MATHS

JEE (MAIN AND ADVANCED MATHEMATICS) FOR BOARD AND COMPETITIVE EXAMS

APPLICATION OF DERIVATIVES

Example

1. Find the rate of change of total surface are of a right circular cone w.r.t. radius . (Cone angle remain constant), when radius=5 cm.

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2. The total cost C(x) in rupees, associated with the production of x units of an item is given by $C(x) = 0.05x^3 - 0.01x^2 + 20x + 1000.$ Find the marginal cost when 3 units are produced .

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3. If a ladder which is 10m long rests against a vertical wall. If the base of ladder slides away from the wall at 1 m/s then how fast is the top of the ladder sliding down along the wall when the base is 6 m from the wall ?

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4. A balloon is rising vertically from a level field , suppose an on-looker sees it rising at 0.14 rad/min. when $\theta = \frac{\pi}{4}$ (when the on -looker is 500 m away from the launch spot), how fast is balloon rising ?

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5. The volume of a spherical balloon is increasing at the rate of 20 $cm^3/\sec$. Find the rate of change of its surface area at the instant when radius is 5 cm.

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6. A water tank has the shape of an inverted righ circular cone with its axis vertical and vertex lowermost . Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 4 cubic meter per hour . Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 2 m.

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7. Show that the function $f(x) = 2x + 3$ is strictly increasing function on R.

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8. Show that function f given by $f(x)=\frac{1}{5}x^5-3x^4+12x^3+4x, x \in R$ is stictly increasing on R.

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9. Prove that function given by f(x)=sin x is

(i) Strictly increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(ii) Strictly decreasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

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10. Find the intervals in which $f(x)=4x^3-45x^2+168x-16$ is (i) Strictly increasing (ii) Strictly decreasing

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11. Find the equation of tangent to the curve $y = x^3 - x$, at the point at which slope of tangent is equal to zero.

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12. Find the point at which the tangent to the curve $y = \sqrt{4x - 3} - 2$ is vertical ?

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13. Find the equation of all the line having slope equal to 3 and being tangent to the curve $x^2 + y^2 = 4$

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14. Find the equation of tangent and normal to the curve $x^{\frac{4}{3}} + y^{\frac{4}{3}}$ = 32 at the point (8,8) .

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15. Find the equation of tangent to the curve by x = $t^2 + t + 1, y = t^2 - t + 1$ at a point where t = 1

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16. Find the equation of tangent to the curve xy = 16 which passes through (0,2) .

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17. Find out square root of 4.01 using differential .

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18. Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.

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19. Find the approximate change in the volume V of a cube of side one metre caused by increasing the side by 40% .

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20. Find maximum and minimum values , if any , of the function f given by f(x) = $x^4 + 1, x \in R.$

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21. Find the maximum and minimum value of f if any of the function $f(x) = |x - 1|, x \in R$.

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22. Find the maximum and minimum values , if any , of the function f given by $f(x) = 3 - |x|$

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23. Find all the points of local maxima and minima of the function f given by $f(x) = \sin x - \cos x, x \in [0, 2\pi]$

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24. Find all the points of local maxima and minima of the function f given by $f(x) = 4x^3 - 12x^2 + 12x + 10$

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25. Find the points of local maxima and minima of the function f given by $f(x) = x^3 - 12x^2 + 36x + 5$.

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26. Find the points of maximum (local maximum) and minimum (local minimum) of the function $f(x) = x^3 - 3x^2 - 9x$?

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27. Find the maximum and minimum values of any of the function f given by $f(x) = x^4, x \in R$.

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28. Find the maximum and minimum values of the function $y = x^3$, in $x \in [-2, 3]$?

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29. $f(x) = 2x^2 - 6x^2 + 6x + 5$ द्वारा प्रदत के लिए स्थानीय उच्चतम और स्थानीय निम्नतम के सभी बिंदुओं को ज्ञात कीजिए|

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30. Find the extrema points of $f(x) = 3x^4 - 4x^3 - 36x^2 + 28$

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31. सिद्ध कीजिए कि एक शंकु के अंतर्गत महत्तम वर्कपृष्ठ वाले लंब वृत्तीय बेलन की त्रिज्या शंकु की त्रिज्या की आधी होती है

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32. Find the shortest distance of the point $(0, c)$ from the parabola $y = x^2, where 0 \leq c \leq 5.$

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33. Find the absolute maximum and absolute minimum vlaues of a function f given by $f(x) = 2x^3 - 15x^2 + 36x + 3$ on the interval [0,6] .

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34. Show that for the curve $y = be^{x/a}$, the subtangent is of constant length and the subnormal varies as the square of ordinate.

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35. The curves $x^3 - 3xy^2 = a$ and $3x^2y - y^3 = b$, where a and b are constants, cut each other at an angle of

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36. Show that 2sinx+tanx≥3x, where 0≤x< π/2

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37. Use the function $f(x) = x^{\frac{1}{x}}, x > 0,$ to determine the bigger of the two numbers $e^{\pi} and \pi^{e}$.

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Try Yourself

1. The formula giving the altitude y(m) in terms of time t (seconds) of an object moving vertically is $y = y_0 + v_0 t - 16t^2$, where y_0 is initial altitude and v_0 is initial velocity .

If $y_0 = 8 \text{ and } v_0 = 48m/s$, what are the altitude , velocity and acceleration of the object when t=2s ?

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2. The vessel is filled with 1000 litres of water at 8 a.m. The volume of water in this vessel at t hours after it is filled is observed to be $v(t = 1000 - 200t + 5t^2$. What is the rate of change of volume (i) at 8:a .m. (ii) at noon (12:00 p.m.)?

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3. A manufacturer's total cost of production (in Rs.) of x things is $f(x) = 1000 + 10^{-10}x^4$. What is the marginal cost when 1200 things are produced ?

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4. The total revenue (in rupees) received from the sale of x units of LCD is given by $R(x) = 1000x^2 + 50x + 10$. Find the marginal revenue , when 10 units of LCD is sold.

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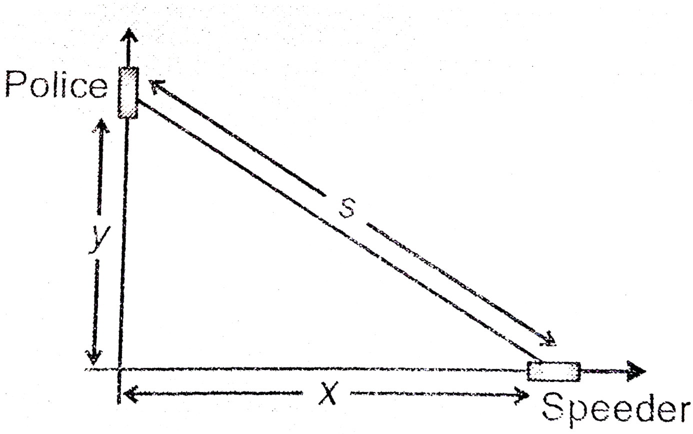
5. Consider a police going 60km/hour sees speeder to be going 20 km/hour away from him when x=0.8 km and y=0.6 km. Find the original speed of speeder at that time ? (see the diagram)



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6. 2 m ऊंचाई का आदमी 6 m ऊंचे बिजली के खंभे से दूर 5 km/h की समान चाल से चलता है| उसकी छाया की लमबायी की वृद्धि की दर ज्ञात कीजिए |

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7. Volume of a cube is increasing at the rate of $9cm^3/s$ if the length of its side is equal to 10 cm than at what rate its surface area is increasing?

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8. A stone is dropped into a well and progressive waves are moving in circles at a speed of 6cm per second . At the instantt when the radius of circular wave is 15 cm, how fast is enclosed area changing ?

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9. Show that $f(x) = [x]$, ([] implies greatest integer function) is increasing function on R.

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10. Show that $f(x) = \frac{1}{x}$ is a decreasing function on $(0, \infty)$.

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11. Prove that funciton given by

(i) $f(x) = x^2$ is

(a) Strictly increasing in $x \in (0, \infty)$

(b) Strictly decreasing in $x \in (-\infty, 0)$

(ii) $f(x) = \tan x$ is strictly increasing in $x \in \left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}\right)$, where $n \in I$.

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12. Find the interval in which the function f given by $f(x) = x^2 - 2x + 3$ is

(a) Strictly increasing

(b) Strictly decreasing

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13. Prove that $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$

$f\left(0, \frac{\pi}{4}\right)$ is increasing function

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14. Find the slope of tangent to curve `y=x+2log x at x=1.

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15. Distance between the points on the curve $4x^2 + 9y^2 = 1$, where tangent is parallel to the line 8x = 9y, is less than

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16. Find the point at which the tangent to the curve $y = \frac{3}{5}x^5 - \frac{5}{3}x^3 - x$ has its slope equal to -3

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17. Find pints on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

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18. Find the equation of a line having slope is equal to 2,which touching the curve$y + \frac{2}{x-3} = 0$

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19. Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

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20. Find the equations of the tangent and the normal to the curve $x^{2/3} + y^{2/3} = 2$ at $(1, 1)$ at indicated points.

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21. Find the equation of tangent to the curve given by $x = 2\sin^3 t, y = 2\cos^3 t$ at a point where $t = \frac{\pi}{2}$?

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22. The normal to the curve $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ at any θ is such that

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23. Prove that the segment of tangent to $xy = c^2$ intercepted between the axes is bisected at the point at contact .

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24. Find the equations of tangent to the curve $y = x^4$ which are drawn from the point (2,0).

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25. Find the approximate value of $\sqrt{36.6.}$

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26. If the radius of a sphere is measured as 9cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

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27. Find the approximate value of $\sin\left(\frac{5\pi}{16}\right)$.

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28. Find the maximum and minimum value if any , of the function f given by $f(x) = 5 - |x - 1|$.

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29. Find the maximum and minimum value if any , of the function f given by $f(x) = x^6 - 5$.

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30. Find the points of local maxima and minima of the function $f(x) = x^2 - 4x$.

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31. Find the points of local maxima and local minima of the function $f(x) = 2x^3 - 3x^2 - 12x + 8$.

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32. Find the maximum , minimum values of following functions (using graphical approach)

(a) $y=|x|, x \in R$

(b) $y=|\sin x|, x \in R$

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33. Find local minimum value of the function f given by $f(x)=3+|x|, x \in R.$

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34. Let $f(x) = 2x^3 - 3(a+b)x^2 + 6abx$. If $a > b$, determine the local maximum / minimum points of f(x). If a=b, how will the answer change ?

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35. Find local maximum and local minimum values of the function f given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$.

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36. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5$.

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37. The operating cost of a truck is $12 + \left(\frac{x}{6}\right)$ per km when the truck travels x km/hour. If the driver earns 6 Rs. per hour, what is the most economical speed to operate the truck on a 400 km road? Also due to construction, the truck can travel only between 35 and 60 km/hour?

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38. Let AP and BQ be two vertical poles at points A and B, respectively. If $AP = 16m, BQ = 22mandAB = 20m$, then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.

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39. Find the absolute maximum and absolute minimum vlaues of a function f given by $f(x) = 2x^3 - \frac{33x^2}{2} + 36x + 2$ on the interval $[-1, 9]$.

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Assignment Section A Competition Level Questions

1. suppose $x_1,$ and x_2 are the point of maximum and the point of minimum respectively of the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ respectively, $(a > o)$ then for the equality $x_1^2 = x_2$ to be true the value of $'a'$ must be

A. 0

B. 2

C. 1

D. $\frac{1}{4}$

Answer: B

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2. Point A lies on curve $y=e^{-x^2}$ and has the coordinates $\left(x, e^{-x^2}\right)$ where $x>0$ Point B has coordinates (x,o) If 'O' is the origin, then the maximum area of ΔAOB is :

A. $\dfrac{1}{\sqrt{2e}}$

B. $\dfrac{1}{\sqrt{4e}}$

C. $\dfrac{1}{\sqrt{e}}$

D. $\dfrac{1}{\sqrt{8e}}$

Answer: D

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3. The radius of a right circular cylinder increases at the rate of 0.1 cm/min, and the height decreases at the rate of 0.2 cm/min. The rate of change of the volume of the cylinder, in $cm^2 / m \in$, when the radius is $2cm$ and the height is 3cm is (a) -2π (b) $-\frac{8\pi}{5}$ $-\frac{3\pi}{5}$ (d) $\frac{2\pi}{5}$

A. -2π

B. $-\frac{8\pi}{5}$

C. $-\frac{3\pi}{5}$

D. $\frac{2\pi}{5}$

Answer: D

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4. The number of points of maxima/minima of $f(x) = x(x+1)(x+2)(x+3)$ is

A. 0

B. 1

C. 2

D. 3

Answer: D

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5. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

A. π

B. $\left(\sqrt{3} - \frac{\pi}{3}\right)$

C. $\frac{\sqrt{3}}{2} + \frac{\pi}{3}$

D. $-\frac{\sqrt{3}}{2} + \frac{2\pi}{3}$

Answer: A

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6. If a variable tangent to the curve $x^2y = c^3$ makes intercepts $a, bonx - andy - axes,$ respectively, then the value of a^2b is $27c^3$ (b) $\frac{4}{27}c^3$ (c) $\frac{27}{4}c^3$ (d) $\frac{4}{9}c^3$

A. $27c^3$

B. $\frac{4}{27}c^3$

C. $\frac{27}{4}c^3$

D. $\frac{4}{9}c^3$

Answer: C

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7. Difference between the greatest and the least values of the function $f(x) = x(\ln x - 2)$ on $[1, e^2]$ is

A. 2

B. e

C. e^2

D. 1

Answer: B

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8. If the sum of the lengths of the hypotenuse and another side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between these sides is $\frac{\pi}{3}$.

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{5\pi}{12}$

Answer: C

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9. Let C be the curve $y = x^3$ (where x takes all real values). The tangent at A meets the curve again at B. If the gradient at B is K times the gradient at A, then K is equal to (a) 4 (b) 2 (c) -2 (d) $\frac{1}{4}$

A. 4

B. 2

C. -2

D. $\frac{1}{4}$

Answer: A

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10. The interval on which $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing in

A. $(-1, \infty)$

B. $(-2, -1)$

C. $(-\infty, -2)$

D. $(-1, 1)$

Answer: B

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11. The function $f(x) = \cot^{-1} x + x$ increases in the interval (a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$

A. $(1, \infty)$

B. $(-1, \infty)$

C. $(-\infty, \infty)$

D. $(0, \infty)$

Answer: C

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12. Divide 64 into two parts such that the sum of the cubes of two parts is minimum.

A. 44,20

B. 16,48

C. 32,32

D. 50,14

Answer: C

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13. If $f(x) = x^5 - 5x^4 + 5x^3 - 10$ has local maximum and minimum at $x = p$ and $x = q$, respectively, then $(p, q) =$ (a) $(0, 1)$ (b) (1,3) (c) (1,0) (d) none of these

A. (0,1)

B. (1,3)

C. (1,0)

D. (2,4)

Answer: B

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14. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(a) (2,6) (b) $(2, -6)$ $\left(\frac{9}{8}, -\frac{9}{2}\right)$ (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$

A. $(2, 4)$

B. (2,-4)

C. $\left(-\frac{9}{8}, \frac{9}{2}\right)$

D. $\left(\frac{9}{8}, \frac{9}{2}\right)$

Answer: D

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15. The real number x when added to its inverse given the minimum value of the sum at x equal to (a) 1 (b) -1 (c) -2 (d) 2

A. 1

B. -1

C. -2

D. 2

Answer: A

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16. Which of the following statement is true for the function $f(x)=\begin{cases}\sqrt{x} & ,x\geq 1\\ x^{3} & ,0\leq x<1\\ \frac{x^{3}}{3}-4x & ,x<0\end{cases}$

A. It is monotonic increasing $\forall x\in R$

B. f'(x) fails to exist for 3 distinct real values of x

C. f'(x) changes its sign twice as x varies from $(-\infty,\infty)$

D. Function attains its extreme values at x_1 and x_2, such that $x_1x_2>0$

Answer: C

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17. The function $f(x) = x^3 - 3x$ is

A. Increasing in $(-\infty, -1) \cup (1, \infty)$ and decreasing in (-1,1)

B. Decreasing in $(-\infty, -1) \cup (1, \infty)$ and increasing in (-1,1)

C. Increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$

D. Decreasing in $(0, \infty)$ and increasing in $(-\infty, 0)$

Answer: A

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18. If $f(x)=x^3+x^2+kx+4$ is always increasing then least positive integral value of k is

A. 4

B. 3

C. 2

D. 1

Answer: D

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19. A curve is represented by the equations $x=\sec^2 tandy=\cot t,$ where t is a parameter. If the tangent at the point P on the curve where $t=\frac{\pi}{4}$ meets the curve again at the point Q, then $|PQ|$ is equal to $\frac{5\sqrt{3}}{2}$ (b) $\frac{5\sqrt{5}}{2}$ (c) $\frac{2\sqrt{5}}{3}$ (d) $\frac{3\sqrt{5}}{2}$

A. $\frac{5\sqrt{3}}{2}$

B. $\frac{5\sqrt{5}}{2}$

C. $\frac{2\sqrt{5}}{3}$

D. $\frac{3\sqrt{5}}{2}$

Answer: D

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20. Statement 1: For all $a, b \in R$, the function $f(x) = 3x^4 - 4x^3 + 6x^2 + ax + b$ has exactly one extremum. Statement 2: If a cubic function is monotonic, then its graph cuts the x-axis only one.

A. No extremum

B. Exactly one extremum

C. Exactly two extremum

D. Three extremum

Answer: B

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21. Maximum area of a reactangle which can be inscribed in a circle of a given radius R is

A. $\frac{\pi R^2}{3}$

B. $\frac{3R^2}{\sqrt{2}}$

C. $2R^2$

D. $3R^2$

Answer: C

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22. If atmosphere pressure at a height of h units is given by the function $P(h) = he^{-h}$ then pressure is maximum at the height of

A. 1 unit

B. $\frac{1}{\sqrt{e}}$ units

C. e units

D. 2 units

Answer: A

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23. The radius of cylinder of maximum volumne which can be inscribed in a right circular cone of radius R and height H (axis of cylinder and cone are same) is given by

A. $\frac{R}{2}$

B. $\frac{R}{3}$

C. $\frac{2R}{3}$

D. $\frac{2R}{5}$

Answer: C

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24. Let $f(x) = \begin{cases} -|x-2| & x \leq 3 \\ x^2 - 2x - 4 & x > 3 \end{cases}$

Then the number of critical points on the graph of the function is

A. 1

B. 2

C. 3

D. 4

Answer: B

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25. The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point:

A. (1,1)

B. (0,1)

C. (1,0)

D. No point

Answer: C

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26. Consider the function $f(x) = x\cos x - \sin x$. Then identify the statement which is correct. f is neither odd nor even. f is monotonic decreasing at $x = 0$ f has a maxima at $x = \pi$ f has a minima at $x = -\pi$

A. f is neither odd nor even

B. f is monotonic increasing in $\left(0, \frac{\pi}{2}\right)$

C. f has a maxima at $x = -\pi$

D. f has a minima at $x = -\pi$

Answer: C

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27. If $x = 2t - 3t^2$ and $y = 6t^3$ then $\frac{dy}{dx}$ at point $(-1, 6)$ is

A. $\frac{1}{3}$

B. $-\frac{9}{2}$

C. $\frac{8}{3}$

D. 0

Answer: B

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28. For the cubic function $f(x) = 2x^3 + 9x^2 + 12x + 1,$ which one of the following statement/statements hold good? $1.$ $f(x)$ is non-monotonic. $2.$ $f(x)$ increases in $(-\infty, -2) \cup (-1, \infty)$ and decreases in $(-2, -1)$ $3.$ $f : \overrightarrow{RR}$ is bijective. $4.$ Inflection point occurs at $x = -\frac{3}{2}.$

A. f(x) is increasing

B. Increasing in $(-\infty, -2) \cup (-1, \infty)$ and decreasing in (-2,-1)

C. f(x) is decreasing

D. f(x) is constant

Answer: B

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29. The function f is defined by $f(x) = x^p(x-1)^q \text{for all } x \in R,$ where p,q are positive integers, has a maximum value for x equal to

A. $\dfrac{pq}{p+q}$

B. (-1)

C. $\frac{1}{2}$

D. $\frac{p}{p+q}$

Answer: D

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30. Let h be a twice continuosly differentiable positive function on an open interval H. Let $g(x) = \log_e(h(x))$ for each $x \in H$

Suppose $(h'(x))^2 > h''(x)h(x)$ for each $x \in H$.

Then, we can conclude that

A. g is increasing on J

B. g is decreasing on J

C. g is concave up on J

D. g'(x) is decreasing function

Answer: D

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31. The least area of the circle circumscribing and right triangle of area S is

A. πS

B. πS

C. $\sqrt{2\pi}S$

D. $4\pi S$

Answer: A

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32. If $f(x) = x(1-x)^3$ then which of following is true ?

A. f(x) has local maxima at x=1

B. f(x) has local minima at x=1

C. f(x) has local minima at $x = \frac{1}{4}$

D. f(x) has local maxima at $x = \frac{1}{4}$

Answer: D

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33. Discuss monotonocityt of y =f(x) which is given by $x = \frac{1}{1+t^2}$ and $y = \frac{1}{t}\left(l + t^{22}\right), t > 0$

A. Increasing for $t \in \left(0, \frac{3}{2}\right)$ and decreasing for $t \in \left(\frac{3}{2}, \infty\right)$

B. Decreasing for $t \in \left(0, \frac{1}{2}\right)$

C. Increasing for $t \in (-, \infty)$

D. Decreasing for $t \in (0, 1)$

Answer: C

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34. The point on the curve $y^2 = x$ where tangent makes 45° angle with x-axis, is

A. $x - y = -\frac{1}{4}$

B. $x - y = \frac{1}{4}$

C. $x + y = \frac{1}{2}$

D. $x - y = \frac{1}{2}$

Answer: A

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35. The radius of a right circular cylinder increases at a constant rate. Its altitude is a linear function of the radius and increases three times as fast as the radius when the radius is $1\ cm$ and the altitude is $6\ cm$. When the radius is $6cm$, the volume is increasing at the rate of $1\ \frac{cm^3}{s}$. When the radius is $36cm$, the volume is increasing at a rate of $n\ \frac{cm^3}{s}$. What is the value of n?

A. 12

B. 22

C. 30

D. 33

Answer: D

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36. Two sides of a triangle are to have lengths 'a'cm & 'b'cm. If the triangle is to have the maximum area, then the length of the median from the vertex containing the sides 'a' and 'b'is

A. $\frac{1}{2}\sqrt{a^2+b^2}$

B. $\frac{2a+b}{3}$

C. $\sqrt{\frac{a^2+b^2}{2}}$

D. $\frac{a+2b}{3}$

Answer: A

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37. The cost function at American Gadget is $C(x) = x^3 - 6x^2 + 15x$ (x is in thousands of units and $x > 0$) . The production level at which average cost is minimum is:

A. (a) 2

B. (b) 3

C. (c) 5

D. (d) 4

Answer: A

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38. A particle moves along the curve $y = x^{\frac{3}{2}}$ in the first quadrant in such a way that its distance from the origin increases at the rate of 11 units per second. Then the value of $\frac{dx}{dt}$ when $x = 3$ is given by

A. 4

B. $\frac{9}{2}$

C. $\frac{3\sqrt{3}}{2}$

D. 12

Answer: A

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39. Let $f(x) = ax^2 - b|x|$, where $aandb$ are constants. Then at $x = 0$, $f(x)$ has a maxima whenever $a > 0, b > 0$ a maxima whenever $a > 0, b < 0$ minima whenever $a > 0, b < 0$ neither a maxima nor a minima whenever $a > 0, b < 0$

A. A maxima whenever $a > 0, b > 0$

B. A maxima whenever $a > 0, b < 0$

C. Minima whenever $a > 0, b > 0$

D. Neither maxima nor minima whenever $a > 0, b < 0$

Answer: A

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40. Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts on the axes.

A. $\left(1, \frac{1}{3}\right)$

B. $(3, \sqrt{3})$

C. $\left(4, \frac{8}{3}\right)$

D. $\left(\frac{6}{5}, \frac{2}{5}\sqrt{\frac{6}{5}}\right)$

Answer: C

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41. The angle made by the tangent of the curve x = a (t + sint cosf), y = a $(1+\sin t)^2$ with the x-axis at any point on it is

A. $\frac{1}{4}(\pi+2t)$

B. $\frac{1+\sin t}{\cos t}$

C. $\frac{1}{4}(2t-\pi)$

D. $\frac{1+\sin t}{\cos 2t}$

Answer: B

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42. A cube of ice melts without changing its shape at the uniform rate of $4\frac{cm^3}{\min}$. The rate of change of the surface area of the cube, in $\frac{cm^2}{\min}$, when the volume of the cube is $125cm^3$, is (a) -4 (b) $-\frac{16}{5}$ (c) $-\frac{16}{6}$ (d) $-\frac{8}{15}$

A. -4

B. $-\frac{16}{5}$

C. $-\frac{16}{6}$

D. $-\frac{8}{15}$

Answer: B

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43. Consider the curve represented parametrically by the equation $x = t^3 - 4t^2 - 3t$ and $y = 2t^2 + 3t - 5$ where $t \in R$.If H denotes the number of point on the curve where the tangent is horizontal and V the number of point where the tangent is vertical then

A. H=2 and V=1

B. H=1 and V=2

C. H=2 and V=2

D. H=1 and V=1

Answer: B

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44. The point on the curve $y = 6x - x^2$ where the tangent is parallel to x-axis is

A. (0,0)

B. (2,8)

C. (6,0)

D. (3,9)

Answer: D

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45. For the curve $y = 3\sin\theta\cos\theta, x = e^{\theta}\sin\theta, 0 \leq \theta \leq \pi,$ the tangent is parallel to x-axis when θ is :

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: C

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46. The slope of normal to the curve $x^3 = 8a^2y, a > 0$ at a point in the first quadrant is $-\frac{2}{3}$, then point is

A. (2a,-a)

B. (2a,a)

C. (a,2a)

D. (a,a)

Answer: B

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47. Find the euation of normal to the curve $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ at any point $'\theta'$

A. It passes through $\left(\frac{a\pi}{2}, -a\right)$

B. Is at a constant distance from origin

C. It passes through origin

D. It makes an angle $\left(\frac{\pi}{2}+\theta\right)$ with the '+' x-axis

Answer: B

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48. The equation of tangent to the curve $x=a\cos^3 t, y=a\sin^3 t$ at 't' is

A. $x\sec t - y\text{cosec}t = a$

B. $x\sec t + y\text{cosec}t = a$

C. $x\text{cosec}t + y\cos t = a$

D. $x\sec t + y\cos t = -a$

Answer: B

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49. If the tangent to the curve $x = t^2 - 1, y = t^2 - t$ is parallel to x-axis , then

A. t=0

B. $t \to \infty$

C. $t = \frac{1}{\sqrt{3}}$

D. $t = -\frac{1}{\sqrt{3}}$

Answer: A

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50. For the function $f(x) = x^2 - 6x + 8, 2 \leq x \leq 4,$ the value of x for which f'(x) vanishes is

A. 3

B. $\frac{5}{2}$

C. $\frac{9}{4}$

D. $\frac{7}{2}$

Answer: A

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Assignment Section B Objective Type Questions One Option Is Correct

1. The slope of the tangent of the curve $y = \int_0^x \frac{dx}{1 + x^3}$ at the point where $x = 1$ is

A. $\frac{1}{2}$

B. 1

C. $\frac{1}{4}$

D. $\frac{1}{5}$

Answer: A

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2. If at each point of the curve $y = x^3 - ax^2 + x + 1$, the tangent is inclined at an acute angle with the positive direction of the x-axis, then (a)$a > 0$ (b) $a < -\sqrt{3}$ (c)$-\sqrt{3} \leq a \leq \sqrt{3}$ (d) $none of these$

A. $a > 0$

B. $a \leq \sqrt{3}$

C. $(-\sqrt{3} < a < \sqrt{3})$

D. $2 \leq a \leq 3$

Answer: C

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3. The curve given by $x + y = e^{xy}$ has a tangent parallel to the y-axis at the point (a)$(0, 1)$ (b) $(1, 0)$ (c) $(1, 1)$ (d) none of these

A. (0,1)

B. (1,0)

C. (1,1)

D. (2,0)

Answer: B

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4. If m is the slope of a tangent to the curve $e^y = 1 + x^2$, then (a)$|m| > 1$ (b) $m > 1$ (c)$m \geq -1$ (d) $|m| \leq 1$

A. $|m| > 1$

B. $m < 1$

C. $|m| < 1$

D. $|m| \leq 1$

Answer: D

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5. The normal to the curve $2x^2 + y^2 = 12$ at the point $(2, 2)$ cuts the curve again at $\left(-\frac{22}{9}, -\frac{2}{9}\right)$ (b) $\left(\frac{22}{9}, \frac{2}{9}\right)$ $(-2, -2)$ (d) none of these

A. $\left(-\frac{22}{9}, -\frac{2}{9}\right)$

B. $\left(\frac{22}{9}, \frac{2}{9}\right)$

C. $(-2, -2)$

D. $(0, \sqrt{12})$

Answer: A

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6. If $1^0 = \alpha$ radians, then find the approximate value of $\cos 60^0 1'$.

A. $\frac{1}{2} + \frac{\alpha\sqrt{3}}{120}$

B. $\frac{1}{2} - \frac{\alpha}{120}$

C. $\frac{1}{2} - \frac{\alpha\sqrt{3}}{120}$

D. $\frac{1}{2}$

Answer: C

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7. The sum of the intercepts made on the axes of coordinates by any tangent to the curve

$\sqrt{x} + \sqrt{y} = \sqrt{a}$ is equal to

A. 2a

B. a

C. $\frac{a}{2}$

D. $2\sqrt{a}$

Answer: B

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8. The slope of the tangent to the curve $x = t^2 + 3t - 8, \ y = 2t^2 - 2t - 5$ at the point $(2, \ -1)$ is (a)$22/7$

(b) $6/7$

(c) $7/6$

(d) $-6/7$

A. $\frac{7}{6}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. $\frac{6}{7}$

Answer: D

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9. The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is(are) ? (a) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (b) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$ (c) $(0, 0)$ (d) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

A. $(\pm 4\sqrt{3}, -2)$

B. $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$

C. (0,0)

D. $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

Answer: D

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10. If $f(x) = x^3 + 4x^2 + \lambda x + 1$ is a monotonically decreasing function of x in the largest possible interval $\left(-2, -\frac{2}{3}\right)$. Then (a)$\lambda = 4$ (b) $\lambda = 2$ $\lambda = -1$ (d) λ has no real value

A. $\lambda = 4$

B. $\lambda = 2$

C. $\lambda = -1$

D. λ has no real value

Answer: A

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11. If a function $f(x)$ increases in the interval (a, b). then the function $\phi(x) = [f(x)]^n$ increases in the same interval and $\phi(x) \neq f(x)$ if

A. $n = -1$

B. $n = 0$

C. $n = 3$

D. $n = 4$

Answer: C

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12. The function f, defined by $f(x) = \frac{x^2}{2} + Inx - 2\cos x$ increases for $x \in$

A. R^-

B. R^+

C. $R - \{0\}$

D. $[1, \infty)$

Answer: B

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13. If $f(x) = x.\, e^{x(1-x)}$, then f(x) is

A. Increasing on R

B. Increasing on $\left[-\frac{1}{2}, 1\right]$

C. Decreasing on R

D. Decreasing non $\left[-\frac{1}{2}, 1\right]$

Answer: B

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14. Which of the following is correct ?

A. $In(a+x) < x \forall -a < x \leq 0$

B. $In(1+x) < x \forall 0 < x$

C. $In(1+x) > x \forall x > 0$

D. $In(1+x) < x \forall x > -1$

Answer: B

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15. Find the value of a, if the equation $x - \sin x = a$ has a unique root in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

A. $a \in \left[-\frac{\pi}{2} + 1, \infty\right)$

B. $a \in \left(-\infty, \frac{\pi}{2} - 1\right]$

C. $a \in \left[1 - \frac{\pi}{2}, \frac{\pi}{2} - 1\right]$

D. $a \in R - \left(1 - \frac{\pi}{2}, \frac{\pi}{2} - 1\right)$

Answer: C

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16. Number of real roots of the equation $e^{x-1} - x = 0$ is

A. 1

B. 2

C. 3

D. 0

Answer: A

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17. If

$$f(x)=\frac{x}{\sin x}\ and\ g(x)=\frac{x}{\tan x},\ where\ 0<x\le 1,$$

then in this interval

A. Both f(x) and g(x) are increasing functions

B. Both f(x) and g(x) are decreasing functions

C. f(x) is an increasing function

D. g(x) is an increasing function

Answer: C

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18. If $f(x)=\frac{p^2-1}{p^2+1}x^3-3x+\log 2$ is a decreasing function of x.in, R then the set of possible values of p (independent of x) is

A. $[-1, 1]$

B. $(1, \infty)$

C. $(-\infty, 1]$

D. $(2, \infty)$

Answer: A

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19. Let $f(x) = (x-a)^2 + (x-b)^2 + (x-c)^2$. Then, $f(x)$ has a minimum at $x = \dfrac{a+b+c}{3}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{8}$ (d) none of these

A. $\dfrac{p+q+r}{3}$

B. $\sqrt[3]{pqr}$

C. $\dfrac{3}{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$

D. $\dfrac{3}{p+q+r}$

Answer: A

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20. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$ is

A. e

B. $e^{\left(\frac{1}{e}\right)}$

C. 1

D. e^{e}

Answer: B

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21. If $f(x) = a\log|x| + bx^2 + x$ has extreme values at $x = -1\, and\, at\, x = 2,$ then find a and b .

A. a=2, b=-1

B. a=2 , $b = -\frac{1}{2}$

C. $a = -2, b = \frac{1}{2}$

D. $a = \frac{1}{2}, b = -\frac{1}{2}$

Answer: B

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22. If $x \in [-1, 1]$ then the minimum value of $f(x) = x^2 + x + 1$ is

A. $\frac{3}{4}$

B. 1

C. 3

D. $-\frac{3}{4}$

Answer: A

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23. If $ax+\frac{b}{x}\geq c, \forall a>0 \text{ and } a, b, c$ are positive constant then

A. $ab\geq\frac{c^2}{4}$

B. $ab\leq\frac{c^2}{4}$

C. $bc\geq\frac{a^2}{4}$

D. $ac\geq\frac{b^2}{4}$

Answer: A

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24. The number which exceeds its square by the greatest possible quantity is $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) none of these

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{3}{4}$

D. 1

Answer: A

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25. The point (0,3) is nearest to the curve $x^2 = 2y$ at

A. $(2\sqrt{2}, 0)$

B. $(0, 0)$

C. (2,2)

D. (2,-2)

Answer: C

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26. Let $f(x)$ be a function defined as follows: $f(x) = \sin\left(x^2 - 3x\right), x \leq 0; and 6x + 5x^2, x > 0$

Then at $x = 0$, $f(x)$ (a)has a local maximum (b)has a local minimum (c)is discontinuous (d) none of these

A. Ha a local maximum

B. Has a local minimum

C. Is discontinuous

D. Point of inflexion

Answer: B

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27. If λ, μ are real numbers such that , $x^3 - \lambda x^2 + \mu x - 6 = 0$ has its real roots and positive, then the minimum value of $\mu,$ is

A. $3 \times \sqrt[3]{36}$

B. 11

C. 0

D. 1

Answer: A

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28. The value of c in Lagrange's mean value theorem for the function $f(x) = |x|$ in the interval $[-1, 1]$ is

A. 0

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. Non-existent in the interval

Answer: D

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29. If a, b, c, d are real numbers such that $\frac{3a + 2b}{c + d} + \frac{3}{2} = 0$ then the equation

$ax^3 + bx^2 + cx + d = 0$ has

A. At least one root in $[-2, 0]$

B. At least one root in $[0, 2]$

C. At least two root in [-2,2]

D. No root in [-2,2]

Answer: B

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30. If the least area of triangle formed by tangent,normal at any point P on the curve $y = f(x)$

and X-axis is 4 sq. unit. Then the ordinate of the point P (P lies in first quadrant) is

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. 2

Answer: D

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31. Let $f''(x) > 0 \forall x \in R$ and let $g(x) = f(x) + f(2 - x)$ then interval of x for which g(x) is increasing is

A. $(1, \infty)$

B. R

C. $[-1, 1]$

D. $[-2, \infty)$

Answer: A

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Assignment Section C Objective Type Questions More Than One Option Are Correct

1. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then $a > 0, b > 0$ $a > 0, b < 0$ $a \langle 0, b \rangle 0$ (d) $a < 0, b < 0$ none of these

A. $a < 0, b > 0$

B. $a > 0, b < 0$

C. $a > 0, b > 0$

D. $a < 0, b < 0$

Answer: A::B

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Assignment Section C Objective Type Questions More Than One Option Is Correct

1. The equation(s) of the tangent(s) to the curve $y = x^4$ from the point (2, 0) not on the curve is given by

A. y=0

B. y-1=5(x-1)

C. $y - \frac{4096}{81} = \frac{2048}{27}\left(x - \frac{8}{3}\right)$

D. $y - \frac{32}{243} = \frac{80}{81}\left(x - \frac{2}{3}\right)$

Answer: A::C

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2. Suppose f'(x) exists for each x and $h(x) = f(x) - (f(x))^2 + (f(x))^3 \forall x \in R$, then

A. h is increasing whenever f is increasing

B. h is increasing whenever f is decreasing

C. h is decreasing whenever f is decreasing

D. Nothing can be said general

Answer: A::C

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3. If $f'(x) = g(x)(x-a)^2$, where g(a) $\neq 0$ and g is continuous at x = a, then :

A. f is increasing near a if $g(a) < 0$

B. f is decreasing near a if $g(a) > 0$

C. f is decreasing near a if $g(a) < 0$

D. f is increasing near a if $g(a) > 0$

Answer: C::D

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4. let $f(x) = (x^2 - 1)^n (x^2 + x - 1)$ then $f(x)$ has local minimum at $x = 1$ when

A. n=2

B. n=3

C. n=4

D. n=5

Answer: A::C

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5. The critical points of $f(x) = (x-2)^{\frac{2}{3}}(2x+1)$ are

A. $-1 \text{ and } 2$

B. 1

C. $1 \text{ and } -\frac{1}{2}$

D. $1 \text{ and } 2$

Answer: D

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Assignment Section D Linked Comprehension Type Questions

1. Let the solution set of the inequation $n\left(\sin x-\frac{1}{2}\right)\left(\sin x-\frac{1}{\sqrt{2}}\right)\leq 0\in\left[\frac{\pi}{2},\pi\right]$ be A and let solution set of equation $\sin^{-1}(3x-4x^3)=3\sin^{-1}x$ be B. Now define a function $f:A\to B$.

A. (a) $\frac{12x}{\pi}-\frac{19}{2}$

B. (b)$-\frac{12x}{\pi}-\frac{19}{2}$

C. (c)$\frac{12x}{\pi}+\frac{19}{2}$

D. (d)$12\pi+\frac{19}{2}x$

Answer: A

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2. Let the solution set of the inequation $n\left(\sin x-\frac{1}{2}\right)\left(\sin x-\frac{1}{\sqrt{2}}\right)\leq 0\in\left[\frac{\pi}{2},\pi\right]$ be A and let solution set of equation $\sin^{-1}(3x-4x^3)=3\sin^{-1}x$ be B. Now define a function $f: A\rightarrow B$.

A. (a)$\cos\theta = f(x)$, for some $s \in A, \theta \in R$

B. (b)$\sin\theta + \cos\theta = f(x)$, for some $x \in A, \theta \in R$

C. (c)$\tan\theta = f(x)$ for some $x \in A, \theta \in R$

D. (d)$\sec\theta = f(x)$ for some $x \in A$ and $\theta \in R$

Answer: D

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3. Let the solution set of the inequation $n\left(\sin x - \frac{1}{2}\right)\left(\sin x - \frac{1}{\sqrt{2}}\right) \leq 0 \in \left[\frac{\pi}{2}, \pi\right]$ be A and let solution set of equation

$\sin^{-1}(3x - 4x^3) = 3\sin^{-1} x$ be B. Now define a function $f: A \to B$.

A. (a) $\left[\frac{3\pi}{4}, \frac{1}{2}\right]$

B. (b) $\left[\frac{5\pi}{6}, 3\right]$

C. (c) $\left[\frac{3\pi}{4}, 3\right]$

D. (d) $(3, \infty)$

Answer: C

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Assignment Section E Assertion Reason Type Questions

1. Statement-1 : Let $f(x)=|x^2-1|, x \in [-2, 2] \Rightarrow f(-2)=f(2)$ and hence there must be at least one $c \in (-2, 2)$ so that $f'(c)=0$, Statement 2: $f'(0)=0$, where f(x) is the function of S_1

A. (a)Statement-1 is True , Statement-2 is True , Statement-2 is a correct explanation for Statement-1 .

B. (b)Statement-1 is True , Statement-2 is True , Statement-2 is NOT a correct explanation for Statement-1 .

C. (c)Statement-1 is True , Statement-2 is False

D. (d)Statement-1 is False , Statement-2 is True

Answer: D

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2. STATEMENT-1: Let f(x) and g(x) be two decreasing function then f(g(x)) must be an increasing function .

and

STATEMENT-2 : $f(g(2)) > f(g(1))$, where f and g are two decreasing function .

A. Statement-1 is True , Statement-2 is True , Statement-2 is a correct explanation for

Statement-1 .

B. Statement-1 is True , Statement-2 is True , Statement-2 is NOT a correct explanation for Statement-1 .

C. Statement-1 is True , Statement-2 is False

D. Statement-1 is False , Statement-2 is True

Answer: B

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3. Statement 1: $f(x) = x^3$ is a one-one function.

Statement 2: Any monotonic function is a one-one function.

A. (a) Statement 1 and Statement 2 are true and Statement 2 is the correct explanation for Statement 1.

B. (b) Statement 1 and Statement 2 are true but Statement 2 is not the correct explanation for Statement 1.

C. (c) Statement 1 is true but Statement 2 is false

D. (d) Statement 2 is true but Statement 1 is false

Answer: A

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4. STATEMENT-1: $f(x) = |x-1| + |x+2| + |x-3|$ has a local minima . and STATEMENT-2 : Any differentiable function f(x) may have a local maxima or minima if f'(x)=0 at some points

A. Statement-1 is True , Statement-2 is True , Statement-2 is a correct explanation for Statement-1 .

B. Statement-1 is True , Statement-2 is True , Statement-2 is NOT a correct explanation for Statement-1 .

C. Statement-1 is True , Statement-2 is False

D. Statement-1 is False , Statement-2 is True

Answer: A

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Assignment Section F Matrix Match Type Questions

1. Match the greatest value of the function

Column-I	Column-II
(A) $y=\sqrt{100-x^2}$ on $[-6, 8]$	(p) No greatest value
(B) $y = 2\tan x - \tan^2 x$ on $\left[0, \frac{\pi}{2}\right)$	(q) 10
(C) $y=\tan^{-1}\frac{1-x}{1+x}$ on $[0, 1]$	(r) $\frac{\pi}{4}$
(D) $y=\frac{a^2}{x}+\frac{b^2}{1-x}$ on $(0, 1)$, $a > 0$, $b > 0$	(s) 1

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2. Let the function defined in Colomn-I have domain $\left(0, \dfrac{\pi}{2}\right)$ then match the following

Column-I	Column-II
(A) $x^2 + 2\cos x + 2$ has	(p) Local maximum at $\cos^{-1}\left(\frac{2}{3}\right)$
(B) $9x - 4\tan x$ has	(q) Maximum at $x = \frac{1}{2}$
(C) $\left(\frac{1}{2} - x\right)\cos x + \sin x - \frac{x^2 - x}{4}$ has	(r) No local extremum
(D) $\left(\frac{1}{2} - x\right)\cos \pi(x+3) + \frac{1}{\pi}\sin \pi(x+3)$ has	(s) Minima at $x = 1$

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3. Match the following

Column I	Column II
(A) The length of interval in which $f(x) = x^2e^{-x}$ is monotonic increasing is	(p) 1
(B) If $y = kx^3 + 3x^2 + (2k+1)x + 1000$ is strictly increasing for all values of x, then the least of positive integral value which k can attain, is	(q) 2
(C) If the sum of the squares of intercepts on axes made by a tangent at any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is a^k then k is	(r) Irrational number
(D) If $\dfrac{\log x}{x}$ attains maximum value at $x = k$ then k is	(s) Greater than 1
	(t) An integer

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4. Match the following

Column I	Column II
(A) Let k be the number which can be the slope of a tangent to the curve $x^2 + y^2 - 2x + 2y + 2xy = 1$, then k is	(p) Less than 1
(B) $f(x) = 2\sec x - \sqrt{3}\tan x$, then minimum value of $f(x)$ is	(q) 1
(C) If the function $f(x) = \dfrac{ax+b}{(x-1)(x-4)}$ has a local minima at $(2, -1)$, then a is	(r) Whole number
(D) Let $f(x) = \max\{\lvert\sin x\rvert, \lvert\cos x\rvert\}\ \forall x \in R$ then minimum value of $f(x)$ is	(s) Irrational number
	(t) $\dfrac{1}{\sqrt{2}}$

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Assignment Section G Integer Answer Type Questions

1. .The ratio of absolute maxima and minima of $f(x) = \frac{x^2 - x + 1}{x^2 + x + 1} x \varepsilon R$ is

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2. Consider the function $f(x) = x + \frac{1}{x}, x \in \left(\frac{1}{2}, \frac{9}{2}\right)$. If α is the length of interval of decreasing and β be the length of internal of increase , then $\frac{\beta}{\alpha}$ is __________

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Assignment Section H Multiple True False Type Questions Identify The Correct Statement Of True T And False F Of The Given Three Statements

1. STATEMENT-1 : If two sides of a triangel are given , then its area will be maximum if the angle between the given side be $\frac{\pi}{2}$.

STATEMENT-2 : The function $f(x) = \frac{1}{1+x^2}$ is increases in the interval $(0, \infty)$

STATEMENT-3: The lenght of the subnormal of the curve $y^3 = 12ax$ at y=1 is 4a

A. TFT

B. TTT

C. FFF

D. FFT

Answer: A

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Assignment Section I Subjective Type Questions

1. Let the circle $x^2 + y^2 = 4$ divide the area bounded by tangent and normal at $(1, \sqrt{3})$ and x-axis in A_1 and A_2. Then $\frac{A_1}{A_2}$ equals to

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2. The minimum value of 64 $\sec\theta + 27\text{cosec}\theta$ where $\theta \in \left(0, \frac{\pi}{2}\right)$ is ________

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3. If the tangent to the curve $2y^3 = ax^2 + x^3$ at the point (a,a) cuts off intercept α and β on the co-ordinate axes , (where $\alpha^2 + \beta^2 = 61$) then a^2 equals

______

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4. If the curves $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ intersect at right angles , then $3a^2$ is equal to _______

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5. Least natural number a for which

$x + ax^{-2} > 2, \forall x \in (0, \infty)$ is

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Assignment Section J Aakash Challengers Questions

1. The area of the triangle formed by the tangent to the curve $y = \frac{8}{4 + x^2}$ at x=2 and the co-ordinate axes is

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2. Any tangent at a point p(x,y) to the ellipse $\frac{x^2}{8}+\frac{y^2}{18}=1$ meets the co-ordinate axes in the points A and B such that the area of the ΔOAB is least , then the point P is of the form (m,n) where $m+n+10$ is

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3. If the tangent at P(1,1) on the curve $y^2=x(2-x)^2$ meets the curve again at A , then the points A is of the form $\left(\frac{3a}{b},\frac{a}{2b}\right)$, where a^2+b^2 is

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4. The curve $y = ax^3 + bx^2 + cx$ is inclined at 45° to x-axis at (0,0) but it touches x-axis at (1,0) , then a+b+c+10 is

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5. Let $f(x) = 2x^3 + ax^2 + bx - 3\cos^2 x$ is an increasing function for all $x \in R$ such that $ma^2 + nb + 18 < 0$ then the value of m+n+7 is

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6. Let $g(x) = 2f\left(\frac{x}{2}\right) + f(2 - x)$ and $f''(x) < 0 \forall x \in (0, 2)$.

If $g(x)$ increases in (a, b) and decreases in (c, d), then the value of $a + b + c + d - \frac{2}{3}$ is

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7. If the equation $3x^2 + 4ax + b = 0$ has at least one root in $(0, 1)$ such that $La + Mb + N = 0$, where L, M, N are co-prime numbers, then the value of $L + M + N + LMN$ is

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8. The set of all values of 'b' for which the function

$f(x) = (b^2 - 3b + 2)(\cos^2 x - \sin^2 x) + (b - 1)x + \sin 2$

does not possess stationary points is

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