

MATHS

BOOKS - IA MARON MATHS (HINGLISH)

APPLICATION OF DIFFERENTIAL CALCULUS TO INVESTIGATION OF FUNCTIONS

Basic Theorems On Differentiable Functions

1. Does the function $f(x) = 3x^2 - 1$ satisfy the condition of the

Fermat theorem in the interval $\begin{bmatrix} 1 & 2 \end{bmatrix}$?

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2. Do the following functions satisfy the conditions of the Rolle

theorem ?

(a) $f(x) = 1 - \sqrt[3]{x^2} in[-1,1],$ (b) $f(x) in \sin x in [\pi/6, 5\pi/6],$ (c) f(x) = 1 - |x| in[-1,1].



3. Prove that the equation

 $3x^5 + 15x - 8 = 0$

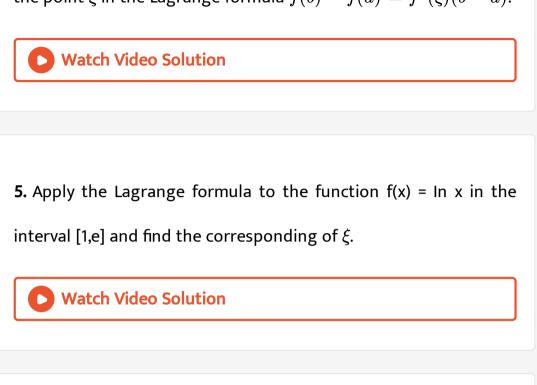
has only one real root .

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4. Does the function $f(x) = 3x^2 - 5$ satisfy the conditions of the

Lagrange theorem in the interval $\left[-2,0
ight]$? If it does then find

the point ξ in the Lagrange formula $f(b) - f(a) = f'(\xi)(b-a).$



6. On the curve $y = x^3$ find the point at which the tangent line

parallel to the chord through the points A(-1, 1) and B(2, 8).



7. Prove the inequality

arc $an x_2 - arc an x_1 < x_2 - x_1$ where $x_2 > x_1$

8. Using the Rolle theorem prove that the derivative f.(x) of the

function.

 $f(x)=egin{cases} x\sin\pi x & ext{at} & x>0\ 0 & ext{at} & x=0 \end{cases}$ vanishes on an infinite set of

points of the interval (0,1).



9. Check whether the Lagrange formula is applicable to following

functions :

 $f(x)=x^2\mathrm{on}[3,4]$



10. Check whether the Lagrange formula is applicable to following

functions :

 $f(x) = -\ln x$ on [1, 3]

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11. Check whether the Lagrange formula is applicable to following

functions

$$f(x) = 4x^3 - 5x^2 + x - 2 \;\; {
m on} \;\; [0,1]$$

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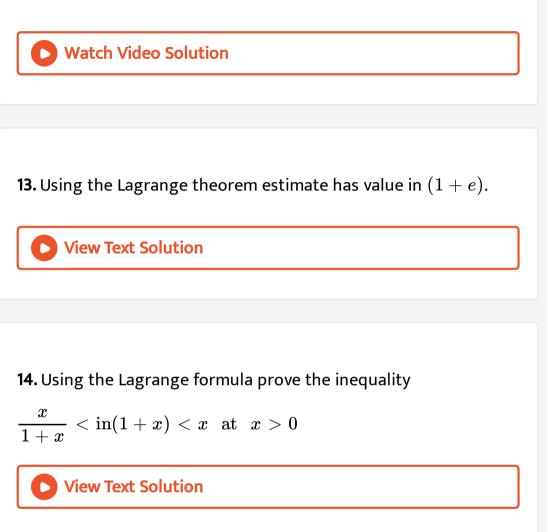
12. Check whether the Lagrange formula is applicable to following

functions :

 $f(x)=x^2\mathrm{on}[3,4]$

$$f(x)=\sqrt[5]{x^4(x-1)} ~~{
m on}~~[\,-1/2,1/2]$$

If it is , find the values of ξ appearing in this formula .



Evaluation Of Indeterminate Forms

1. Applying the L.Hospital rule , find the limits of the following

functions :

$$\lim_{x
ightarrow 0} rac{e^{ax}-e^{-2ax}}{\ln(1+x)}$$

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2. Applying the L.Hospital rule , find the limits of the following

functions :

 $\lim_{x \to -1} \frac{\sqrt[3]{1+2x+1}}{\sqrt{2+x+x}}$

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3. Applying the L.Hospital rule , find the limits of the following

functions :

$$\lim_{x\to 0}\;\frac{e^x-e^{-x}-2x}{x-\sin x}$$

4. Applying the L.Hospital rule , find the limits of the following

functions :

$$\lim_{x
ightarrow 0} \ rac{\ln(1+x^2)}{\cos 3x - e^{-x}}$$



5. Applying the L.Hospital rule , find the limits of the following

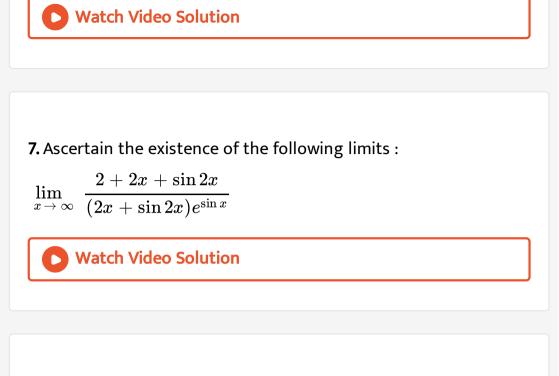
functions :

$$\lim_{x
ightarrow\infty} \; rac{e^{1/\,x^2}-1}{2\; rc an x^2-\pi}$$

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6. Ascertain the existence of the following limits :

$$\lim_{x \to 0} \, \frac{x^2 \sin(1/x)}{\sin x}$$



8. Ascertain the existence of the following limits :

 $\lim_{x o \pi/2} rac{ an x}{\sec x}$

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9. Using the L .Hospital rule find limits of the following functions :

 $\lim_{x
ightarrow 2} \, rac{Inig(x^2-3ig)}{x^2+3x-10}$

10. Using the L .Hospital rule find limits of the following functions

$$\lim_{x \to 1} \frac{a^{\ln x} - x}{\ln x}$$

:

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11. Using the L .Hospital rule find limits of the following functions :

 $\lim_{x
ightarrow 0} rac{ an x-x}{x-\sin x},$

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12. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 1} \frac{1 - 4\sin^2(\pi x \, / \, 6)}{1 - x^2}$$

13. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to a} \arcsin \frac{x - a}{a} \cot(x - a)$$

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14. Using the L .Hospital rule find limits of the following functions

$$\lim_{x \to 0} \left(\frac{1}{x}\right)^{\tan x}$$

:

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15. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x
ightarrow\infty} \ \Big(a^{1\,/\,x}-1\Big)x(a>0),$$

16. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 0} (\cos mx)^{n/x^2}$$
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17. Using the L .Hospital rule find limits of the following functions :
$$\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{x}{\ln x}\right)$$
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18. Using the L .Hospital rule find limits of the following functions :

 $\lim_{x\,\to\,\infty}\ x^{1\,/\,In\,(\,e^x\,-\,1\,)}$

19. Using the L .Hospital rule find limits of the following functions :

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x \right)$$
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20. Using the L .Hospital rule find limits of the following functions

:
$$\lim_{x \to 0} \left(\frac{5}{2 + \sqrt{9 + x}} \right)^{1/\sin x}$$

21. Using the L .Hospital rule find limits of the following functions :

 $\lim_{x
ightarrow\infty} \; rac{e^{1\,/\,x^2\,-\,1}}{2{
m arc}\,{
m tan}x^2\,-\,\pi}$

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1. Expand the polynomial $P(x)=x^5-2x^4+x^3-x^2+2x-1$

in powers of the binomial x - 1 using Taylor formula .

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2. Compute the approximate value of $\sqrt[4]{83}$ accurate to six decimal

places .

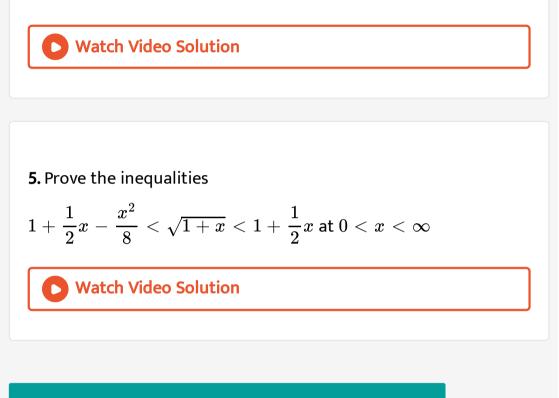
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3. Prove the inequalities

$$x-x^2 < In(1+x) \mathrm{at} x > 0$$

4. Prove the inequalities

 $an x > x + x^3/3 \;\; {
m at} 0 < x < \pi/2$



Application Of Taylor S Formula To Evaluation Of Limits

1. Applying the Taylor formula with the remainder in Peano.s form

compute the limits :

$$\lim_{x
ightarrow 0} rac{\sqrt[3]{1+3x}-\sqrt{1+2x}}{x^2}$$

2. Applying the Taylor formula with the remainder in Peano.s form

compute the limits :

 $\lim_{x
ightarrow 0} \ rac{\cos x - e^{-x^2/2}}{x^4}$



3. Applying the Taylor formula with the remainder in Peano.s form

compute the limits :

 $\lim_{x
ightarrow 0} \, rac{e^x \sin x - x(1+x)}{x^3}$



4. Applying the Taylor formula with the remainder in Peano.s form

compute the limits :



5. Expand the following function in positive integral powers of the variable x up to the terms of the inficated order , inclusive $f(x) = e^{2x - x^2}$ up to the terms containing x^5

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6. Expand the following function in positive integral powers of the

variable x up to the terms of the inficated order, inclusive

In cos x up to the terms containing x^6

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Testing A Function For Monotonicity

1. Find the intervals of monotonicity of the following $f(x)=2x^2-\ln \lvert x
vert$

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2. Determine the intervals of monotonicity for the following functions :

$$f(x) = 2x^3 - 9x^2 - 24x + 7,$$

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3. Determine the intervals of monotonicity for the following functions :

$$f(x) = x^2 e^{-x}$$

4. Determine the intervals of monotonicity for the following

functions :

 $f(x) = \ln |x|,$

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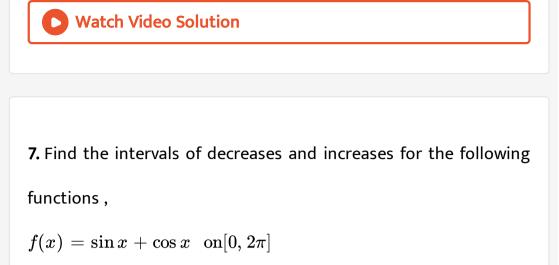
5. Determine the intervals of monotonicity for the following functions :

 $f(x) = 4x^3 - 21x^2 + 18x + 20,$

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6. Determine the intervals of monotonicity for the following functions :

$$f(x) = e^x + 5x$$



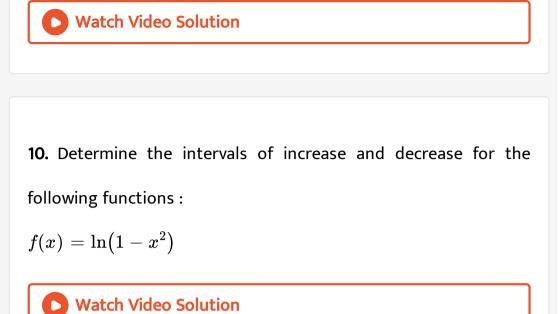
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8. Prove that the function $y = x^5 + 2x^3 + x$ increases everywhere and the function $y = 1 - x^3$ decreases everywhere .

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9. Determine the intervals of increase and decrease for the following functions :

$$f(x) = x^3 + 2x - 5$$



11. Determine the intervals of increase and decrease for the following functions :

 $f(x) = \cos x - x$



12. Determine the intervals of increase and decrease for the following functions :

$$f(x)=rac{1}{3}x^3-rac{1}{x}$$

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13. Determine the intervals of increase and decrease for the

following functions :

$$f(x) = rac{2x}{\ln x}$$

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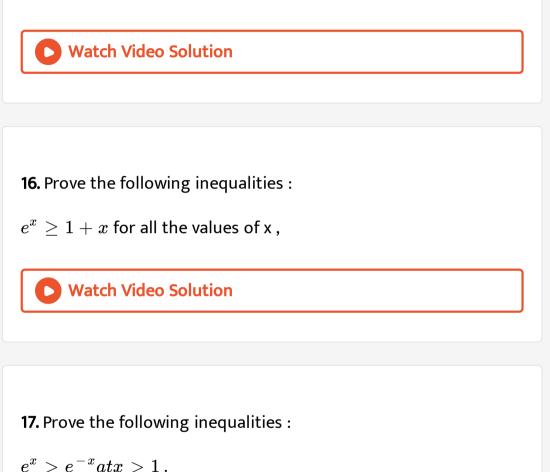
14. Determine the intervals of increase and decrease for the

following functions :

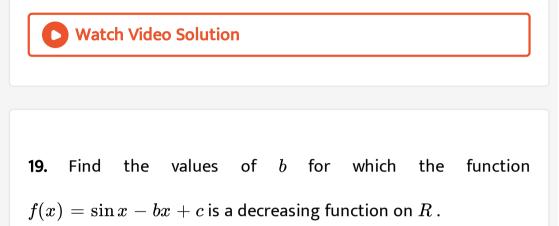
$$f(x)=rac{2x}{1+x^2}$$

15. Prove the following inequalities :

 $an x > x + x^3/3 ext{if} \ \ (0 < x < \pi/2)$



18. At what values of the coefficient a does the function $f(x) = a^3 - ax$ increase along the entire number scale ?



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Maxima And Minima Of Function

1. Using the first derivative , find the extreme of the following

functions :

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 12,$$



2. Using the first derivative , find the extreme of the following functions :

$$f(x) = x(x+1)^3(x-3)^2$$
,

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3. Using the first derivative , find the extreme of the following

functions :

$$f(x)=rac{x^2-3x+2}{x^2+2x+1}$$



4. Using the first derivative , find the extreme of the following functions :

$$f(x)=3\sqrt[3]{x^2}-x^2,$$

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5. Using the second derivative , find the out the character of the extrema of the following functions :

 $f(x) = 2x^3 - 15x^2 - 84x + 8$

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6. Investigate the following functions for an extremum at the

point x = 0:

$$y=\cos x-1+rac{x^2}{2}$$

7. Investigate the following functions for extrema :

$$f(x)=x^4e^{\,-x^2}$$



8. Investigate the following functions for extrema :

 $f(x) = \sin 3x - 3 \sin x$

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9. Find the maxima and minima of the following functions :

 $f(x)=x^3e^{\,-\,x}$

10. Find the maxima and minima of the following functions :

$$f(x) = \frac{4x}{x^2 + 4}$$

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11. Find the maxima and minima of the following functions :
$$f(x) = \sqrt[3]{2x^3 + 3x^2 - 36x}$$

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12. Find the maxima and minima of the following functions :

$$f(x)=x^2\ln x$$
 ,

13. Investigate the following functions for an extremum at the

point x = 0: $f(x) = \left\{egin{array}{cc} e^{1/x}, ext{if} & x
eq 0 \ 0, & ext{if} & x = 0 \end{array}
ight.$

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Finding The Greatest And The Least Values Of A Function

1. Find the greatest and the least values of the following functions

on the indicated intervals :

$$f(x)=2x^3-3x^2-12x+1 \;\; {
m on} \;\; [\,-2,5/2],$$

2. Find the greatest and the least values of the following functions on the indicated intervals :

 $f(x) = x^3 \; \ln \; \mathrm{x} \; \mathrm{on}[1,e]$

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3. Find the greatest and the least values of the following functions on the indicated intervals :

$$f(x) = \sqrt{\left(1-x
ight)^2 \left(1+2x^2
ight)} \;\; \mathrm{on}[\,-1,1]$$

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4. Find the greatest and the least values of the following functions on the indicated intervals ,

$$y=~~{
m sin}\,{
m x}\,2{
m x}\,{
m on}~~(~-\infty,\infty),$$

5. Find the greatest and the least values of the following functions on the indicated intervals ,

 $y = x + \sqrt{x}$ on [0, 4]

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6. Find the greatest and the least values of the following functions on the indicated intervals :

$$f(x)=rac{1}{4}x^4-rac{2}{3}x^3-rac{3}{2}x^2+2 ~~{
m on}~~[-2,4]$$

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7. Find the greatest and the least values of the following functions on the indicated intervals :

$$f(x)=\sqrt{4-x^2} \;\; {
m on} \;\; [\,-2,2]$$

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8. Find the greatest and the least values of the following functions on the indicated intervals :

 $f(x) = rc an \ x - rac{1}{2} \ ext{ in x on } \left[rac{1}{\sqrt{3}}, \sqrt{3}
ight]$

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9. Find the greatest and the least values of the following functions on the indicated intervals :

$$f(x)=2~\sin\mathrm{x}+\sin2\mathrm{x}~\mathrm{on}iggl[0,rac{3}{2}\piiggr]$$

10. Find the greatest and the least values of the following functions on the indicated intervals :

f(x) = x-2lnx on [1,e]



11. Find the greatest and the least values of the following functions on the indicated intervals :

$$f(x) = igg\{ igg(2x^2 + rac{2}{x^2} ext{for} - 2 \leq x < 0, 0 < x \leq 2 igg), (1 \;\; ext{for}\;\; x = 0)$$

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Solving Problems In Geometry And Physics

1. The force of a circular electric current acting on a small magnet with the axis perpendicular to the plane of the circle and passing through its centre is expressed by the formula

$$F = rac{C(x)}{\left(a^2 + x^2
ight)^{3\,/\,2}}$$

where a = radius of the circle

x = distance from the centre of the circle to the magnet $(0 < x < \infty)$

C = constant

At what x will the value of F be the greatest ?



2. Determine the most economial dimensions of an open - air swimming pool of volume $32m^3$ with a square bottom so that the facing of its walls and bottom require the least quantity of material.



3. A tin of a given volume V has the form of a cylinder . What must be the ratio of its height h to diameter 2R so as to use the least amount of material for its manufacture ?

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Convexity And Concavity Of A Curve Points Of Inflection

1. Find the intervals in which the graphs of the following functions

are concave or convex and locate the points of inflection :

$$y = x^4 + x^3 - 18x^2 + 24x - 12,$$

2. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of inflection :

$$y = 3x^4 - 8x^3 + 6x^2 + 12$$

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3. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of inflection :

$$y = rac{x}{1+x^2}$$



4. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of

inflection :

$$y = x + x^{2/3}$$

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5. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of inflection :

$$y=rac{In^2x}{x}(x>0)$$

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6. Find the intervals in which the graphs of the following functions are concave or convex and locate the points of inflection :

$$y=2-\left|x^{5}-1
ight|$$

7. What conditions must be coefficients a,b,c satisfy for the curve

 $y=ax^4+bx^3+cx^2+dx+e$ to have points of inflection ?

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8. At what values of a will the curve

$$y=x^4+ax^3+\ +\ rac{3}{2}x^2+1$$

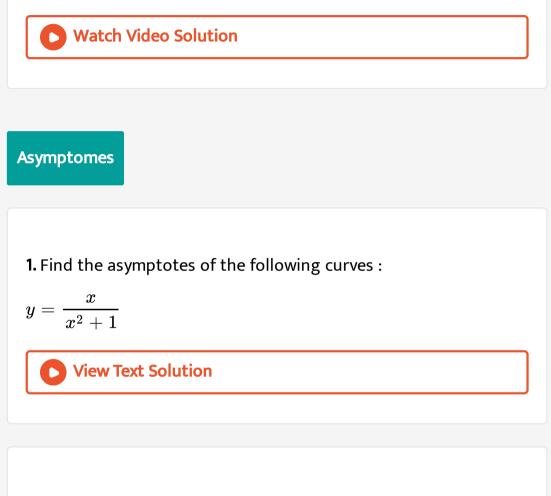
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9. Show that the curve $y = rac{x+1}{x^2+1}$ has three points of influence

lying in a straight line .

10. Show that the points of inflection of the curve $y = x \sin x$ lie on

the curve
$$y^2ig(4+x^2ig)=4x^2$$



2. Find the asymptotes of the following curves :

$$y=rac{1}{x}+4x^2$$

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3. Find the asymptotes of the following curves :

$$y = \frac{x^2 - 6x + 3}{x - 3}$$

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4. Find the asymptotes of the following curves :

$$y = \arctan x$$

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5. Find the asymptotes of the following curves :

 $y=x+\left(\sin x
ight)/x$

6. Find the asymptotes of the following curves :

$$y = \text{In } (4 - x^2)$$
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7. Find the asymptotes of the following curves :
$$y = 2x - \arccos \cdot \frac{1}{x}$$
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General Plan For Incestigating Functions And Sketching Graphs

1. Investigate and graph the following functions :

$$y=rac{2x^3}{x^2-4}$$

2. Investigate and graph the following functions :

$$y=rac{1-x^3}{x^2}$$

3. Investigate and graph the following functions :

$$y=x+Inig(x^2-1ig)$$

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4. Investigate and graph the following functions :

$$y=x^2e^{1\,/\,x}$$

5. Investigate and graph the following functions :

$$y=1+x^2-rac{x^4}{2}$$

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Approximate Solution Of Algebraic And Transcendental Equations

1. Determine the number of real roots of the equation ,

$$f(x) = x + e^x$$

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2. Find the least positive root of the equation $\tan x = x$ with an

accuracy up to 0.0001 applying Newton.s method .



3. Using the combined method find all roots of the equation $f(x)\equiv x^3-5x+1=0$ accurate to three decimal places.

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4. Applying the method of chords , find the positive root of the equation

$$f(x) = x^3 + 1.1x^2 + 0.9x - 1.4 = 0$$

with an accuracy of 0.005 .

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5. Using the method of chords , find approximate values of the real roots of the following with an accuracy up to 0.01 the positive roots of the following equations :

(a)
$$(x - 1)^2 - 2\sin x = 0$$

(b) $e^x - 2(1 - x)^2 = 0$
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6. Applying Newton.s method , find the an accuracy up to 0.01 the positive roots of the following equations :

(a)
$$x^3 + 50x - 60 = 0$$

(b) $x^3 + x - 32 = 0$



7. Using the combined method find the valus of the root of the equatin

 $x^3-x-1=0$

On the interval [1,2] with an accuracy up to 0.005 .



8. Applying the iteration method , find all roots of the equation

 $4x-5 \ln x$ = 5 accurate to four decimal places .

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9. Does the function
$$f(x) = \left\{egin{array}{cc} x & ext{if} & x < 1 \ 1/x & ext{if} & x \geq 1 \end{array}
ight.$$

satisfy the conditions of the Lagrange theorem on the interval

[0,2] ?

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Additional Problems

1. Prove that for the function $y = \alpha x^2 + \beta x + \gamma$ the number ξ in the Lagrange formula ,used on an arbitrary interval [a,b] is the arithmetic mean of the numbers a and b : $\xi = (a + b)/2$



2. Prove that if the equation

 $a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x = 0$

has a positive root x_0 , then the equation

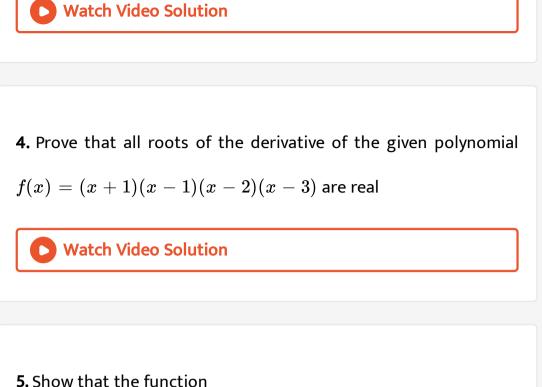
 $na_0x^{n-1}+(n-1)a_1x^{n-2}+\ldots+a_{n-1}=0$ has a positive

root less than x_0 .

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3. Prove that the equation $x^4 - 4x - 1 = 0$ has two different real

roots .



1 / 2 / 2 0)

 $y = egin{array}{ccc} 1/x^2 & (x > 0) \ 3x^2 & (x \le 0) \end{array}$

has a minimum at the point x = 0, though its first derivative doex

not change sign when passing through this point .



6. For what choice of the parameter h does the " curve of probabilities "

$$y=rac{n}{\sqrt{\pi}}e^{-h^2x^2}(h>0)$$
 have points of inflection $x=~\pm\,\sigma$?

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7. Taking the function $y=x^4+8x^3+18x^2+8$ as an example ,

ascertain that there any be no points of extremum between the

abscissas of the points of inflection on the graph of a function .

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8. Prove that any polynomial with positive coefficients ,which is an even function , is concave everywhere and has only one point of minimum .

9. Proceeding directly from the definition ,ascertain that the straight line y=2x+1 is an asymptote of the curve $y=rac{2x^4+x^3+1}{x^3}$