a^2+b^2}{2}$

D. $\frac{a^2-b^2}{2}$

Answer: A

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73. To find the numberical value of $\int_{-2}^{2}(px^3+qx+s)dx$ it is necessary to know the values of the constants:

A. p

B. q

C. s

D. p and s

Answer: C

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74. Let $f:R\overrightarrow{a}ndg:R\overrightarrow{R}$ be continuous function. Then the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[f(x)+f(-x)][g(x)-g(-x)]dxis$ (a)π (b) 1 (c) -1 (d) 0

A. π

B. 1

C. -1

D. 0

Answer: D

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75. The value of $\int_{-1/2}^{1/2} \left| x \cos\left(\frac{\pi x}{2}\right) \right| dx$ is

A. $\frac{\pi\sqrt{2} + 4\sqrt{2} - 8}{\pi^2}$

B. $\frac{\sqrt{2} + 4\pi\sqrt{2} - 8}{\pi^2}$

C. $\frac{\pi\sqrt{2} + 4\sqrt{2} + 8}{\pi^2}$

D. none of these

Answer: A

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76. The value of the integral $\int_0^{\pi/2} \frac{f(x)}{f(x)+f\left(\frac{\pi}{2}-x\right)} dx$ is

A. $\pi/4$

B. $\pi/2$

C. π

D. 0

Answer: A

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77. The value of $\int_{\pi/2}^{0} \frac{1}{9\cos x + 12\sin x} dx$ is

A. $\frac{1}{15}\log_{10} 6$

B. $\frac{1}{15}\log_e 6$

C. $\log\left(\frac{6}{15}\right)$

D. $\log\left(\frac{15}{6}\right)$

Answer: B

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78. If $I = \int_3^4 \frac{1}{3\sqrt{\log x}} dx$then

A. $I < 1$

B. $I > 1$

C. $I < 0.92$

D. none of these

Answer: A

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79. If $I = \int_0^{1/2} \frac{1}{\sqrt{1 - x^{2n}}} dx$then which one of the following is not true

?

A. $I < \frac{\pi}{6}$

B. $I \geq \frac{1}{2}$

C. $I > 0$

D. none of these

Answer: D

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80. Q. $\int_0^{\pi} e^{\cos^2 x}\left(\cos^3(2n+1)x dx, n \in I\right.$

A. π

B. 1

C. 0

D. none of these

Answer: C

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81. The value of the integral $\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx$ is equal to

A. 0

B. 2a

C. a

D. none of these

Answer: C

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82. If `int_(0)^(1) (log(1+x)/(1+x^(2))dx=

A. 4

B. 8

C. π

D. 2π

Answer: B

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83. $If \int_{\log 2}^{x} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}, \text{then } \ x \ \text{ is equal to}$

(a)4 (b) 1n 8 (c) 1n 4 (d) none of these

A. e^2

B. $1/e$

C. log 4

D. none of these

Answer: C

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84. The value of the integral $\int_0^{\pi} \log(1+\cos x)\,dx$ is

A. $\frac{\pi}{2}\log 2$

B. $-\pi\log 2$

C. $\pi\log 2$

D. none of these

Answer: B

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85. The value of the integral $\int_0^{\pi} \frac{1}{a^2-2a\cos x+1}dx(a>1)$, is

A. $\frac{\pi}{1-a^2}$

B. $\frac{\pi}{a^2-1}$

C. $\frac{2\pi}{a^2-1}$

D. $\frac{3\pi}{4}$

Answer: A

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86. The integral $\int_0^{\pi/2} f(\sin 2x)\sin x\,dx$ is equal to

A. $\int_0^{\pi/2} f(\cos 2x)\sin x\,dx = \sqrt{2}\int_0^{\pi/2} f(\cos 2x)\sin x\,dx$

B. $\int_0^{\pi/2} f(\sin 2x)\cos x\,dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x\,dx$

C. $\int_0^{\pi/2} f(\cos 2x)\cos x\,dx = \sqrt{2}\int_0^{\pi/2} f(\cos 2x)\cos x\,dx$

D. $\int_0^{\pi/2} f(\sin 2x)\cos x\,dx = \sqrt{2}\int_0^{\pi/2} f(\cos 2x)\cos x\,dx$

Answer: B

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87. $\int_0^{\pi} k\left(\pi x-x^2\right)^{100} \sin 2x$ dx is equal to

A. π^{100}

B. $\frac{1}{2}\left(\pi^{100}-\pi^{97}\right)$

C. $\frac{1}{2}\left(\pi^{100}+\pi^{97}\right)$

D. 0

Answer: D

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88. The value of the integral $\int_2^4 \frac{\sqrt{x^2-4}}{x^4} dx$ is

A. $\sqrt{\frac{3}{32}}$

B. $\frac{\sqrt{3}}{32}$

C. $\frac{32}{\sqrt{3}}$

D. $-\frac{\sqrt{3}}{32}$

Answer: B

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89. The value of the integral $\int_0^{\pi} \frac{1}{a^2 - 2a\cos x + 1} dx (a > 1)$, is

A. $\frac{\pi}{1 - a^2}$

B. $\frac{\pi}{a^2 - 1}$

C. $\frac{2\pi}{a^2 - 1}$

D. $\frac{2\pi}{1 - a^2}$

Answer: B

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90. If $fandg$ are continuous function on $[0, a]$ satisfying $f(x) = f(a - x) and g(x)(a - x) = 2,$ then show that $\int_0^a f(x)g(x)dx = \int_0^a f(x)dx \cdot$

A. $\int_0^a g(x)dx$

B. $\int_0^a f(x)dx$

C. 0

D. none of these

Answer: B

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91. $\int_0^{\pi/2} x\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$ equals

A. $\frac{\pi}{2\sqrt{2}}$

B. $\frac{\pi^2}{2}$

C. $\frac{\pi^2}{2\sqrt{2}}$

D. $\frac{\pi^2}{2\sqrt{3}}$

Answer: C

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92. Choose the correct answer The value of the integral $\int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$ is (A) 6 (B) 0 (C) 3 (D) 4

A. 6

B. 0

C. 3

D. 4

Answer: A

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93. Evaluate: $\int_0^{100\pi} \sqrt{(1 - \cos 2x)} dx.$

A. $100\sqrt{2}$

B. $200\sqrt{2}$

C. 0

D. $400\sqrt{2}$

Answer: B

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94. Evaluate: $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left[\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right]^{\frac{1}{2}}dx$

A. $\log\left(\frac{4}{3}\right)$

B. $4\log\left(\frac{3}{4}\right)$

C. $4\log\left(\frac{4}{3}\right)$

D. $\log\left(\frac{3}{4}\right)$

Answer: C

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95. The value of the integral $\int_{1/e}^{e} |\log x| dx$, is

A. $2\left(\frac{e-1}{e}\right)$

B. $2\left(\frac{1-e}{e}\right)$

C. $2-\frac{1}{e}$

D. none of these

Answer: A

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96. The value of $\int_0^{\pi/2} \frac{\sin 8x \log \cot x}{\cos 2x} dx$, is

A. 0

B. π

C. $\frac{5\pi}{2}$

D. $\frac{3\pi}{2}$

Answer: A

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97. The value of $\int_0^{\pi/2} x^{10} \sin x \quad dx$, is then the value of $\mu_{10} + 90\mu_8$, is

A. $9\left(\frac{\pi}{2}\right)^8$

B. $\left(\frac{\pi}{2}\right)^9$

C. $10\left(\frac{\pi}{2}\right)^9$

D. $9\left(\frac{\pi}{2}\right)^9$

Answer: C

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98. $Thevalueof \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan^3 x} is$ 0 (b) 1 (c) $\frac{\pi}{2}$ (d) π

A. 0

B. 1

C. $\pi/2$

D. $\pi/4$

Answer: D

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99. The value of $\int_0^{\pi} \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)}dx$ is, (a) $n \in I, n \geq 0\frac{\pi}{2}$ (b) 0 (c) π (d)

2π

A. $\frac{\pi}{2}$

B. 0

C. π

D. 2π

Answer: C

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100. If $\frac{d(f(x))}{dx} = g(x) \, \forall x \in [a, b]$ then $\int_a^b f(x). \, g(x) dx$ is equal to :

A. $f(b) - f(a)$

B. $g(b) - g(a)$

C. $\frac{[f(b)]^2 - [f(a)]^2}{2}$

D. $\frac{[g(b)]^2 - [g(a)]^2}{2}$

Answer: C

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101. For any integer n,the integral $\int_0^{\pi} e^{\sin^2 x} \cos^3(2n+1)x \, \text{ dx}$ has the value

A. π

B. 1

C. 0

D. none of these

Answer: C

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102. The value of the integral $\int_0^3 \sqrt{3+x^3}dx$ lies in the interval

A. (1,3)

B. (2,30)

C. $\left(4, 2\sqrt{30}\right)$

D. none of these

Answer: C

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103. The value of the integral $\int_0^1 \frac{1}{(1+x^2)^{3/2}} dx$ is

A. $1/2$

B. $1/\sqrt{2}$

C. 1

D. $\sqrt{2}$

Answer: B

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104. If $I = \int_0^{2\pi} \sin^2 x dx$, then

A. $I = 4\int_0^{\pi} \sin^2 x \quad \mathrm{dx} = 4\int_0^{\pi/2} \sin^2 x \quad \mathrm{dx}$

B. $I = \int_0^{\pi/2} \cos^2 x \quad \mathrm{dx}$

C. $I = 8\int_0^{\pi/4} \sin^2 x \quad \mathrm{dx}$

D. none of these

Answer: D

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105. If $\int_0^1 f(x) = M, \int_0^1 g(x)dx = N$, then which of the following is correct ?

A. $\int_0^1 (f(x) + g(x)dx = M + N$

B. $\int_0^1 (f(x) + g(x)dx = MN$

C. $\int_0^1$ (1)/(f(x))dx=(1)/(M)`

D. $\int_0^1 \frac{f(x)}{g(x)} dx = \frac{M}{N}$

Answer: A

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106. The value of $\int_0^{\pi/4}(\pi x-4x^2)\log(1+\tan x)dx$ is

A. $\frac{\pi^3}{192}\log_e 2$

B. $\frac{\pi^3}{192}\log\sqrt{2}$

C. $\frac{\pi^3}{36}\log 2$

D. $\frac{\pi^3}{48}\log\sqrt{2}$

Answer: A

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107. The value of $\int_{-\pi/2}^{\pi/2}\sin\left\{\log\left(x+\sqrt{x^2+1}\right\}dx\right.$ is

A. 1

B. -1

C. 0

D. none of these

Answer: C

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108. The value of $\int_0^{2\pi} \cos^{99} x dx$, is

A. 1

B. -1

C. 99

D. 0

Answer: D

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109. If f(a+x)=f(x), then $\int_0^{na} f(x)dx$ is equal to $(n \in N)$

A. $(n-1)\int_0^a f(x)dx$

B. $n\int\limits_0^a f(x)dx$

C. $\int\limits_0^{(n-1a)} f(x)dx$

D. none of these

Answer: B

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110. If $f(t)$ is an odd function, then prove that $\varphi(x) = \int_a^x f(t)dt$ is an even function.

A. is an odd function

B. is an even function

C. is an increasing function on [a,b]

D. none of these

Answer: B

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111. If f(x) is an integrable function over every interval on the real line such that f(t+x)=f(x) for every x and real t, then

$\int_a^{a+t} f(x)dx$ is equal to

A. $\int_0^a f(x)dx$

B. $\int_0^t f(x)dx$

C. $\int_a^t f(x)dx$

D. none of these

Answer: B

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112. If $I_1 = \int_{3\pi}^{0} f(\cos^2 x)dx$ and $I_2 = \int_{\pi}^{0} f(\cos^2 x)$ then

A. $I_1 = I_2$

B. $I_1 = 2I_2$

C. $I_1 = 5I_2$

D. none of these

Answer: D

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113. If f(x) is a quadratic polynomial in x such that $6\int_0^1 f(x)dx - \left\{f(0) + 4f\left(\frac{1}{2}\right)\right\} = kf(1)$, then k=`

A. -1

B. 0

C. 1

D. 2

Answer: C

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114. The value of integral $\int_{-2}^{4} x[x]dx$ is

A. $41/2$

B. 20

C. $21/2$

D. none of these

Answer: A

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115. If h(a)=h(b), the value of the integral

$\int_{a}^{b} \left[f(g(h(x))]^{-1} f'(g(h(x)))g'(h(x))h'(x)dx\right.$ is equal to

A. 0

B. f(a)-f(b)

C. f(g(a))-f(g(b))

D. none of these

Answer: A

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116. Given that, $F(x) = \frac{1}{x^2}\int_4^x \left(4t^2 - 2F'(t)\right)dt$, find $F'(4)$.

A. 32

B. $32/3$

C. $32/9$

D. none of these

Answer: C

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117. It is known that $f(x)$ is an odd function in the interval $\left[\frac{p}{2}, \frac{p}{2}\right]$ and has a period p, Prove that $\int_q^x (t)dt$ is also periodic function with the same period.

A. a periodic function with period $T/2$

B. a periodic function with period T

C. not a periodic function

D. a periodic function with period$T/4$

Answer: B

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118. Suppose for every integer n, . $\int_n^{n+1} f(x)dx = n^2$. The value of $\int_{-2}^{4} f(x)dx$ is :

A. 16

B. 14

C. 19

D. none of these

Answer: C

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119. $\int_{-\pi+4}^{\pi/4} \frac{\tan^2 x}{1+a^x} dx$ is equal to

A. $\frac{\pi+4}{4}$

B. $\frac{\pi-4}{4}$

C. $\frac{a\pi}{4}$

D. $\frac{a+\pi}{4}$

Answer: B

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120. The value of $\int_0^{\pi/2} \cos ec(x-\pi/3)\cos ec(x-\pi/6)dx$ is

A. 2 log 3

B. $-2\log 3$

C. log 3

D. none of these

Answer: B

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121. The value of $\int_{-1}^{1}(x|x|)dx$ is equal to

A. 2

B. 1

C. 0

D. none of these

Answer: C

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122. $\int_0^3 \left|x^3 + x^2 + 3x\right| dx$ is equal to

A. $\frac{171}{2}$

B. $\frac{171}{4}$

C. $\frac{170}{4}$

D. $\frac{170}{3}$

Answer: B

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123. Evaluate: $\int_{-1}^{1} \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$

A. In 3

B. 2 In 3

C. $\frac{1}{2}$ In 3

D. none of these

Answer: D

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124. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left\{\frac{ax^2+bx+c}{ax^2-bx+c}(a+b)|\sin x|\right\}dx$

A. $\pi \log_e(a+b)$

B. $\pi \log_e\left(\frac{a+b}{2}\right)$

C. $\frac{\pi}{2}\log_e(a+b)$

D. none of these

Answer: B

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125. For any natural number n, the value of the integral

$$\int_0^{\sqrt{n}} [x^2]\,dx \text{ is}$$

A. $n\sqrt{n}+\sum_{r=1}^{n}\sqrt{r}$

B. $n\sqrt{n}-\sum_{r=1}^{n}\sqrt{r}$

C. $\sum_{r=1}^{n}\sqrt{r}-n\sqrt{n}$

D. none of these

Answer: B

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126. For any $n \in R^{+}$, the value of the integral

$$\int_0^{n[x]} (x-[x])\,dx \text{ is}$$

A. n[x]

B. [x]

C. $\frac{n}{2}[x]$

D. none of these

Answer: C

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127. Let d/(dx)F(x)=((e^(sinx))/x),x > 0. If int_1^4 3/x e^sin (x^3)dx=F(k)-F(1), then one of the possible values of k , is: (a) 15 (b) 16 (c) 63 (d) 64

A. 27

B. 18

C. 9

D. none of these

Answer: A

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128. The equation

$$\int_{-\pi/4}^{\pi/4}\left\{a|\sin x| + \frac{b\sin x}{1+\cos x} + c\right\}dx = 0$$ where a,b,c are constants,

gives a relation between

A. a,b and c

B. a and c

C. a and b

D. b and c

Answer: B

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129. Let $f(x)$ be a continuous function such that $f(a-x)+f(x)=0$ for all $x \in [0, a]$. Then the value of the integral $\int_0^a \frac{1}{1+e^{f(x)}}dx$ is equal to

A. a

B. $\frac{a}{2}$

C. f(a)

D. $\frac{1}{2}f(a)$

Answer: B

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130. The value of $\int_{\alpha}^{\beta} x|x|dx$, where $a < 0 < \beta$, is

A. $\frac{1}{2}(\alpha^2 + \beta^2)$

B. $\frac{1}{3}(\beta^2 - \alpha^2)$

C. $\frac{1}{3}(\alpha^3 + \beta^3)$

D. none of these

Answer: C

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131. $\int_{-\pi/2}^{\pi/2} \frac{|x|}{8\cos^2 2x + 1} dx$has the value

A. $\frac{\pi^2}{6}$

B. $\frac{\pi^2}{12}$

C. $\frac{\pi^2}{24}$

D. none of these

Answer: B

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132. If [.] denotes the greatest integer function and

$$f(x) = \begin{cases} 3(x) - \frac{5|x|}{x}, x \neq 0 \\ 2, x = 0 \end{cases}$$

then $\int_{-3/2}^{2} f(x) dx$ is equal to

A. $-\frac{11}{2}$

B. $-\frac{7}{2}$

C. -6

D. $-\frac{17}{2}$

Answer: A

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133. Find the value of $\int_{-1}^{1}\left[x^2+\{x\}\right]dx$, where [.] and $\{.\}$ denote the greatest function and fractional parts of x, respectiely.

A. $\frac{5+\sqrt{5}}{2}$

B. $\frac{5-\sqrt{5}}{2}$

C. $-\frac{5+\sqrt{5}}{2}$

D. none of these

Answer: B

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134. The value of

$$\int_{-1}^{1} \sin^{-1}\left[x^2 + \frac{1}{2}\right]dx + \int_{-1}^{1} \cos^{-1}\left[x^2 - \frac{1}{2}\right]dx,$$ where [.] denotes the greatest integer function , is

A. π

B. 2π

C. 4π

D. 0

Answer: B

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135. Let $\Delta(y) = \begin{vmatrix} y+a & y+b & y+a-c \\ y+b & y+c & y-1 \\ y+c & y+d & y-b+d \end{vmatrix}$

and, $\int_0^2 \Delta(y)dy = -16$, where a,b,c,d are in A.P., then the common difference of the A.P. is equal to

A. ± 1

B. ± 2

C. ± 3

D. none of these

Answer: B

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136. If $I = \int_0^1 \frac{1}{1 + x^{\pi/2}} dx$, then\

A. $I > \text{In}2$

B. $I < \text{In}2$

C. $I < \frac{\pi}{4}$

D. $I > \frac{\pi}{4}$

Answer: A

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137. If $\int_0^x f(t)dt = x^2 + 2x - \int_0^x tf(t)dt, x \in (0, \infty)$. Then, f(x) is

A. Periodic

B. Periodic but fundamental does not exist

C. Periodic but fundamental period exits

D. Nothing can be said

Answer: A::B

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138. If $f(x) = \min\left(|x|, 1 - |x|, \frac{1}{4}\right) \forall x \in R,$ then find the value of $\int_{-1}^{1} f(x)dx\cdot$

A. 40

B. 50

C. 1

D. $\frac{2}{5}$

Answer: A

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139. If $I_n = \int_0^{\pi} e^x (\sin x)^n dx$, then $\frac{I_3}{I_1}$ is equal to

A. $\frac{3}{5}$

B. $\frac{1}{5}$

C. 1

D. $\frac{2}{5}$

Answer: A

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140. Given that

$$\lim_{n \to \infty} \sum_{r=1}^{n} \frac{\log(n^2 + r^2) - 2\log n}{n} = \log 2 + \frac{\pi}{2} - 2, \text{ then}$$

$$\lim_{n \to \infty} \frac{1}{n^{2m}} \left[(n^2 + 1^2)^m (n^2 + r^2)^m \ldots\ldots (n^2)^m \right]^{1/n}$$ is equal to

A. $2^m e^{m\left(\frac{\pi}{2} - 2\right)}$

B. $2^m e^{m\left(2 - \frac{\pi}{2}\right)}$

C. $e^m\left(\frac{\pi}{2} - 2\right)$

D. $e^{2m\left(\frac{\pi}{2} - 2\right)}$

Answer: A

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141. Let f be a differentiable function such that $f'(x) = f(x) + \int_0^2 f(x)dx$ and $f(0) = \frac{4 - e^2}{3}$. Find $f(x)$.

A. $e^x - \left(\frac{e^2 - 1}{3}\right)$

B. $e^x + \left(\frac{e^2 - 1}{3}\right)$

C. $e^x - \left(\frac{e^2+1}{3}\right)$

D. none of these

Answer: A

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142. Let f be a differentiable function such that $f'(x) = f(x) + \int_0^2 f(x)dx$ and $f(0) = \frac{4-e^2}{3}$. Find $f(x)$.

A. 0

B. 1

C. 2

D. 3

Answer: B

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143. The value of the integral $\int_{-10}^{0} \frac{\left|\frac{2|x|}{[x]-3x}\right|}{\left(\frac{2|x|}{3x-[x]}\right)} dx$ where [.] denotes GIF

A. $\frac{28}{3}$

B. 0

C. 10

D. -10

Answer: A

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Chapter Test 1

1. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x (\sin x \cos x) dx =$

A. $2/15$

B. $4/15$

C. $2/15$

D. 0

Answer: D

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2. $\lim_{n \to \infty} \frac{1}{n^2} \sum_{r=1}^{n} re^{r/n} =$

A. 0

B. 1

C. e

D. 2e

Answer: B

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3. The value of the integral $\int_{-1}^{1} \sin^{11} x \quad dx$ is

A. $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3}$

B. $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{\pi}{2}$

C. 1

D. 0

Answer: D

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4. The value of $\int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x \quad dx$, is

A. 0

B. $\pi - \frac{\pi^3}{3}$

C. $2\pi - \pi^3$

D. $\frac{\pi}{2} - 2\pi^3$

Answer: A

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5. $\int_{-\pi}^{\pi}[\cos px - \sin qx]^2$ dx where p,q are integers is equal to

A. $-\pi$

B. 0

C. π

D. 2π

Answer: D

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6. about to only mathematics

A. $1/2$

B. $1/3$

C. $1/4$

D. $1/8$

Answer: C

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7. The value of $\int_3^5 \frac{x^2}{x^2-4}$dx, is

A. $2 - \log_e\left(\frac{15}{7}\right)$

B. $2 + \log_e\left(\frac{15}{7}\right)$

C. $2 + 4\log_e 3 - 4\log_e 7 + 4\log_e 5$

D. $2 - \tan^{-1}\left(\frac{15}{7}\right)$

Answer: B

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8. The greater value of (x) $= \int_{-1/2}^{x} |t| dt$ on the interval $[-1/2, 1/2]$, is

A. $\frac{3}{8}$

B. $\frac{1}{4}$

C. $-\frac{3}{8}$

D. $-\frac{1}{2}$

Answer: B

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9. $f(x) = \begin{cases} 1-x, & 0 \leq c \leq 1 \\ 0, & 1 \leq x \leq 2 \\ (2-x)^2, & 2 \leq x \leq 3 \end{cases}$ and $\phi(x) = \int_0^x f(t)\,$ dt. Then for any $x \in [2, 3]$, $\phi(x)$ equals

A. $\frac{(x-2)^3}{3}$

B. $\frac{1}{2} - \frac{(x-2)^3}{3}$

C. $\frac{1}{2} + \frac{(x-2)^3}{3}$

D. none of these

Answer: C

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10. If $f(x)=\int_{-1}^{x}|t|dt$, then for any $x\geq 0$, $f(x)$ equals

A. $\frac{1}{2}(1-x^2)$

B. $\frac{1}{2}x^2$

C. $\frac{1}{2}(1-x^2)$

D. none of these

Answer: C

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11. $\int_{0}^{2}(x-\log_2 a)dx=2\log\left(\frac{2}{a}\right)$, if

A. a=2

B. $a > 2$

C. 3

D. 4

Answer: A

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12. If $\int_1^a (a - 4x)dx \geq 6 - 5a, \, a > 1$, then a equals

A. 1

B. 2

C. 3

D. 4

Answer: B

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13. The value of $\int_1^2 \{f(g(x))\}^{-1} f'(g(x))g'(x)$ dx , where g(1)=g(2), is equal to

A. 1

B. 2

C. 0

D. none of these

Answer: C

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14. If $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}=0$, where C_0C_1, C_2 are all real, the equation $C_2x^2+C_1x+C_0=0$ has

A. at least one root in (0,1)

B. one root in (1,2) and the other In(3,4)

C. one root in (-1,1) and the other in (-5,2)

D. both roots imaginary

Answer: A

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15. The solution of the equation $\int_{\log_2}^{x} \frac{1}{e^x - 1} dx = \frac{\log(3)}{2}$ is given by x=

A. e^2

B. $1/e$

C. log 4

D. none of these

Answer: C

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16. If $\int_a^b \frac{x^n}{x^n + (16-x)^n} dx = 6$, then

A. $a = 4, b = 12, n \in R$

B. $a = 2, b = 14, n \in R$

C. $a = -4, b = 20, n \in R$

D. $a = 2, b = 8, n \in R$

Answer: B

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17. Let m be any integer. Then, the integral $\int_0^\pi \frac{\sin 2mx}{\sin x} dx$ equals

A. 0

B. π

C. 1

D. none of these

Answer: A

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18. $\int_{-\pi/4}^{\pi/4} e^{-x}\sin x \quad \text{dx is}$

A. $-\frac{\sqrt{2}}{2}e^{-\pi/4}$

B. $\frac{\sqrt{2}}{2}e^{-\pi/4}$

C. $\sqrt{2}\left(e^{-\pi/4}-e^{\pi/4}\right)$

D. zero

Answer: A

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19. If $\frac{d(f(x))}{dx}=g(x)\,\forall x\in[a,b]$ then $\int_a^b f(x).\,g(x)dx$ is equal to :

A. $f(b)-f(a)$

B. $\phi(b) - \phi(a)$

C. $\dfrac{[f(b)]^2 - [f(a)]^2}{1+e^x}$

D. $\dfrac{[\phi(b)]^2 - [\phi(a)]^2}{2}$

Answer: C

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20. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx$

A. 1

B. 0

C. -1

D. none of these

Answer: A

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21. $\int_0^{\infty} \frac{dx}{\left[x + \sqrt{x^2 + 1}\right]^3} dx =$

A. $\frac{3}{8}$

B. $\frac{1}{8}$

C. $-\frac{3}{8}$

D. none of these

Answer: A

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22. $\int_0^1 \frac{x}{(1 - x)^{3/4}} dx =$

A. $\frac{12}{5}$

B. $\frac{16}{5}$

C. $-\frac{16}{5}$

D. none of these

Answer: B

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23. $\int_0^{\pi} x \sin x \cos^4 x dx =$

A. $\frac{\pi}{10}$

B. $\frac{\pi}{5}$

C. $-\frac{\pi}{5}$

D. none of these

Answer: B

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24. $\int_0^{\pi} [2 \sin x] dx =$

A. $2\pi/3$

B. $-5\pi/3$

C. $-\pi$

D. -2π

Answer: A

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25. $\int_{\pi/2}^{3\pi/2}[2\cos x]dx$ is equal to

A. $\frac{5\pi}{3}$

B. $-\frac{5\pi}{3}$

C. $-\pi$

D. -2π

Answer: B

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26. If $f(x)$ satisfies the condition of Rolles theorem in [1, 2] then $\int_1^2 f'(x)dx$ is equal to (A) 1 (B) 3 (C) 0 (D) none of these

A. 3

B. 0

C. 1

D. 2

Answer: B

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27. The tangent lines for the curve $y = \int_0^x 2|t|dt$ which are parallel to the bisector to the bisector of the first coordinate angle, is given by

A. $y = x \pm \frac{1}{4}$

B. $y = x \pm \frac{3}{2}$

C. $y = x \pm \frac{1}{2}$

D. none of these

Answer: A

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28. If f(x) $= ae^{2x} + be^{x} + cx$, satisfies the conditions f(0)=-1, f'(log 2)=31, $\int_0^{\log 4} (f(x) - cx)dx = \frac{39}{2}$, then

A. a=5,b=6,c=3

B. a=5,b=-6,c=3

C. a=-5,b=6, c=3

D. none of these

Answer: B

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29. $\int_{\pi}^{2\pi}\left[\sqrt{2}\cos x\right]dx =$

A. $-\pi/2$

B. $\pi/2$

C. π

D. none of these

Answer: A

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30. $\int_{0}^{\pi/3}\left[\sqrt{3}\tan x\right]dx =$

A. $\frac{5\pi}{6}$

B. $\frac{5\pi}{6} - \tan^{-1}\left(\frac{2}{\sqrt{2}}\right)$

C. $\frac{\pi}{2} - \tan^{-1}\left(\frac{2}{\sqrt{2}}\right)$

D. none of these

Answer: D

Watch Video Solution

31. $\int_{3\pi/2}^{5\pi/3}[2\cos x]dx =$

A. $\frac{5\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{2\pi}{3}$

D. none of these

Answer: D

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32. $\int_{0}^{50\pi}|\cos x|dx =$

A. 100

B. 50

C. 0

D. none of these

Answer: A

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33. The values of 'a' for which $\int_0^a (3x^2 + 4x - 5)dx < a^3 - 2$ are

A. $\frac{1}{2} < a < 2$

B. $\frac{1}{2} \leq a \leq 2$

C. $a \leq \frac{1}{2}$

D. $a \geq 2$

Answer: A

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34. If (-1,2) and and (2,4) are two points on the curve y=f(x) and if g(x) is the gradient of the curve at point (x,y) then the value of the integral $\int_{-1}^{2} g(x)dx$ is

A. 2

B. -2

C. 0

D. 1

Answer: A

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35. If $I_1 = \int_{1-x}^{x} x \sin\{x(1-x)\}dx$ and $I_2 = \int_{1-x}^{x} \sin\{x(1-x)\}dx$, then

A. $I_1 2I_2$

B. $2I_1 = I_2$

C. $I_1 = I_2$

D. none of these

Answer: B

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36. If $\int_{-\pi/3}^{\pi/3}\left(\frac{a}{3}|\tan x| + \frac{b\tan x}{1+\sec x} + c\right)dx = 0$ where a, b, c are constants, then c=

A. a In 2

B. $\frac{a}{\pi}$In 2

C. $-\frac{a}{\pi}$ In 2

D. $\frac{2a}{\pi}$ In 2`

Answer: C

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37. Estimate the absolute value of the integral $\int_{10}^{19} \frac{\sin x}{1+x^8} dx$

A. 10^{-10}

B. 10^{-11}

C. 10^{-7}

D. 10^{-9}

Answer: C

Watch Video Solution

38. The smallest interval [a,b] such that

$\int_0^1 \frac{1}{\sqrt{1+x^4}} dx \in [a, b]$, is

A. $\left[\frac{1}{\sqrt{2}}, 1\right]$

B. [0,1]

C. $\left[\frac{1}{2}, 1\right]$

D. $\left[\frac{3}{4}, 1\right]$

Answer: A

Watch Video Solution

39. Let $I_n = \int_0^{\pi/2} \sin^n x\,dx, n \in N$. Then

A. $I_n : I_{n-2} = n : (n-1)$

B. $I_n > I_{n-2}$

C. $n(I_{n-2} - I_n) = I_{n-2}$

D. none of these

Answer: C

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40. If f(x) $= \int_0^x \sin^4 t\,dt$, then $f(x+2\pi)$ is equal to

A. f(x)

B. $f(x) + f(2\pi)$

C. $gf(x) - f(2\pi)$

D. $f(x).\ f(2\pi)$

Answer: B

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41. $\int_0^{\pi} \frac{dx}{1 + 3^{\cos x}}$ is equal to:

A. π

B. 0

C. $\pi / 2$

D. none of these

Answer: C

Watch Video Solution

42. Let $\int_0^a f(x)dx = \lambda$ and $\int_0^a f(2a-x)dx = \mu$. Then $\int_0^{2a} f(x)dx$ is :

A. $\lambda + \mu$

B. $\lambda - \mu$

C. $2\lambda - \mu$

D. $\lambda - 2\mu$

Answer: A

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43. Evaluate : $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx$

A. $(\sqrt{2}-1)\pi$

B. $(\sqrt{2+1})\pi$

C. π

D. none of these

Answer: A

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44. Let $I_n = \int_0^{\pi/2} \cos^n x \cos nx dx$. Then, $I_n : I_{n+1}$ is equal to

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45. The value of $\int_{-1}^{1} \max [2 - x, 2, 1 + x]$ dx is

A. 4

B. $9/2$

C. 2

D. none of these

Answer: B

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46. $\int_{0}^{\pi/4} \sin(x - [x])$ dx is equalto

A. $\frac{1}{2}$

B. $1 - \frac{1}{\sqrt{2}}$

C. 1

D. none of these

Answer: B

Watch Video Solution

47. The value of the integral $\int_{-1}^{1} (x - [2x])$dx,is

A. 1

B. 0

C. 2

D. 4

Answer: A

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48. Let $f: R \in R$ be a continuous function such that f(1)=2. If $\lim_{x \to 1} \int_2^{f(x)} \frac{2t}{x-1} dt = 4$, then the value of f'(1) is

A. 1

B. 2

C. 4

D. none of these

Answer: A

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49. Let $f: R \in R$ be a continuous function such that f(x) is not identically equal to zero. If $\int_0^x |x-2| dx, x \geq 0$. Then, f'(x) is

A. an even function

B. an odd function

C. a periodic function

D. none of these

Answer: D

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50. Let f(x) $= \int_0^x | \times \ - 2| dx, \ \geq 0$. Then, f'(x) is

A. continuous and non differentiable at x=2

B. discontinuous at x=4

C. neither continuous nor differentiable at x=2

D. non-differentiable at x=4

Answer: A

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51. $Lt_{n \to \infty}\left\{\frac{n!}{(kn)^n}\right\}^{\frac{1}{n}}$, $k \neq 0$, is equal to (A) $\frac{k}{e}$ (B) $\frac{e}{k}$ (C) $\frac{1}{ke}$ (D) none of these

A. ke

B. $\frac{e}{k}$

C. $\frac{k}{e}$

D. $\frac{1}{ke}$

Answer: D

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52. $\int_0^{\sqrt{2}}\left[x^2\right]dx$, is

A. $2-\sqrt{2}$

B. $2+\sqrt{2}$

C. $\sqrt{2}-1$

D. $\sqrt{2}-2$

Answer: C

Watch Video Solution

53. Let $f(x)$ be a function satisfying $f'(x)=f(x)$, $f(0)=1$ and g be a function satisfying $f(x)+g(x)=x^2$ then $\int_0^1 f(x)g(x)dx$ equals

A. $e+\frac{e^2}{2}+\frac{5}{2}$

B. $e-\frac{e^2}{2}-\frac{5}{2}$

C. $e+\frac{e^2}{2}-\frac{3}{2}$

D. $e-\frac{e^2}{2}-\frac{3}{2}$

Answer: D

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54. $\left(\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}(x)dx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}(x)dx\right)$

A. 27^{2}

B. -54

C. 54

D. 0

Answer: D

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55. If $f(y)=e^{y}, g(y)=y, y>0$ and $F(t)=\int_{0}^{1}f(t-y)g(y)dt$ then

A. $F(t)=te^{-t}$

B. $F(t)=1-e^{-t}(t+1)$

C. $F(t) = e^t - (1+t)$

D. $F(t) = te^t$

Answer: D

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56. If $I_n = \int_0^{\pi/2} x^n \sin x dx$, then $I_4 + 12I_2$ is equal to\

A. 4π

B. $3\left(\frac{\pi}{2}\right)^3$

C. $\left(\frac{\pi}{2}\right)^2$

D. $4\left(\frac{\pi}{2}\right)^3$

Answer: C

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57. $\int_0^1 \sin\left\{2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right\}dx =$

A. $\pi/6$

B. $\pi/4$

C. $\pi/2$

D. π

Answer: B

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Chapter Test 2

1. The value of the integral $\int_0^2 x[x]dx$

A. $\frac{7}{2}$

B. $\frac{3}{2}$

C. $\frac{5}{2}$

D. none of these

Answer: B

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2. The value of integral $\sum_{k=1}^{n} \int_0^1 f(k-1+x)dx$ is

A. $\int\limits_0^1 f(x)dx$

B. $\int\limits_0^2 f(x)dx$

C. $\int\limits_0^2 f(x)dx$

D. $n\int\limits_0^2 f(x)dx$

Answer: C

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3. Let f (x) be a function satisfying $f(x) = f(x)$ with f(0) = 1 and g be the function satisfying $f(x) + g(x) = x^2$

The value of integral $\int f(x)g(x)dx$ is

A. $\frac{1}{4}(e-7)$

B. $\frac{1}{4}(e-2)$

C. $\frac{1}{4}(e-3)$

D. none of these

Answer: D

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4. If $I = \int_0^1 \cos\left(2\cot^{-1}\sqrt{\left(\frac{1-x}{1+x}\right)}\right)dx$ then :

A. $I > \frac{1}{2}$

B. $I = -\frac{1}{2}$

C. $0 < I < \frac{1}{2}$

D. $I = \frac{1}{2}$

Answer: B

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5. The value of $\int_a^{a+(\pi/2)} (\sin^4 x + \cos^4 x)\,dx$ is

A. $\frac{3\pi}{8}$

B. $a\left(\frac{\pi}{2}\right)^2$

C. $\frac{3\pi a^2}{8}$

D. none of these

Answer: A

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6. The vaue of $\int_{-1}^{2} \frac{|x|}{x} dx$ is

A. 0

B. 1

C. 3

D. none of these

Answer: B

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7. The value of $\int_{0}^{1} \frac{x^3}{1+x^8} dx$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{16}$

D. none of these

Answer: C

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8. The value of $\int_0^3 x\sqrt{1+x}dx$, is

A. $\frac{9}{2}$

B. $\frac{27}{4}$

C. $\frac{126}{15}$

D. $\frac{116}{15}$

Answer: D

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9. Evaluate $\int_0^1 \log\left(\sin\left(\frac{\pi x}{2}\right)\right)dx$

A. log 2

B. $-\log 2$

C. $\frac{\pi}{2}\log 2$

D. $-\frac{\pi}{2}\log 2$

Answer: B

Watch Video Solution

10. Evaluate $\int_0^{\pi} x \log \sin x dx$

A. $\frac{\pi}{2}\log 2$

B. $\frac{\pi^2}{2}\log 2$

C. $-\frac{\pi^2}{2}$ log 2\`

D. none of these

Answer: C

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11. If $I_1 = \int_0^{\infty} \frac{dx}{1+x^4} dx$ and $I_2 = \int_0^{\infty} x^2 \frac{dx}{1+x^4}$ thenn $\frac{I_1}{I_2} =$

A. 1

B. 2

C. $1/2$

D. none of these

Answer: A

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12. If $f(x) = \begin{cases} x & x < 1 \\ x - 1 & x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x)$ dx is equal to :

A. 1

B. $\frac{4}{3}$

C. $\frac{5}{3}$

D. $\frac{5}{2}$

Answer: C

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13. The value of the integral $\int_0^1 \frac{1}{(1+x^2)^{3/2}}dx$ is

A. $1/2$

B. $\sqrt{2}/2$

C. 1

D. $\sqrt{2}$

Answer: B

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14. Prove that: $\int_0^{2a} f(x)dx = \int_0^{2a} f(2a-x)dx$.

A. $f(2a - x) = - f(x)$

B. $f(2a - x) = f(x)$

C. f(x) is an odd function

D. f (x) is an even function

Answer: B

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15. If $\int_0^{36} \frac{1}{2x + 9} dx = \log k$, is equal to

A. 3

B. $9/2$

C. 9

D. 81

Answer: A

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16. The value of the integral $\int_0^{\pi/2} \sin^6 x dx$, is

A. $\dfrac{3\pi}{4}$

B. $\dfrac{5}{32}\pi$

C. $\dfrac{3}{16}\pi$

D. none of these

Answer: B

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17. If $\int_0^{\infty} e^{-x^2} dx = \sqrt{\dfrac{\pi}{2}} \text{then} \int_0^{\infty} e^{-ax^2} dx, a > 0$, s

A. $\dfrac{\sqrt{\pi}}{2}$

B. $\dfrac{\sqrt{\pi}}{2a}$

C. $2\dfrac{\sqrt{\pi}}{a}$

D. $\frac{1}{2}\left(\sqrt{\frac{\pi}{a}}\right)$

Answer: D

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18. The value of the integral $\int_0^\infty \frac{1}{1+x^4} dx$ is

A. $\frac{\pi}{2}$

B. $\frac{\pi}{\sqrt{2}}$

C. $\frac{\pi}{2\sqrt{2}}$

D. none of these

Answer: C

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19. The value of $\alpha \in [0, 2\pi]$ which does not satify the equation $\int_{\pi/2}^{\alpha} \sin x dx = \sin 2\alpha$, is

A. π

B. $\frac{3\pi}{2}$

C. $\frac{7\pi}{6}$

D. $\frac{11\pi}{6}$

Answer: A

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20. $\lim_{x \to 0} \frac{\int_0^{x^2} \sin \sqrt{t} dt}{x^3}$ is equl to

A. $1/3$

B. 1

C. $2/3$

D. none of these

Answer: C

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21. If

$$\left[\int_0^1 \frac{dt}{t^2+2t\cos\alpha+1}\right]x^2-\left[\int_{-3}^{3}\frac{t^2\sin 2t}{t^2+1}dt\right]x-2=0(0<\alpha<\pi)$$

then the value of x is

A. $\pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$

B. $\pm\sqrt{\frac{\sin\alpha}{\alpha}}$

C. $\pm 4\sqrt{\frac{\sin\alpha}{\alpha}}$

D. none of these

Answer: A

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22. The number of value of α in the interval $[-\pi, 0]$ satisfying $\sin\alpha + \int_{\alpha}^{2\alpha} \cos 2x dx = 0$, then

A. $-\pi/2$

B. $-\pi$

C. $-\pi/3$

D. π

Answer: C

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23. The value of $\int_0^{\pi/2} \frac{\sin^3 x \cos x}{\sin^4 x + \cos^4 x} dx$ is

A. $\pi/8$

B. $\pi/4$

C. $\pi/2$

D. π

Answer: A

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24. The value of $\int_0^{\pi} \frac{1}{5+3\cos x} dx$ is

a. $\pi/2$

b. $\pi/4$

c. 0

d. $\pi/8$

A. π

B. $2\pi/3$

C. $\pi/4$

D. 2

Answer: C

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25. $\lim_{n \to \infty} [\sin'(\pi)/(n)+\sin'(2pi)/(n) + \ldots\ldots + \frac{\sin((n-1))}{n}$pi`] is equal to :

A. 0

B. π

C. ∞

D. none of these

Answer: C

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26. $\lim_{n \to 0} \sum_{r=1}^{n} \left(\frac{r^3}{r^4 + n^4}\right)$ equals to :

A. log 2

B. $\frac{1}{2}\log 2$

C. $\frac{1}{3}\log 2$

D. $\frac{1}{4}\log 2$

Answer: D

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27. The value of $\lim_{n \to \infty} \left\{ \left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)...(2) \right\}^{1/n}$, is

A. $e/4$

B. $4/e$

C. $2/e$

D. none of these

Answer: B

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28. Evaluate: $(\lim)_{n \xrightarrow{} \infty} n\left[\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}++\frac{1}{nb}\right]$

A. $\log\left(\frac{b}{a}\right)$

B. $\log\left(\frac{a}{b}\right)$

C. log a

D. log b

Answer: A

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29. If $I_n = \int_0^{\pi/4} \tan^n x dx, (n > 1$ is an integer), then (a) $I_n + I_{n-2} = \frac{1}{n+1}$ (b) $I_n + I_{n-2} = \frac{1}{n-1}$ (c) $I_2 + I_4, I_6, \ldots\ldots$ are in H.P. (d) $\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}$

A. $I_n = I_{n-2}$

B. $I_n + I_{n-2} = \frac{1}{n-1}$

C. $I_n - I_{n-2} = \frac{1}{n-1}$

D. none of these

Answer: B

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30. If $I_m = \int_1^x (\log x)^m dx$ satisfies the relation $(I_m) = k - lI_{m-1} then$

A. k=e

B. l=m

C. $k = \frac{1}{e}$

D. none of these

Answer: B

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31. If $I_m = \int_0^\infty e^{-x} x^{n-1} dx, \text{ then} \int_0^\infty e^{-\lambda x} x^{n-1} dx$

A. λI_n

B. $\frac{1}{\lambda} I_n$

C. $\frac{I_n}{\lambda^n}$

D. $\lambda^n I_n$

Answer: C

Watch Video Solution

32. If $I(mn)=\int_0^1 x^{m-1}(1-x)^{n-1}dx, (m, n\varepsilon I, m, n \geq 0)$, then

A. $I(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx=\int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}dx$

B. $I(m,n)=\int_0^\infty \frac{x^{m}}{(1+x)^{m+n}}dx=\int_0^\infty \frac{x^{n}}{(1+x)^{m+n}}dx$

C. $I(m,n)=\int_0^\infty \frac{x^{n}}{(1+x)^{m+n-1}}dx=\int_0^\infty \frac{x^{n}}{(1+x)^{m+n-1}}dx$

D. none of these

Answer: A

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33. Find the points of maxima /minima of $\int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt.$

A. $x = 0, \ \pm 1, \ +1$

B. $X = \ \pm 1, \ \pm 2, \ \pm 3$

C. $x = 0, 1, 2, 3$

D. none of these

Answer: A

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34. Evaluate the following definite integral: $\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$

A. $\pi^2 / 4$

B. π^2

C. 0

D. $\pi / 2$

Answer: B

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35. The value of the inntegral $\int_{\alpha}^{\beta} \frac{1}{\sqrt{(x-\alpha)(\beta-x)}} dx$ is

A. 0

B. $\pi / 2$

C. `pi

D. none of these

Answer: C

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36. The value of the integral $\int_{\alpha}^{\beta} \sqrt{(x-\alpha)(\beta-x)} dx$, is

A. $\frac{\pi}{4}(\beta-\alpha)^2$

B. $\frac{\pi}{2}(\beta - \alpha)^2$

C. $\frac{\pi}{8}(\beta - \alpha)^2$

D. none of these

Answer: C

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37. If $\int_0^{x^2} \sqrt{1+t^2}\, dt$, then f'(x)n equals

A. $\sqrt{1+x^2}$

B. $\sqrt{1+x^4}$

C. $2x\sqrt{1+x^4}$

D. none of these

Answer: C

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38. The value of integral $\int_1^e (\log x)^3 dx$, is

A. $6 + 2e$

B. $6 - 2e$

C. $2e - 6$

D. none of these

Answer: B

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39. If $\int_{x^2}^{x^4} \sin \sqrt{t} dt,$ f'(x) equals

A. $\sin x^2 - \sin x$

B. $4x^3 \sin x^2 - 2 \sin x$

C. $x^4 \sin x^2 - x \sin x$

D. none of these

Answer: B

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40. $\lim_{n->\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{n}{n}\right)\right]^{\frac{1}{n}}$

A. $4/e$

B. $e/4$

C. 4e

D. none of these

Answer: A

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41. $\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right).....\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$ is equal to :

A. $\frac{e^{\pi/2}}{2e^2}$

B. $2e^2 e^{\pi/2}$

C. $\frac{2}{e^2} e^{\pi/2}$

D. none of these

Answer: C

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42. If $\int_0^1 e^{x^2}(x-\alpha)dx=0$, then (a)$\alpha<2$ (b)$\alpha<0$ $(c)0<\alpha<1$ (d) $\alpha=0$

A. $1<\alpha<2$

B. $\alpha<0$

C. $0<\alpha<1$

D. $\alpha=0$

Answer: C

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43. If f(x) satisfies the requirements of Rolle's Theorem in [1,2] and f(x) is continuous in [1,2] then $\int_1^2 f'(x)$ dx is equal to

A. 0

B. 1

C. 3

D. -1

Answer: A

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44. The value of the integral $\int_0^1 \cot^{-1}(1 - x + x^2)\,dx$, is

A. $\pi - \log 2$

B. $\frac{\pi}{2} - \log 2$

C. $\pi + \log 2$

D. $\frac{\pi}{2} + \log 2$

Answer: B

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45. The integral $\int_{-1}^{1} \frac{|x+2|}{x+2} dx$ is equal to

A. 1

B. 2

C. 0

D. -1

Answer: B

Watch Video Solution

46. Let $I=\int_0^1 \frac{e^x}{x+1}$ dx, then the vlaue of the intergral $\int_0^1 \frac{xe^{x^2}}{x^2+1}$ dx, is

A. I^2

B. $\frac{1}{2}I$

C. $2I$

D. $\frac{1}{2}I^2$

Answer: B

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47. Evaluate $\int_0^\pi \frac{xdx}{1+\cos\alpha\sin x}$,where $0<\alpha<\pi$.

A. $\frac{\pi\alpha}{\sin\alpha}$

B. $\frac{\pi\alpha}{1+\sin\alpha}$

C. $\frac{\pi\alpha}{\cos\alpha}$

D. $\dfrac{\pi\alpha}{1+\cos\alpha}$

Answer: A

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48. $\int_{\pi}^{10n} |\sin x| dx$ is equla to

A. 20

B. 8

C. 10

D. 18

Answer: D

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49. about to only mathematics

A. $\dfrac{\pi a}{\left(a^2 - b^2\right)^{3/2}}$

B. $\dfrac{\pi b}{\left(a^2 - b^2\right)^{3/2}}$

C. $\dfrac{\pi}{\left(a^2 - b^2\right)^{3/2}}$

D. none of these

Answer: A

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50. If $\int_0^{\infty} e^{-ax} dx = \frac{1}{a}, \quad \text{then} \quad \int_0^{\infty} x^n e^{-ax} dx$ is

A. $\dfrac{(-1)^n n!}{a^{n+1}}$

B. $\dfrac{(-1)^n (n-1)!}{a^n}$

C. $\dfrac{n!}{a^{n+1}}$

D. none of these

Answer: C

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51. The value of $\int_0^{2\pi}[2\sin x]dx$, where [.] represent the greatest integral function, is

A. $\frac{-5\pi}{3}$

B. $-\pi$

C. $\frac{5\pi}{2}$

D. -2π

Answer: A

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52. $If f(x) = A\sin\left(\frac{\pi x}{2}\right) + b, f'\left(\frac{1}{2}\right) = \sqrt{2} and \int_0^1 f(x)dx = \frac{2A}{\pi}$ the constants A and B are (a) $\frac{\pi}{2} and \frac{\pi}{2}$ (b) $\frac{2}{\pi} and \frac{3}{\pi}$ (c) $0 and -\frac{4}{\pi}$ (d) $\frac{4}{\pi} and 0$

A. $\frac{\pi}{2}$ and $\frac{\pi}{2}$

B. $\dfrac{2}{\pi}$ and $\dfrac{3}{\pi}$

C. 0 and $-\dfrac{4}{\pi}$

D. $\dfrac{4}{\pi}$ and 0

Answer: D

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53. If $I_{m,n} = \int_0^1 x^m (\ln x)^n$dx then $I_{m,n}$ is also equal to

A. $\dfrac{n}{n+1} I_{m,n-1}$

B. $\dfrac{-m}{n+1} I_{m,n-1}$

C. $\dfrac{-n}{m+1} I_{m,n-1}$

D. $\dfrac{m}{n+1} I_{m,n-1}$

Answer: C

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54. $\lim_{n\to\infty}\dfrac{1^{99}+2^{99}+3^{99}+.......n^{99}}{n^{100}}=$

A. $\dfrac{99}{100}$

B. $\dfrac{1}{100}$

C. $\dfrac{1}{99}$

D. $\dfrac{1}{101}$

Answer: B

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55. $I_n=\int_0^{\pi/4}\tan^n x dx$, then $\lim_{n\to\infty} n[I_n+I_{n+2}]$ equals :

A. 1

B. 2

C. $\pi/4$

D. π

Answer: A

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56. Let $\int_0^a f(x)dx = \lambda$ and $\int_0^a f(2a - x)dx = \mu$. Then $\int_0^{2a} f(x)dx$ is :

A. $\lambda + \mu$

B. $\lambda - \mu$

C. $2\lambda + \mu$

D. $\lambda + 2\mu$

Answer: B

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