MATHS

JEE (MAIN AND ADVANCED MATHEMATICS) FOR BOARD AND COMPETITIVE EXAMS

CONTINUITY AND DIFFERENTIABILITY

Example

1. Check the continuity of the function given by f(x) = 3x -5 t at 5 =1

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2. Examine whether the function f given by $f(x) = x^3$ is continuous at $x = 0$

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3. Discuss the continuity of the function $f(x) = |x - 1|$ at $x = 1$

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4. Show that the function f given by $f(x) = \begin{cases} x^2 + 5 & \text{if} \quad x \neq 0 \\ 3 & \text{if} \quad x = 0 \end{cases}$ is not continuous at x=0

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5. Discuss the continuity of the function f defined by $f(x) = \frac{1}{x - 1}, x \neq 1$

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6. Discuss the continuity of the function f defined by $f(x) = \begin{cases} x + 3 & \text{if} \quad x \leq 1 \\ x - 3 & \text{if} \quad x > 1 \end{cases}$

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7. Find all the points of discontinuity of the function $f(x)$ defined by

$$f(x)=\begin{cases}x+1 & \text{if } x<1\\ 1 & \text{if } x=1\\ x-1 & \text{if } x>1\end{cases}$$

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8. Disucss the continuity of the function f given by

$$f(x)=\begin{cases}-x & \text{if } x\geq 0\\ -x^2 & \text{if } x<0\end{cases}$$

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9. Find all the points of discontinuity of the greatest integer function defined by $f(x)=[x]$, where [x] denotes the greatest integer less than or equal to x.

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10. Show that the function f defined by $f(x) = |1 - x + |x||$ is everywhere continuous.

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11. Show that $f(x) = |x|$ is not differentiable at $x = 0$.

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12. Find the derivative of the following functions w.r.t x

(i) $\sqrt{3x - 2}$ for $x > \frac{2}{3}$

(ii) $\left(2x^2 + 3\right)^{\frac{5}{3}}(x + 5)^{-\frac{1}{3}}$

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13. Differentiate w.r.t x : sin $\left(x^3\right)$

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14. Differentiate w.r.t x f (x) sin 3x $\sin^3 x$

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15. Find $\frac{dy}{dx}$, when $y = e^{-x^2} \sin(\log x)$

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16. Find $\frac{dy}{dx}$ if $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

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17. If $y = x \sin y$, prove that $x. \frac{dy}{dx} = \frac{y}{1 - x \cos y}$

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18. Find the derivative of $\cos^{-1} x$ assuming that it exists.

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19. Differentiate $\sqrt{\sin^{-1}\sqrt{x}}$ w.r.t. x, $(0 < x < 1)$

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20. Differentiate w.r.t x , $y = \tan^{-1}\left(\frac{x}{\sqrt{1+x^2}-1}\right)$

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21. Differentaite function w.r.t x : $f(x) = \frac{x\sqrt{x^2+4}}{(3x+4)^{\frac{2}{3}}}, x > 0$

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22. Find $\frac{dy}{dx}$ when $y = \left(x^{\log x}\right)(\log x)^x, x > 1$

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23. Differentiate $x^x + (\sin x)^{\log x}$ w.r.t. x ,$x > 0$

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24. find $\frac{dy}{dx}$, when $y = \sin u,, u = e^{\sqrt{t}}$ and$t = \log x$

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25. If $x = \frac{1 - t^2}{1 + t^2}$ and $y = \frac{2t}{1 + t^2}$, prove that $\frac{dy}{dx} + \frac{x}{y} = 0$

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26. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

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27. Find $\frac{d^2y}{dx^2}$, when $y = e^x \sin x$

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28. If $y = \log\left[x + \sqrt{x^2 + 1}\right]$, prove that $\left(x^2 + 1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

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29. if $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \quad \text{prove that } \frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$

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30. If $x = 2\cos\theta - \cos 2\theta$

$y = 2\sin\theta - \sin 2\theta$

Find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$

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31. Verify Rolle's theorem for the function f(x) = $x^2 + x - 6$ in the interval [-3,2]

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32. Verify Lagrange's mean theorem for the function $f(x) = x^2 + x - 1$ in the interval [0,4]

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33. Evaluate $\lim_{x \to -\infty} \frac{\sqrt{x^2 + 3x + 1}}{2x + 4}$

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34. Evaluate the following using :L ' Hospital 's rule .

(i) if f(x) be a twice differentiable function and f'' (0) =2 , then find

$$\lim_{x \to 0} \frac{2f(x) - 3f(3x) + f(4x)}{x^2}$$

(ii) if f(a) =2, f' (a) = 1, g (a) =2 , then find $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$

(iii) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

(iv) $\lim_{x \to 0+} xInx$

(v) $\lim_{x \to 0} (\cot x)^{\sin x}$

(vi) $\lim_{x \to 0} \frac{\tan x + 4\tan 2x - 3\tan 3x}{x^2 \tan x}$

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35. (i) Let $h(x) = \lim_{x \to \infty} \frac{x^{2n} f(x) + g(x)}{1 + x^{2n}}$, find h(x) in terms of f(fx) and g(x)

(ii) without using L Hospital rule or series expansion for e^xevaluate

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$$

(iii) $\lim_{n \to \infty} \left[\frac{e^{\frac{1}{n}}}{n^2} + 2\frac{\left(e^{\frac{1}{n}}\right)^2}{n^2} + 3.\frac{\left(e^{\frac{1}{n}}\right)^3}{n^2} + \ldots\ldots + n\frac{\left(e^{\frac{1}{n}}\right)^n}{n^2}\right]$

(iv) $\lim_{x \to 0} \left[\frac{a\sin x}{x}\right] + \left[\frac{b\tan x}{x}\right]$ Where a,b are inegers and [] denotes integral part.

(v) $\lim_{x \to a}\left(\frac{\sin x}{\sin a}\right)^{\frac{1}{x-a}}$

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36. Evaluate $\lim_{x \to \infty}\frac{[x]}{2x}$

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37. if $f(x) = \begin{cases} 1+x & 0 \le x \le 2 \\ 3-x & 2 < x \le 3 \end{cases}$

Determine the points of discontinuity of the function f (f(x))

(ii) Check the continuity of the function f(x) = $[x]^2 - [x^2]$

(iii) find the values of a and b if f is continuous at $x = \frac{\pi}{2}$

$$f(x) = \begin{cases} \left(\frac{8}{5}\right)\frac{\tan 8x}{\tan 5x} & 0 < x < \pi/2 \\ a+4 & x = \pi/2 \\ (1+|\cot x|)\frac{b|\tan x|}{a} & \frac{\pi}{2} < x < \pi \end{cases}$$

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38. (i) if $f\left(\frac{x+2y}{3}\right)] = \frac{f(x)+2f(y)}{3}, x, y \in R$ and f (0) exists and is finite ,show that f(x) is continuous on the entire number line.

(ii) Let f : $R \rightarrow R$ satisfying f(x) -f (y)| $\leq |x-y|^3$, $\forall x, y \in R$, The prove that f(x) is a constant function.

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39. If $x = a\sin 2\theta(1+\cos 2\theta), y = b\cos 2\theta(1-\cos 2\theta)$, then $\frac{dy}{dx}$=

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40. (i) If $y^{1/m} + y^{-1/m} = 2x,$ then prove that $(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$

(ii) If y = ln $\left(x+\sqrt{1+x^2}\right)$, then prove that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

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41. Find the values of a and b if f is continuous at x=0, where

$$f(x) = \begin{cases} (\sin x + \cos x)^{\cos ecx} & -\frac{\pi}{2} < x < 0 \\ a & x = 0 \\ \frac{e^{1/x} + e(2/x) + e^{3/x}}{ae^{-2+1/x} + be^{-1+3/x}} & 0 < x < \frac{\pi}{2} \end{cases}$$

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42. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants.

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Try Youself

1. Prove that the function $f(x) = \begin{cases} \frac{\sin x}{x} & x < 0 \\ x^2 + 1 & x \geq 0 \end{cases}$ in $x \in \mathbb{R}$ is continuous

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2. Show that the function $f(x) = \begin{cases} x + \lambda & x < 1 \\ \lambda x^2 + 1 & x \geq 1 \end{cases}$

is a continuous function, regardless of the choice of $\lambda \in R$

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3. Show that the given function f(x) = |2 - 3x + |x|| is continuous for every

$x \in R$

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4. Find the domain and range of $f(x) = \sin^{-1}(x - [x])$, where [.]

represents the greatest integer function.

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5. Differentiate $f(x) = (x + 2)^{\frac{2}{3}}(1 - x)^{\frac{1}{3}}$ with respect to x

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6. Find the derivative of $\cos(\sin x^2)at x = \sqrt{\frac{\pi}{2}}$

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7. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{x}{y}\right)$ then show that $\frac{dy}{dx} = \frac{y - x}{y + x}$

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8. Find $\frac{dy}{dx}$ when $y = \sin^{-1}\sqrt{\frac{1 + x^2}{2}}$

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9. Differentiate the following functions w.r.ts

(i) $\frac{e^{x^2 \tan^{-1} x}}{\sqrt{1 - x^2}}$

(ii) $x^{\sin x} \quad (x > 0)$

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10. Find $\frac{dy}{dx}$ of $x^y + y^x = a^b$

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11. If

$x = a(\sin\theta - \theta\cos\theta)$ and $y = a(\cos\theta + \theta\sin\theta)$ find $\frac{dy}{dx} at \theta = \frac{\pi}{4}$

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12. If $x = a\left(\cos t + \log\left(\tan\left(\frac{t}{2}\right)\right)\right), y = a\sin t, then \frac{dy}{dx} =$

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13. If $y = x^x$, find $\frac{d^2y}{dx^2}$.

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14. If $y = A\cos(\log x) + B\sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

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15. Verify Rolle's theorem for the function $f(x) = x^3 - 9x^2 + 26x - 24$ in the interval [2.4]

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16. Verify Lagrange's mean value theorem for the function $f(x) = 2x^2 - 10x + 29$ in the interval [2.7]

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Assignment Section A

1. Let $f(x) = \begin{cases} x & \text{for} \quad 0 \leq x < 1 \\ 3 - x & \text{for} \quad 1 \leq x \leq 2 \end{cases}$

Then f(x) is

A. continuous at x =1

B. Right continuous at x =1

C. Left continuous at x =1

D. Limit exists at x =1

Answer: B

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2. If the function f(x) = (1-x) $\tan \frac{\pi x}{2}$ is continuous at x =1 then f (1) =

A. $\frac{2}{\pi}$

B. $\frac{\pi}{2}$

C. 0

D. 2π

Answer: A

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3. Let $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & x \neq 0 \\ k & x = 0 \end{cases}$

then f(x) is continuous at x = 0 if

A. k =1

B. k =0

C. k = 2

D. k = -1

Answer: B

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4. Let f(x)= $\begin{cases} \frac{3|x|+4\tan x}{x} & x \neq 0 \\ k & x = 0 \end{cases}$

Then f(x) is continuous at x = 0 for ,

A. k =7

B. k =1

C. No k

D. k = 2

Answer: C

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5. Let $f(x) = \begin{cases} (x+a) & x < 1 \\ ax^2 + 1 & x \geq 1 \end{cases}$ then f(x) is continuous at x =1 for

A. a = 0

B. a = 1

C. All a $\in R$

D. No value of a

Answer: C

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6. If the function $f(x)=\frac{x\left(e^{\sin x}-1\right)}{1-\cos x}$ is continuous at x =0 then f(0)=

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7. Let $f(x)=\frac{x(2^{x}-1)}{1-\cos x}$ for $x\neq 0$ what choice of f(0) , if any, will make f (x) continuous at x = 0 ?

A. log 2

B. $\frac{1}{2}\log 2$

C. 1/(2 log 2)`

D. 2 log 2

Answer: D

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8. If $f(x) = \begin{cases} [x] + [-x] & x \neq 0 \\ \lambda & x = 0 \end{cases}$ where [.] denotes the greatest function , then f(x) is continuous at $x = 0, \quad \text{for} \lambda$

A. -1

B. 0

C. 1

D. No value is possible

Answer: A

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9. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]} & [x] \neq 0 \\ 0 & [x] = 0 \end{cases}$

where [.] denotes the greatest integer less than or equal to x then

A. Continuous at x =0

B. Left continuous at x =0

C. Discontinuous at x =0

D. Right continuous at x = 0

Answer: C

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10. Let $f(x) = \sin\frac{1}{x}, x \neq 0$ Then f(x) can be continuous at x =0

A. If f(0) =1

B. If f(0) =0

C. If f(0) = -1

D. For no value of (0)

Answer: D

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11. If $f(x) = \begin{cases} px^2 - q & x \in [0, 1) \\ x + 1 & x \in (1, 2] \end{cases}$

and $f(1) = 2$, then the value of the pair (p, q) for which $f(x)$ cannot be continuous at $x = 1$ is:

A. (a) $(2, 0)$

B. (b) $(1, -1)$

C. (c) $(4, 2)$

D. (d) $(1, 1)$

Answer: D

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12. let $f(x) = \begin{cases} x^2 & x \leq 0 \\ ax & x > 0 \end{cases}$ then f (x) is derivable at x = 0 if

A. a = 0

B. a =1

C. $a \neq 1$

D. Not possible

Answer: A

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13. If f is derivable at x =a,then $\lim_{x \to a} \left(\frac{xf(a) - af(x)}{x - a} \right)$

A. f'(a)

B. af'(a) - f(a)

C. f(a) - af'(a)

D. af'(a)

Answer: C

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14. Let f(x) = x|x| then f'(0) is equal to

A. 1

B. -1

C. 0

D. ± 1

Answer: C

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15. If f(x) =|x| , then f'(0) is

A. 0

B. 1

C. -1

D. Not in existence

Answer: D

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16. Let $f(x) = \begin{cases} x + a & x \geq 1 \\ ax^2 + 1 & x < 1 \end{cases}$ then f(x) is derivable at x =1 , if

A. a=1

B. a=0

C. a =2

D. $a = \dfrac{1}{2}$

Answer: D

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17. If $f(x) = \sqrt{25 - x^2}$, then what is $\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$ equal to

A. $\frac{1}{24}$

B. $\frac{1}{5}$

C. $-\sqrt{24}$

D. $\frac{1}{\sqrt{24}}$

Answer: D

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18. if $f(x) = e^{-\frac{1}{x^2}}, x \neq 0$ and $f(0) = 0$ then $f'(0)$ is

A. not defined

B. 1

C. e

D. 2

Answer: A

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19. If $f(x) = \log|x|, x \neq 0$ then f'(x) equals

A. $\frac{1}{|x|}$

B. $\frac{1}{x}$

C. $-\frac{1}{x}$

D. $\pm\frac{1}{x}$

Answer: D

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20. $\frac{d}{dx}\left(\sin^{-1}\frac{2x}{1+x^2}\right)$ is equal to

A. $\frac{2}{1+x^2}$

B. $-\frac{2}{1+x^2}$

C. $\frac{3(1-x^2)}{|1-x^2|(1-x^2)}, x \neq 1$

D. $\frac{2}{1-x^2}$

Answer: B

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21. Differential coefficient of $\log_{10} x w.r.t \log_x 10$ is

A. $-\frac{(\log x)^2}{(\log 10)^2}$

B. $\frac{(\log_{10} x)^2}{(\log 10)^2}$

C. $\frac{(\log_x 10)^2}{(\log 10)^2}$

D. $-\frac{(\log 10)^2}{(\log x)^2}$

Answer: A

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22. Find $\frac{dy}{dx}$ if $y = \log\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$

A. $\frac{x^2-4}{x^2-4}$

B. $\frac{3}{x^2-4}$

C. $\frac{x^2-1}{x^2-4}$

D. $\frac{3x^2}{x^2-4}$

Answer: C

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23. If $y = \frac{e^x - e^{-x}}{e^x + e^{-1}}, \text{prove that} \frac{dy}{dx} = 1 - y^2$

A. $1 + y^2$

B. $1 - y^2$

C. $y^2 - 1$

D. $y^2 - 2$

Answer: B

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24. If $y = ae^{mx} + be^{-mx}$ then $\frac{d^2y}{dx^2}$ is

A. $-m^2y$

B. $-m^2y^2$

C. my

D. $-my$

Answer: A

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25. If $y^2 = ax^2 + b, \quad \text{then } \frac{d^2y}{dx^2}$

A. $\frac{ab}{x^3}$

B. $\dfrac{x^3}{ab}$

C. $\dfrac{ab}{y^2}$

D. $\dfrac{ab}{y^3}$

Answer: D

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26. If $y = \dfrac{\log x}{x}$ then $\dfrac{d^2y}{dx^2} =$

A. $\dfrac{3 - 2\log x}{x^3}$

B. $\dfrac{2\log x - 3}{x^3}$

C. $\dfrac{2\log x - 3}{x^4}$

D. $\dfrac{2 - 3\log x}{x^4}$

Answer: B

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27. Differentiate the following w.r.t.x. The differentiation coneffiecient of $f(\log_e x) w.r.t.x,$ where $f(x) = \log_e x,$ is

A. $\dfrac{x}{\log_e x}$

B. $\dfrac{1}{x}\log_e x$

C. $\dfrac{1}{x\log_e x}$

D. x log x

Answer: C

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28. If $y = (1 + x)(1 + x^2)(1 + x^4)(1 + x^{2n}),$ then find $\dfrac{dy}{dx} at x = 0.$

A. 1

B. -1

C. 0

D. 2

Answer: A

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29. If

$$f(x) = \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x, \quad \text{then find} \quad f'\left(\frac{\pi}{4}\right).$$

A. $\sqrt{2}$

B. $\frac{1}{\sqrt{2}}$

C. 1

D. -1

Answer: A

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30. If $y = \cos^{-1}(\cos x)$,then $\frac{dy}{dx}$ at x=$\frac{5\pi}{4}$ is equal to

(a)1 (b)-1 (c)0 (d)4

A. 1

B. -1

C. $\frac{1}{\sqrt{2}}$

D. $\sqrt{2}$

Answer: B

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31. If $x = e^{y + e^{y + e^{y + \dots \infty}}}, x > 0$, then $\frac{dy}{dx}$ is equal to

A. $\frac{x}{1+x}$

B. $\frac{1}{x}$

C. $\frac{1-x}{x}$

D. $\frac{x}{1-x}$

Answer: B

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32. if $x^y . y^x = 16$ then $\frac{dy}{dx}$ at $(2, 2)$ is equal to

A. 1

B. -1

C. 0

D. 2

Answer: C

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33. If $y = \sin x^\circ$ and $u = \cos x$ then $\frac{dy}{du}$ is equal to

A. $-\cos ecx \cos x$

B. $\frac{\pi}{180} \cos ec \frac{\pi x}{180} \cos x$

C. $-\frac{\pi}{180} \cos ecx \cos \frac{\pi x}{180}$

D. $\frac{\pi}{180}\sin x\frac{\cos\pi x}{180}$

Answer: B

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34. Let the function y = f(x) be given by

$x = t^5 - 5t^3 - 20t + 7$

and $y = 4t^3 - 3t^2 - 18t + 3$

where $t \in (-2, 2)$ then f'(x) at t = 1 is

A. $\frac{5}{2}$

B. $\frac{2}{5}$

C. $\frac{7}{5}$

D. $\frac{5}{7}$

Answer: B

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35. If $u = f(x^3), v = g(x^2), f'(x) = \cos x, and g'(x) = \sin x, then \frac{du}{dv}$ is $\frac{3}{2}x\cos x^3 \cos ecx^2$ $\frac{2}{3}\sin x^3 \sec x^2 \tan x$ (d) none of these

A. $\frac{3}{2}x\cos x^3 \cos ecx^2$

B. $\frac{2}{3}\sin x^3 \sec x^2$

C. $\tan x$

D. $\frac{3}{2}x\sin x^2 \sec x^3$

Answer: A

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36. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ w.r.t. $\sqrt{1 - x^2}$ at $x - \frac{1}{2}$.

A. -4

B. 4

C. -1

D. -2

Answer: A

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37. If $t(1+x^2) = x$ and $x^2 + t^2 = y$ then at x = 2, the value of $\frac{dy}{dx}$ is

A. $\frac{88}{125}$

B. $\frac{488}{125}$

C. 1

D. $\frac{244}{125}$

Answer: B

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38. If $x = t\cos t,\ y = t + \sin t$. Then $\dfrac{d^2x}{dy^2}$ at $t = \dfrac{\pi}{2}$ is

(a) $\dfrac{\pi+4}{2}$ (b) $-\dfrac{\pi+4}{2}$ (c) -2 (d) none of these

A. $\dfrac{\pi+4}{2}$

B. $-\dfrac{\pi+4}{2}$

C. 64

D. 128

Answer: B

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39. if y =$\sin 2x,\quad \text{then}\ \dfrac{d^6y}{dx^6}\ \text{at} x = \dfrac{\pi}{2}$ is equal to

A. -64

B. 0

C. 64

D. 128

Answer: B

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40. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant. Then $\frac{d^3}{dx^3}(f(x))$ at $x = 0$ is

(a)p (b) $p - p^3$ (c)$p + p^3$ (d) independent of p

A. p

B. $p + p^2$

C. $p + p^3$

D. Independent of p

Answer: D

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41. If $x = 2at$, $y = at^2$, where a is a constant, then find $\frac{d^2y}{dx^2}$ at $x = \frac{1}{2}$

A. 8a

B. 1

C. 2a

D. $\frac{1}{2a}$

Answer: D

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42. If $y = x^{\frac{1}{x}}$, the value of $\frac{dy}{dx}$ at x =e is equal to

A. 1

B. 0

C. -1

D. $e^{\left(\frac{1}{e}\right)-2}$

Answer: B

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43. If $y = \tan^{-1}\left(\sqrt{\frac{x+1}{x-1}}\right) \text{ for } |x| > 1 \text{ then } \frac{dy}{dx}$ =

A. $\frac{-1}{2|x|\sqrt{x^2-1}}$

B. $\frac{-1}{2x\sqrt{x^2-1}}$

C. $\frac{1}{2x\sqrt{x^2-1}}$

D. $\frac{1}{2x\sqrt{x^2-1}}$

Answer: A

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44. If $y = \log_2 \log_2(x)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{x}\log_2 e.\ \log_x e$

B. $\frac{1}{x}\log_2 x$

C. $\frac{1}{x}\log_e x$

D. $\frac{1}{x}\log_x e$

Answer: A

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45. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2)$, $then \frac{dy}{dx}$ at x = 1 is

A. 2

B. 1

C. -2

D. -1

Answer: A

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46. Let the function f(x) be defined as $f(x) = \begin{cases} \frac{\log x - 1}{x - e} & x \neq e \\ k & x = e \end{cases}$

The value of k , for which the function is continuous at x =e , is equal to

A. e

B. $\frac{1}{e}$

C. e^2

D. $-e$

Answer: B

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47. Rolle's theorem is not applicable to f(x) = |x| in [-2,2] because

A. f(x) is not continuous in [-2,2]

B. f(x) is not derivable in (-2,2)

C. $f(2) \neq f(-2)$

D. it is applicable

Answer: B

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48. Lagrange's mean value theorem is not applicable to f(x) in [1,4] where f(x) =

A. $x^2 - 2x$

B. |x-2|

C. x|x|

D. x^3

Answer: B

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49. The value of C (if exists) in Lagrange's theorem for the function |x| in the interval [-1,1] is

A. 0

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. Nonexistent in the interval

Answer: D

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50. If f be a function such that $f(9) = 9$ and $f'(9) = 3$, then $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ is equal to

A. 9

B. 3

C. 1

D. $\frac{1}{3}$

Answer: B

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51. If $f(x) = \begin{cases} \frac{1}{1+e^{1/x}} & x \neq 0 \\ 0 & x = 0 \end{cases}$ then f(x) is

A. continuous at x =0

B. continuous and differnetiable at x =0

C. continuous but not differentiable at x=0

D. Discontinuous at x=0

Answer: D

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52. $f(x) = \sqrt{1-\sqrt{1-x^2}}$ then at $x = 0$,value of f(x) is

A. continuous on [-1,1] and differentiable on (-1,1)

B. continuous on [-1,1] and differentiable on $(-1, 0) \cup (0, 1)$

C. continuous and differentiable on (-1,1)

D. Discontinuous on [-1,1]

Answer: B

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53. Domain of differentiations of the function f(x) = |x -2| cos x is

A. R

B. $R - \{2\}$

C. $(0, \infty)$

D. {2}

Answer: B

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54. Let $f(x) = \frac{\sin(\pi[x+\pi])}{1+[x]^2}$ where [] denotes the greatest integer function then f(x) is

A. continuous and differentiable at all $x \in R$

B. continuous but not differentiable at some x

C. Differentiable but not continuous at x =0

D. Neither continuous nor differentiable at x =0

Answer: A

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55. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + ... \infty$ term then at $x = 0, f(x)$

A. $\lim_{x \to 0} f(x)$ does not exist

B. f(x) is continuous but not differentiable at x=0

C. f(x) is discontinuous at x =0

D. f(x) is differentiable at x=0

Answer: C

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56. Let $f(x) = \begin{cases} \dfrac{|x+1|}{\tan^{-1}(x+1)}, x \neq -1, 1, x \neq -1 \end{cases}$ Then $f(x)$ is

A. continuous at x =-1

B. Differentiable at x=-1

C. Discontinuous at x =-1

D. Continuous but not derivable at x =1

Answer: C

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57. The value of $\lim_{h \to 0} \dfrac{f(x+h) + f(x-h)}{h}$ is equal to

A. (a)f(x)

B. (b)0

C. (c)2f'(x)

D. $(d) - f'(x)$

Answer: C

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58. If $y = \dfrac{e^x + 1}{e^x - 1}$, then $\dfrac{y^2}{2} + \dfrac{dy}{dx}$ is equal to

A. 1

B. -1

C. $-\dfrac{1}{2}$

D. $\dfrac{1}{2}$

Answer: D

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59. If $f(x) = e^x g(x), g(0) = 2, g'(0) = 1, then f'(0)$ is

A. 1

B. 3

C. 2

D. 0

Answer: B

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60. If $ax^2 + 2hxy + by^2 = 0, \text{ show that } \frac{d^2y}{dx^2} = 0$

A. $\frac{h^2 - ab}{(hx + by)^2}$

B. $\dfrac{h^2 - ab}{(hx + by)^3}$

C. $\dfrac{ab - h^2}{(hx + by)^3}$

D. $\dfrac{ab - h^2}{(hx + by)^2}$

Answer: B

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61. Derivative of the function $f(x) = \log_5(\log_8 x)$, where x > 7 is

A. $\dfrac{1}{x(\log 5)(\log 7)(\log_7 x)}$

B. $\dfrac{1}{x(\log 5)(\log 7)}$

C. $\dfrac{1}{x(\log x)\log_e 5}$

D. $\dfrac{1}{(\log 5)(\log 7)(x \log x)}$

Answer: C

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Section B

1. The value of $\lim_{x \to 0}\left(1-\frac{1}{2^x}\right)\left(\frac{1}{\sqrt{\tan x+4}-2}\right)$

A. $\log_a 16$

B. Cannot exist

C. 3l n2

D. 4 ln 2

Answer: D

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2. If $\lim_{x \to 0}\frac{\sin x}{\tan 3x}=a, \lim_{x \to \infty}\frac{\sin x}{x}=b, \lim_{x \to \infty}\frac{\log x}{x}=c$ then value of a + b + c is

A. 1

B. 1/3

C. 3

D. 4

Answer: B

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3. Let $a = \lim_{x \to 0} x \cot x$ and $b = \lim_{x \to 0} x \log x$, then

A. a =b

B. b > a

C. a =b+1

D. b =a +1

Answer: C

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4. $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ equal

A. 1

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

Answer: D

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5. $\lim_{x \to \infty} \frac{(1 + x + x^3)}{(\ln x)^3}$ is equal to

A. 2

B. e^2

C. e^{-2}

D. Not defined

Answer: D

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6. Find $\lim_{x \to 0} \frac{\sin x^n}{(\sin x)^m}$ where, $m, n \in Z^+$ equal

A. (a)$1 \quad \text{if} \quad n < m$

B. (b)$0, \quad \text{if} \quad n = m$

C. (c)$\frac{n}{m}$

D. (d)$0, \quad \text{if} \quad n > m$

Answer: D

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7. Let $f(2) = 4$ and $f'(2) = 4$. Then $\lim_{x \to 2} \frac{x f(2) - 2 f(x)}{x - 2}$ is equal to

A. $-\frac{1}{3}$

B. -2

C. -4

D. 3

Answer: C

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8. If f(4)= 4, f'(4) =1 then $\lim_{x \to 4} 2\left(\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}\right)$ is equal to

A. 0

B. 2

C. -1

D. 2

Answer: B

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9. Evaluate: $(\lim)_{x \to 0} \frac{2^x - 1}{\sqrt{1+x} - 1}$

A. 2

B. $\log_e 2$

C. $\frac{\log_e 2}{2}$

D. $2 \log_e 2$

Answer: D

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10. Let α and β be the distinct roots of $ax^2 + bx + c = 0$. Then $\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ equal to

A. $\frac{1}{2}(\alpha - \beta)^2$

B. $-\frac{a^2}{2}(\alpha - \beta)^2$

C. 0

D. $\frac{a^2}{2}(\alpha - \beta)^2$

Answer: D

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11. $(\lim)_{x \overrightarrow{0}} \left(\frac{\sin\left(\pi \cos^2 x\right)}{x^2} is equa < o - \pi$ (b) π (c) $\frac{\pi}{2}$ (d) 1

A. $-\pi$

B. π

C. $\frac{\pi}{2}$

D. e

Answer: B

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12. $\lim_{x \to \infty} \left(\sqrt{x + \sqrt{x}} - \sqrt{x} \right)$ equals

A. $\frac{1}{2}$

B. 2

C. 3

D. 4

Answer: A

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13. $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\tan^2 x}\right)$

A. 0

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{2}$

Answer: C

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14. $\lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x$ is equal to :

A. e

B. e^{-1}

C. e^{-5}

D. e^5

Answer: C

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15. $\lim_{x \to 0} \left(\frac{1+\tan x}{1+\sin x}\right)^{\cos ecx}$ is equal to

A. -1

B. e

C. 1

D. e^{-1}

Answer: C

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16. $\lim_{x \to 0} \left(\frac{4^x + 9^x}{2}\right)^{\frac{1}{x}}$

A. 2

B. 6

C. 16

D. 112

Answer: B

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17. If $\lim_{x \to 0} (\cos x + a \sin bx)^{\frac{1}{x}} = e^2$ then the possible values of a and b are

A. a =1 , b = -2

B. $a = 2\sqrt{2}, b = \sqrt{2}$

C. $a = 2\sqrt{2}, b = \frac{1}{\sqrt{2}}$

D. a= -2 , b = 1

Answer: C

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18. $\underset{x \to 0}{Lt} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2}x} =$

A. 1

B. -1

C. zero

D. Does not exist

Answer: D

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19. $\lim_{x \to 2^+} \left(\frac{[x]^3}{3} - \left[\frac{x}{3}\right]^3 \right)$ is where [x] represents the integral part of x

A. 0

B. $\frac{64}{27}$

C. $\frac{8}{3}$

D. None of these

Answer: C

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20. The value of $\lim_{x \to 0} \frac{(4^x - 1)^3}{\sin\frac{x}{4} \log\left(1 + \frac{x^2}{3}\right)}$ equals

A. $3(\log 4)^3$

B. $4(\log 4)^3$

C. $12(\log 4)^3$

D. $15(\log 4)^3$

Answer: C

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21. If $f(x) = \begin{cases} \frac{x^2+3x+p}{2(x^2-1)} & x \neq 1 \\ \frac{5}{4} & x = 1 \end{cases}$ is continuous at x =1 then

A. p =2

B. p=0

C. p=-4

D. None of these

Answer: C

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22. In order that the function $f(x) = (x+1)^{\cot x}$ is continuous at x = 0, f(0) must be defined as

A. 0

B. $\frac{1}{e}$

C. e

D. 1

Answer: C

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23. The number of points at which the function $f(x) = \frac{1}{\log|2x|}$ is discontinuous is

A. 1

B. 2

C. 3

D. 4

Answer: C

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24. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x \neq 0 \\ 0 & x = 0 \end{cases}$ then f(x) is

A. (a)Continuous for all x but not differentiable

B. (b)Neither differentiable nor continuous

C. (c)Discontinuous everywhere

D. (d)Continuous as well as differentiable for all x

Answer: A

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25. The set of points where $f(x) = \dfrac{x}{1+|x|}$ is differentiable is

A. $(-\infty, -1), \cup (-1, \infty)$

B. $(-\infty, \infty)$

C. $(0, \infty)$

D. $(-\infty, 0) \cup (0, \infty)$

Answer: B

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26. The function $\left|x^2 - 3x + 2\right| + \cos|x|$ is not differentiableat $x =$

A. $-1, 2$

B. $-1, -2$

C. 1,2

D. `-2,1

Answer: C

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27. At x = 0 , the function $y = e^{-|x|}$ is

A. Continuous

B. Continuous and differentiable

C. Differentiable with derivative = 1

D. Differentiable with derivative =- 1

Answer: A

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28. Let $f(x) = \lambda + \mu|x| + \nu|x|^2$, where $\lambda, \mu, \nu \in R$, then $f'(0)$ exists if

A. $\mu = 0$

B. v =0

C. $\lambda = 0$

D. $\mu = v$

Answer: A

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29. If {x} denotes the fractional part of x, then $\lim_{x \to 0} \frac{\{x\}}{\tan\{x\}}$ is equal to

A. 1

B. 0

C. -1

D. Limit doesn't exist

Answer: D

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30. Let $f(x) = [x], g(x) = |x|$ and $f\{g(x)\} = h(x)$,where [.] is the greatest integer function . Then h(-1) is

A. 0

B. $-\infty$

C. 3

D. None of these

Answer: D

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31. A function f is defined by $f(x^2) = x^3 \forall x > 0$ then f(4) equals

A. (a)1

B. (b)2

C. (c)8

D. (d) Not differentiable

Answer: C

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32. If $3\sin(xy) + 4\cos(xy) = 5$, then $\frac{dy}{dx} =$

(a) $\frac{y}{x}$

(b) $\frac{3\sin(xy) + 4\cos(xy)}{3\cos(xy) - 4\sin(xy)}$

(c) $\frac{3\cos(xy) + 4\sin(xy)}{4\cos(xy) - 3\sin(xy)}$

(d) none

A. $\frac{y}{x}$

B. $-\frac{y}{x}$

C. $\frac{x}{y}$

D. $-\frac{x}{y}$

Answer: B

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33. Let f(x) = maximum $\{4, 1+x^2, x^2-1\}, \forall x \in R$. Then, the total number of points, where f(x) is not differentiable,.......

A. 2

B. 4

C. 6

D. 0

Answer: A

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34. Let g(x) be the inverse of f(x) and $f'(x) = \frac{1}{1+x^3}$.Find g'(x) in terms of g(x).

A. $\frac{1}{(1+g^3)}$

B. $\frac{1}{1+f^3}$

C. $1+g^3$

D. $1 + f^3$

Answer: C

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35. If f is a real- valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2, x, y, \in R$ and $f(0) = 0$ then f(1) equals

A. 1

B. 2

C. 0

D. -1

Answer: C

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Section C More Than One Options Are Correct

1. $\lim_{x \to 0} (1 + ax)^{\frac{b}{x}} = e^2$, " where" a, b in N` such that a+ b = 3 , then the value of (a,b) is

A. (16,8)

B. (8,4)

C. (2,1)

D. (1,2)

Answer: C::D

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2. Number of integral values of λ for which $(\lim)_{x \to 1} \sec^{-1}\left(\frac{\lambda^2}{(\log)_e x} - \frac{\lambda^2}{x - 1}\right)$ does not exist is a. 1 b. 2 c. 3 d. 4

A. $\left(- \infty, \sqrt{2}\right]$

B. $\left[\sqrt{2}, \infty\right)$

C. $\left(-\infty, \sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$

D. None of these

Answer: A::B::C

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3. Given the function $f(x) = \frac{1}{1-x}$, The points of discontinuity of the composite function f[f{f(x)}] are given by

A. 0

B. 1

C. 2

D. -1

Answer: A::B

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4. A function is defined as follows

$$f(x) = \begin{cases} x^3 & x^2 < 1 \\ x & x^2 \geq 1 \end{cases}$$ then function is

A. Continuous at x=1

B. Differentiable at x=1

C. Continuous but not differentiable at x=1

D. None of these

Answer: A::C

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5. Which of the following function(s) defined at x = 0 has/have removable discontinuity at the origin ?

A. $\dfrac{1}{\left(1 + 2^{\cot x}\right)}$

B. $\cos\left(\dfrac{|\sin x|}{x}\right)$

C. $x \sin \dfrac{\pi}{x}$

D. $\frac{1}{\ln|x|}$

Answer: B::C::D

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Section D

1. A square is inscribed in circle of radius R, a circle is inscribed in the square, a new square in the circle and so on for n times.

Sum of the areas of all circles is

A. $4\pi R^2\left(1-\left(\frac{1}{2}\right)^n\right)$

B. $2\pi R^2\left(1-\left(\frac{1}{2}\right)^n\right)$

C. $3\pi R^2\left(1-\left(\frac{1}{3}\right)^n\right)$

D. $\pi R^2\left(1-\left(\frac{1}{2}\right)^n\right)$

Answer: B

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2. A square is inscribed in a circle of radius R, a circle is inscribed in this square then a square in this circle and so on n times. Find the limit of the sum of areas of all the squares as $n \to \infty$.

A. $2R^2$

B. $3R^2$

C. $4R^2$

D. $8R^2$

Answer: C

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3. A square is inscribed in a circle of radius R, a circle is inscribed in this square then a square in this circle and so on n times. Find the limit of the sum of areas of all the squares as $n \to \infty$.

A. $2\pi R^2$

B. $3\pi R^2$

C. $4\pi R^2$

D. $8\pi R^2$

Answer: A

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4. A function f : $R \rightarrow R$ Satisfies the following conditions

(i) $f(x) \neq 0 \forall x \in R$

(ii) $f(x+y) = f(x)f(y) \forall x, y, \in R$

(iii) f(x) is differentiable

(iv) f'(0) =2

The derivative of f(x) satisfies the equation

A. f'(x +y) =f'(x) + f'(y)

B. f'(x +y) = f'(x) $\times$ f'(y)

C. f'(x+y) = f'(x) f(y)

D. f'(x +y) = f'(x) + f(y)

Answer: B

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5. A function f : $R \rightarrow R$ Satisfies the following conditions

(i) $f(x) \neq 0 \forall x \in R$

(ii) $f(x+y) = f(x) f(y) \forall x, y, \in R$

(iii) f(x) is differentiable

(iv) f'(0) =2

The derivative of f(x) satisfies the equation

A. 1

B. -1

C. 2

D. $\frac{1}{2}$

Answer: A

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6. A function f : $R \rightarrow R$ Satisfies the following conditions

(i) $f(x) \neq 0 \forall x \in R$

(ii) $f(x+y) = f(x) f(y) \forall x, y, \in R$

(iii) f(x) is differentiable

(iv) f'(0) =2

$$\lim_{x \rightarrow 0} \frac{f(x) - f(-x)}{x}$$

A. 1

B. 2

C. 3

D. 4

Answer: D

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Section E

1. STATEMENT -1 : $f(x) = \frac{1-\cos(1-\cos x)}{x^4}$ is continuous if f(0)=1/8

and

STATEMENT -2 : $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = \frac{1}{8}$

A. Statement -1 is True, Statement -2 is True, Statement -2 is a correct explantion for Statement -1

B. Statement -1 is True, Statement -2 is True, Statement -2 is NOT a correct explanation for Statement -1

C. Statement -1 is True , Statement -2 is Flase

D. Statement -1 is Flase , Statement - 2 is True

Answer: A

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2. STATEMENT -1 : If f(x) $= \log_{x^2}(\log x)$, $\quad \text{then} f'(e) = \frac{1}{e}$

STATEMENT -2 : If a gt 0 , b gt 0 and $a \ne b$ then $\log_a b = \frac{\log b}{\log a}$

A. Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1

B. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct expanation for statement-1

C. Statement-1 is True, Statement-2 is false

D. Statement-1 is False, Statement-2 is true

Answer: D

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3. STATEMENT - 1 : Let f be a twice differentiable function such that $f'(x) = g(x)$ and $f''(x) = -f(x)$. If $h'(x) = [f(x)]^2 + [g(x)]^2, h(1) = 8 \text{ and } h(0) = 2 \Rightarrow h(2) = 14$ and

STATEMENT - 2 : $h''(x) = 0$

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4. STATEMENT -1 : for the function y= f(x),

$$f(x), \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = -\frac{\left\{1+\left(\frac{dx}{dy}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2x}{dy^2}}$$

STATEMENT -2 : $\frac{dy}{dx} = \frac{\frac{1}{dx}}{dy}$ and $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

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Section F

1. For f to be continuous at x = f(0) is given by

Column- I	Column - II
(A) $f(x) = \frac{\ln(1+4x)}{x}$	(p) $\frac{1}{4}$
(B) $f(x) = \frac{\ln(4+x) - \ln 4}{x}$	(q) 0
(C) $f(x) = \frac{1}{\sin x} - \frac{1}{\tan x}$	(r) 4
(D) $f(x) = \frac{1-\cos^3 x}{x \sin 2x}$	(s) $\frac{3}{4}$

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2. Match the following :

Column-I	Column - II
(A) $\lim_{x \to \frac{\pi}{4}} (\sin 2x)^{\tan^2 2x}$	(p) $\frac{1}{2}$
(B) $\lim_{x \to \infty} \left(\frac{2x-1}{2x+1}\right)^x$	(q) $e^{-\frac{1}{2}}$
(C) $\lim_{x \to \frac{\pi}{2}} (\tan x)^{\tan 2x}$	(r) e^{-1}
(D) $\lim_{x \to \frac{\pi}{4}} \tan 2x \tan\left(\frac{\pi}{4} - x\right)$	(s) 1

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3. Match the following :

Column I	Colum
(A) if $\lim_{x \to 1} (1-x)\tan\frac{\pi x}{2} = k$ then $\sin\left(\frac{1}{k}\right)$ is	(p) 4
(B) if $\lim_{x \to 5} \frac{x^k - 5^k}{x-5} = 500$ then k is	(q) 1
(C) $\lim_{x \to \infty} \left(1 + \frac{4}{x+1}\right)^{\frac{3x-1}{3}}$ is equal to e^k, then k is	(r) A pe
(D) $\frac{d^{20}}{dx^{20}}(2\cos x, \cos 3x) = 2^{4k}\left[\cos 2x + 2^{20}. \cos 4k\right]$ then k is	(s) 5
	(t) An oc

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Section G

1. The number of points where f(x) = max {|sin x| , | cos x|} , $x \in (-2\pi, 2\pi)$ is not differentiable is _____

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2. The number of points where g(x) = min { ||x| -3|, $9 - x^2\}(x \in R)$ is not differentiable is ______

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3. if the right hand derivative of h(x) = {x} ({.} is fractional of x) exists at x =1 and it is equal to ____\

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Section H

1. Statement -1 : y = [x] , ([.] denotes greatest integer function) is not a continuous function .

Statement -2 : {x} ({.} denotes fractional fractional function) is discontinuous at integral points.

Statement -3 : $y = 7^x$ is continuous in its domain.

A. TFT

B. TTT

C. FFF

D. FFT

Answer: B

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2. Statement -1 : $\lim_{x \to 0} \frac{\sin x}{x}$ exists ,.

Statement -2 : |x| is differentiable at x=0

Statement -3 : If $\lim_{x \to 0} \frac{\tan kx}{\sin 5x} = 3$, then k = 15

A. T F T

B. TTT

C. FFF

D. FFT

Answer: A

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Section I

1. Let f(x + y) = f(x) .f(y) $\forall x, y \in R$ suppose that f(k) =3, $k \in R$ and $f'(0) = 11$ then find f'(k)

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2. If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3} = b$ (finite), then $(ab)^2$ equals.....

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3. $f(x) = \begin{cases} x + a\sqrt{2}\sin x & 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x & \frac{\pi}{2} < x \le \pi \end{cases}$ continuous function $\forall x \in [0, \pi]$ then $5\left(\frac{a}{b}\right)^2$ equals ....

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4. Find the value of f(1) that the function $f(x) = \dfrac{9\left(x^{\frac{2}{3}} - 2x^{\frac{1}{3}} + 1\right)}{(x-1)^2}, x \ne 1$ is continuous at x =1

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5. find the value of $\lim_{x \to \infty} 48x\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{x+1}{x+2}\right)\right)$

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6. Let f(x) be a continuous function defined for $\forall x \in R$, if f(x) take rational values $\forall x \in R$ and $f(2) = 198$, then $f(2^2)$ = ...

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7. ABC is an isosceles triangle inscribed in a circle of radius r , if AB = AC and h is the altitude from A to BC . If the $\triangle ABC$ has perimeter P and $\triangle$ is area then $\lim_{h \to 0} 512r\frac{\Delta}{p^3}$ equals

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8. Let $F(x) = f(x)g(x)h(x)$ for all real x, where f(x), g(x), and h(x) are differentiable functions. At some point $x_0, F'(x_0) = 21F(x_0), f'(x_0) = 4f(x_0), g'(x_0) = -7g(x_0),$ and $h'(x_0)$

Then k = -

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Section J

1. the value of $\lim_{x \to y} \frac{\sin^2 x - \sin^2 y}{x^2 - y^2}$ equals

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2. Let $f(x) = \frac{x + x^2 + \ldots + x^n - n}{x - 1}$, $g(x) = (4^n + 5^n)^{\frac{1}{n}}$ and α and β are the roots of equation $\lim_{x \to 1} f(x) = \lim_{n \to \infty} g(x)$ then the value of $\sum_{n=0}^{\infty} \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is

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3. Let $f(x) = \frac{x + x^2 + \ldots\ldots + x^n}{x - 1}$ and $g(x) = (4^n + 5^n)^{1/n}$ such that $\lim_{x \to 1} f(x) = \lim (n \to \infty) g(x)$

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4. If $g(x) = (\lim)_{m \to \infty} \frac{x^m f(x) + h(x) + 3}{2x^m + 4x + 1}$ when $x \neq 1 and g(1) = e^3$ such that $f(x), g(x) and h(x)$ are continuous functions at $x = 1$ then the value of $5f(1) - 2h(1)$ is 7 b. 6 c. 9 d. 8

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5. Let $f(x)$ is a polynomial satisfying $f(x).\, f(y) = f(x) + f(y) + f(xy) - 2$ for all x, y and $f(2) = 1025$, then the value of $\lim_{x \to 2} f'(x)$ is

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6. Let f(x) = ||x| -1| , g(x) = |x| + |x-2| , h (x) = $\max\left\{1, x, x^3\right\}$ If a, b,c are the no .of points where f(x), g(x) and h(x) , are not differentiable then the value of a+ b + c is .....

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