MATHS

BOOKS - OBJECTIVE RD SHARMA ENGLISH

DIFFERENTIATION

Illustration

1. If $f(x) = e^x g(x), g(0) = 2, g'(0) = 1, \quad \text{then} \quad f'(0) =$

A. 1

B. 3

C. 2

D. 0

Answer: B

Watch Video Solution

Watch Video Solution

2. If $f(x) = (ax + b)\sin x + (cx + d)\cos x$, then the values of a b, c and d such that $f(x) = x\cos x$ for all x, then (a + b + c + d)is

A. a=0, b=c1, d=0

B. a=b=c=d=1

C. a=d=0, b=c=-1

D. none of these

Answer: A

Watch Video Solution

3. If f(x) $= x\tan^{-1} x$, then f '(1) is equal to

A. $\frac{1}{2} + \frac{\pi}{4}$

B. $-\frac{1}{2} + \frac{\pi}{4}$

C. $-\frac{1}{2}-\frac{\pi}{4}$

D. $\frac{1}{2}-\frac{\pi}{4}$

Answer: A

Watch Video Solution

4. If $x=e^{y+e^{y+\ldots \text{to}\infty}}$, where $x>0$, then find $\frac{dy}{dx}$.

A. $\frac{1+x}{x}$

B. $\frac{1}{x}$

C. $\frac{1-x}{x}$

D. $\frac{x}{1+x}$

Answer: C

Watch Video Solution

5. If $f(x) = \log|2x|, x \neq 0$ then $f'(x)$ is equal to

A. $\frac{1}{x}$

B. $-\frac{1}{x}$

C. $\frac{1}{|x|}$

D. none of these

Answer: A

Watch Video Solution

6. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then prove that $\frac{dy}{dx} = \frac{1}{x^3 y}$

A. $\frac{y}{x}$

B. $-\frac{y}{x}$

C. $\frac{x}{y}$

D. $-\frac{x}{y}$

Answer: B

Watch Video Solution

7. If $y = \{f(x)\}^{\phi(x)}$, then $\frac{dy}{dx}$ is

A. $f^{\phi \log f}.\ \frac{\phi}{f}\frac{df}{dx} + \log f.\ \frac{d\phi}{dx}\Big\}$

B. $\frac{\phi}{f}\left(\frac{df}{dx}\right) + \frac{d\phi}{dx}\log f$

C. $e^{\phi \log f}.\ \phi\frac{f'}{f} + \phi' \log f'\Big\}$

D. none of these

Answer: A

Watch Video Solution

8. If $y = \log_{\cos x} \sin x,$ then $\frac{dy}{dx}$ is equal to

A. $\frac{\cot x \log \cos x + \tan x \log \sin x}{(\log \cos x)^2}$

B. $\frac{\tan x \log \cos x + \cot x \log \sin x}{(\log \cos x)^2}$

C. $\frac{\cot x \log \cos x + \tan x \log \sin x}{(\log \sin x)^2}$

D. none of these

Answer: A

Watch Video Solution

9. If $y = x^{x^{x^{x^{\cdots}}}}$, then $\frac{dy}{dx}$ is equal to

A. yx^{y-1}

B. $\frac{y^2}{x(1 - y \log x)}$

C. $\frac{y}{x(1 + y \log x)}$

D. none of these

Answer: C

Watch Video Solution

10. If $x = a\left\{\cos\theta + \log\tan\frac{\theta}{2}\right\}$ and $y = a\sin\theta$, then $\frac{dy}{dx}$ is equal to

A. $\cot\theta$

B. $\tan\theta$

C. $\sin\theta$

D. $\cos\theta$

Answer: B

Watch Video Solution

11. If $y = f(x^3), z = g(x^2), f'(x), = \cos x$ and $g'(x) = \sin x$, then $\frac{dy}{dz}$ is

A. $\frac{3x}{2}\cos x^3 \cos ecx^2$

B. $\frac{2}{3}\sin x^3 \sec x^2$

C. $\tan x$

D. none of these

Answer: A

Watch Video Solution

12. Differentiate $\sec^{-1}\frac{1}{2x^2-1}$ with respect to $\sqrt{1-x^2}$

A. -4

B. 4

C. 2

D. -2

Answer: B

Watch Video Solution

13. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, if `-1

A. 1 for all x

B. $1 \text{ for } |x| > 1 \text{ and } -1 \text{ for } |x| < 1$

C. $1 \text{ for } |x| < 1 \text{ and } -1 \text{ for } |x| > 1$

D. $1 \text{ for } |x| \leq 1 \text{ and } -1 \text{ for } |x| > 1$

Answer: C

Watch Video Solution

14. The derivative of $\log_{10} x$ with respect to x^2, is

A. $\frac{1}{2x^2}\log_e 10$

B. $\frac{2}{x^2}\log_{10} e$

C. $\frac{1}{2x^2}\log_{10}^{e}$

D. none of these

Answer: B

Watch Video Solution

15. If $f_r(x), g_r(x)h_{,r}(x), r = 1, 2, 3$ are polynomials in x such that $f_r(a) = g_r(a) - h_r(a) r = 1, 2, 3$ and $F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$, then $F'(x)$ at $x = a$ is .....

A. 0

B. $f_1(a)g_2(a)h_3(a)$

C. 1

D. none of these

Answer: A

Watch Video Solution

16. If $f(x) = \begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ \theta\sec x & \tan x & x \\ 1 & \tan x - \tan\theta & 0 \end{vmatrix}$, then $f'(\theta)$ is

A. 0

B. -1

C. independent of θ

D. none of these

Answer: B

Watch Video Solution

17. $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix}$, $x \in \mathbb{R}$, then $\frac{d^2y}{dx^2} + y$ is equal to :

A. 6

B. 4

C. -10

D. 0

Answer: A

Watch Video Solution

18. If $y = x + e^x$, find $\frac{d^2x}{dy^2}$.

A. $\frac{1}{(1+e^x)^2}$

B. $-\frac{e^x}{(1+e^x)^2}$

C. $-\frac{e^x}{(1+e^x)^3}$

D. e^x

Answer: C

Watch Video Solution

19. The second order derivative of $a\sin^3 t$ w.r.t, $a\cos^3 t$ at $t = \frac{\pi}{4}$ is

A. $\frac{4\sqrt{2}}{3a}$

B. 2

C. $\frac{1}{12a}$

D. 0

Answer: A

Watch Video Solution

20. Suppose f and g are functions having second derivatives f' and g' every where, if $f(x).g(x) = 1$ for all x and f'', g'' are never zero then $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)}$ equals

A. $3\left(\frac{f''}{g} - \frac{g''}{f}\right)$

B. $3\left(\frac{f''}{f} - \frac{g''}{g}\right)$

C. $3\left(\frac{g''}{g} - \frac{f''}{g}\right)$

D. $3\left(\frac{f''}{g} - \frac{g''}{f}\right)$

Answer: B

Watch Video Solution

21. If $x = \phi(t), y = \psi(t),$ then $\frac{d^2y}{dx^2} =$

A. $\frac{\phi'\Psi'' - \Psi'\phi''}{(\phi)^2}$

B. $\frac{\phi'\Psi'' - \Psi'\phi''}{(\phi')^3}$

C. $\frac{\phi''}{\Psi''}$

D. $\frac{\Psi''}{\phi''}$

Answer: B

Watch Video Solution

22. If $y = \sin x + e^x,$ then $\frac{d^2x}{dy^2} =$

A. $(-\sin x + e^x)^{-1}$

B. $\frac{\sin x - e^x}{(\cos + e^x)^2}$

C. $\frac{\sin x - e^x}{(\cos x + e^x)^3}$

D. $\frac{\sin x + e^x}{(\cos x + e^x)^3}$

Answer: C

Watch Video Solution

23. If $y = \left\{x + \sqrt{x^2+1}\right\}^m$, show that $\left(x^2+1\right)y_2 + xy_1 - m^2 y = 0$

A. $m^2 y$

B. my^2

C. $m^2 y^2$

D. none of these

Answer: A

Watch Video Solution

24. If $f(x) = x^n, n\varepsilon N,$ then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f(1)}{2!} - (f'''')\frac{1}{3!} + \ldots + (-1)^n \frac{f^n(1)}{n!}$ is

A. n

B. 2^n

C. 2^{n-1}

D. $\frac{n(n+1)}{2}$

Answer: B

Watch Video Solution

Section I Solved Mcqs

1. If $f(x) = \sqrt{x^2 + 9}$, write the value of $(\lim)_{x \to 4} \frac{f(x) - f(4)}{x - 4}$.

A. $5/4$

B. $-4/5$

C. $4/5$

D. none of these

Answer: C

Watch Video Solution

2. Let f be a twice differentiable function such that

$f''(x) = -f(x)$ and $f'(x) = g(x)$. If $h'(x) = [f(x)]^2 + [g(x)]^2$, $h(1) = 8$ and $h(0) = 2$, then $h(2) =$

A. 1

B. 2

C. 3

D. none of these

Answer: D

Watch Video Solution

3. If f(x) = log |x| then for $x \neq 0$ f(x) is equal to

A. $\frac{1}{|x|}$

B. $\frac{1}{x}$

C. $-\frac{1}{x}$

D. none of these

Answer: B

Watch Video Solution

4. If $f(x) = |\log x|$, then for $x \neq 1$, $f'(x)$ equals

A. $1/x$

B. $1/|x|$

C. $-1/x$

D. none of these

Answer: D

Watch Video Solution

5. If $f(9) = 9$ and $f'(9) = 4$ then $Lt_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ is equal to

A. 9

B. 4

C. 36

D. none of these

Answer: B

Watch Video Solution

6. If $f(x) = |\log_e |x||, \quad \text{then} \quad f'(x)$ equals

A. $\frac{1}{|x|}, x \neq 0$

B. $\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

C. $-\frac{1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

D. $\frac{1}{x}$ for $x > 0$ and $-\frac{1}{x}$ for $x < 0$

Answer: B

Watch Video Solution

7. If $f(x)$ is given by, $f(x) = (\cos x + i \sin x)$ $(\cos 3x + i \sin 3x).....(\cos(2n-1)x + i \sin(2n-1)x)$, then $f''(x)$ is equal to

A. $n^2 f(x)$

B. $-n^4 f(x)$

C. $-n^2 f(x)$

D. $n^4 f(x)$

Answer: B

Watch Video Solution

8. If $y = (\log_{\cos x} \sin x)(\log_{\sin x} \cos x) + \sin^{-1} \frac{2x}{1+x^2}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is equal to

A. $\frac{8}{\pi^2 + 4}$

B. 0

C. $\frac{-8}{\pi + 4}$

D. 1

Answer: C

Watch Video Solution

9. Let $f(x)$ be a polynomial. Then, the second order derivative of $f(e^x)$ is

$f(e^x)e^{2x} + f'(e^x)e^x$ (b) $f(e^x)e^x + f'(e^x)$ (c) $f(\hat{}x)e\hat{}(2x)+f(e^x)e^x$ (d)

$f(e^x)$

A. $f''(e^x).e^x + f'(e^x)$

B. $f''(e^x).e^{2x} + f'(e^x).e^{2x}$

C. $f''(e^x)e^{2x}$

D. $f''(e^x)e^{2x} + f'(e^x).e^x$

Answer: D

Watch Video Solution

10. Let $f(x) = x^n$,n being a non negative integer. The value of n for which the equality$f'(a+b) = f'(a) + f'(b)$ is valid for all $a.\ b > 0$ is

A. 0,1

B. 1,2

C. 2,4

D. none of these

Answer: D

Watch Video Solution

11. Let $f(x) = \sin x$, $g(x) = x^2$ and $h(x) = \log_e x$.

If $F(x) = (\text{hog of })(x)$, then $F''(x)$ is equal to

A. $2\cos ec^3 x$

B. $2\cot x^2 - 4x^2 \cos ec^2 x^2$

C. $2x\cot x^2$

D. $-2\cos ec^2 x$

Answer: D

Watch Video Solution

12. If $f(x) = \sin\left\{\frac{\pi}{3}[x] - x^2\right\}$ for $2 < x < 3$ and [x] denotes the greatest integer less than or equal to x, then $f'(\sqrt{\pi/3})$ is equal to

A. $\sqrt{\pi/3}$

B. $-\sqrt{\pi/3}$

C. $-\sqrt{\pi}$

D. none of these

Answer: B

Watch Video Solution

13. If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ equals

A. -1

B. 1

C. $\log_e 2$

D. $-\log_e 2$

Answer: A

Watch Video Solution

14. The function $u = e^x \sin x; v = e^x \cos x$ satisfy the equation a. $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$ b. $\frac{d^2u}{dx^2} = 2v$ c. $\frac{d^2v}{dx^2} = -2u$ d. $\frac{du}{dx} + \frac{dv}{dx} = 2v$

A. $v\frac{du}{dx}u\frac{dv}{dx} = u^2 + v^2$

B. $\frac{d^2u}{dx^2} = 2v$

C. $\frac{d^2v}{dx^2} = -2u$

D. all the above

Answer: D

Watch Video Solution

15. If $f(x) = |x - 2|$ and $g(x) = f(f(x))$, then for $x > 20, g'(x)$ equals

A. -1

B. 1

C. 0

D. none of these

Answer: B

Watch Video Solution

16. If $f(x) = |x - 2|$ and $g(x) = f(f(x))$, then for $2 < x < 4, g'(x)$ equals

A. -1

B. 1

C. 0

D. none of these

Answer: B

Watch Video Solution

17. If $f(x) = \log_x(\ln x)$ then $f'(x)$ at x=e is

A. e

B. $-e$

C. e^2

D. e^{-1}

Answer: D

Watch Video Solution

18. Let $f(t) = \ln(t)$. Then, $\frac{d}{dx}\left(\int_{x^2}^{x^3} f(t)\ \mathrm{dt}\right)$

A. has value 0 when x=0

B. has value 0 when x = 1 and $x = 4/9$

C. has value 9 $e^2 - 4e$ when x=e`

D. has differential coefficient $27e - 8$ for x=e

Answer: A

Watch Video Solution

19. If g is the inverse of f and $f'(x) = \frac{1}{1+x^n}$, prove that $g'(x) = 1 + (g(x))^n$

Watch Video Solution

20. If $f(x) = (|x|)^{|\sin x|}$, then $f'(-\pi/4)$ is equal to

A. $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

B. $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

C. $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{\pi}{4} - \frac{2\sqrt{2}}{\pi}\right)$

D. $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{\pi}{4} + \frac{2\sqrt{2}}{\pi}\right)$

Answer: A

Watch Video Solution

21.

If $y = (1+x)\left(1+x^2\right)\left(1+x^4\right)\ldots\left(1+x^{2^n}\right)$, then find $\frac{dy}{dx} atx = 0.$

A. 1

B. -1

C. 0

D. none of these

Answer: A

Watch Video Solution

22. Let $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. 0

D. none of these

Answer: D

Watch Video Solution

23. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{2}\right)$ is equal to

A. 1

B. -1

C. 0

D. none of these

Answer: A

Watch Video Solution

24. If $y = |x - x^2|$, then $\frac{dy}{dx}$ at $x = 1$.

A. -1

B. 1

C. does not exist

D. none of these

Answer: C

Watch Video Solution

25. If $y = |\cos x| + |\sin x|, then \frac{dy}{dx} at x = \frac{2\pi}{3}$ is $\frac{1-\sqrt{3}}{2}$ (b) 0 (c) $\frac{1}{2}(\sqrt{3}-1)$ (d) none of these

A. $\frac{1-\sqrt{3}}{2}$

B. 0

C. $\frac{\sqrt{3}-1}{2}$

D. none of these

Answer: C

Watch Video Solution

26. If f(x)= |cosxl, then $f'\left(\frac{3\pi}{4}\right)$ equal to -

A. $\frac{-1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{2}}$

C. 1

D. none of these

Answer: B

Watch Video Solution

27. If $f(x) = |x|^{|\tan x|}$ then $f'\left(-\frac{\pi}{6}\right)$ is equal to

A. $\left(\frac{\pi}{6}\right)^{1/\sqrt{3}}\left\{\frac{2\sqrt{3}}{\pi}-\frac{4}{3}\log\frac{6}{\pi}\right\}$

B. $\left(\frac{\pi}{6}\right)^{1/\sqrt{3}}\left\{\frac{-2\sqrt{3}}{\pi}+\frac{4}{3}\log\frac{6}{\pi}\right\}$

C. $\left(\frac{\pi}{6}\right)^{1/\sqrt{3}}\left\{\frac{2\sqrt{3}}{\pi}+\frac{4}{3}\log\frac{6}{\pi}\right\}$

D.

Answer: B

Watch Video Solution

28. If $x^2+y^2=1$then

A. $yy''-2(y')+1=0$

B. $yy''+(y')^2+1=0$

C. $yy''+(y')^2-1=0$

D. $yy''+2(y')^2+1=0$

Answer: B

Watch Video Solution

Watch Video Solution

29. If $y = \cos^{-1}(\cos x), then \frac{dy}{dx}$ is equal to $\frac{x}{y}$ (b) $\frac{y}{x^2}$ $\frac{x^2 - y^2}{x^2 + y^2}$ (d) $\frac{y}{x}$

A. 1

B. -1

C. $\frac{1}{\sqrt{2}}$

D. none of these

Answer: B

Watch Video Solution

30. If $y = \sin^{-1}(\sin x),$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is

A. 1

B. -1

C. non-exisent

D. none of these

Answer: C

Watch Video Solution

31. If $y = \sec\left(\tan^{-1} x\right), \quad \text{then} \quad \frac{dy}{dx} \quad \text{at} \quad x = 1$ is equal to

A. $\frac{1}{\sqrt{2}}$

B. $-\frac{1}{\sqrt{2}}$

C. 1

D. none of these

Answer: A

Watch Video Solution

32. Let f be a differentiable function satisfying $[f(x)]^n = f(nx)$ for all $x \in R$ Then $f'(x)f(nx)$ equals

A. $f(x)$

B. 0

C. $f(x)f'(nx)$

D. none of these

Answer: C

Watch Video Solution

33. If $f(x) = |x-1|$ and $g(x) = f(f(f(x)))$, then $g'(x)$ is equal to:

A. 1 for all $x > 2$

B. 1 for $2 < x < 3$

C. -1 for $2 < x < 3$

D. none of these

Answer: C

Watch Video Solution

34. Let $F(x) = f(x)g(x)h(x)$ for all real x, where f(x), g(x), and h(x) are differentiable functions. At some point $x_0, F'(x_0) = 21F(x_0), f'(x_0) = 4f(x_0), g'(x_0) = -7g(x_0),$ and $h'(x_0)$

Then k = -

A. 12

B. 12

C. 24

D. -24

Answer: C

Watch Video Solution

35. If g is the inverse function of and f'(x) = sin x then prove that g'(x) = cosec (g(x))

A. $\cos ec\{g(x)\}$

B. $\sin\{g(x)\}$

C. $\frac{1}{\sin\{g(x)\}}$

D. none of these

Answer: C

Watch Video Solution

36. Let $f(x)$ be a second degree polynomial function such that $f(-1) = f(1)$ and α, β, γ are in A.P. Then, $f'(\alpha), f'(\beta), f'(\gamma)$ are in

A. AP

B. GP

C. HP

D. none of these

Answer: A

Watch Video Solution

37. Find the derivative of f(tan x) w.r.t. g (sec x) at $x = \frac{\pi}{4}$, where f'(1)=2 and $g'\left(\sqrt{2}\right) = 4$.

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{2}$

C. 1

D. 0

Answer: A

Watch Video Solution

38. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x-\frac{1}{2}$.

A. -4

B. 4

C. -1

D. none of these

Answer: A

Watch Video Solution

39. The derivative of $\sin^{-1}\left(3x-4x^3\right)$ with respect to $\sin^{-1}x,$ is

A. $3,\quad$ for $\quad|x|<1$

B. $3,\quad$ for $\quad|x|<\frac{1}{2}\quad$ and $\quad-3\quad$ for $\quad\frac{1}{2}<|x|<1$

C. $-3,\quad$ for $\quad|x|<12$

D. -3, for $|x| \leq \frac{1}{2}$ and 3 for $\frac{1}{2} < |x| < 1$

Answer: B

Watch Video Solution

40. If $f(x) = \cos\left\{\frac{\pi}{2}[x] - x^3\right\}, 1 < x < 2$, and [x] denotes the greatest integer less than or equal to x, then the value of $f'\left(\sqrt[3]{\frac{\pi}{2}}\right)$, is

A. 0

B. $3\left(\frac{\pi}{2}\right)^{2/3}$

C. $-3\left(\frac{\pi}{2}\right)^{2/3}$

D. none of these

Answer: A

Watch Video Solution

41. Let $f(x) = \sin x, g(x) = 2x$ and $h(x) = \cos x.$ If $\phi(x) = [\text{go}(fh)](x),$ then $\phi''\left(\frac{\pi}{4}\right)$ is equal to

A. 4

B. 0

C. -4

D. none of these

Answer: C

Watch Video Solution

42. Let f(x) be a polynomial function satisfying $f(x) + f\left(\frac{1}{x}\right) = f(x)f\left(\frac{1}{x}\right)$ for all $x \neq 0.$ If $f(5) = 126$ and a,b,c are in G.P., then $f'(a), f'(b), f'(c)$ are in

A. AP

B. GP

C. HP

D. none of these

Answer: B

Watch Video Solution

43. If $f(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$, then the value of $\frac{d^n}{dx^n}(f(x))$ at $x = 0$ for $n = 2m + 1$ is

A. -1

B. 0

C. 1

D. independent of a

Answer: B

Watch Video Solution

44. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, then find $\frac{dy}{dx}$

A. $\sin\left(\frac{2x-1}{x^2+1}\right)^2\left\{\frac{x^2+2x+2}{(x^2+1)}\right\}$

B. $\sin\left(\frac{2x-1}{x^2+1}\right)^2\left\{\frac{2+2x-2x^2}{(x^2+1)^2}\right\}$

C. $\sin\left(\frac{2x-1}{x^2+1}\right)^2\left\{\frac{2+2x-x^2}{(x^2+1)^2}\right\}$

D. none of these

Answer: B

Watch Video Solution

45. Let f be a differentiable function defined for all $x \in R$ such that $f(x^3) = x^5$ fol all $x \in R, x \neq 0$. Then the value of $f'(8)$, is

A. 20

B. $\frac{20}{3}$

C. $\frac{5}{3}$

D. none of these

Answer: B

Watch Video Solution

46. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$ then find $f'\left(\frac{\pi}{4}\right)$

A. $\sqrt{2}$

B. $\frac{1}{\sqrt{2}}$

C. 1

D. none of these

Answer: A

Watch Video Solution

47. If $f(x) = \cos x \cos 2x \cos 2^2 x \cos 2^3 x \ldots . \cos 2^{n-1} x$ and $n > 1,$ then $f'\left(\frac{\pi}{2}\right)$ is

A. 1

B. 0

C. -1

D. (-1)^n-1

Answer: A

Watch Video Solution

48. $f'(x) = \varphi'(x) = f(x)$ for all x. Also, $f(3) = 5 and f'(3) = 4.$ Then the value of $[f(10)]^2$ -$[\varphi(10)]^2$ is _____

A. 0

B. 9

C. 41

D. none of these

Answer: B

Watch Video Solution

49. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2)$, $then \frac{dy}{dx}$ at x = 1 is

A. 2

B. 1

C. -2

D. none of these

Answer: A

Watch Video Solution

50. Let f be a one-one function satisfying $f'(x) = f(x)$ then $\left(f^{-1}\right)''(x)$ is equal to

A. $-\frac{1}{x^3}$

B. $-\frac{1}{x^2}$

C. f(x)

D. $f^{-1}(x)$

Answer: B

Watch Video Solution

51. Differentiate $\sec^{-1}\frac{1}{2x^2-1}$ with respect to $\sqrt{1-x^2}$

A. -4

B. 4

C. 2

D. -2

Answer: B

Watch Video Solution

52. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x=1$, is

A. 2

B. -2

C. non-exisent

D. none of these

Answer: C

Watch Video Solution

53. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x=0$, is

A. 2

B. -2

C. 1

D. none of these

Answer: D

Watch Video Solution

54. $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$. Find $\frac{dy}{dx}$.

A. $\frac{3}{1 + 9x^2}$ for all $x \in R$

B. $\frac{3}{1 + x^2}$ for all $x \in R$

C. $\frac{3}{1 + x^2}$ for all $x \in R - \left\{-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}$

D. $\frac{1}{9 + x^2}$

Answer: C

Watch Video Solution

55. If $5f(x) + 3f\left(\frac{1}{x}\right) = x + 2$ and $y = xf(x)$, then find $\frac{dy}{dx}$ at $x = 1$.

A. 14

B. $\frac{7}{8}$

C. 1

D. none of these

Answer: B

Watch Video Solution

56. Let f and g be differentiable functions satisfying g'(a) = 2 g(a) = b and fog = I (Identity function). Then f'(b) is equal to

A. 2

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. none of these

Answer: C

Watch Video Solution

57. If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2, then|f'(-3)|$ equals_____.

A. 1

B. 2

C. -2

D. 0

Answer: C

Watch Video Solution

58. If $P(x)$ is a polynomial such that

$P\left(x^2+1\right)=\{P(x)\}^2+1$ and P(0)=0, then P'(0) is equal to

A. 1

B. 0

C. -1

D. none of these

Answer: A

Watch Video Solution

59. Let f(x) be a differentiable function such that

$f'(x)=\sin x+\sin 4x\cos x.$ Then $f'\left(2x^2+\frac{\pi}{2}\right)$ at $x=\sqrt{\frac{\pi}{2}}$ is equal to

A. 0

B. -1

C. $-2\sqrt{\pi}$

D. none of these

Answer: C

Watch Video Solution

60. Let $f(x) = \frac{x^2 - x}{x^2 + 2x}$ then $d\frac{f^{-1}x}{dx}$ is equal to

A. $\frac{-3}{(1-x)^2}$

B. $\frac{3}{(1-x)^2}$

C. $\frac{1}{(1-x)^2}$

D. none of these

Answer: D

Watch Video Solution

61. let $f(x)$ be a polynomial function of degree 2 and $f(x) > 0$ for all $x \in R$. if $g(x) = f(x) + f'(x) + f''(x)$, then for any x show that $g(x) > 0$

A. $g(x) < 0$ for all x

B. $g(x) > 0$ for all x

C. $g(x) = 0$ for all x

D. $g(x) \geq 0$ for all x.

Answer: B

Watch Video Solution

62. If f is a bijection satisfying $f'(x) = \sqrt{\left(1 - \{f(x)\}^2\right)}$, then $\left(f^1\right)'x$

A. is equal to $\frac{1}{\sqrt{1-x^2}}$

B. may not exist for every $x \in R$

C. may not be known explicitly

D. is equal $\sin^{-1}(f(x))$

Answer:

Watch Video Solution

63. If $f(x) = \cos^{-1}\left\{\frac{1-(\log_e x)^2}{1+(\log_e x)^2}\right\}$, then $f'\left(\frac{1}{e}\right)$ is equal to

A. e

B. $-e$

C. 1

D. $2e$

Answer: B

Watch Video Solution

64. Let $f(x) = x^n$,n being a non negative integer. The value of n for which the equality$f'(a+b) = f'(a) + f'(b)$ is valid for all $a.\ b > 0$ is

A. 1,2

B. 0,2

C. 0,1

D. none of these

Answer: B

Watch Video Solution

65. Let f be a differentiable function satisfying

$f(x) + f(y) + f(z) + f(x)f(y)f(z) = 14 \quad \text{for all} \quad x, y, z \in R$ Then,

A. $f'(x) < 0 \quad \text{for all} \quad x \in R$

B. $f'(x) = 0 \quad \text{for all} \quad x \in R$

C. $f'(x) > 0 \quad \text{for all} \quad x \in R$

D. none of these

Answer: B

Watch Video Solution

66. $If f(x) = 2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right), x > 1$. Then, f(5) is equal to

A. $f'(2) = f'(3)$

B. $f'(2) = 0$

C. $f'\left(\frac{1}{2}\right) = \frac{16}{5}$

D. all the above

Answer: D

Watch Video Solution

67. Let f(x) be a polynomial of degree 3 such that $f(3) = 1, f'(3) = -1, f''(3) = 0,$ and $f'''(3) = 12.$ Then the value of f'(1) is

A. 12

B. 23

C. -13

D. none of these

Answer: B

Watch Video Solution

68. If $f(x) = \tan^{-1} x + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then

A. $f'(-2) = \frac{4}{5}$

B. $f'(-1) = -1$

C. $f'(x) = 0$ for all $x < 0$

D. none of these

Answer: C

Watch Video Solution

69. Let

$f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = x^2 + xf'(2) + f''(3).$

Then

A. $f'(1) = 4 + f'(2)$

B. $g'(2) = 8 + g0(1)$

C. $g''(2) + f''(3) = 4$

D. all the above

Answer: D

Watch Video Solution

70.

$f: R \to R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3) \quad \text{for all} \quad x \in R.$

The value of f(1) is

A. $f(0) + f(2) = f(1)$

B. $f(0) + f(3) = 0$

C. $f(1) + f(3) = f(2)$

D. all the above

Answer: D

Watch Video Solution

71. Let $y = \frac{x^8 + x^4 + 1}{x^4 + x^2 + 1}$. If $\frac{dy}{dx} = ax^3 + bx$. Then,

A. $a = 4, b = 2$

B. $a = 4, b = -2$

C. $a = -2, b = 4$

D. none of these

Answer: B

Watch Video Solution

72. If $f(x-y), f(x)f(y), \text{ and } f(x+y)$ are in A.P. for all x, y, and $f(0) \neq 0$, then

A. $f(1)+f(-1)=0$

B. $f(2)+f(-2)=0$

C. $f'(3)+f'(-3)=0$

D. $f'(2)=f'(-2)$

Answer: C

Watch Video Solution

73. Let $f(x) = 3x^2 + 4xg'(1) + g''(2)$

and, $g(x) = 2x^2 + 3xf'(2) + f''(3)$ for all $x \in R$. Then,

A. $f'(1) = 22 + 12f'(2)$

B. $g,(2) = 44 + 12g'(1)$

C. $f''(3) + g''(2) = 10$

D. all the above

Answer: D

Watch Video Solution

74. If $f(x) = x^n$, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \frac{f''''(1)}{4!} - .... + \frac{(-1)^n f^n(1)}{n!}$ is

A. 1

B. 2^n

C. 2^{n-1}

D. 0

Answer: D

Watch Video Solution

75. let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a_1, a_2, a_3 are in AP, then show that $f'(a_1), f'(a_2), f'(a_3)$ are in AP.

A. Arthmetic-Geometric Progression

B. AP

C. GP

D. HP

Answer: B

Watch Video Solution

76. Let $f(x) = \sqrt{x-1} + \sqrt{x+24-10\sqrt{x-1}}, 1 < x < 26$ be real valued function, then $f'(x)$ for $1 < x < 26$ is

A. 0

B. $\frac{1}{\sqrt{x-1}}$

C. $2\sqrt{x-1}$

D. none of these

Answer: A

Watch Video Solution

77. $\frac{d^2x}{dy^2}$ equals:

$(1.)\left(\frac{d^2y}{dx^2}\right)^{-1}$

$(2)-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(3) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$

(4) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{3}$

A. $\left(\frac{d^2x}{dy^2}\right)^{-1}$

B. $\left(\frac{d^2x}{dy^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

C. $\left(\frac{d^2x}{dy^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

D. $-\left(\frac{d^2x}{dy^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Answer: D

Watch Video Solution

78. $f(x)$ and $g(x)$ are two differentiable functions in $[0, 2]$ such that $f(x)=g(x) = 0, f'(1) = 2, g'(1) = 4, f(2) = 3, g(2) = 9$ then $f(x) - g(x)$ at $x = \frac{3}{2}$ is

A. 0

B. 2

C. 10

D. 5

Answer: D

Watch Video Solution

79. If $f(1) = 3, f'(1) = -\frac{1}{3}$, then the derivative of $\left\{x^{11} + f(x)\right\}^{-2}$ at $x = 1,$ is

A. $-\frac{1}{2}$

B. -1

C. 1

D. $f'(1)$

Answer: D

Watch Video Solution

80. If y is a function of x and $\log(x+y)-2xy=0,$ then the value of $y'(0)$ is

(a)1 (b) -1 (c) 2 (d) 0

A. 1

B. -1

C. 2

D. 0

Answer: A

Watch Video Solution

81. Let y be an implicit function of x defined by $x^{2x}-2x^{x}\cot y-1=0.$ Then y' (1) equals

A. -1

B. 1

C. $\log 2$

D. $-\log 2$

Answer: A

Watch Video Solution

82. If the functions $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, the value of g'(1) is ..... .

A. $\frac{1}{2}$

B. 2

C. 1

D. $-\frac{1}{2}$

Answer: B

Watch Video Solution

83. Let g(x) = log f(x), where f(x) is a twice differentiable positive function on $(0, \infty)$ such that $f(x+1) = xf(x)$. Then, for $N = 1, 2, 3, \ldots\ldots, g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right)$ is equal to

A. $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N-1)^2}\right\}$

B. $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N-1)^2}\right\}$

C. $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N+1)^2}\right\}$

D. $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N+1)^2}\right\}$

Answer: A

Watch Video Solution

84. Consider the function $f: (-\infty, \infty) \to (-\infty, \infty)$ defined by $f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}; 0 < a < 2$. Which of the following is true ?

A. $(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$

B. $(2-a)^2 f''(1) - (2+a)^2 f''(-1) = 0$

C. $f'(1)f'(-1) = (2-a)^2$

D. $f'(1)f'(-1) = -(2+a)^2$

Answer: A

Watch Video Solution

85. If $f(x) = \frac{x}{1+|x|}$ for $x \in R$, then f'(0)`=

A. 0

B. 1

C. 2

D. 3

Answer: B

Watch Video Solution

86. If $f(x) = \frac{1}{1-x}$, then the derivative of the composite function f[f{f(x)}]` is equal to

A. 0

B. $1/2$

C. 1

D. 2

Answer: C

Watch Video Solution

87.

Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then th

is -

A. 1

B. 2

C. 3

D. 4

Answer: A

Watch Video Solution

88. if $y^x - x^y = 1$ then the value of $\frac{dy}{dx}$ at $x = 1$ is

A. $2(1 - \log 2)$

B. $2(1 + \log 2)$

C. $2 - \log 2$

D. $2 + \log 2$

Answer: A

Watch Video Solution

89. If $y = \sec\left(\tan^{-1} x\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. 1

D. $\sqrt{2}$

Answer: A

Watch Video Solution

90. Let $y(x) = \cos\left(3\cos^{-1} x\right), x \in [-1, 1], x \neq \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{\left(x^2 - 1\right)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equals

A. 1

B. 2

C. 8

D. 9

Answer: D

Watch Video Solution

91. If a curve is represented parametrically by the equation $x = 4t^3 + 3$ and $y = 4 + 3t^4$ and $\frac{\left(\frac{d^2x}{dy^2}\right)}{\left(\frac{dx}{dy}\right)^n}$ is constant then value of $\frac{n}{2}$ is

A. 3

B. 4

C. 5

D. 6

Answer: C

Watch Video Solution

92.

If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then g(x) equals

A. $\frac{9}{1+9x^3}$

B. $\frac{3x\sqrt{x}}{1-9x^3}$

C. $\frac{3x}{1-9x^3}$

D. $\frac{3}{1+9x^3}$

Answer: A

Watch Video Solution

Exercise

1. If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ is

A. $\frac{-2}{1+x^2}$ for all x

B. $\frac{-2}{1+x^2}$ for all $|x| > 1$

C. $\frac{2}{1+x^2}$ for $|x| < 1$

D. none of these

Answer: D

Watch Video Solution

2. If $x = \int_0^y \frac{1}{\sqrt{1+4t^2}}$ dt, then $\frac{d^2y}{dx^2}$, is

A. 2y

B. 4y

C. 8y

D. 6y

Answer: B

Watch Video Solution

3. If $f(x) = \sqrt{x^2 - 2x + 1}$, then f' (x) ?

A. $f'(x) = 1 \quad \text{for all} \quad x$

B. $f'(x) = -1 \quad \text{for all} \quad x \leq 1$

C. $f'(x) = 1 \quad \text{for all} \quad x > 1$

D. none of these

Answer: D

Watch Video Solution

4. If $f(x) = \sqrt{1 - \sin 2x}$, then $f'(x)$ equals

A. $-(\cos x + \sin x). \quad \text{for} \quad x \in (\pi/4, \pi/2)$

B. $\cos x + \sin x, \quad \text{fro} \quad x \in (0, \pi/4)$

C. $-(\cos x + \sin x), \quad \text{for} \quad x \in (0, \pi/4)$

D. $\cos x - \sin x, \quad \text{for} \quad x \in (\pi/4, \pi/2)$

Answer: C

Watch Video Solution

5. If $f(x) = \left|x^2 - 5x + 6\right|, then f'(x) equals$

A. $2x - 5 \quad \text{for} \quad 2 < x < 3$

B. $5 - 2x \quad \text{for} \quad 2 < x < 3$

C. $2x - 5 \quad \text{for} \quad 2 \leq x \leq 3$

D. $5 - 2x \quad \text{for} \quad 2 \leq x \leq 3$

Answer: B

Watch Video Solution

6. If $x^2 + y^2 = a^2 \quad \text{and} \quad k = 1/a$ then k is equal to

A. $\frac{y''}{\sqrt{1 + y'}}$

B. (1)/y(sqrt(1+yy'')

C. $\frac{2y''}{\sqrt{1+y'^2}}$

D. $\frac{y''}{2\sqrt{(1+y'^2)^3}}$

Answer: B

Watch Video Solution

7. If $f(x) = \sin x$ and $g(x) = sgn \sin x$, then $g'(1)$ equals

A. 0

B. $-\cos 1$

C. $\cos 1$

D. none of these

Answer: C

Watch Video Solution

8. If $y = \sin^{-1}\left(\frac{x}{2}\right) + \cos^{-1}\left(\frac{x}{2}\right)$ then $\frac{dy}{dx} =$

A. 1

B. -1

C. 0

D. 2

Answer: C

Watch Video Solution

9. If $y = \cos^{-1}\left(\frac{2\cos x - 3\sin x}{\sqrt{13}}\right)$, then $\frac{dy}{dx}$, is

A. zero

B. constant=1

C. constant $\neq 1$

D. none of these

Answer: B

Watch Video Solution

10. If $y = x + e^x$, find $\frac{d^2x}{dy^2}$.

A. e^x

B. $-\frac{e^x}{(1+e^x)^3}$

C. $-\frac{e^x}{(1+e^x)^2}$

D. $\frac{1}{(1+e^x)^2}$

Answer: B

Watch Video Solution

11. Given that, $F(x) = \frac{1}{x^2}\int_4^x \left(4t^2 - 2F'(t)\right)dt$, find $F'(4)$.

A. $\frac{32}{9}$

B. $\frac{64}{3}$

C. $\frac{64}{9}$

D. $\frac{32}{3}$

Answer: A

Watch Video Solution

12. If $y^2 = p(x)$ is a polynomial of degree 3, then $2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$ is equal to

A. $P(x) + P''(x)$

B. $P(x)$

C. $P(x)P'''(x)$

D. a constant

Answer: C

Watch Video Solution

13. if $2^x + 2^y = 2^{x+y}$ then the value of $\frac{dy}{dx}$ at $x = y = 1$

A. 0

B. -1

C. 1

D. 2

Answer: B

Watch Video Solution

14. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = 0$ is $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

A. $1/8$

B. $1/4$

C. $1/2$

D. 1

Answer: B

Watch Video Solution

15. If $y = \tan^{-1}\left\{\frac{(\log)_e(e/x^2)}{(\log)_e(ex^2)}\right\} + \tan^{-1}\left(\frac{3 + 2(\log)_e x}{1 - 6(\log)_e x}\right)$, then $\frac{d^2y}{dx^2} =$ (a) 2 (b) 1 (c) 0 (d) -1

A. 2

B. 1

C. 0

D. -1

Answer: C

Watch Video Solution

16. The expression of $\frac{dy}{dx}$ of the function $y = a^{x^{a^{x \ldots \infty}}}$, is

A. $\frac{y^2}{x(1 - y \log x)}$

B. $\frac{y^2 \log y}{x(1 - y \log x)}$

C. $\frac{y^2 \log y}{x(1 - y \log x \log y)}$

D. $\frac{y^2 \log y}{x(1 + y \log x \log y)}$

Answer: C

Watch Video Solution

17. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then $\frac{dy}{dx}$ equals

A. $\sqrt{(1 - x^2)(1 - y^2)}$

B. $\sqrt{\frac{1 - y^2}{1 - x^2}}$

C. $\sqrt{\frac{1 - x^2}{1 - y^2}}$

D. none of these

Answer: B

Watch Video Solution

18. If $y = e^{1+\log_e x}$, then the value of $\frac{dy}{dx}$ is equal to

A. e

B. 1

C. 0

D. $\log_e xe \log_{e^{ex}}$

Answer: A

Watch Video Solution

19. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to

A. $(1+\log x)^{-1}$

B. $(1+\log x)^{-2}$

C. $\log x(1+\log x)^{-2}$

D. none of these

Answer: C

Watch Video Solution

20. Let $f(x)=\frac{x^2}{1-x^2}, x \neq 0, \pm 1$, then derivative of f(x) with respect to x, is

A. $\frac{2x}{(1-x^2)^2}$

B. $\frac{1}{(2+x^2)^3}$

C. $\frac{1}{(1-x^2)^2}$

D. $\frac{1}{(2-x^2)^2}$

Answer: A

Watch Video Solution

21. If $y = e^{\sin^{-1}x}$ and $u = \log x$, then $\frac{dy}{du}$, is

A. $\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$

B. $xe^{\sin^{-1}x}$

C. $\frac{xe^{\sin^{-1}x}}{\sqrt{1-x^2}}$

D. $\frac{e^{\sin^{-1}x}}{x}$

Answer: C

Watch Video Solution

22. The differential coefficient of $f(x) = \log(\log x)$ with respect to x is

A. $\frac{x}{\log x}$

B. $\frac{\log x}{x}$

C. $(x\log x)^{-1}$

D. $x \log x$

Answer: C

Watch Video Solution

23. If $y = \left(\tan^{-1}\right) \frac{\sqrt{1+x^2}-1}{x}$, then $y'(1)$ is equal to

A. `(1)/(4)

B. 0

C. 1

D. -1

Answer: A

Watch Video Solution

24. The derivative of $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$ with respect to x is

A. $-\frac{1}{2\sqrt{1-x^2}}$

B. $\frac{1}{2\sqrt{1-x^2}}$

C. $\frac{2}{\sqrt{1-x^2}}$

D. $\frac{-2}{\sqrt{1-x^2}}$

Answer: A

Watch Video Solution

25. If $f(x) = \log_a(\log_a x)$, then f'(x), is

A. $\frac{\log_a e}{x \log_e x}$

B. $\frac{\log_e a}{x \log_a x}$

C. $\frac{\log_e a}{x}$

D. $\frac{x}{\log_e a}$

Answer: A

Watch Video Solution

26. The differential coefficient of $f\left((\log)_e x\right)$ with respect to x, where $f(x) = (\log)_e x$, is

(a) $\frac{x}{(\log)_e x}$ (b) $\frac{1}{x}(\log)_e x$ (c) $\frac{1}{x(\log)_e x}$ (d) none of these

A. $\frac{x}{\log x}$

B. $(x \log x)^{-1}$

C. $\frac{\log x}{x}$

D. $x \log x$

Answer: B

Watch Video Solution

27. If $x^m . y^n = (x + y)^{m+n}$, prove that

(i) $\frac{dy}{dx} = \frac{y}{x}$ and (ii) $\frac{d^2y}{dx^2} = 0$

A. $\frac{y}{x}$

B. $\frac{py}{qx}$

C. $\frac{x}{y}$

D. $\frac{qy}{px}$

Answer: A

Watch Video Solution

28. The value of $\frac{d}{dx}(|x-1|+|x-5|)$ at x=3, is

A. -2

B. 0

C. 2

D. 4

Answer: B

Watch Video Solution

29. $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$, $x > 0$. Find dy/dx

A. 1

B. $\frac{x-1}{x+1}$

C. 0

D. $\frac{x+1}{x-1}$

Answer: C

Watch Video Solution

30. If $f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ equals

A. $\sin(\log x).\ \frac{1}{x \log x}$

B. $\frac{12}{(3-2x)^2}\sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$

C. $\sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$

D. none of these

Answer: B

Watch Video Solution

31. If $f(x) = (\log_{\cot x} \tan x)(\log_{\tan x} \cot x)^{-1} + \tan^{-1}\left(\frac{x}{\sqrt{4-x^2}}\right)$, then f'(0) is equal to

A. 2

B. 0

C. $1/2$

D. -2

Answer: C

Watch Video Solution

32. If $y = x^{x^{x^{x \ldots \infty}}}$, then $x(1 - y\log x)\frac{dy}{dx}$

A. x^2

B. y^2

C. xy^2

D. xy

Answer: B

Watch Video Solution

33. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a, then \frac{dy}{dx}$ is equal to (a) $\frac{x}{y}$ (b) $\frac{y}{x^2}$ (C) $\frac{x^2-y^2}{x^2+y^2}$ (d) $\frac{y}{x}$

A. $\frac{x}{y}$

B. $\frac{y}{x^2}$

C. $\frac{x^2-y^2}{x^2+y^2}$

D. $\frac{y}{x}$

Answer: D

Watch Video Solution

34. If $y = \sec^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x-1}}\right) + \sin^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{x+1}}\right)$ then $\frac{dy}{dx} =$

A. 1

B. 0

C. $\frac{\sqrt{x}+1}{\sqrt{x}-1}$

D. $\frac{\sqrt{x}-1}{\sqrt{x}+1}$

Answer: B

Watch Video Solution

35. If $x^2 + y^2 = \left(t + \frac{1}{t}\right)$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $x^3 y \frac{dy}{dx} =$

A. 0

B. 1

C. -1

D. none of these

Answer: B

Watch Video Solution

36. $y = \dfrac{\tan^{-1}\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}{\sqrt{1+x^2}-\sqrt{1-x^2}}$ then $\dfrac{dy}{dx}$

A. $\dfrac{1}{\sqrt{1-x^4}}$

B. $-\dfrac{1}{\sqrt{1-x^4}}$

C. $\dfrac{x}{\sqrt{1-x^4}}$

D. $-\dfrac{x}{\sqrt{1-x^4}}$

Answer: D

Watch Video Solution

37. $If y = \int_0^x f(t)\sin\{k(x-t)\}dt, then prove \widehat{\frac{dt^2y}{dx^2}} + k^2y = kf(x)$

A. 0

B. y

C. $kg(x)$

D. $k^2f(x)$

Answer: C

Watch Video Solution

38. If $f(x) = \begin{vmatrix} x^3 & x^4 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant, then $\frac{d^3}{dx^3}(f(x))$, is

A. proportional to x^2

B. proportional to x

C. proportional to x^3

D. a constant

Answer: B

Watch Video Solution

39. If $f(x) = x + 2, \quad \text{then} \quad f'(f(x)) \quad \text{at} \quad x = 4$, is

A. 8

B. 1

C. 4

D. 5

Answer: B

Watch Video Solution

40. If $y^2 = ax^2 + bx + c, \quad \text{then} \quad y^3 \frac{d^2y}{dx^2}$ is

A. a constant

B. a function of x

C. a function of y

D. a function of x and y both

Answer: A

Watch Video Solution

41. If $x = a\cos\theta, y = b\sin\theta, \quad \text{then}\frac{d^3y}{dx^3}$ is equal to

A. $-\frac{3b}{a^3}\text{cosec}^4\theta\cot^4\theta$

B. $\frac{3b}{a^3}\text{cosec}^4\theta\cot\theta$

C. $-\frac{3b}{a^3}\text{cosec}^4\theta\cot\theta$

D. none of these

Answer: C

Watch Video Solution

42. If $f(1)=1, f'(1)=2,$ then write the value of $(\lim)_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$

A. 2

B. 4

C. 1

D. $1/2$

Answer: C

Watch Video Solution

43. If variables x and y are related by the equation $x=\int_0^y \frac{1}{\sqrt{1+9u^2}}\,du$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{\sqrt{1+9y^2}}$

B. $\sqrt{1+9y^2}$

C. $1+9y^2$

D. $\dfrac{1}{1+9y^2}$

Answer: B

Watch Video Solution

44. The differential coefficient of $a^{\log 10 \ \operatorname{cosec}^{-1}x}$, is

A. $\dfrac{a^{\log 10\left(\operatorname{cosec}^{-1}x\right)}}{\operatorname{cosec}^{-1}x}\dfrac{1}{x\sqrt{x^2-1}}\log_{10} a$

B. $-\dfrac{a^{\log 10\left(\operatorname{cosec}^{-1}x\right)}}{\operatorname{cosec}^{-1}x}.\dfrac{1}{|x|\sqrt{x^2-1}}\log_{10} a$

C. $\dfrac{-a^{\log 10\left(\operatorname{cosec}^{-1}x\right)}}{\operatorname{cosec}^{-1}x}.\dfrac{1}{|x|\sqrt{x^2-1}}\log_{a} 10$

D. $\dfrac{a^{\log 10\operatorname{cosec}^{-1}x}}{\operatorname{cosec}^{-1}x}.\dfrac{1}{x\sqrt{x^2-1}}\log_{a} 10$

Answer: B

Watch Video Solution

45. about to only mathematics

A. $\tan^{-1}\{(\log x)^n\}$

B. .

C. $1/2$

D. none of these

Answer: B

Watch Video Solution

46. If $y = \sin^2\alpha + \cos^2(\alpha+\beta) + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$, then $\frac{d^3y}{d\alpha^3}$, is

A. $\frac{\sin^3(\alpha+\beta)}{\cos\alpha}$

B. $\cos(\alpha+3\beta)$

C. 0

D. none of these

Answer: C

Watch Video Solution

47. If $y = \cos 2x \cos 3x$, then y_n is equal to

A. $6^n \cos\left(2x + \frac{n\pi}{2}\right)\cos\left(3x + \frac{n\pi}{2}\right)$

B. $6^n \sin\left(2x + \frac{n\pi}{2}\right)\cos\left(\frac{3x + n\pi}{2}\right)$

C. $\frac{1}{2}\left\{5^n \sin\left(5x + \frac{n\pi}{2}\right) + \sin\left(x + \frac{n\pi}{2}\right)\right\}$

D. none of these

Answer: D

Watch Video Solution

48. If $f(x) = (x+1)\tan^{-1}\left(e^{-2x}\right)$, then f'(0) is

A. $\frac{\pi}{2} + 1$

B. $\frac{\pi}{4} - 1$

C. $\frac{\pi}{6} + 5$

D. $\frac{\pi}{4} + 1$

Answer: B

Watch Video Solution

49. if $f(x) = 3e^{x^2}$ then $f'(x) - 2xf(x) + \frac{1}{3}f(0) - f'(0)$

A. 0

B. 1

C. $\left((7/3)e^{x^2}\right.$

D. e^{x^2}

Answer: B

Watch Video Solution

50. If $y=ce^{x/(x-a)}$, then $\frac{dy}{dx}$ equals

A. $a(x-a)^2$

B. $-\frac{ay}{(x-a)^2}$

C. $a^2(x-a)^2$

D. $a(x-a)$

Answer: B

Watch Video Solution

51. If $y=\frac{\sin^{-1}((\sin\alpha\sin x))}{1-\cos\alpha\sin x}$, then y'(0), is

A. 1

B. $2\tan\alpha$

C. $(1/2)\tan\alpha$

D. $\sin\alpha$

Answer: D

Watch Video Solution

52. If $y = \log_{x^2+4}(7x^2 - 5x + 1)$, then $\frac{dy}{dx}$ is equal to

A. $\log_e(x^2+4).\left\{\frac{14x-5}{7x^2-5x+1} - \frac{2xy}{x^2+4}\right\}$

B. $\frac{1}{\log_e(x^2+4)}\left\{\frac{14x-5}{7x^2-5x+1} - \frac{2xy}{x^2+4}\right\}$

C. $\log_e(7x^2-5x+1)\left\{\frac{2x}{x^2+4} - \frac{(14x-5)y}{7x^2-5x+1}\right\}$

D. $\frac{1}{\log_e(7x^2-5x+1)}\left\{\frac{2x}{x^2+4} - \frac{(14x-5)y}{7x^2-5x+1}\right\}$

Answer: B

Watch Video Solution

53. If a curve is given by $x = a\cos t + \frac{b}{2}\cos 2t$ and $y = a\sin t + \frac{b}{2}\sin 2t$, then the points for which $\frac{d^2y}{dx^2} = 0$, are given by

A. $\sin t = \dfrac{2a^2 + b^2}{3ab}$

B. $\cos t = -\dfrac{a^2 + 2b^2}{3ab}$

C. $\tan t = a/b$

D. none of these

Answer: B

Watch Video Solution

54. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \ldots\infty}}}}$, then $\dfrac{dy}{dx}$ is equal to

A. $\dfrac{y + x}{y^2 - 2}$

B. $\dfrac{y^3 - x}{2y^2 - 2xy - 1}$

C. $\dfrac{y^3 + x}{2y^2 - x}$

D. none of these

Answer: D

Watch Video Solution

Watch Video Solution

55. If $x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$ then $\frac{dy}{dx} =$

A. $2x\{1+\tan(\log_e x)\}+x\sec^2(\log_e x)$

B. $x\{1+\tan(\log_e x)\}+\sec^2(\log_e x)$

C. $2x\{1+\tan(\log_e x)\}+x^2\sec^2(\log_e x)$

D. $2x\{1+\tan(\log_e x)\}+\sec^2(\log_e x)$

Answer: A

Watch Video Solution

56. $\frac{d}{dx}\left[\sin^2\cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right]\right.$ is equal to

A. -1

B. $1/2$

C. $-1/2$

D. 1

Answer: B

Watch Video Solution

57. If $\sin y + e^{-x\cos y} = e$, then $\frac{dy}{dx}$ at $(1, \pi)$, is

A. $\sin y$

B. $-x\cos y$

C. e

D. $\sin y - x\cos y$

Answer: C

Watch Video Solution

58. If $\sqrt{1-x^6}+\sqrt{1-x^6}=a\left(x^3-y^3\right)$, then prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

A. $\frac{y}{x}$

B. $\frac{x^2}{y^2}$

C. $\frac{2y^2}{x^2}$

D. $\frac{y^2}{x^2}$

Answer: B

Watch Video Solution

Chapter Test

1. If $f(x)=\log_e[\log_e x]$, then what is f' (e) equal to?

A. 0

B. 1

C. $1/e$

D. $e/2$

Answer: C

Watch Video Solution

2. If $e^y + xy = e$ then the value of $\frac{d^2y}{dx^2}$ for $x = 0$ is

A. $1/e$

B. $1/e^2$

C. $1/e^3$

D. e

Answer: B

Watch Video Solution

3. If $\sqrt{x+y}+\sqrt{y-x}=5$, then $\frac{d^2y}{dx^2}=$

A. $2/c$

B. $-2/c^2$

C. $2/c^2$

D. $-2/c$

Answer: C

Watch Video Solution

4. If $ax^2+2hxy+by^2=1$, then $\frac{d^2y}{dx^2}$ is

A. $\frac{h^2+ab}{(hx+by)^3}$

B. $\frac{h^2-ab}{(hx+by)^2}$

C. $\frac{h^2+ab}{(hx+by)^3}$

D. $\frac{h^2-ab}{(hx+by)^3}$

Answer: D

Watch Video Solution

5. If $f(x) = \sin\left\{\frac{\pi}{2}[x] - x^5\right\}, 1 < x < 2$ and [.] denotes the greatest integer function, then $f'\left(5\sqrt{\frac{\pi}{2}}\right)$ is equal to

A. 0

B. $5(\pi/2)^{4/5}$

C. $-5(\pi/2)^{4/5}$

D. none of these

Answer: C

Watch Video Solution

6. f(x) is a polynomial of degree

A. n

B. $n-1$

C. $n-2$

D. none of these

Answer: B

Watch Video Solution

7. If $y=\sin(\log_e x)$, then $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is equal to

A. $\sin(\log_e x)$

B. $\cos(\log_e x)$

C. y^2

D. $-y$

Answer: D

Watch Video Solution

8. If $f(x) = (1-x)^n$, then the value of $f(0) + f'(0) + \frac{f''(0)}{2!} + \frac{f'''(0)}{3!} + \ldots\ldots \frac{f^n(0)}{n!}$.

A. $2n$

B. 0

C. 2^{n-1}

D. none of these

Answer: B

Watch Video Solution

9. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

A. $\frac{1}{(1+x)^2}$

B. $-\frac{1}{(1+x)^2}$

C. $\frac{1}{1+x^2}$

D. $\frac{1}{1-x^2}$

Answer: B

Watch Video Solution

10. If $8f(x)+6f\left(\frac{1}{x}\right)=x+5$ and $y=x^2(f(x),$ then $\frac{dy}{dx}$ at $x=-1$ is equal to 0 (b) $-\frac{1}{14}$ (c) $-\frac{1}{4}$ (d) None of these

A. 0

B. $\frac{1}{14}$

C. $-\frac{1}{14}$

D. $\frac{1}{7}$

Answer: C

Watch Video Solution

11. If $y = \sin^{-1}\left\{\frac{5x + 12\sqrt{1 - x^2}}{13}\right\}$, find $\frac{dy}{dx}$.

A. $\frac{1}{\sqrt{1 - x^2}}$

B. $\frac{-1}{\sqrt{1 - x^2}}$

C. $\frac{3}{\sqrt{1 - x^2}}$

D. $\frac{1}{\sqrt{1 + x^2}}$

Answer: A

Watch Video Solution

12. If $f(x) = \cos^{-1}\left\{\frac{1 - (\log_e x)^2}{1 + (\log_e x)^2}\right\}$, then f'(e)

A. does not exist

B. is equal to $\frac{2}{e}$

C. is equal to $\frac{1}{e}$

D. is equal to 1

Answer: C

Watch Video Solution

13. $y = \sin^{-1}[\sqrt{x - ax} - \sqrt{a - ax}]$

A. $\frac{1}{\sin\sqrt{a - ax}}$

B. $\sin\sqrt{x}\sin\sqrt{a}$

C. $\frac{1}{2\sqrt{x(1 - x)}}$

D. 0

Answer: C

Watch Video Solution

14. Let $f(x) = \left(x^3 + 2\right)^{30}$ If $f^n(x)$ is a polynomial of degree 20 where $f^n(x)$ denotes the n^{th} derivativeof $f(x)$ w.r.t x then then value of n is

A. 60

B. 40

C. 70

D. 50

Answer: C

Watch Video Solution

15. If $f(x) = \cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \sin x \sin\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 3$, then $\frac{d}{dx}(gof(x))$=

A. 1

B. 0

C. -1

D. none of these

Answer: B

Watch Video Solution

16. If $f(x) = 10\cos x + (13 + 2x)\sin x$ then $f''(x) + f(x) =$

A. $\cos x$

B. $4\cos x$

C. $\sin x$

D. $4\sin x$

Answer: B

Watch Video Solution

17. Let a function $f: R \to R$ satisfy the equation $f(x + y) = f(x) = f(Y)\,\forall x, y \varepsilon R$. If the function f(x) is continuous at x=0, then

A. $f'(x) = 0$ for all $x \in R$

B. $f'(0) < f'(1)$

C. $f'(x)$ does not exist

D. none of these

Answer: A

Watch Video Solution

18. If $f(x) = \log\left\{\frac{u(x)}{v(x)}\right\}$, $u(1) = v(1)$ and $u'(1) = v'(1) = 2$, then find the value of $f'(1)$.

A. 0

B. 1

C. -1

D. none of these

Answer: A

Watch Video Solution

19. If $f'(x) = arc\tan\left(\frac{x^x - x^{-x}}{2}\right)$, then f'(1) is equal to

A. 1

B. -1

C. $\log 2$

D. none of these

Answer: D

Watch Video Solution

20. Let $f(x) = 2^{2x-1}$ and $g(x) = -2^x + 2x\log 2$. Then the set of points satisfying $f'(x) > g'(x)$, is

A. $(0, 1)$

B. $[0, 1)$

C. $(0, \infty)$

D. $[0, \infty)$

Answer: C

Watch Video Solution

21. If $y = \log_u |\cos 4x| + |\sin x|$,where $u = \sec 2x$ find $\frac{dy}{dx}$ at $x = -\frac{\pi}{6}$

A. $\frac{-6\sqrt{3}}{\log_e 2} - \frac{\sqrt{3}}{2}$

B. $\frac{-6\sqrt{3}}{\log_e 2} + \frac{\sqrt{3}}{2}$

C. $\frac{6\sqrt{3}}{\log_e 2} + \frac{\sqrt{3}}{2}$

D. none of these

Answer: D

Watch Video Solution

22. If f(4)= 4, f'(4) =1 then $\lim_{x \to 4} 2\left(\frac{2 - \sqrt{f(x)}}{2 - \sqrt{x}}\right)$ is equal to

A. -1

B. 1

C. 2

D. -2

Answer: B

Watch Video Solution

23. if $2x^2 - 3xy + y^2 + x + 2y - 8 = 0$ then $\frac{dy}{dx}$

A. $\frac{3y - 4x - 1}{2y - 3x + 2}$

B. $\frac{3y + 4x + 1}{2y + 3x + 2}$

C. $\frac{3y - 4x + 1}{2y - 3x - 2}$

D. $\frac{3y - 4x + 1}{2y + 3x + 2}$

Answer: A

Watch Video Solution

24. If $y = \log\left\{\left(\frac{1+x}{1-x}\right)^{1/4}\right\} - \frac{1}{2}\tan^{-1}x, \quad \text{then} \quad \frac{dy}{dx} =$

A. $\frac{x}{1-x^2}$

B. $\frac{x^2}{1-x^4}$

C. $\frac{x}{1+x^4}$

D. $\frac{x}{1-x^4}$

Answer: B

Watch Video Solution

25. If $x = \cos\theta, y = \sin 5\theta, \quad \text{then} \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} =$

A. $-5y$

B. 5y

C. 25y

D. $-25y$

Answer: D

Watch Video Solution

26. If f : R - R is an even function which is twice differentiable on R and $f''(\pi) = 1$, then $f''(-\pi)$

A. -1

B. 0

C. 1

D. 2

Answer: C

Watch Video Solution

27. Observe the following statements:

I. If $f(x) = ax^{41} + bx^{-40}$, then $\dfrac{f''(x)}{f(x)} = 1640x^2$

II. $\dfrac{d}{dx}\left\{\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)\right\} = \dfrac{1}{1+x^2}$

Which of the following is correct ?

A. I is true, but II is false

B. Both I and II true

C. Neither I nor II is true

D. I is false, but II is true

Answer: A

Watch Video Solution

28. If $x = e^t \sin t, y = e^t \cos t$ then $\dfrac{d^2y}{dx^2}$ at $x = \pi$ is

A. $2e^{\pi}$

B. $\dfrac{1}{2}e^{\pi}$

C. $\frac{1}{2e^{\pi}}$

D. $\frac{2}{e^{\pi}}$

Answer: D

Watch Video Solution

29. The value of $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$, where y is given by $y = x^{\sin x} + \sqrt{x}$, is

A. $1 + \frac{1}{\sqrt{2\pi}}$

B. 1

C. $\frac{1}{2\pi}\Big)$

D. $1 - \frac{1}{\sqrt{2\pi}}$

Answer: A

Watch Video Solution

30. If $2^x + 2^y = 2^{x+y}$ then $\frac{dy}{dx}$ is equal to

A. $\frac{2^x + 2^y}{2^x - 2^y}$

B. $\frac{2^x + 2^y}{1 + 2^{x+y}}$

C. $2^{x-y}\left(\frac{2^y - 1}{1 - 2^x}\right)$

D. $\frac{2^{x+y} - 2^x}{2^y}$

Answer: C

Watch Video Solution