MATHS

BOOKS - IA MARON MATHS (HINGLISH)

APPLICATION OF DIFFERENTIAL CALCULUS TO INVESTIGATION OF FUNCTIONS

Basic Theorems On Differentiable Functions

1. Does the function $f(x) = 3x^2 - 1$ satisfy the condition of the Fermat theorem in the interval $[1 \quad 2]$?

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2. Do the following functions satisfy the conditions of the Rolle theorem ?

(a) $f(x) = 1 - \sqrt[3]{x^2} \text{in}[-1, 1]$,

(b) $f(x) \text{in} \sin x \text{ in } [\pi/6, 5\pi/6]$,

(c) $f(x) = 1 - |x| \text{ in}[-1, 1]$.

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3. Prove that the equation

$3x^5 + 15x - 8 = 0$

has only one real root .

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4. Does the function $f(x) = 3x^2 - 5$ satisfy the conditions of the Lagrange theorem in the interval $[-2, 0]$? If it does then find

the point ξ in the Lagrange formula $f(b) - f(a) = f'(\xi)(b - a)$.

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5. Apply the Lagrange formula to the function f(x) = In x in the interval [1,e] and find the corresponding of ξ.

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6. On the curve $y = x^3$ find the point at which the tangent line parallel to the chord through the points $A(-1, 1)$ and $B(2, 8)$.

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7. Prove the inequality

arc $\tan x_2 - arc\tan x_1 < x_2 - x_1$ where $x_2 > x_1$

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8. Using the Rolle theorem prove that the derivative f.(x) of the function.

$f(x) = \begin{cases} x \sin \pi x & \text{at} \quad x > 0 \\ 0 & \text{at} \quad x = 0 \end{cases}$ vanishes on an infinite set of points of the interval (0,1).

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9. Check whether the Lagrange formula is applicable to following functions :

$f(x) = x^2 \text{on} [3, 4]$

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10. Check whether the Lagrange formula is applicable to following functions :

$f(x) = \ln x \text{ on } [1, 3]$

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11. Check whether the Lagrange formula is applicable to following functions

$f(x) = 4x^3 - 5x^2 + x - 2 \text{ on } [0, 1]$

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12. Check whether the Lagrange formula is applicable to following functions :

$f(x) = x^2 \text{on} [3, 4]$

$f(x) = \sqrt[5]{x^4(x-1)}$ on $[-1/2, 1/2]$

If it is , find the values of ξ appearing in this formula .

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13. Using the Lagrange theorem estimate has value in $(1+e)$.

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14. Using the Lagrange formula prove the inequality

$$\frac{x}{1+x} < \text{in}(1+x) < x \quad \text{at} \quad x > 0$$

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Evaluation Of Indeterminate Forms

1. Applying the L.Hospital rule , find the limits of the following functions :

$$\lim_{x \to 0} \frac{e^{ax} - e^{-2ax}}{\ln(1+x)}$$

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2. Applying the L.Hospital rule , find the limits of the following functions :

$$\lim_{x \to -1} \frac{\sqrt[3]{1+2x+1}}{\sqrt{2+x+x}}$$

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3. Applying the L.Hospital rule , find the limits of the following functions :

$$\lim_{x \to 0} \frac{e^{x} - e^{-x} - 2x}{x - \sin x}$$

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4. Applying the L.Hospital rule , find the limits of the following functions :

$$\lim_{x \to 0} \frac{\ln(1+x^2)}{\cos 3x - e^{-x}}$$

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5. Applying the L.Hospital rule , find the limits of the following functions :

$$\lim_{x \to \infty} \frac{e^{1/x^2} - 1}{2 \ \arctan x^2 - \pi}$$

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6. Ascertain the existence of the following limits :

$$\lim_{x \to 0} \frac{x^2 \sin(1/x)}{\sin x}$$

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7. Ascertain the existence of the following limits :

$$\lim_{x \to \infty} \frac{2 + 2x + \sin 2x}{(2x + \sin 2x)e^{\sin x}}$$

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8. Ascertain the existence of the following limits :

$$\lim_{x \to \pi/2} \frac{\tan x}{\sec x}$$

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9. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 2} \frac{In\left(x^2 - 3\right)}{x^2 + 3x - 10}$$

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10. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 1} \frac{a^{\ln x} - x}{\ln x}$$

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11. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 0} \frac{\tan x - x}{x - \sin x},$$

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12. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 1} \frac{1 - 4\sin^2(\pi x / 6)}{1 - x^2}$$

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13. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to a} \text{arc}\sin\frac{x-a}{a}\cot(x-a)$$

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14. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 0} \left(\frac{1}{x}\right)^{\tan x}$$

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15. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to \infty} \left(a^{1/x} - 1\right)x(a > 0),$$

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16. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 0} (\cos mx)^{n/x^2}$$

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17. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{x}{\ln x} \right)$$

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18. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to \infty} x^{1/In(e^x - 1)}$$

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19. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x \right)$$

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20. Using the L .Hospital rule find limits of the following functions

:

$$\lim_{x \to 0} \left(\frac{5}{2 + \sqrt{9 + x}} \right)^{1/\sin x}$$

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21. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to \infty} \frac{e^{1/x^2 - 1}}{2 \text{arc} \tan x^2 - \pi}$$

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Taylor S Formula Application To Approximate Calculations

1. Expand the polynomial $P(x) = x^5 - 2x^4 + x^3 - x^2 + 2x - 1$ in powers of the binomial x - 1 using Taylor formula .

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2. Compute the approximate value of $\sqrt[4]{83}$ accurate to six decimal places .

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3. Prove the inequalities

$x - x^2 < In(1 + x) \text{at} x > 0$

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4. Prove the inequalities

$\tan x > x + x^3/3 \quad \text{at} 0 < x < \pi/2$

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5. Prove the inequalities

$1 + \frac{1}{2}x - \frac{x^2}{8} < \sqrt{1+x} < 1 + \frac{1}{2}x \text{ at } 0 < x < \infty$

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Application Of Taylor S Formula To Evaluation Of Limits

1. Applying the Taylor formula with the remainder in Peano.s form compute the limits :

$\lim_{x \to 0} \frac{\sqrt[3]{1+3x} - \sqrt{1+2x}}{x^2}$

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2. Applying the Taylor formula with the remainder in Peano.s form compute the limits :

$$\lim_{x \to 0} \frac{\cos x - e^{-x^2/2}}{x^4}$$

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3. Applying the Taylor formula with the remainder in Peano.s form compute the limits :

$$\lim_{x \to 0} \frac{e^x \sin x - x(1+x)}{x^3}$$

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4. Applying the Taylor formula with the remainder in Peano.s form compute the limits :

$$\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$$

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5. Expand the following function in positive integral powers of the variable x up to the terms of the inficated order , inclusive

$f(x) = e^{2x - x^2}$ up to the terms containing x^5

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6. Expand the following function in positive integral powers of the variable x up to the terms of the inficated order , inclusive

In cos x up to the terms containing x^6

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Testing A Function For Monotonicity

1. Find the intervals of monotonicity of the following

$f(x) = 2x^2 - \ln|x|$

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2. Determine the intervals of monotonicity for the following functions :

$f(x) = 2x^3 - 9x^2 - 24x + 7,$

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3. Determine the intervals of monotonicity for the following functions :

$f(x) = x^2 e^{-x}$

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4. Determine the intervals of monotonicity for the following functions :

$f(x) = \ln|x|,$

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5. Determine the intervals of monotonicity for the following functions :

$f(x) = 4x^3 - 21x^2 + 18x + 20,$

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6. Determine the intervals of monotonicity for the following functions :

$f(x) = e^x + 5x$

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7. Find the intervals of decreases and increases for the following functions ,

$f(x) = \sin x + \cos x \quad \text{on}[0, 2\pi]$

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8. Prove that the function $y = x^5 + 2x^3 + x$ increases everywhere and the function $y = 1 - x^3$ decreases everywhere .

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9. Determine the intervals of increase and decrease for the following functions :

$f(x) = x^3 + 2x - 5$

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10. Determine the intervals of increase and decrease for the following functions :

$f(x) = \ln(1 - x^2)$

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11. Determine the intervals of increase and decrease for the following functions :

$f(x) = \cos x - x$

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12. Determine the intervals of increase and decrease for the following functions :

$f(x) = \frac{1}{3}x^3 - \frac{1}{x}$

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13. Determine the intervals of increase and decrease for the following functions :

$f(x) = \frac{2x}{\ln x}$

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14. Determine the intervals of increase and decrease for the following functions :

$f(x) = \frac{2x}{1 + x^2}$

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15. Prove the following inequalities :

$\tan x > x + x^3/3 \text{if} \quad (0 < x < \pi/2)$

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16. Prove the following inequalities :

$e^x \geq 1 + x$ for all the values of x ,

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17. Prove the following inequalities :

$e^x > e^{-x} atx > 1$.

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18. At what values of the coefficient a does the function $f(x) = a^3 - ax$ increase along the entire number scale ?

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19. Find the values of b for which the function $f(x) = \sin x - bx + c$ is a decreasing function on R .

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Maxima And Minima Of Function

1. Using the first derivative , find the extreme of the following functions :

$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 12,$

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2. Using the first derivative , find the extreme of the following functions :

$f(x) = x(x+1)^3(x-3)^2,$

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3. Using the first derivative , find the extreme of the following functions :

$f(x) = \frac{x^2 - 3x + 2}{x^2 + 2x + 1}$

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4. Using the first derivative , find the extreme of the following functions :

$f(x) = 3\sqrt[3]{x^2} - x^2,$

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5. Using the second derivative , find the out the character of the extrema of the following functions :

$f(x) = 2x^3 - 15x^2 - 84x + 8$

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6. Investigate the following functions for an extremum at the point x = 0 :

$y = \cos x - 1 + \frac{x^2}{2}$

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7. Investigate the following functions for extrema :

$f(x) = x^4 e^{-x^2}$

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8. Investigate the following functions for extrema :

$f(x) = \sin 3x - 3\sin x$

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9. Find the maxima and minima of the following functions :

$f(x) = x^3 e^{-x}$

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10. Find the maxima and minima of the following functions :

$$f(x) = \frac{4x}{x^2 + 4}$$

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11. Find the maxima and minima of the following functions :

$$f(x) = \sqrt[3]{2x^3 + 3x^2 - 36x}$$

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12. Find the maxima and minima of the following functions :

$f(x) = x^2 \ln x$,

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13. Investigate the following functions for an extremum at the point x = 0:

$$f(x) = \begin{cases} e^{1/x}, \text{ if } x \neq 0 \\ 0, \quad \text{if } x = 0 \end{cases}$$

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Finding The Greatest And The Least Values Of A Function

1. Find the greatest and the least values of the following functions on the indicated intervals :

$$f(x) = 2x^3 - 3x^2 - 12x + 1 \text{ on } [-2, 5/2],$$

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2. Find the greatest and the least values of the following functions on the indicated intervals :

$f(x) = x^3 \ \ln \ \mathrm{x} \ \mathrm{on}[1, e]$

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3. Find the greatest and the least values of the following functions on the indicated intervals :

$f(x) = \sqrt{(1-x)^2(1+2x^2)} \ \ \mathrm{on}[-1, 1]$

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4. Find the greatest and the least values of the following functions on the indicated intervals ,

$y = \ \ \sin \mathrm{x} \ 2\mathrm{x} \ \mathrm{on} \ \ (-\infty, \infty),$

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5. Find the greatest and the least values of the following functions on the indicated intervals ,

$y = x + \sqrt{x} \quad \text{on} \quad [0, 4]$

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6. Find the greatest and the least values of the following functions on the indicated intervals :

$f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2 + 2 \quad \text{on} \quad [-2, 4]$

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7. Find the greatest and the least values of the following functions on the indicated intervals :

$f(x) = \sqrt{4 - x^2}$ on $[-2, 2]$

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8. Find the greatest and the least values of the following functions on the indicated intervals :

$f(x) = \arctan\ x - \frac{1}{2}$ in x on $\left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$

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9. Find the greatest and the least values of the following functions on the indicated intervals :

$f(x) = 2\ \sin x + \sin 2x$ on $\left[0, \frac{3}{2}\pi\right]$

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10. Find the greatest and the least values of the following functions on the indicated intervals :

f(x) = x-2lnx on [1,e]

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11. Find the greatest and the least values of the following functions on the indicated intervals :

$$f(x) = \left\{\left(2x^2 + \frac{2}{x^2} \text{for} -2 \leq x < 0, 0 < x \leq 2\right), (1 \quad \text{for} \quad x = 0)\right.$$

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Solving Problems In Geometry And Physics

1. The force of a circular electric current acting on a small magnet with the axis perpendicular to the plane of the circle and passing through its centre is expressed by the formula

$$F = \frac{C(x)}{(a^2 + x^2)^{3/2}}$$

where a = radius of the circle

x = distance from the centre of the circle to the magnet $(0 < x < \infty)$

C = constant

At what x will the value of F be the greatest ?

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2. Determine the most economial dimensions of an open - air swimming pool of volume $32m^3$ with a square bottom so that the facing of its walls and bottom require the least quantity of material .

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3. A tin of a given volume V has the form of a cylinder . What must be the ratio of its height h to diameter 2R so as to use the least amount of material for its manufacture ?

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Convexity And Concavity Of A Curve Points Of Inflection

1. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of inflection :

$y = x^4 + x^3 - 18x^2 + 24x - 12,$

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2. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of inflection :

$y = 3x^4 - 8x^3 + 6x^2 + 12$

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3. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of inflection :

$y = \frac{x}{1 + x^2}$

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4. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of

inflection :

$y = x + x^{2/3}$

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5. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of inflection :

$y = \frac{In^2 x}{x} (x > 0)$

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6. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of inflection :

$y = 2 - |x^5 - 1|$

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7. What conditions must be coefficients a,b,c satisfy for the curve $y = ax^4 + bx^3 + cx^2 + dx + e$ to have points of inflection ?

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8. At what values of a will the curve

$y = x^4 + ax^3 + \ + \frac{3}{2}x^2 + 1$

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9. Show that the curve $y = \frac{x+1}{x^2+1}$ has three points of influence lying in a straight line .

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10. Show that the points of inflection of the curve y = x sin x lie on the curve $y^2(4+x^2) = 4x^2$

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Asymptomes

1. Find the asymptotes of the following curves :

$$y = \frac{x}{x^2+1}$$

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2. Find the asymptotes of the following curves :

$$y = \frac{1}{x} + 4x^2$$

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3. Find the asymptotes of the following curves :

$$y = \frac{x^2 - 6x + 3}{x - 3}$$

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4. Find the asymptotes of the following curves :

$$y = \quad \arctan \quad x$$

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5. Find the asymptotes of the following curves :

$$y = x + (\sin x) / x$$

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6. Find the asymptotes of the following curves :

$y = \text{In } (4 - x^2)$

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7. Find the asymptotes of the following curves :

$y = 2x - arc\cos. \frac{1}{x}$

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General Plan For Incestigating Functions And Sketching Graphs

1. Investigate and graph the following functions :

$y = \frac{2x^3}{x^2 - 4}$

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2. Investigate and graph the following functions :

$$y = \frac{1 - x^3}{x^2}$$

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3. Investigate and graph the following functions :

$$y = x + In\left(x^2 - 1\right)$$

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4. Investigate and graph the following functions :

$$y = x^2 e^{1/x}$$

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5. Investigate and graph the following functions :

$$y = 1 + x^2 - \frac{x^4}{2}$$

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Approximate Solution Of Algebraic And Transcendental Equations

1. Determine the number of real roots of the equation ,

$$f(x) = x + e^x$$

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2. Find the least positive root of the equation tan x = x with an accuracy up to 0.0001 applying Newton.s method .

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3. Using the combined method find all roots of the equation $f(x) \equiv x^3 - 5x + 1 = 0$ accurate to three decimal places.

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4. Applying the method of chords , find the positive root of the equation

$f(x) = x^3 + 1.1x^2 + 0.9x - 1.4 = 0$

with an accuracy of 0.005 .

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5. Using the method of chords , find approximate values of the real roots of the following with an accuracy up to 0.01 the positive roots of the following equations :

(a) $(x-1)^2 - 2\sin x = 0$

(b) $e^x - 2(1-x)^2 = 0$

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6. Applying Newton.s method , find the an accuracy up to 0.01 the positive roots of the following equations :

(a) $x^3 + 50x - 60 = 0$

(b) $x^3 + x - 32 = 0$

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7. Using the combined method find the valus of the root of the equatin

$x^3 - x - 1 = 0$

On the interval [1,2] with an accuracy up to 0.005 .

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8. Applying the iteration method , find all roots of the equation $4x - 5$ In x = 5 accurate to four decimal places .

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9. Does the function $f(x) = \begin{cases} x & \text{if } x < 1 \\ 1/x & \text{if } x \geq 1 \end{cases}$ satisfy the conditions of the Lagrange theorem on the interval [0,2] ?

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Additional Problems

1. Prove that for the function $y = \alpha x^2 + \beta x + \gamma$ the number ξ in the Lagrange formula ,used on an arbitrary interval [a,b] is the arithmetic mean of the numbers a and b : $\xi = (a+b)/2$

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2. Prove that if the equation

$a_0 x^n + a_1 x^{n-1} + \ldots. + a_{n-1} x = 0$

has a positive root x_0 , then the equation

$n a_0 x^{n-1} + (n-1) a_1 x^{n-2} + \ldots. + a_{n-1} = 0$ has a positive root less than x_0.

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3. Prove that the equation $x^4 - 4x - 1 = 0$ has two different real roots .

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4. Prove that all roots of the derivative of the given polynomial $f(x) = (x+1)(x-1)(x-2)(x-3)$ are real

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5. Show that the function

$$y = \begin{matrix} 1/x^2 & (x > 0) \\ 3x^2 & (x \leq 0) \end{matrix}$$

has a minimum at the point x = 0 , though its first derivative doex not change sign when passing through this point .

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6. For what choice of the parameter h does the " curve of probabilities "

$y = \frac{n}{\sqrt{\pi}} e^{-h^2 x^2} (h > 0)$ have points of inflection $x = \pm \sigma$?

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7. Taking the function $y = x^4 + 8x^3 + 18x^2 + 8$ as an example , ascertain that there any be no points of extremum between the abscissas of the points of inflection on the graph of a function .

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8. Prove that any polynomial with positive coefficients ,which is an even function , is concave everywhere and has only one point of minimum .

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9. Proceeding directly from the definition ,ascertain that the straight line $y = 2x + 1$ is an asymptote of the curve $y = \frac{2x^4 + x^3 + 1}{x^3}$

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