MATHS

BOOKS - OBJECTIVE RD SHARMA ENGLISH

MAXIMA AND MINIMA

Illustration

1. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let m(b) be the minimum value of f (x). As b varies, the range of m (b) is

A. [0,1]

B. (0, 1/2]

C. [1 /2 , 1]

D. [0, 1]

Answer: D

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2. if $f(x) = \int_0^x (t^2 + 2t + 2)$ dt where $x \in [2, 4]$ then

A. the minimum value of f(x) is $\frac{32}{3}$

B. the minimum value of f(x)Is 10

C. the maxium value of f(x) is 10

D. none of these

Answer: A

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3. The minimum value that

$f(x) = 4x^2 - 4x + 11 + \sin 3\pi x$ attains is

A. 12

B. 10

C. 8

D. none of these

Answer: D

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4. If ma and M respectively denote the minimum and maximum of $f(x) = (x - 1)^2 + 3$ for $x \in [-3, 1]$ then the ordered pair (m,M)=

A. (-3,19)

B. (3,19)

C. (-19 , 3)

D. (-19 ,-3)

Answer: B

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5. If m and M are the minimum and the maximum values of $4+\frac{1}{2}\sin^2 2x-2\cos^4 x, x \in R$ then

A. $\frac{7}{4}$

B. $\frac{15}{4}$

C. $\frac{9}{4}$

D. $\frac{1}{4}$

Answer: C

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6. Let $f(x) = |x - x_1| + |x - x_2|$ where x_1 and x_2 are disinct real numbers of points at which f(x) is mimum is

A. More than 3

B. 1

C. 2

D. 3

Answer: A

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7. The number of points in the interval $\left[-\sqrt{13}\sqrt{13}\right]$ at which f(x) $= \sin x^2 + \cos x^2$ attains its maximum value is

A. 2

B. 8

C. 0

D. 4

Answer: D

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8. Let the tangent to the graph of $y = f(x)$ at the point $x = a$ be parallel to the x-axis and let $f'(a - h) > 0$ and $f(a + h) < 0$, where h is a very small positive number.Then, the ordinate of the points is

A. a maximum

B. a maximum

C. both a maximum and a mimum

D. neither a maximum nor a minimum

Answer: A

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9. The condition

$f(x) = x^3 + px^2 + qx + r(x \in R)$ to have no extreme value is

A. $p^2 < 3q$

B. $2p^2 < q$

C. $p^2 < \frac{q}{4}$

D. $p^2 > 3q$

Answer: A

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10. In the interval $[0, 1]$, the function $x^{25}(1-x)^{75}$ takes its maximum value at the point (a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

A. 0

B. $1/4$

C. $1/2$

D. $1/3$

Answer: B

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11. The value of a so that the sum of the cubes of the roots of the equation $x^2ax+(2a-3)=0$ assumes the minimum vlaue's

A. a=1

B. a=3

C. a=0

D. non of these

Answer: B

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12. If $f(x) = 2x^3 - 21x^2 + 36x - 30$, then

A. f(x) has minimum at x=1

B. f(x) has maximum at x=6

C. f(x) has maximum at x=1

D. f(x) has no maximum of minimum

Answer: C

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13. The maximum ordinate of a point on the graph of the function f(x) = sin x(1+ cos x) is

A. $\frac{2+\sqrt{3}}{4}$

B. $\frac{3\sqrt{3}}{4}$

C. 1

D. non of these

Answer: D

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14. Find the value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assumes the least value.

A. 2

B. 0

C. 3

D. 1

Answer: D

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15. The minimum distance of a point on the curve $y = x^2 - 4$ from origin ,

A. $\frac{\sqrt{5}}{2}$

B. $\frac{\sqrt{19}}{2}$

C. $\sqrt{\frac{15}{2}}$

D. $\sqrt{\frac{19}{2}}$

Answer: C

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16. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in $sq\dot{m}$) of the flower-bed is: (1) 25 (2) 30 (3) 12.5 (4) 10

A. 12.5

B. 10

C. 25

D. 30

Answer: C

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17. f(x)= $\begin{matrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{matrix}$

A. f(x) =0 at exactly three points in $(-\pi, \pi)$

B. f(x) =0 at more than three points in $(-\pi, \pi)$

C. f(x) attains its minimum at x=0

D. f(x) attains its minimums at x=0

Answer: B::C

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18. The minmumu value of the fucntion

$f(x) = \frac{a^2}{x} + \frac{b^2}{a-x}, a > 0, b > 0$, in (0,a) is

A. a+b

B. $\frac{1}{a+b}$

C. $\frac{(a+b)^2}{a}$

D. $\frac{a+b}{a^2}$

Answer: C

19. A wire of length 2 units is cut into two parts which are bent respectively to from a square ofside c units and a circle of radius r units if the sum of the sum of the areas of the square and the circle so fromed is minimum then

A. $2x = (\pi 4)r$

B. $(4 - \pi)x = \pi r$

C. x=2r

D. 2x=r

Answer: C

20. The minimum value of a $\tan^2 x + b\cot^2 x$ equals the maximum value of a $\sin^2\theta + b\cos^2\theta$ where $a > b > 0$ when

A. a=b

B. a=2b

C. a=3b

D. a=4b

Answer: D

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21. The number of critical points of $f(x) = \dfrac{|x-1|}{x^2}$ is

A. 1

B. 2

C. 3

D. none of these

Answer: C

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22. All possible value of $f(x) = (x+1)^{\frac{1}{3}} - (x-1)^{\frac{1}{3}}$ on [0,1] is 1 (b) 2 (c) 3 (d) $\frac{1}{3}$

A. 1

B. 2

C. 3

D. $1/3$

Answer: B

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23. The difference between the greatest between the greatest and least value of the function $f(x) = \sin 2x - x \quad \text{on}[-\pi/2, \pi/6]$, is

A. $\dfrac{\sqrt{3}+\sqrt{2}}{2}$

B. $\dfrac{\sqrt{3}+\sqrt{2}}{2}+\dfrac{\pi}{6}$

C. $\dfrac{\sqrt{3}}{2}+\dfrac{\pi}{2}$

D. $\dfrac{\sqrt{3}+\sqrt{2}}{2}-\dfrac{\pi}{3}$

Answer: C

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24. Let $f(x) = \cos \pi x + 10x + 3x^2 + x^3, \ -2 \leq x \leq 3.$ The absolute minimum value of $f(x)$ is 0 (b) -15 (c) $3 - 2\pi$ none of these

A. 0

B. -15

C. $3 - 2\pi$

D. none of these

Answer: B

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Section I Solved Mcqs

1. The value of a for which the function $f(x) = a \sin x + \left(\frac{1}{3}\right) \sin 3x$ has an extremum at $x = \frac{\pi}{3}$ is (a) 1 (b) -1 (c) 0 (d) 2

A. 1

B. -1

C. 0

D. 2

Answer: D

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2. If $f(x) = a \log|x| + bx^2 + x$ has extreme values at $x = -1 \, and \, at \, x = 2,$ then find a and b .

A. a=2 , b=-1

B. $a = 2, b = -1/2$

C. $a = -2, b = 1/2$

D. none of these

Answer: B

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3. The critical points of $f(x) = \frac{|2-x|}{x^2} is / are$

A. x=0,2

B. x=2,4

C. x=2,-4

D. none of these

Answer: D

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4. The set of all values of a for which the function $f(x)=\left(a^2-3a+2\right)\left(\cos^2\frac{x}{4}-\sin^2\frac{x}{4}\right)+(a-1)x+\sin 1$ does not possess critical points is (A) $[1,\infty)$ (B) $(0,1)\cup(1,4)$ (C) $(-2,4)$ (D) $(1,3)\cup(3,5)$

A. $[1,\infty]$

B. $(0,1)\cup(1,4)$

C. (-2,4)

D. $(1, 3) \cup (3, 5)$

Answer: B

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5. The value of a for which the function $f(x) = (4a-3)(x+\log 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right)$ does not possess critical points is (a) $\left(-\infty, -\frac{4}{3}\right)$ (b) $(-\infty, -1)$ (c) $[1, \infty)$ (d) $(2, \infty)$

A. $(\infty, 4/3)$

B. $(\infty, 1)$

C. $(1, \infty)$

D. $(2, \infty)$

Answer: A::D

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6. For $a \in [\pi, 2\pi]$ and $n \in Z$ the critical points of g

$f(x) = \frac{1}{3}\sin a \tan^3 x + (\sin a - 1)\tan x + \frac{\sqrt{a-2}}{8-a}$ are

A. $x = n\pi$

B. $x = 2n\pi$

C. $x = (2n+1)\pi$

D. none of these

Answer: D

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7. Suppose the cubic $x^3 - px + q$ has three real roots where $p > 0$ and $q > 0$. Then which one of the following holds ?

A. The cubic has minimum at both $\sqrt{\frac{p}{3}}$ and -$\sqrt{\frac{p}{3}}$

B. The cubic has maximum at both $\sqrt{\frac{p}{3}}$ and -$\sqrt{\frac{p}{3}}$

C. The cubic has minimum at $\sqrt{\frac{p}{3}}$ and maximum at $-\sqrt{\frac{p}{3}}$

D. The cubic has minimum at $-\sqrt{\frac{p}{3}}$ and maximum at $\sqrt{\frac{p}{3}}$

Answer: C

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8. The critical points of the function $f(x) = (x-2)^{2/3}(2x+1)$ are

A. 1 and 2

B. 1 and $-\frac{1}{2}$

C. $-1 \text{ and } 2$

D. 1

Answer: A

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9. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is :

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{2}$

C. 2

D. $\frac{1}{2}$

Answer: B

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10. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$

A. P(-1) is the minimum and p(1)the maximum of P

B. (-1) is not minimum but p(1) the maximum of P

C. p(-1) is the minimum but P(1) is not the maximum of P

D. Niether p(-1) is the maxiumum nor P(1) is the maximum of P

Answer: B

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11. The difference between the greatest and least value of the functions, $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$ is

A. $2/3$

B. $8/7$

C. $9/4$

D. $3/8$

Answer: C

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12. A straight line through the point (h, k) where $h > 0$ and $k > 0$, makes positive intercepts on the coordinate axes. Then the minimum length of line intercepted between the coordinate axes is

A. $\left(h^{2/3} + k^{2/3}\right)^{3/2}$

B. $\left(h^{3/2} + k^{3/2}\right)^{2/3}$

C. $\left(h^{2/3} - k^{2/3}\right)^{2/3}$

D. $\left(h^{3/2} - k^{3/2}\right)^{2/3}$

Answer: A

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13. The total number of local maxima and local minima of the function

$$f(x) = \begin{cases} (2+x)^3, & -3 < x \leq -1 \\ x^{\frac{2}{3}}, & -1 < x < 2 \end{cases} is$$

A. 0

B. 1

C. 2

D. 3

Answer: C

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14. Consider the function $f:(-\infty, \infty) \rightarrow (-\infty, \infty)$ defined by $f(x) = \dfrac{x^2 - ax + 1}{x^2 + ax + 1}; 0 < a < 2$. Which of the following is true ?

A. f(x) is decreasing on (-1,1) and has local minimum at x=1

B. f(x) is increasing on (-1,1) and has local minimum at x=1

C. f(x) is increasing on(-1,1) and has neither a local maximum nor a local minimum at x=1

D. f(x) is crdeeasing on(-1,1) but has neither a local maximum nor a local minimum at x=1

Answer: A

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15. If $f(x) = \begin{cases} |x|, & \text{for} \quad 0 < |x| \leq 2 \\ 1, & f \text{ or} \quad x = 0 \end{cases}$. Then, at x = 0, f has

A. f(x) is decreasing on (-1,1) and has local minimum at x=1

B. f(x) is increasing on (-1,1) and has local minimum at x=1

C. f(x) is increasing on(-1,1) and has neither a local maximum nor a local minimum at x=1

D. f(x) is crdeeasing on(-1,1) but has neither a local maximum nor a local minimum at x=1

Answer: A

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16. If f(x) is a cubic polynomial which has local maximum at $x = -1, If f(2) = 18, f(1) = -1$ and $f'(x)$ has local minimum at x=0, then

A. the distance between (-1,2) and $(\alpha, f(\alpha)$ where $x = \alpha$ is the point of local minima is $2\sqrt{5}$

B. f(x) is increasing for `x in [1,2sqrt5] and has a local minima at x=1

C. the value of f(0) is 5

D. none of these

Answer: B

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17. If $f(x) = \begin{cases} e^x & ,0 \le x < 1 \\ 2 - e^{x-1} & ,1 < x \le 2 \\ x - e & ,2 < x \le 3 \end{cases}$ and $g(x) = \int_0^x f(t)dt$,

$x \in [1, 3]$, then

A. g(x) has a local maxima at $x = 1 + \log_e 2$ and local minima at x=e

B. f(x) has a local maxima at x=1 and local minima at x=2

C. f(x) and f(x) have same points of local maxima and local minima

D. none of these

Answer: C

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18. For the functions $f(x) = \int_0^x \frac{\sin t}{t} dt$ where $x > 0$. At $x = n\pi$ f(x) attains

A. maximum or minimum according as n is odd or even respectively .

B. minimum or maximum according as n is odd or even respectively

C. maximum at x= n π

D. minimum at x= n π

Answer: A

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19. Let

$$f(x)=\int_0^x (\sin t-\cos t)\left(e^t-2\right)(t-1)^3(t-1)^3(t-2)^5dt, 0<x\leq 4$$

Then , the number of points where f(x) assumes local maximum value , is

A. 1

B. 2

C. 3

D. none of these

Answer: C

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20. Let $f(x)$ be a function defined as follows: $f(x) = \sin(x^2 - 3x), x \leq 0; and 6x + 5x^2, x > 0$ Then at $x = 0, f(x)$ (a)has a local maximum (b)has a local minimum (c)is discontinuous (d) none of these

A. has a local maxima

B. has a local minimum

C. is discontinuous

D. none of these

Answer: B

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21. Let f(x) be a function defined by

$$f(x) = \int_1^x t(t^2 - 3t + 2)\,dt, \; x \in [1, 3]$$

Then the range of f(x), is

A. [0,2]

B. $\left[-\frac{1}{4}, 4\right]$

C. $\left[-\frac{1}{4}, 2\right]$

D. none of these

Answer: C

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22. The function $f(x) = (4\sin^2 x - 1)^n (x^2 - x + 1), n \in N,$ has a local minimum at $x = \frac{\pi}{6}$. Then n is any even number n is an odd number n is odd prime number n is any natural number

A. can be any odd natural number

B. can only be an odd prime number d

C. can be any even natural number

D. can only be a multiple of 4.

Answer: C

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23. Find the set of critical points of the function

$$f(x) = x - \log x + \int_2^x \left(\frac{1}{z} - 2 - 2\cos 4z\right) dz.$$

A. $\left\{\frac{\pi}{6} + \frac{n\pi}{2} : n = 0, 1, 2. \dots\right\}$

B. $\{n\pi : n \in N\}$

C. $\left\{n\pi + \frac{\pi}{6} : n \in N\right\} \cup \left\{\frac{\pi}{2}\right\}$

D. none of these

Answer: A

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24. If h(x)=f(x)+f(-x), " then " h(x) has got and extreme value at a point where f'(x) is

A. an even function

B. an odd function

C. zero

D. none of these

Answer: A

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25. Let $f(x) = (x-2)^2 x^n, n \in N$ Then f(x) has a minimum at

A. $x = 2 \quad \text{for all} \quad n \in N$

B. x=2 ifn is odd

C. x=0 ifn is even

D. x=0 is if n is odd

Answer: A::C

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26. The difference between the greateset ahnd least vlaue of the function $f(x) = \int_0^x \left(6t^2 - 24\right) \text{ dt on } [1, 3] \text{ dt on } [1, 3]$ is

A. 14

B. 10

C. 5

D. 4

Answer: A

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27. Set of values of b for which local extrema of the function f(x) are positive where $f(x)=\frac{2}{3}a^2x^3-\frac{5a}{2}x^2+3x+b$ and maximum occurs at $x\frac{1}{3}$ is -

A. $(-4, \infty)$

B. $(-3/8, \infty)$

C. $(-10, 3/8)$

D. non of these

Answer: B

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28. if $f(x) = \left(\frac{\sin(x+\alpha)}{\sin(x+\beta)}\right), \alpha \neq \beta$ then f(x) has

A. maximum at x=0

B. minimum at x=0

C. neither maximum nor minimum

D. non of these

Answer: C

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29. if $f(x) = \left(\frac{\sin(x+\alpha)}{\sin(x+\beta), \alpha \neq \beta}\right.$ then f(x) has

A. maximum at x=0

B. minimum at x=0

C. neith maximum nor minimum

D. none of these

Answer: B

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30. Let $f(x)=1+2x^2+2^2x^4+\ldots..+2^{10}x^{20}$. The , $f(x)$ has

A. more than one minimum

B. exactly one minimum

C. at least one maximum

D. neither a maximum nor a minimum

Answer: B

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31. The function $f(x)=\frac{x}{1+x\tan x}$

A. one point of minimum in the interval $(0, \pi/2)$

B. one point of maxmimum $(0, \pi/2)$

C. no points of maximum , no point of minimum in the interval $(0, \pi/2)$

D. two points of maxima in the interval $(0, \pi/2)$

Answer: B

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32. A polynomial function f(x) is such that f'(4)= f''(4)=0 and f(x) has minimum value 10 at x=4 .Then

A. $f''(x) = 4 + (x - 4)^4$

B. $f(x) = 10 + (x - 4)^4$

C. $f(x) - (x - 4)^4$

D. non of these

Answer: B

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33. about to only mathematics

A. 0

B. 1

C. 2

D. Infinite

Answer: B

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34. In the interval $(0, \pi/2)$ the fucntion $f(x) = \tan^n x + \cot^n$x attains

A. the minimum value which is independent of n

B. a minimum vlaue which is a fuction of n

C. the minimum vlaue which is a function of 1

D. non of these

Answer: A

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35. The fraction exceeding its pth power by the greatest number possible, where $p \geq 2$, is

A. $\left(\frac{1}{n}\right)^{\frac{1}{n-1}}$

B. $\left(\frac{1}{n}\right)^{n-1}$

C. $n^{\frac{1}{n}-1}$

D. non of these

Answer: A

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36. The greatest value of the fucntion $f(x)=\sin^{-1}x^2$ in interval $\left[-1/\sqrt{2}, 1/\sqrt{2}\right]$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $-\frac{\pi}{2}$

D. $\frac{\pi}{6}$

Answer: D

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37. The minimum value of the fuction $f(x) = 2|x-2| + 5|x-3|$ for all $x \in R, is$

A. 3

B. 2

C. 5

D. 7

Answer: B

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38. The minimum value of the fuction f(x) given by $f(x) = \frac{x^m}{m} + \frac{x^{-n}}{n}$ where $\frac{1}{m} + \frac{1}{n} = 1$ and $m > 1$ is

A. 1

B. 0

C. 2

D. non of these

Answer: A

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39. The largest term in the sequence $a_n = \frac{n^2}{n^3 + 200}$ is given by $\frac{529}{49}$ (b) $\frac{8}{89}$ $\frac{49}{543}$ (d) none of these

A. $\frac{49}{543}$

B. $\frac{8}{89}$

C. $\frac{1}{52}$

D. non of these

Answer: A

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40. Let $f(x) = ax^3 + bx^2 + cx + 1$ has exterma at $x = \alpha, \beta$ such that $\alpha\beta < 0$ and $f(\alpha)f(\beta) < 0$ f . Then the equation $f(x) = 0$ has (a)three equal real roots (b)one negative root if $f(\alpha)$ < 0 and $f(\beta)$ >0 (c)one positive root if $f(\alpha)$ < 0 and $f(\beta)$ >0 (d) none of these

A. three distinct real roots

B. one positive root if $f(\alpha) < 0$ and $f(\beta) > 0$

C. on negative root if $f(\alpha) > 0$ and $(\beta) < 0$

D. all the above

Answer: D

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41. $P = x^3 - \frac{1}{x^3}, Q = x - \frac{1}{x}$ $x \in (1, \infty)$ then minimum value of $\frac{P}{\sqrt{3}Q^2}$

A. $2\sqrt{2}$

B. $-2\sqrt{3}$

C. non-existent

D. non of these

Answer: A

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42. Let $f(x) = \cos 2\pi x + x - [x]([,]$ denote the greatest integer function). Then number of points in [0,10] at which f(x) assumes its local maximum value, is

A. 0

B. 10

C. 9

D. Infinite

Answer: B

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43. Let $f(x) = a - (x - 3)^{8/9}$ then greatest value of f(x) is

A. 3

B. a

C. no maximum vlaue

D. non of these

Answer: B

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44. A function f such that $f'(a) = f''(a) = \ldots. = f^{2n}(a) = 0$, and f has a local maximum value b at x=a ,if f (x) is

A. $(x-a)^{2n-2}+b$

B. $b-1-(x+a)^{2n+1}$

C. $b-(x-a)^{2n+2}$

D. $(x-a)^{2n+2}+b$

Answer: C

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45. Let $f(x) = \begin{cases} 3x^2-2x+10 & x<1 \\ -2 & x>1 \end{cases}$

The set of values of b for which f(x) has greatest value at x=1 is

A. $(-6, -2)$

B. (2,6)

C. $(-6, -2) \cup (2, 6)$

D. $(-6, 6)$

Answer: C

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46. The maximum value of cos $\left(\int_{2x}^{x^2} e^t \sin^2 \text{ t dt}\right)$

A. $\frac{1}{2}$

B. 0

C. 1

D. non -existent

Answer: C

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47. Let $f(x) = \begin{cases} 1 + \sin x, & x < 0 \\ x^2 - x + 1, & x \geq 0 \end{cases}$, then:

A. f has a local maximum at x=0

B. f has a local minimum at x=0

C. f is increasing in (0, 1/2)

D. f is decreasing in (0,1/2)

Answer: B

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48. Let f(x) $= x^{n+1} + ax^n$, where $a > 0$. Then, x=0 is point of

A. local minimum for any integer n

B. local minimum if n is an even integer

C. local maximum if n is an even integer

D. local minimum if n is am odd interger

Answer: C

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49. The greph of $y = x^3 + ax^2 + bx + c$ has no extemun if and only if

A. $a^2 = b$

B. $a^2 < 3b$

C. $a^2 > 2b$

D. $a^2 > 2b^2$

Answer: B

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50. If f(x) $= \int_{x}^{x^2} (t-1)dt, 1 \leq x \leq 2$ then the greatest value of ϕ (x) , is

A. 2

B. 4

C. 8

D. none of these

Answer: B

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51. If the parabola $y = ax^2 + bx + c$ has vertex at(4,2)and $a \in [1, 3]$ then the difference beteween the extreme value of abc is equal to

A. 3600

B. 144

C. 3456

D. none of these

Answer: C

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52. Let $f(x) = In(2x - x^2) + \sin\frac{\pi x}{2}$. Then

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53. Find a quadratic polynomial $\varphi(x)$ whose zeros are the maximum and minimum values of the function

$$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$$

A. $\alpha + \beta^9 = 4$

B. $\alpha^3 - \beta^7 = 26$

C. $\alpha^{2n} - \beta^{2n}$ is always and even integer for $n \in N$

D. a triangle can be costructed having its sides as α, β and $\alpha - \beta$

Answer: D

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54. Let $f(x) = \begin{cases} x^2 + 4x, -3 \leq x \leq 0 \\ -\sin x, 0 < x \leq \pi/2 \\ -\cos x - 1, \pi/2 < x \leq \pi \end{cases}$

which one of the following is not true ?

A. x=-2 is the point of global minimum

B. x=π is the point of global maximum

C. f(x) is not differentiable at $x = \frac{\pi}{2}$

D. f(x) is discontinunous at x=0

Answer: A,D

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55. If α be the number of solutions of the equation [sin x] =|x|) and β be the greatest value of the function

$f(x) = \cos\left(x^2 - \left[x^2\right]\right)$ in the interval][-1,1] the

A. $\alpha = \beta$

B. $\alpha > \beta$

C. $\alpha < \beta$

D. non of these

Answer: A

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56. Let

$f(x_1, x_2, x_3, x_4) = x_1^2 + x_2^2 + x_3^2 + x_4^2 - 2(x_1 + x_2 + x_3 + x_4) + 10$

and $x_1, x_3 \in [-1, 2]$ and $x_2, x_4 \in [1, 4]$ then the maximum value of f is

A. 24

B. 20

C. 32

D. none of these

Answer: C

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57. Let f,g and h be real-valued functions defined on the interval [0,1] by $f(x) = e^{x^2} + e^{-x^2}, g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2 e^{x^2} + e^{-x^2}$, If a, b and c denote respectively the absolute maximum of f,g and h on [0,1], then

A. $a = b$ and $c \neq b$

B. $a = c$ and $a \neq b$

C. $a \neq b$ and $c \neq b$

D. a=b=c

Answer: D

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58. Let f be a function defined on R (the set of all real numbers) such that $f'(x) = 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4$, for all $x \in R$. If g is a function defined on R with values in the interval $(0, \infty)$ such that $f(x) = \ln(g(x))$, for all $x \in R$, then the number of point is R at which g has a local maximum is ___

A. 1

B. 2

C. 3

D. 4

Answer: A

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59. Let $f:R\overrightarrow{R}$ be defined by $f(x)=\{k-2x, \quad \text{if} \quad x\leq -12x+3, fx\succ 1\}$. If f has a local minimum at $x=1$, then a possible value of k is (1) 0 (2) $-\frac{1}{2}$ (3) -1 (4) 1

A. $-1/2$

B. -1

C. 1

D. 0

Answer: B

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60. For $x\varepsilon\left(0, \frac{5\pi}{2}\right)$, definite $f(x) = \int_0^x \sqrt{t}\sin t dt$. Then f has

A. local maximum at π and 2π

B. local manimum at π and 2π

C. local minimum at π and maximum at2π

D. local maximum at π and minimum at 2π

Answer: D

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61. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6 and p(3) = 2$, then $p'(0)$ is____

A. 8

B. 9

C. 3

D. 6

Answer: B

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62. Let $IR\overrightarrow{I}R$ be defined as $f(x) = |x| + + x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is______

A. 2

B. 4

C. 5

D. 6

Answer: C

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63. If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)dt$ for all $x \in (0, \infty)$, then

A. f has a local maximum at x=2 and local minimum at x=3

B. f is dereasing on (2,3)

C. there exists $c \in (0, \infty)$ such that f $(c) = 0$

D. f is inceresing on R^+

Answer: D

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64. The funciton f(X) =2|x|+|x+2|-||x+2|-|x|| has a local mimimum or a local maximum at x=

A. -2 and $-\frac{2}{3}$

B. (b)-2 and 0`

C. $-\frac{2}{3}$ and 2

D. 2 and -2

Answer: A

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65. Let $f:[0,1] \to R$ be a function. Suppose the function f is twice differentiable, $f(0)=f(1)=0$ and satisfies $f''(x)-2f'(x)+f(x) \geq e^{x}, x \in [0,1]$ Which of the following is true for $0 < x < 1$?

A. $0 < f(x) < \infty$

B. $-\frac{1}{2} < f(x) < \frac{1}{2}$

C. $-\frac{1}{4} < f(x) < 1$

D. $-\infty < f(x) < 0$

Answer: D

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66. Let f:[0,1] $\rightarrow$ R be a function.such that $f(0) = f(1) = 0$ and $f''(x) + f(x) \geq e^x$ for all $x \in [0, 1]$.If the fucntion $f(x)e^{-x}$ assumes its minimum in the interval [0,1] at $x = \frac{1}{4}$ which of the following is true ?

A. $f(x) < 0f(x) \text{ for } \frac{1}{4} < x < \frac{3}{4}$

B. $f(x) \geq f(x)f \text{ or } 0 < x < \frac{1}{4}$

C. $f(x) < f(x) \text{ for } 0 < x < \frac{1}{4}$

D. $f(x)f \text{ or } \frac{3}{4} < x <$

Answer: C

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67. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio $8:15$ is converted into anopen rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the length of the sides of the rectangular sheet are 24 (b) 32 (c) 45 (d) 60

A. 24,45

B. 32,65

C. 24,60

D. 32,60

Answer: A

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68. Let f(X) be a polynomila of degree four having extreme values at x =1 and x=2.If $\lim_{x \to 0}\left[1+\frac{f(x)}{x^2}\right]=3$ then f(2) is equal to

A. 0

B. 4

C. -8

D. -4

Answer: A

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69. A cylindrical container is to be made from certain solid material with the following constraints: It has a fixed inner volume of Vm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2mm and is of radius equal to the outer radius of the container. If the volume the material used to make the container is minimum when the inner radius of the container is $10mm$. then the value of $\frac{V}{250\pi}$ is

A. 6

B. 8

C. 7

D. 4

Answer: D

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70. The minimum value of the function,

$f(x) = x^{3/2} + x^{-3/2} - 4\left(x + \frac{1}{x}\right)$. For all permissible real values of x is

A. -10

B. -6

C. -7

D. -8

Answer: A

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71. The least value of $\alpha \in R$ for which $4ax^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is

A. $\frac{1}{64}$

B. $\frac{1}{32}$

C. $\frac{1}{27}$

D. $\frac{1}{25}$

Answer: C

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72. The abscissae of a point, tangent at which to the curve $y = e^x \sin x, x \in [0, \pi]$ has maximum slope is

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

Answer: C

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Section Ii Assertion Reason Type

1. Statement -1 The maximum value of $f(x) = \frac{1}{3x^4 + 8x^3 - 18x^2 + 60}$ is $\frac{1}{53}$

Statement -2 : The function g(x) $= \frac{1}{f(x)}$ attains its minimum value at x=1 and x=-3

A. Statement-1 is True, Statement-2 is True,Statement -2 is a correct explanation for Statement -1

B. Statement -1 True ,Statement -2 is True ,Stament -2 is not a correct explanation for Statement -!

C. Statement -1 is True Statement -2 is False

D. Statement -1 is Flase,Statement -2 is True

Answer: A

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2. $f(x) = \begin{cases} e^x + 1, & -1 \leq x > 0 \\ e^x, & x = 0 \\ e^x - 1, & 0 < x \leq 1 \end{cases}$

Statement -1 is bounded but never attains its macimum and minimum values

Statement-2 x=0 is the point of discontinuity of f(x)

A. Statement-1 is True, Statement-2 is True,Statement -2 is a correct explanation for Statement -2

B. Statement -1 True ,Statement -2 is True ,Stament -2 is not a correct explanation for Statement -!

C. Statement -1 is True Statement -2 is False

D. Statement -1 is Flase,Statement -2 is True

Answer: A

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3. Statement-1 . The critical points of f(x)=xcosx occur in $(\pi/4, \pi/3)$

Statement-2 : The functions g(x)=xtanx increase ion $(0, \pi/2)$

A. Statement-1 is True, Statement-2 is True,Statement -2 is a correct explanation for Statement -3

B. Statement -1 True ,Statement -2 is True ,Stament -2 is not a correct explanation for Statement -!

C. Statement -1 is True Statement -2 is False

D. Statement -1 is Flase,Statement -2 is True

Answer: B

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4. Let f(x)=2sinx+tax-3x

Statement-1: f(x) does not attain extreme in $(-\pi/2, \pi/2)$

Statement-2 : f(x) is strictly increasing on $(-\pi/2, \pi/2)$

A. Statement-1 is True, Statement-2 is True,Statement -2 is a correct explanation for Statement -4

B. Statement -1 True ,Statement -2 is True ,Stament -2 is not a correct explanation for Statement -!

C. Statement -1 is True Statement -2 is False

D. Statement -1 is Flase,Statement -2 is True

Answer: A

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5. Let $\tan^{-1}\frac{1-x}{1+x}$ Stament-1: The difference of the greatest and smaallest values of $f(x)$ on $[0, 1] is f(0) - f(1) = \pi/4$

Statement-2 : $g(x) = \tan^{-1} x$ is an increasing functions on $[0, \infty]$

A. Statement-1 is True, Statement-2 is True,Statement -2 is a correct explanation for Statement -5

B. Statement -1 True ,Statement -2 is True ,Stament -2 is not a correct explanation for Statement -!

C. Statement -1 is True Statement -2 is False

D. Statement -1 is Flase,Statement -2 is True

Answer: A

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6. Let $f: R \rightarrow R$ be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$. Statement-1: $f(c) = \frac{1}{3}$, for some $c \in R$. Statement-2: $0 < f(x) \leq \frac{1}{2\sqrt{2}}$, for all $x \in R$. (1) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation for Statement-1 (2) Statement-1 is true, Statement-2 is false (3) Statement-1 is false, Statement-2 is true (4) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1

A. Statement-1 is True, Statement-2 is True,Statement -2 is a correct explanation for Statement -6

B. Statement -1 True ,Statement -2 is True ,Stament -2 is not a correct explanation for Statement -!

C. Statement -1 is True Statement -2 is False

D. Statement -1 is Flase,Statement -2 is True

Answer: A

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7. Let be a function defined by $f(x) = \begin{cases} \frac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

Statement-1: x=0 is a point on minima of f

Statement-2: f'(0)=0

A. Statement-1 is True, Statement-2 is True,Statement -2 is a correct explanation for Statement -7

B. Statement -1 True ,Statement -2 is True ,Stament -2 is not a correct explanation for Statement -!

C. Statement -1 is True Statement -2 is False

D. Statement -1 is Flase,Statement -2 is True

Answer: B

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Exercise

1. For the curve $y = xe^{x}$, the point

A. x=-1 is a point of minimum

B. x=0 is a point of minimum

C. x=-1 is a point of maximum

D. x=0 is a point of maximum

Answer: A

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2. The greatest value of the function $f(x) = \dfrac{\sin 2x}{\sin\left(x + \frac{\pi}{4}\right)}$ on the interval $\left(0, \dfrac{\pi}{2}\right) is$

A. $1/\sqrt{2}$

B. $\sqrt{2}$

C. 1

D. $-\sqrt{2}$

Answer: B

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3. Let $P(x) = a_0 + a_1x^2 + a_2x^4 + \, + a_nx^{2n}$ be a polynomial in a real variable x with 0 $<\ a_0\ <\ a_1\ <\ a_2 \langle\ a_n$. The function $P(x)$ has a. neither a maximum nor a minimum b. only one maximum c. only one minimum d. only one maximum and only one minimum e. none of these

A. niether a maximum nor a minimum

B. only one maximum

C. only one minimum

D. none of these

Answer: C

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4. A differentiable function $f(x)$ has a relative minimum at $x=0.$ Then the function $f=f(x)+ax+b$ has a relative minimum at $x=0$ for (a)all a and all$b$ (b) all b if $a=0$ (c)all $b>0$ (d) all $a>0$

A. all a and all b

B. all b if a=0

C. all b gt 0

D. all a gt 0

Answer: B

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5. Investigate for the maxima and minima of the function

$$f(x) = \int_1^x \left[2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\right] dt$$

A. 1

B. 2

C. 3

D. 4

Answer: A

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6. If the function $f(x)=x^3+3(a-7)x^2+3(a^2-9)x-1$ has a positive point Maximum , then

A. $a \in (3, \infty) \cup (-\infty, -3)$

B. $a \in (-\infty, -3) \cup (3, 29/7)$

C. $(-\infty, 7)$

D. $(-\infty, 29)$

Answer: B

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7. Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$.

A. e

B. e^e

C. $e^{1/e}$

D. $(1/e)^{1/e}$

Answer: C

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8. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1,$ where $a > 0,$ attains its maximum and minimum at p and $q,$ respectively, such that $p^2 = q,$ then a equal to (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 3

A. 0

B. 1

C. 2

D. none of these

Answer: C

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9. The maximum distance of the point (k,0) from the curve $2x^2 + y^2 - 2x = 0$ is equal to

A. $\sqrt{1 - 2a + a^2}$

B. $\sqrt{1 + 2a + 2a^2}$

C. $\sqrt{1 + 2a - a^2}$

D. $\sqrt{1 - 2a + 2a^2}$

Answer: D

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10. A cubic function $f(x)$ vanishes at $x = -2$ and has relative minimum/maximum at $x = -1 and x = \frac{1}{3}$ if $\int_{-1}^{1} f(x) dx = \frac{14}{3}$. Find the cubic function $f(x)$.

A. $x^3 + x^2 - x$

B. $x^3 + x^2 - x + 1$

C. $x^3 + x^2 - x + 2$

D. $x^3 + x^2 - x - 2$

Answer: C

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11. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a . Show that the area of the triangle is maximum when $\theta = \frac{\pi}{6}$

.

A. $\pi / 6$

B. $\pi / 4$

C. $\pi / 3$

D. $\pi / 2$

Answer: A

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12. Find minimum value of $px + qy$ where $p > 0, q > 0, x > 0, y > 0$ when $xy = r,^2$ without using derivatives.

A. $2r\sqrt{pq}$

B. $2pq\sqrt{r}$

C. $-2r\sqrt{pq}$

D. none of these

Answer: A

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13. The maximum slope of curve y $= -x^3 + 3x^2 + 9x - 27$ is

A. 0

B. 12

C. 16

D. 32

Answer: B

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14. If $\frac{x+c}{1+x^2}$ where c is a constant , then when y is stationary , xy is equal to

A. $1/2$

B. $3/4$

C. $5/8$

D. 1

Answer: A

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15. N Characters of information are held on magnetic tape, in batches of x characters each, the batch processing time is $\alpha + \beta x^2$seconds, α and β are constants. The optical value of x for fast processing is

A. α / β

B. β / α

C. $\sqrt{\alpha / \beta}$

D. $\sqrt{\beta / \alpha}$

Answer: C

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16. Statement I If $A > 0, B > 0$ and $A + B = \frac{\pi}{3}$, then the maximum value of tan A tan B is $\frac{1}{3}$.

Statement II If $a_1 + a_2 + a_3 + \ldots + a_n = k$(constant), then the value $a_1 a_2 a_3 \ldots a_n$ is greatest when

$a_1 = a_2 = a_3 = \ldots + a_n$

A. $1/\sqrt{3}$

B. $1/3$

C. 3

D. $\sqrt{3}$

Answer: B

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17. The largest value of $2x^3 - 3x^2 - 12x + 5$ for $-2 \leq x \leq 2$ occurs when

A. -2

B. -1

C. 2

D. 4

Answer: D

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18. The first and second order derivatives of a function f(x) exit at all point in (a,b) with f'(c) =0 , where $a < c < b$, of c and $f'(x) > 0$ for all points on the immediate right of c, and $f'(x) < 0$ for all points on the immediate left of c then at x=c , , f(x) has a

A. local maximum

B. local minimum

C. point of inflexion

D. none of these

Answer: B

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19. The minimum value of 2^{x^2-3} ^ (3) +27 is 2^{27} (b) 2 (c) 1 (d) none of these

A. 2^{27}

B. 2

C. 1

D. 4

Answer: C

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20. Let f(x) =cosx sin2x. Then , min $(f(x): -\pi \leq x \leq \pi)$ is

A. $-9/7$

B. $9/7$

C. $-1/9$

D. $-2/9$

Answer: A

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21. If $f(x) = \sin^6 x + \cos^6 x$, then which one of the following is false

A. $f(x) \leq 1$

B. $f(x) \leq 2$

C. $f(x) > \frac{1}{4}$

D. $f(x) \leq \frac{1}{8}$

Answer: D

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22. The value of a for which the function $f(x) = a\sin x + \left(\frac{1}{3}\right)\sin 3x$ has an extremum at $x = \frac{\pi}{3}$ is (a) 1 (b) -1 (c) 0 (d) 2

A. 3

B. $1/3$

C. 2

D. $1/2$

Answer: C

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23. If $ax + \frac{b}{x} \geq c$ for all positive x, where $a, b, c > 0$, then-

A. $ab \geq \frac{c^2}{4}$

B. $ab < \frac{c^2}{4}$

C. $bc \geq \frac{a^2}{41}$

D. $ac \geq \frac{b^2}{4}$

Answer: A

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24. The greatest value of $f(x) = \cos\left(xe^{[x]} + 7x^2 - 3x\right), x \in [-1, \infty],$ is (where [.] represents the greatest integer function). -1 (b) 1 (c) 0 (d) none of these

A. -1

B. 1

C. 0

D. cos1

Answer: B

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25. The points of extremum of $\phi(x) = \int_1^x e^{-t^{2/2}}(1-t^2)\,dt$ are

A. x=0,1

B. x=1,-1

C. x=$1/2$

D. $x = -1/2$

Answer: B

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26. Let $f(x)=\int_0^x \frac{\cos t}{t}dt$ Then at $x=(2n+1)\frac{\pi}{2}$ f(x) has

A. maxima when n=-2,-4,-6,.. and minima when n=-1,3,-5,..

B. maxima when n=-1,-3,-5,... and minima when n=1,3,5,...

C. minima when n=0,2,4,... and maxima when n=1,3,5,...

D. none of these

Answer: B

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27. It is given that at $x=1$, the function x^4-62x^2+ax+9 attains its maximum value on the interval $[0,\ 2]$. Find the value of a .

A. 120

B. -120

C. 52

D. 60

Answer: A

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28. The minimum value of $\left(1+\frac{1}{\sin^n \alpha}\right)\left(1+\frac{1}{\cos^n \alpha}\right)$ is

A. 1

B. 2

C. $\left(1+2^{n/2}\right)^2$

D. 4

Answer: C

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29. The minimum value of (x-a)(x-b) is

A. ab

B. $\frac{(a-b)^2}{4}$

C. 0

D. $\frac{-(a-b)^2}{4}$

Answer: D

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30. The altitude of a right circular cone of minimum volume circumscired about a sphere of radius r is

A. 2r

B. 3r

C. 5r

D. $\frac{3}{2}r$

Answer: D

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31. If $(x-a)^{2m}(x-b)^{2n+1}$, where m and n are positive integers and $a > b$, is the derivative of a function f then-

A. x=a is a point of minimum

B. x=b is a point of maximum

C. x=a is not a point of maximum or minimum

D. none of these

Answer: C

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32. If $(x-a)^{2m}(x-b)^{2n+1}$, where m and n are positive integers and $a > b$, is the derivative of a function f then-

A. x=b is point of minimum

B. x=b is a point of maximum

C. x=b is a point of inflextion

D. none of these

Answer: A

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33. In a $\triangle ABC, \angle B = 90^0$ and $b + a = 4$. The area of the triangle is maximum when $\angle C$ is (a)$\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c)$\frac{\pi}{3}$ (d) none of these

A. $\pi/4$

B. $\pi/6$

C. $\pi/3$

D. none of these

Answer: C

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34. The function f(x) given by

$$f(x)=\begin{vmatrix} x-1, & x+1, & 2x+1 \\ x+1, & x+3, & 2x+3 \\ 2x+1, & 2x-1, & 4x+1 \end{vmatrix}$$ has

A. one point of maximum and one point of minimum

B. one point of maximum only

C. one point of maximum only

D. none of these

Answer: D

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35. Maximum area of a reactangle which can be inscribed in a circle of a given radius R is

A. πr^2

B. r^2

C. $\pi r^2 / 4$

D. $2r^2$

Answer: D

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36. If $f(x) = \begin{cases} 3x^2 + 12x - 1, -1 \leq x \leq 2 \\ 37 - x, 2 < x \leq 3 \end{cases}$ then

A. f(x) is increasing in [-1,2]

B. f(x) is continuous in [-1,3]

C. f(x) is maximum at x=2

D. all of the above

Answer: D

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37. The perimeter of a sector is p. The area of the sector is maximum when its radius is

A. $p/2$

B. $1/\sqrt{p}$

C. $\sqrt{p}$

D. $p/4$

Answer: D

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38. If $a^2x^4 - b^2y^4 = c^6 (a, b, x, y, c > 0)$ then the maximum value of xy is

A. $\frac{c^3}{2ab}$

B. $\frac{c^3}{\sqrt{2ab}}$

C. $\frac{c^3}{ab}$

D. $\frac{c^3}{\sqrt{ab}}$

Answer: B

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39. The function $\int_{-1}^{x} t(e^t - 1)(t - 1)(t - 2)^3 (t - 3)^5 dt$ has local minimum at x=

A. 0

B. 1

C. 2

D. 3

Answer: D

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40. Let f(x) be a function such that $f'(a) \neq 0$. Then , at x=a, f(x)

A. cannot have a maximum

B. cannot have a minimum

C. must have niether a maximum nor a minimum

D. none of these

Answer: C

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41. Let a,b,c be positive real parameter and $ax^2 + \frac{b}{x^2} \geq c, \forall x \varepsilon R$ then (A) $4ab \geq c^2$ (B) $4c \geq b^2$ (C) $4bc \geq c^2$ (D) $4ac < b^2$

A. $4ab \geq c^2$

B. $4ac \geq b^2$

C. $4bc \geq a^2$

D. $4ac < b^2$

Answer: A

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42. If $xy = a^2$ and $S = b^2x + c^2y$ where a, b and c are constants then the minimum value of S is

A. abc

B. $\sqrt{a}bc$

C. 2abc

D. none of these

Answer: C

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43. Let $f(x) = e^x \sin x$, slope of the curve y=f(x) is maximum at x=a if 'a' equals

A. 0

B. $\pi / 4$

C. $\pi / 2$

D. none of these

Answer: C

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44. If $a > b > 0$ then maximum value of

$$\frac{ab(a^2 - b^2)\sin x \cos x}{a^2 \sin^2 x + b^2 \cos^2 x}, x \in (0, \pi/2)$$ is

A. $a^2 - b^2$

B. $\frac{a^2 - b^2}{2}$

C. $\frac{a^2 + b^2}{2}$

D. none of these

Answer: B

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45. The maximum value of the function $f(x) = \frac{(1+x)^{0.3}}{1+x^{0.3}}$ in [0,1] is

A. 1

B. $2^{0.7}$

C. $2^{-0.7}$

D. $2^{0.3}$

Answer: A

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46. If $g(x) = \max\left(y^2 - xy\right)(0 \le y \le 1)$, then the minimum value of g(x) (for real x) is

A. $\frac{1}{4}$

B. $3 - \sqrt{8}$

C. $3 + \sqrt{8}$

D. $\frac{1}{2}$

Answer: B

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47. If a,b,c are positive constants such that $a > b$ then the maximum value of r , given by $\frac{c^4}{r^2} = \frac{a^2}{\sin^2\theta} + \frac{b^2}{\cos^2\theta}$, must be

A. $\frac{c^2}{a-b}$

B. $\frac{c^2}{a+b}$

C. $\frac{c^2}{\sin^2\theta}$

D. $\frac{c^2}{\sqrt{ab}}$

Answer: B

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Chapter Test

1. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$ is

A. e

B. $\sqrt[e]{e}$

C. `

D. $e^{1/e}$

Answer: B

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2. If $ax^2 + \frac{b}{x} \geq c$ for all positive x where $a > 0$ and $b > 0$, show that $27ab^2 \geq 4c^3$.

A. $27ab^2 \geq 4c^3$

B. $27ab^2 < 4c^3$

C. $4ab^2 \geq 27c^3$

D. none of these

Answer: A

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3. The greatest value of the funxtion $f(x) = xe^{-x}$ in $[0, \infty]$ is

A. 0

B. $1/e$

C. $-e$

D. e

Answer: B

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4. Let $f(x) = x^3 - 6x^2 + 12x - 3$. Then at x=2 f(x) has

A. a maximum

B. a minimum

C. both a maximum and a minimum

D. neither a maximum nor a minimum

Answer: D

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5. In the right triangle $BAC, \angle A = \frac{\pi}{2}$ and $a + b = 8$. The area of the triangle is maximum when $\angle C$, is

A. $\pi / 3$

B. $\pi / 4$

C. $\pi / 6$

D. $\pi / 2$

Answer: A

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6. The range of values of a for which the function

$$f(x) = (a^2 - 7a + 12)\cos x + 2(a - 4)x + 3e^5$$

does not process critical points is

A. (1,5)

B. $(1, 4) \cup (4, 5)$

C. (1,4)

D. none of these

Answer: B

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7. If the function

$$f(x) = (2a - 3)(x + 2\sin 3) + (a - 1)(\sin^4 x + \cos^4 x) + \log 2$$

does not process critical poits , then

A. $a \in (-\infty, 4/3) \cup (2, \infty)$

B. $a \in (4/3, 2)$

C. $a \in (4/3, \infty)$

D. $a \in (2, \infty)$

Answer: A

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8. The function y=$\frac{ax+b}{x-1}(x-4)$ has turning point at P(2,-1) Then find the values of a and b.

A. a=0, b=1

B. a=0,b=-1

C. a=1, b=0

D. a=-1,b=0

Answer: C

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9. Find the least value of the expressions $2\log_{10} x - \log_x 0.01$, where $x > 0, x \neq 1$.

A. 1

B. -1

C. 2

D. $1/2$

Answer: D

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10. The maximum value of the function f(x) given by

$f(x) = x(x-1)^2, 0 < x < 2$, is

A. 0

B. $4/27$

C. -4

D. $1/4$

Answer: B

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11. The least value of a for which the equation $\frac{4}{\sin x} + \frac{1}{1-\sin x} = a$ has at least one solution in the interval $\left(0, \frac{\pi}{2}\right)$ 9 (b) 4 (c) 8 (d) 1

A. `4

B. 1

C. 3

D. 9

Answer: C

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12. The minimum value of $f(x)=e^{(x^4-x^3+x^2)}$ is

A. e

B. e^2

C. 1

D. e^{-1}

Answer: C

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13. Let $f(x) = \frac{a}{x} + x^2$. If it has a maximum at $x = -3$, then find the value of a.

A. -1

B. 16

C. 1

D. 4

Answer: D

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14. Find the maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$.

A. 4

B. $3 + \sqrt{2}$

C. $4 + \sqrt{2}$

D. $2+\sqrt{2}$

Answer: C

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15. The least value of the f(x) given by

$f(x) = \tan^{-1} x - \frac{1}{2}\log_e x$ in the interval $[1/\sqrt{3}, \sqrt{3}]$, is

A. $\frac{\pi}{6} + \frac{1}{4}\log_e 3$

B. $\frac{\pi}{3} - \frac{1}{4}\log_e 3$

C. $\frac{\pi}{6} - \frac{1}{4}\log_e 3$

D. $\frac{\pi}{3} + \frac{1}{4}\log_e 3$

Answer: B

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16. The slope of the tangent to the curve $y = e^x \cos x$ is minimum at $x = a, 0 \leq a \leq 2\pi$, then the value of a is

A. 0

B. π

C. 2π

D. $3\pi/2$

Answer: B

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17. The value of a for which the function

$f(x) = \begin{cases} \tan^{-1} a - 3x^2 & , \quad 0 < x < 1 \\ -6x & , \quad x \geq 1 \end{cases}$ has a maximum at x=1 , is

A. 0

B. 1

C. 2

D. none of these

Answer: D

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18. The minimum value of $27^{\cos 3x}81^{\sin 3x}$ is

A. $1/243$

B. -5

C. $1/5$

D. $1/3$

Answer: A

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19. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$. For every real number x, then the minimum value of $f\cdot$ (a) does not exist because f is unbounded (b) is not attained even through f is bounded (c) is equal to 1 (d) is equal to -1

A. does not exits because f- is unbounded

B. is not attained even though f is bounded

C. is equal to 1

D. is equal to -1

Answer: D

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20. $f(x) = |x| + |x - 1| + |x - 2|$, then which one of the following is not correct ?

A. f(x) has a minimum at x=1

B. f(x) has a maximum at x=0

C. f(x) has niether a maximum nor a minimum at x=0

D. f(x) has niether a maximum nor a minimum x=2

Answer: B

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21. Write the maximum value of $f(x) = \frac{\log x}{x}$, if it exists.

A. $1/e$

B. e

C. $2/e$

D. 1

Answer: A

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22. The function $f(x) = 2x^3 - 3x^2 - 12x - 4$ has

A. no maxima and minima

B. one maximum and one minimum

C. two maxima

D. two minima

Answer: B

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23. In (-4,4) the function $f(x) = \int_{-10}^{x} (t^2 - 4)e^{-4t} dt$, has

A. no extrema

B. one extremum

C. two extrema

D. four ectrema

Answer: C

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24. On [1,e] the greatest value of $x^2 \log_e x$, is

A. e^2

B. $\frac{1}{2}\log\left(\frac{1}{\sqrt{e}}\right)$

C. $e^2 \log \sqrt{e}$

D. e

Answer: A

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25. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$. For every real number x, then the minimum value of $f \cdot$ (a) does not exist because f is unbounded (b) is not attained even through f is bounded (c) is equal to 1 (d) is equal to -1

A. does not exits because f is unbounded

B. is not attained even though f is bounded

C. is equal to 1

D. is equal to -1

Answer: D

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26. If $f: R \rightarrow R$ be defined by f(x) =2x + cosx, then f

A. has a minimum at $x = \pi$

B. has a maximum at x=0

C. is decreasing on R

D. in increasing function on R

Answer: D

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27. The maximum distance from origin of a point on the curve $x = a \sin t - b \sin\left(\frac{at}{b}\right), y = a \cos t - b \cos\left(\frac{at}{b}\right)$, borth a,b>0 is

A. a-b

B. a+b

C. $\sqrt{a^2 + b^2}$

D. $\sqrt{a^2 - b^2}$

Answer: B

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28. The maximum value of $x^{\frac{1}{x}}, x > 0$ is $e^{\frac{1}{e}}$ (b) $\left(\frac{1}{e}\right)^{e}$ (c) 1 (d) none of these

A. $1/e$

B. e

C. $e^{1/e}$

D. $1/e$

Answer: C

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29. The perimeter of a sector is a constant. If its area is to be maximum, the sectorial angle is

A. $\frac{\pi^{c}}{6}$

B. $\frac{\pi^c}{4}$

C. 4^c

D. 2^c

Answer: D

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