



## MATHS

### BOOKS - IA MARON MATHS (HINGLISH)

#### APPLICATIONS OF THE DEFINITE INTEGRAL

##### Computing The Limits Of Sums With The Aid Of Definite Integrals

1. Compute  $\lim_{n \rightarrow \infty} \frac{\pi}{n} \left| \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{(n-1)\pi}{n} \right|$



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##### Computing The Limits Of Sums With The Aid

1. Compute the limit

$$\lim_{n \rightarrow \infty} \left( \frac{1}{\sqrt{4n^2 - 1}} + \frac{1}{\sqrt{4n^2 - 2^2}} + \dots + \frac{1}{\sqrt{4n^2 - n^2}} \right)$$

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2.

$$\lim_{n \rightarrow \infty} \frac{3}{n} \left[ 1 + \sqrt{\frac{n}{n+3}} + \sqrt{\frac{n}{n+6}} + \sqrt{\frac{n}{n+9}} + \dots + \sqrt{\frac{n}{n+3(n-1)}} \right]$$

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3. Compute the limit  $A = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n!}}{n}$

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## Finding Average Values Of A Function

1. Find the average value  $\mu$  of the function  $f(x) = \sqrt[3]{x}$  over the interval  $[0, 1]$

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2. Find the average values of this functions:

(a)  $f(x) = \sin^2 x$  over  $[0, 2\pi]$  (b)  $f(x) = \frac{1}{e^x + 1}$  over  $[0, 2]$

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3. Determine the average length of all vertical chords of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ over the interval } a \leq x \leq 2a$$

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4. Find the average ordinate of the sinusoid  $y = \sin x$  over the interval

$$[0, \pi]$$

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5. Find the average length of all positive ordinates of the circle

$$x^2 + y^2 = 1$$

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6. Show that the average value of the function  $f(x)$ , continuous on the interval  $[a,b]$ , is the limit of the arithmetic mean of the values of this function taken over equal intervals of the argument  $x$ .

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7. Find the average values of pressure ( $p_m$ ) varying from 2 to 10 atm if the pressure  $p$  and the volume  $v$  are related as follows:

$$PV^{\frac{3}{2}} = 160$$

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8. In hydraulics there is Bazrin.s formula expressing the velocity  $v$  of water flowing in a wide rectangular channel as a function of the depth  $h$  at which the point under consideration is situated below the open surface,

$$v = v_0 - 220\sqrt{HL}\left(\frac{h}{H}\right)^2,$$

where  $v_0$  is the velocity on the open surface,  $H$  is the depth of the channel,  $L$  its slope.

Find the average velocity  $v_m$  of flow in the cross-section of the channel.

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9. Determine the average value of the electromotive force  $E_m$  over one period, ie, over the time from  $t = 0$  to  $t = T$ , if electromotive force is computed by the formula

$$E = E_0 \sin \frac{2\pi t}{T}$$

where  $T$  is the duration of the period in seconds,  $E_0$  the amplitude (the maximum value) of the electromotive force corresponding to the value  $t = 0.25T$ . The fraction  $\frac{2\pi t}{T}$  is called the phase.

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10. Find the average value of the square of the electromotive force  $(E^2)_m$  over the interval from  $t = 0$  to  $t = \frac{T}{2}$

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11. If a function  $f(x)$  is defined on an infinite interval  $[0, \infty]$ , then its average

value will be  $\mu = \lim_{b \rightarrow \infty} \frac{1}{b} \int_0^b f(x) dx,$

If this limit exists. Find the average power consumption of an alternating current circuit if the current intensity  $I$  and voltage  $u$  are expressed by the following formulas, respectively:

$$I = I_0 \cos(\omega t + \alpha),$$

$$u = \mu_0 \cos(\omega t + \alpha + \varphi),$$

where  $\varphi$  is the constant phase shift of the voltage as compared with the current intensity (the parameters  $\omega$  and  $\alpha$  will not enter into the expression for the average power)



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12. Find the average value of  $\mu$  of the function  $f(x)$  over the indicated intervals:

(a)  $f(x) = 2x^2 + 1$  over  $[0, 1]$

(b)  $f(x) = \frac{1}{x}$  over  $[1, 2]$

(c)  $f(x) = 3^x - 2x + 3$  over  $[0, 2]$

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**13.** A body falling to the ground from a state of rest acquires a velocity  $v_1 = \sqrt{2gs_1}$  on covering a vertical path  $s = s_1$ . Show that the average velocity  $v_m$  over this path is equal to  $\frac{2v_1}{3}$

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**14.** Find the average value  $I_m$  of alternating current intensity over time interval from 0 to  $\frac{\pi}{\omega}$

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**15.** Find the average value of the function

$f(x) = \frac{\cos^2 x}{\sin^2 x + 4 \cos^2 x}$  over the interval  $\left[0, \frac{\pi}{2}\right]$ . Check directly that

this average, equal to  $\frac{1}{6}$ , is the value of the function  $f(x)$  for a certain  $x = \xi$  lying within the indicated interval.

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## Computing Areas In Rectangular Coordinates

1. Compute the area of the figure bounded by the straight lines  $x=0$ ,  $x=2$  and the curves  $y = 2^x$ ,  $y = 2x - x^2$

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2. Compute the area of the figure bounded by the parabolas  $x = -2y^2$ ,  $x = 1 - 3y^2$

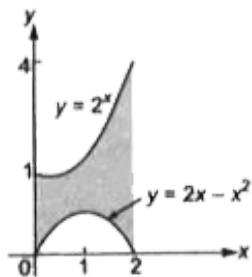


Fig. 66

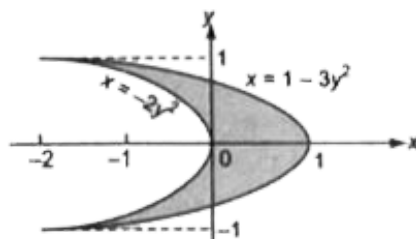


Fig. 67



3. Find the area of the figure contained between the parabola  $x^2 = 4y$  and the witch of Agnesi  $y = \frac{8}{x^2 + 4}$

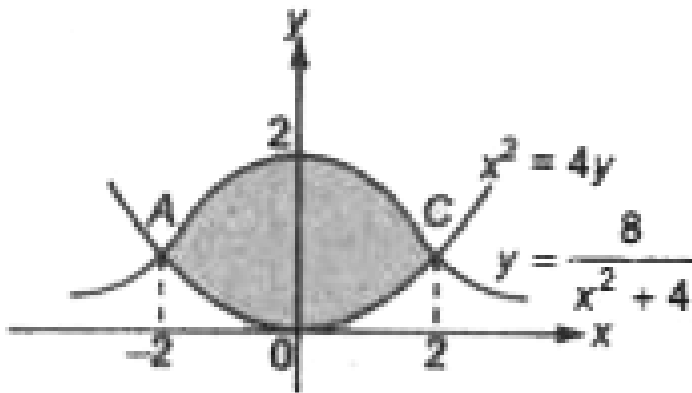
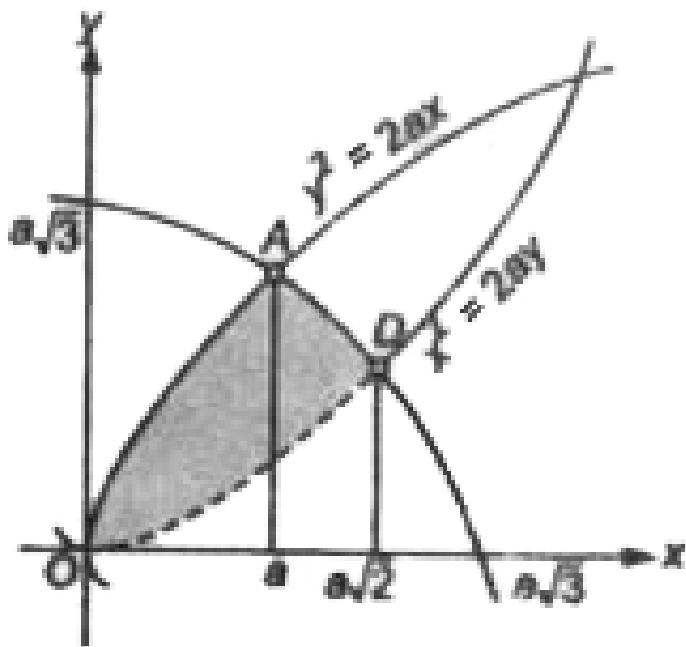


Fig. 68

4. Compute the area of the figure which lies in the first quadrant inside the circle  $x^2 + y^2 = 3a^2$  and is bounded by the parabolas

$$x^2 = 2ay \text{ and } y^2 = ax (a > 0)$$



**Fig. 68**

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5. Compute the area of the figure lying in the first quadrant and bounded by the curves  $y^2 = 4x$ ,  $x^2 = 4y$  and  $x^2 + y^2 = 5$

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6. Compute the area of the figure bounded by the lines  $y = x + 1$ ,  $y = \cos x$  and the x-axis

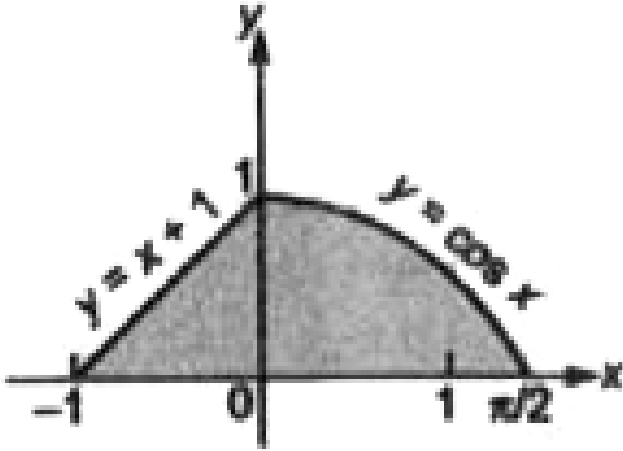


Fig. 70

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7. Find the area of the segment of the curve  $y^2 = x^3 - x^2$  if the line  $x=2$  is the chord determining the segment

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8. Determine the area of the figure bounded by two branches of the curve  $(y - x)^2 = x^3$  and the straight line  $x=1$

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9. Compute the area enclosed by the loop of the curve  $y^2 = x(x - 1)^2$

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10. Find the area enclosed by the loop of the curve  $y^2 = (x - 1)(x - 2)^2$ .

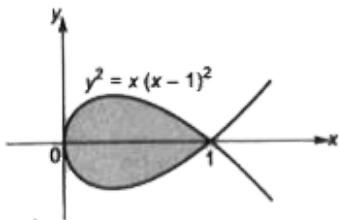


Fig. 72

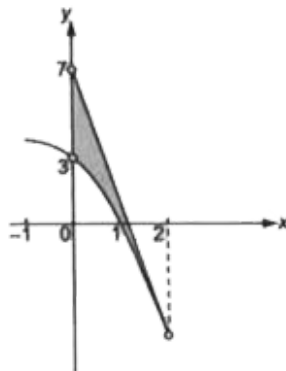


Fig. 73

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11. Find the area of the figure bounded by the parabola  $y = -x^2 + 2x + 3$ , the line tangent to it at the point M (2, -5) and the y-axis



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12. Find the area bounded by the parabola  $y = x^2 - 2x + 2$ , the line tangent to it at the point M(3,5) and the axis of ordinates



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13. We take on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$$

a point M(x,y) lying in the first quadrant.

Show that the sector of the ellipse bounded by its semi-major axis and the focal radius drawn to the point M has an area.

$$S = \frac{ab}{2} \arccos \frac{x}{a}.$$

With the aid of this result deduce a formula for computing the area of

the entire ellipse.

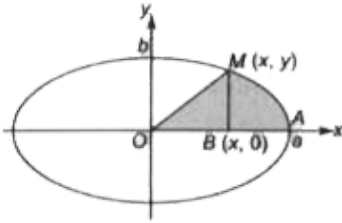


Fig. 74

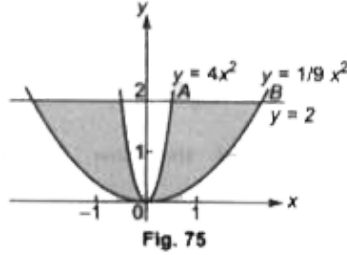


Fig. 75

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14. The area bounded between the parabola  $x^2 = \frac{y}{4}$  and  $x^2 = 9y$  and the straight line  $y = 2$  is

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15. From an arbitrary point  $M(x, y)$  of the curve  $y = x^m (m > 0)$  perpendiculars  $MN$  and  $ML$  ( $x > 0$ ) are dropped onto the coordinate axes. What part of the area of the rectangle  $ONML$  does the area  $ONMO$

constitute ?

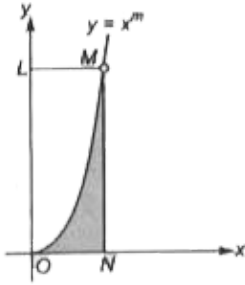


Fig. 76

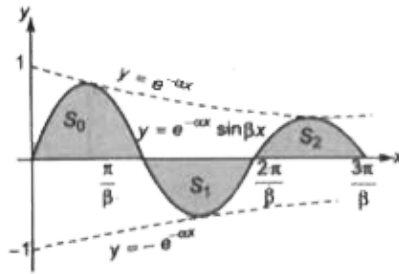


Fig. 77

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16. Prove that the areas  $S_0, S_1, S_2, \dots, S_3, \dots$ , bounded by the x-axis and half-waves of the curve  $y = e^{-\alpha x} \sin \beta x, x \geq 0$ , form a geometric progression with the common ratio  $q = \left(\frac{e^{-\alpha \pi/\beta}}{\beta}\right)$

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17. Find the areas enclosed between the circle  $x^2 + y^2 - 2x + 4y - 11 = 0$  and the parabola

$$y = -x^2 + 2x + 1 - 2\sqrt{3}$$

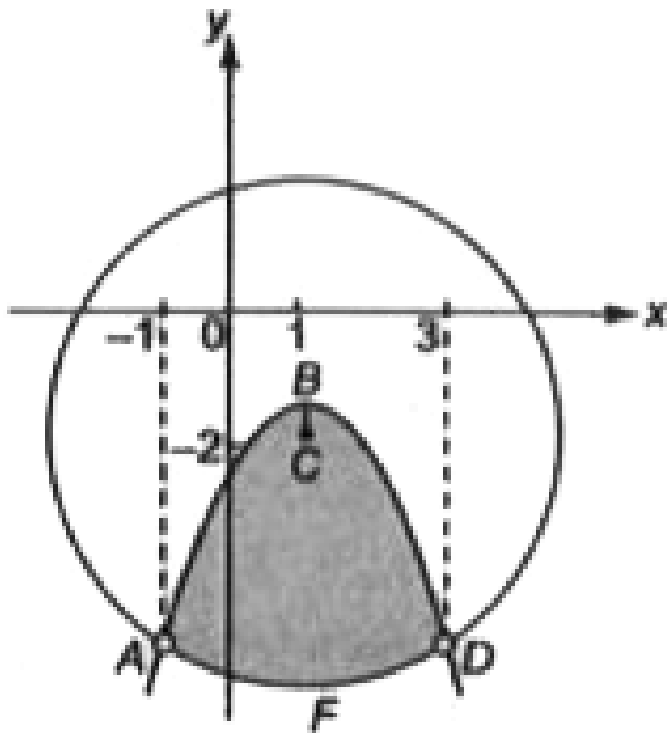


Fig. 78

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18. Find the area of the region bounded by curves  $f(x) = (x - 4)^2$ ,  $g(x) = 16 - x^2$  and the x-axis.

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19. Compute the area enclosed between the parabolas

$$x = y^2, x = \frac{3}{4}y^2 + 1$$

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20. Compute the area enclosed by the curve  $y^2 = (1 - x^2)^3$

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21. Compute the area enclosed by the loop of the curve

$$4(y^2 - x^2) + x^3 = 0$$

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22. Compute the area of the figure bounded by the curve  $\sqrt{x} + \sqrt{y} = 1$

and the straight line  $x + y = 1$

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23. Compute the area of the figure enclosed by the curve

$$y^2 = x^2(1 - x^2)$$

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24. Compute the area enclosed by the loop of the curve

$$x^3 + x^2 - y^2 = 0$$

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25. Compute the area bounded by the axis of ordinates and the curve

$$x = y^2(1 - y)$$

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1. Compute the area enclosed by the ellipse

$$x = a \cos t, y = b \sin t (0 \leq t \leq 2\pi)$$

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2. Find the area enclosed by the astroid  $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$

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3. Find the area of the region bounded by an arc of the cycloid

$$x = a(t - \sin t), y = a(1 - \cos t) \text{ and the x-axis}$$

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4. Compute the area of the region enclosed by the curve

$$x = a \sin t, y = b \sin 2t$$

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5. Find the area of the region enclosed by the loop of the curve

$$x = \frac{t}{3}(6 - t), y = \frac{t^2}{8}(6 - t)$$

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6. Find the area enclosed by the loop of the curve ,  $x = t^2, y = t - \frac{t^3}{3}$

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7. Compute the area enclosed by the cardioid:

$$x = a \cos t(1 + \cos t), y = a \sin t(1 + \cos t)$$

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The Area Of A Curvilinear Sector In Polar Coordinates

1. Find the area of the region situated in the first quadrant and bounded by the parabola  $y^2 = 4ax$  and the straight lines  $y = x - a$  and  $x = a$

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2. Find the area of the regions bounded by the curve  $\rho = 2a \cos 3\varphi$  and the arcs of the circle  $\rho = a$  and situated outside the circle.

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3. Compute the area of the figure bounded by the circle  $\rho = 3\sqrt{2}a \cos \varphi$  and  $\rho = 3a \sin \varphi$

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4. Find the area of the figure out only by the circle  $\rho = \sqrt{3} \sin \varphi$  from the cardioid  $\rho = 1 + \cos \varphi$



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5. Find the area of the region enclosed by the loop of the folium of Descartes  $x^3 + y^3 = 3axy$

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## Computing The Volume Of Solid

1. Find the volume of the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

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2. The axes of two identical cylinders with bases of radius  $a$  intersect at right angles. Find the volume of the solid constituting the common portion of the two cylinders.

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3. On all chords (parallel to one and the same direction) a circle of radius  $R$  symmetrical parabolic segments of the same altitude  $h$  are constructed. The planes of the segments are perpendicular to the plane of the circle. Find the volume of the solid thus obtained

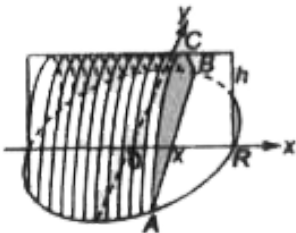


Fig. 88

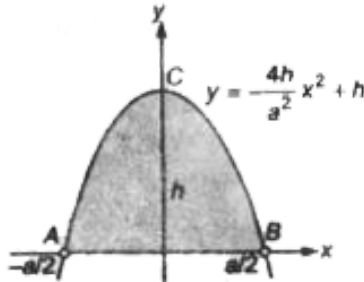


Fig. 90



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4. Compute the volume of the solid generated by revolving about the  $x$ -axis the area bounded by the axes of coordinates and the parabola  $x^{\frac{1}{2}} + y^{\frac{1}{2}} = a^{\frac{1}{2}}$ .



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5. The figure bounded by an arc of the sinusoid  $y = \sin x$ , the axis of ordinates and the straight line  $y=1$  revolves about the  $y$ -axis

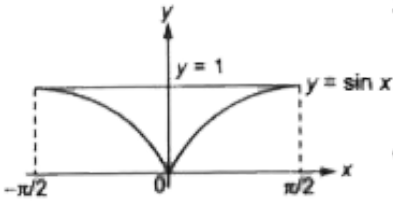


Fig. 91

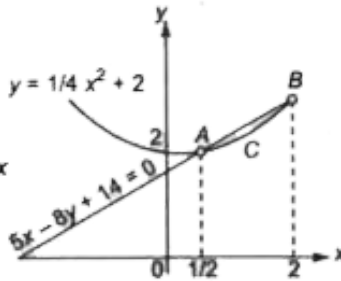


Fig. 92

Compute the volume  $V$  of the solid of revolution thus generated.



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6. Compute the volume of the solid generated by revolving about the  $x$ -axis the figure bounded by the parabola  $y = 0.25x^2 + 2$  and the straight line  $5x - 8y + 14 = 0$



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7. Compute the volume of the solid generated by revolving about the  $y$ -axis the figure bounded by the parabolas  $y = x^2$  and  $8x = y^2$

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8. Find the volume of the solid generated by revolving about the line  $y = -2a$  the figure bounded by the parabola  $y^2 = 4ax$  and the straight line  $x=a$

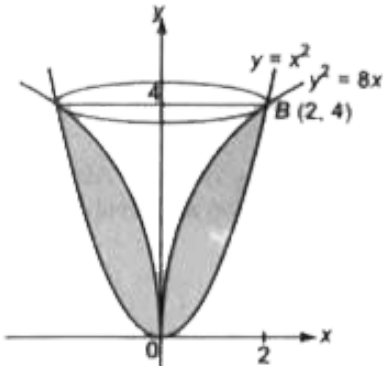


Fig. 93

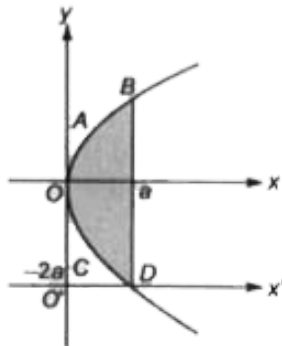


Fig. 94

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9. Find the volume of the solid generated by revolving about the x-axis the figure enclosed by the astroid :  $x = a \cos^3 t, y = a \sin^3 t$ .



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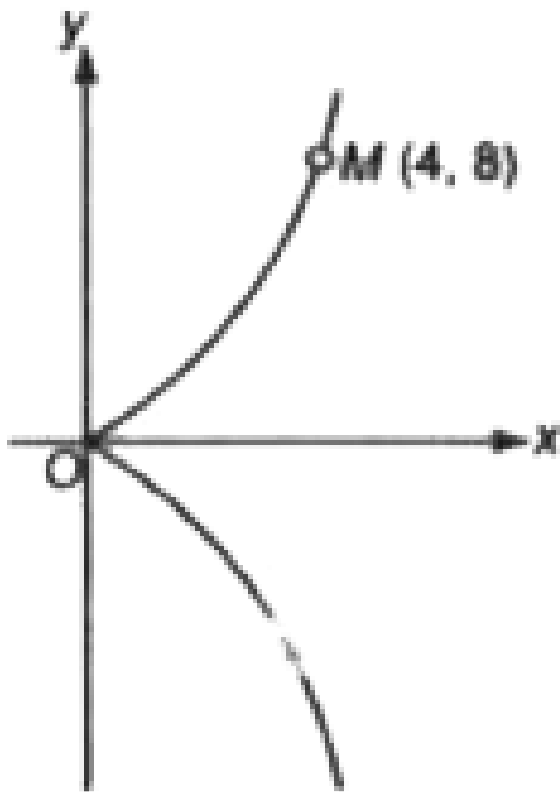
10. Compute the volume of the solid obtained by revolving about the polar axis the cardioid  $\rho = a(1 + \cos \varphi)$  shown in



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## The Arc Length Of A Plane Curve In Rectangular Coordinates

1. Compute the length of the arc of the semicubical parabola  $y^2 = x^3$  between the points  $(0, 0)$  and  $(4, 8)$



**Fig. 96**

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2. Compute the length of the arc cut off from the curve  $y^2 = x^3$  by the straight line  $x = \frac{4}{3}$

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3. Compute the arc length of the curve  $y = \ln \cos x$  between the points with the abscissas  $x = 0, x = \frac{\pi}{4}$

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4. Compute the arc length of the curve  $y = \ln \frac{e^x + 1}{e^x - 1}$  from  $x_1 = a$  to  $x_2 = b (b > a)$

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5. Find the arc length of the curve  $x = \frac{1}{4}y^2 - \frac{1}{2} \ln y$  between the points with the ordinates  $y = 1$  and  $y = 2$

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6. Find the length of the astroid  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

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7. Compute the length of the path OABCO consisting of portions of the curves  $y^2 = 2x^3$  and  $x^2 + y^2 = 20$

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## The Arc Length Of A Curve Represented Parametrically

1. Compute the arc length of the involute of a circle  $x = a(\cos t + t \sin t)$ ,  $y = a(\sin t - t \cos t)$  from  $t=0$  to  $t = 2\pi$ .

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2. Compute the length of the astroid :  $x = a \cos^3 t$ ,  $y = a \sin^3 t$

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3. Compute the length of the loop of the curve  $x = \sqrt{3}t^2, y = t - t^3$

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4. Compute the are length of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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## The Arc Length Of A Curve In Polar Coordinates

1. Find the length of the first turn of the spiral of the Archimedes  $\rho = a\varphi$

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2. Find the length of the logarithmic spiral  $\rho = ae^{m\phi}$  between a certain point  $(\rho_0, \varphi_0)$  and a moving point  $(\rho, \varphi)$

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3. Find the arc length of the cardioid

$$\rho = a(1 + \cos \varphi) \quad (a > 0, 0 \leq \varphi \leq 2\pi)$$

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4. Find the length of the lemniscate  $\rho^2 = 2a^2 \cos 2\varphi$  between the right-hand vertex corresponding to  $\varphi = 0$  and any point with a polar angle

$$\varphi < \frac{\pi}{4}$$

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5. Compute the length of the segment of the straight line

$$\rho = a \sec\left(\varphi - \frac{\pi}{3}\right) \text{ between } \varphi = 0 \text{ and } \varphi = \frac{\pi}{2}$$

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6. Find the length of the closed curve  $\rho = a \sin^4 \frac{\varphi}{4}$

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7. Find the length of the curve  $\varphi = \frac{1}{2}(\rho + 1/\rho)$  between  $\rho = 2$  and  $\rho = 4$

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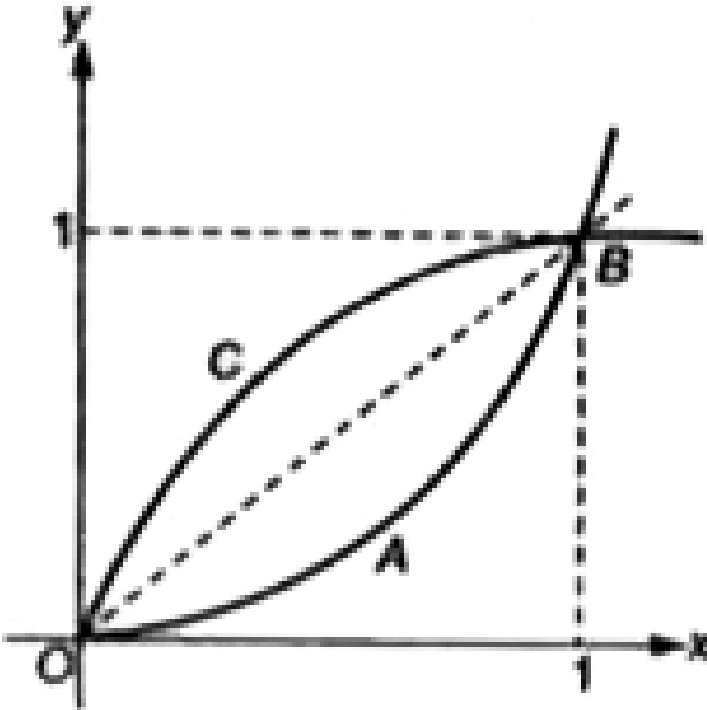
## Area Of Surface Of Revolution

1. Find the area of the surface formed by revolving the astroid  $x^{2/3} + y^{2/3} = a^{2/3}$  about the x-axis

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2. Find the area of the surface generated by revolving about the x-axis a closed contour OABCO formed by the curves  $y = x^2$  and  $x = y^2$



**Fig. 100**

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3. The area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is

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4. An arc of the catenary  $y = \frac{a}{2} \left( e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right) = a \cosh \frac{x}{a}$ ,

whose end-points have abscissas 0 and  $x$ , respectively, revolves about the  $x$ -axis. Show that the surface area  $P$  and the volume  $V$  of the solid thus generated are related by the formula  $P = \frac{2V}{a}$

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5. Find the area of the surface obtained by revolving a loop of the curve  $9ax^2 = y(3a - y)^2$  about the  $y$ -axis

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6. Compute the area of the a surface generated by revolving about the  $x$ -axis an arc of the curve  $x = t^2, y = \frac{t}{3}(t^2 - 3)$  between the points of intersection of the curve and the  $x$ -axis

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7. Compute the surface area of the torus generated by revolving the circle

$$x^2 + [y - b]^2 = r^2 (0 < r < b) \text{ about the x-axis}$$



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8. Compute the area of the surface formed by revolving the lemniscate

$$\rho = a\sqrt{\cos 2\varphi} \text{ about the polar axis.}$$



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9. Compute the area of the surface formed by revolving about the

straight line  $x + y = a$  the quarter of the circle  $x^2 + y^2 = a^2$  between

A(a, 0) and B(0, a).



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10. Computes the area of the surface formed by revolving one branch of the lemniscate  $\rho = a\sqrt{\cos 2\varphi}$  about the straight line  $\varphi = \frac{\pi}{4}$

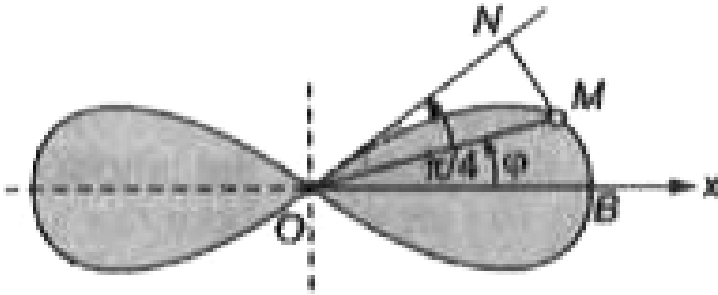


Fig. 102



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## Geometrical Applications Of The Definite Integral

1. Given: the cycloid  $x = a(t - \sin t)$ ,  $y = a(1 - \cos t)$ ,  $0 \leq t \leq 2\pi$

Compute. (a) the areas of the surface formed by revolving the arc OBA about the x and y-axis,

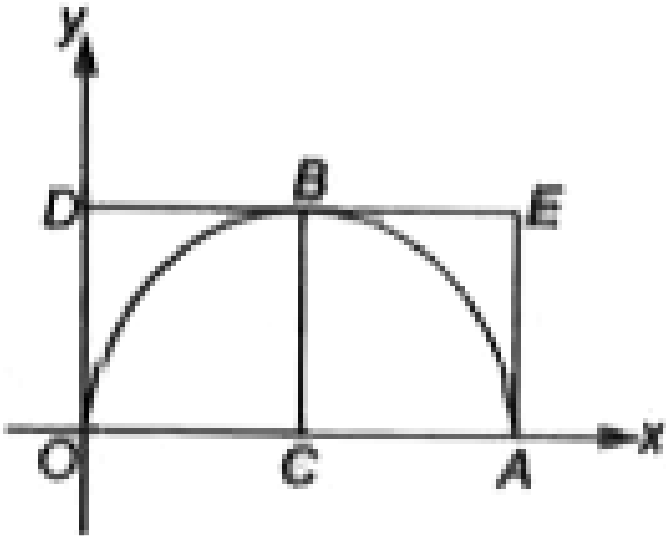
(b) the volumes of the solids generated by revolving the figure OBAO about the y-axis and the axis BC,

(c) the area of the surface generated by revolving the arc BA about the

axis BC,

(d) the volume of the solid generated by revolving the figure ODBEABO about the tangent line DE touching the figure at the vertex B,

(e) the area of the surface formed by revolving the arc of the cycloid (see item (d)]



**Fig. 103**



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2. Find the volume of the solid bounded by the surfaces  $z^2 = 8(2 - x)$  and  $x^2 + y^2 = 2x$ .

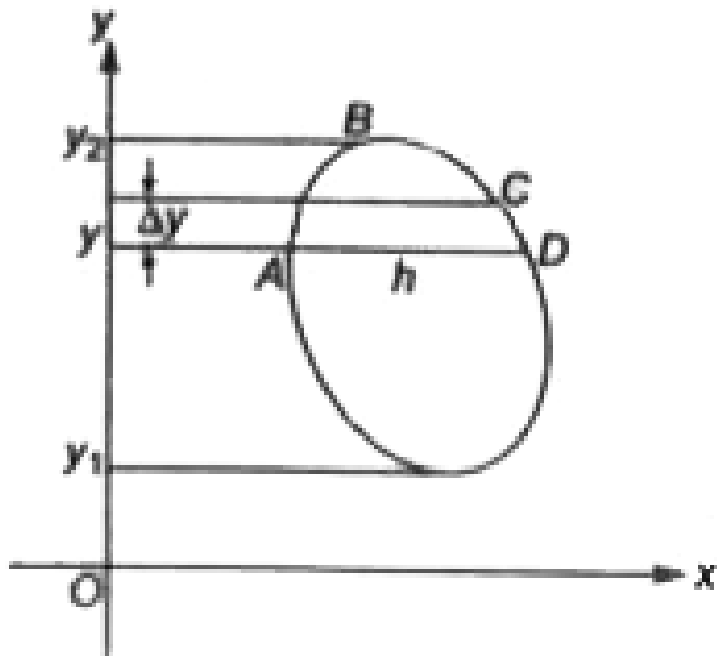


3. Prove that if the figure  $S$  is bounded by a simple convex contour and is situated between the ordinates  $y_1$  and  $y_2$ , then the volume of the solid generated by revolving this figure about the  $x$ -axis can be expressed by the formula

$$V = 2\pi \int_{y_1}^{y_2} y h dy,$$

where  $h = x_2(y) - x_1(y)$ ,  $x = x_1(y)$  being the equation of the left

portion of the contour and  $x = x_2(y)$  that of the right portion.



**Fig. 104**

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4. The planer region bounded by the parabola  $y = 2x^2 + 3$ , the x-axis and the verticals  $x=0$  and  $x=1$  revolves about the y-axis. Compute the volume of the solid of revolution thus generated.

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5. Compute the area of the portion of the cylinder surface  $x^2 + y^2 = ax$  situated inside the sphere

$$x^2 + y^2 + z^2 = a^2$$

$$x^2 + y^2 + z^2 = a^2.$$

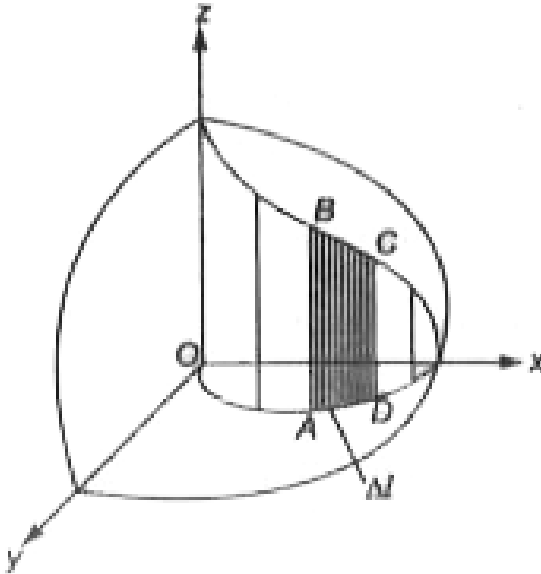


Fig. 105



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6. Find the area of the surface cut off from a right circular cylinder by a plane passing through the diameter of the base and inclined at an angle



of  $45^\circ$  to the base.

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7. The axes of two circular cylinders with equal bases intersect at right angles. Compute the surface area of the solid constituting the part common to both cylinders.

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8. Find the area  $S$  of the ellipse given by the equation  $Ax^2 + 2Bxy + Cy^2 = 1$  ( $\Delta = AC - B^2 > 0, C > 0$ )

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9. Find the length of the arc OA of the curve  $y = a \ln \frac{a^2}{a^2 - x^2}$ , where  $O(0, 0)$ ,  $A\left(\frac{a}{2}, a \ln \frac{4}{3}\right)$

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10. Find the volume of the solid generated by revolving about the x-axis the figure bounded by the straight lines  $y = x + 1$ ,  $y = 2x + 1$  and  $x = 2$ .

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## Computing Pressure Work And Other Physical Quantities By The Definite Integrals

1. Compute the force of pressure experienced by a vertical triangle with base  $b$  and altitude  $h$  submerged base downwards in water so that its vertex touches the surface of the water.

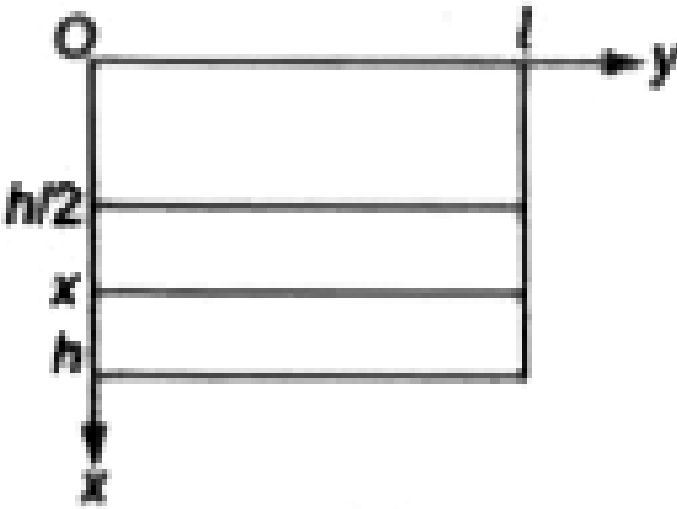
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2. A vertical dam has the form of a trapezoid whose upper base is 70m long, the lower one 50m, and the altitude 20m. Find the force of water

pressure experienced by the dam

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3. A rectangular vessel is filled with equal volumes of water and oil, water is twice as heavy as oil. Show that the force of pressure of the mixture on the wall will reduce by one fifth if the water is replaced by oil.



**Fig. 109**

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4. The electric charge  $E$  concentrated at the origin of coordinates repulses the charge from the point  $(a, 0)$  to the point  $(b, 0)$ . Find the work  $A$  of the repulsive force  $F$ .

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5. Calculate the work performed in launching a rocket of weight  $P$  from the ground vertically upwards to a height  $h$ .

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6. Calculate the work that has to be done to stop an iron sphere of radius  $R$  rotating about its diameter with an angular velocity  $\omega$ .

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7. Find the amount of heat released by an alternating sinusoidal current

$$I = I_0 \sin\left(\frac{2\pi}{T}t - \varphi\right).$$

during a cycle T in a conductor with resistance R.



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## Computing Static Moments And Moments Of Inertia Determining Coordinates Of The Centre Of Gravity

1. Find the static moment of the upper portion of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ about the x-axis.}$$



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2. Find the moment of inertia of a rectangle with base b and altitude h about its base.



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3. Calculate the moment of inertia about the  $y$ -axis of the figure bounded by the parabola  $y^2 = 4ax$  and the straight line  $x=a$ .

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4. In designing wooden girder bridges we often have to deal with logs flattened on two opposite sides. Figure 111. shows the cross-section of such a log. Determine the moment of inertia of this cross-section about the horizontal centre line.

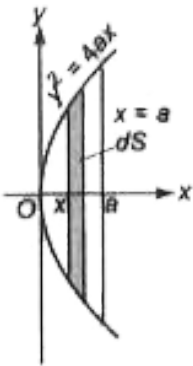


Fig. 110

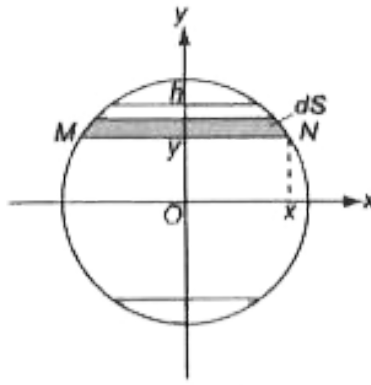


Fig. 111

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5. Find the moment of inertia about the x-axis of the figure bounded by two parabolas with dimensions indicated in.

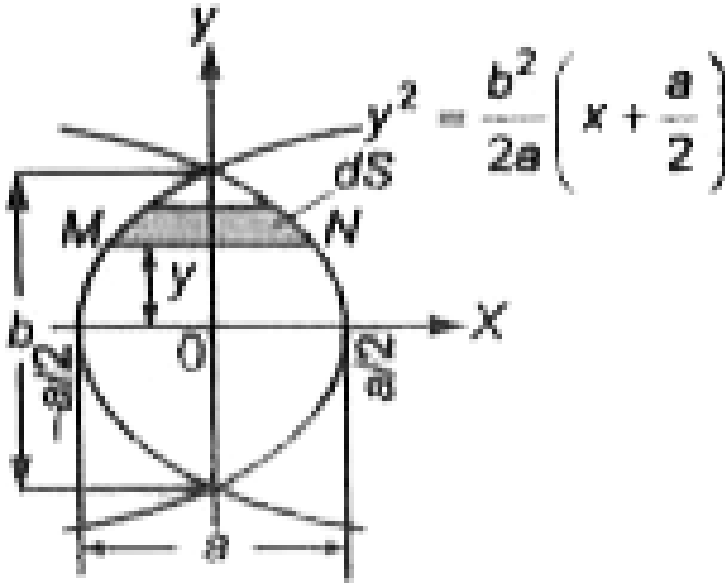


Fig. 112

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6. Find the centre of gravity of the semicircle  $x^2 + y^2 = a^2$  situated above the x-axis.

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7. Find the coordinates of the centre of gravity of the catenary

$$y = \frac{1}{2}(e^x + e^{-x}) = \cosh x \text{ between } A(0, 1) \text{ and } B(a, \cosh a).$$

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8. Find the centre of gravity of the first are of the cycloid:

$$x = a(t - \sin t), y = a(1 - \cos t) (0 \leq t \leq 2\pi).$$

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9. Find the Cartesian coordinates of the center of gravity of the are of the cardioid  $\rho = a(1 + \cos \varphi)$  between  $\varphi = 0$  and  $\varphi = \pi$ .

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10. Find the centre of gravity of the figure bounded by the ellipse  $4x^2 + 9y^2 = 36$  and the circle  $x^2 + y^2 = 9$  and situated in the first quadrant



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11. Find the Cartesian coordinates of the centre of gravity of the figure enclosed by the curve  $r = a \cos^3 \varphi$  ( $a > 0$ ).



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12. Find the coordinates of the centre of gravity of the figure bounded by the straight line  $y = \frac{2}{\pi}x$  and the sinusoid  $y = \sin x$  ( $x \geq 0$ )



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13. Using the first Guldin theorem, find the centre of gravity of a semicircle of radius  $a$ .

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14. Using the second Guldin theorem, find the coordinates of the centre of gravity of the figure bounded by the  $x$ -axis and one arc of the cycloid:  
 $x = a(t - \sin t)$ ,  $y = a(1 - \cos t)$ .

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15. An equilateral triangle with side  $a$  revolves about an axis parallel to the base and situated at a distance  $b > a$  from the base. Find the volume of the solid of revolution.

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