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MATHS

JEE (MAIN AND ADVANCED MATHEMATICS) FOR BOARD AND COMPETITIVE EXAMS

INVERSE TRIGONOMETRIC FUNCTIONS

Examples

1. Find the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

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2. What is the principal value of $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$?

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3. Find the principal value of $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

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4. Find the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

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5. Find the principal value of $\tan^{-1}(1)$

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6. Find the principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.

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7. Find the principal values of $\cot^{-1}\sqrt{3}$ and $\cot^{-1}(-1)$

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8. Find the principal values of $\cot^{-1}\sqrt{3}$ and $\cot^{-1}(-1)$

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9. Find the principal value of $\sec^{-1}(\sqrt{2})$.

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10. Find the principal value of $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

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11. Find the principal value of $\cos ec^{-1}\left(\frac{2}{\sqrt{3}}\right)$.

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12. Write the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

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13. Find the value of $\cot^{-1}(-\sqrt{3}) + \cos ec^{-1}(2) + \sec^{-1}(-\sqrt{2})$.

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14. Find the value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(-1) + \tan^{-1}(-1)$

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15. Find the value of $\sin^{-1}\left(\sin.\ \frac{5\pi}{3}\right)$.

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16. Find the value of $\sec^{-1}(-2)+\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

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17. Find the value of $\tan^{-1}(-x)-\cot^{-1}\left(-\frac{1}{x}\right), x>0$

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18. If $\sin^{-1}x+2\cos^{-1}x=\frac{2\pi}{3}$, then find x.

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19. If $\sin^{-1}\left(\frac{5}{x}\right)+\cos^{-1}\left(\frac{x}{5}\right)=\frac{\pi}{2}$,then find x.

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20. Evaluate the following: $\cos\left(\tan^{-1}3/4\right)$

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21. Simplify $\sin\left(\cot^{-1}\left(\cos\left(\tan^{-1}x\right)\right)\right),\ 0<x<1.$

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22. Prove that : `tan^(-1)(1/7)+tan^(-1)(1/13)=tan^(-1)(2/9)

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23. Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$

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24. Prove that: $\frac{\sin^{-1}(12)}{13} + \frac{\cos^{-1}4}{5} + \frac{\tan^{-1}(63)}{16} = \pi$

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25. Prove the following: $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

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26. Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x$

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27. Express `tan^(-1)((cosx)/(1-sinx)),-pi/2

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28. Solve the following for x: $\tan^{-1} 2x + \tan^{-1} 3x = n\pi + \frac{3\pi}{4}$

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29. Find all possible values of x and y for which $\cos^{-1}\sqrt{x} + \cos^{-1}\sqrt{1-x} + \cos^{-1}\sqrt{1-y} = \left(\frac{3\pi}{4}\right)$

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30. Find the greatest and least values of the function

$f(x) = \left(\sin^{-1} x\right)^3 + \left(\cos^{-1} x\right)^3, \ -1 \leq x \leq 1$

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31. The simplest form of $\left(\cos^{-1} x + \cos^{-1}\left\{\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right\}\right), \forall x \in \left[\frac{1}{2}, 1\right]$ is A. Then the value of tanA is

A. $1\sqrt{3}$

B. $\sqrt{3}$

C. 1

D. -1

Answer: B

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32. Let $f(x) = \tan^{-1}\left(x^2 - 18x + a\right) > 0x \in R$. Then the value of a lies in

A. $(81, \infty)$

B. $\{81, \infty)$

C. $(-\infty, 81)$

D. $(-\infty, 81]$

Answer: A

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33. Let $f(x) = \sin^{-1} 2x + \cos^{-1} 2x + \sec^{-1} 2x$. Then the sum of the maximum and minimum values of f(x) is

A. π

B. 2π

C. 3π

D. $\frac{\pi}{2}$

Answer: B

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34. Solutions of $\sin^{-1}(\sin x) = \sin x$ are , if $x \in (0, 2\pi)$

A. 4 real roots

B. 2 positive real roots

C. 2 negative real roots

D. 5 real roots

Answer: D

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Try Yourself

1. What is the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

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2. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$.

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3. Find the principal value of each of the following: (i) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (ii) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

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4. The Principle value of $\cot^{-1}(-\sqrt{3})$ is

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5. Find the principal values of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ and $\sec^{-1}(-2)$

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6. Find the principal value of $\cos ec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$.

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7. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\cos ec^{-1}(-2)$

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8. Find the principal value of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)+\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\sec^{-1}(2)$

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9. Find the principal value of $\cot^{-1}(-\sqrt{3})+2\cos ec^{-1}(-2)+\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

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10. Find the principal value of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) - 2\tan^{-1}\left(-\sqrt{3}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$.

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11. Find $\tan^{-1}(3) + \cot^{-1}\left(-\frac{1}{3}\right) + \sec^{-1} 2$.

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12. If $\tan^{-1} x + 2\cot^{-1} x = \frac{5\pi}{6}$, then find x.

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13. Evaluate $\tan\left(\cos ec^{-1}\left(\frac{5}{3}\right)\right)$.

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14. Prove that $\cos\left(\tan^{-1}\left(\sin\left(\cot^{-1}x\right)\right)\right)=\sqrt{\frac{x^2+1}{x^2+2}}$

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15. Prove that $\tan^{-1}\left(\frac{1}{70}\right)-\tan^{-1}\left(\frac{1}{99}\right)=\tan^{-1}\left(\frac{1}{239}\right)$

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16. Prove that $\cot^{-1}(13)+\cot^{-1}(21)+\cot^{-1}(-8)=\pi$.

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17. Prove that $\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{15}{17}\right)=\cos^{-1}\left(\frac{36}{85}\right)$

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18. Find the value of

$$\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$$

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19. Prove that

$$3\tan^{-1}x=\begin{cases}\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & \text{if } -\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}\\ \pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & \text{if } x>\frac{1}{\sqrt{3}}\\ -\pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & \text{if } x<-\frac{1}{\sqrt{3}}\end{cases}$$

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20. Solve $\tan^{-1}\left[\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right]$, if $\frac{a}{b}\tan x\leq -1$

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21. Evaluate $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{4}\right)$

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Assignment Section A Objective Type Questions One Option Is Correct

1. Write the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

A. $\frac{\pi}{6}$

B. $\frac{5\pi}{6}$

C. $-\frac{\pi}{6}$

D. Both (1) & (2)

Answer: A

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2. Find the principal value of: $\cos^{-1}\left(-\frac{1}{2}\right)$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: C

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3. Find the principal value of $\cot^{-1}(-\sqrt{3})$

A. $-\frac{\pi}{3}$

B. $-\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: D

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4. Find the principal value of each of the following: (i) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (ii) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

A. $-\frac{\pi}{6}$

B. $-\frac{\pi}{3}$

C. $\frac{5\pi}{6}$

D. Both (1) & (2)

Answer: A

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5. Find the principal value of (a) $\cos ec^{-1}(-1)$ (b) $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

A. $-\frac{\pi}{2}$

B. 0

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{2}$

Answer: A

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6. The principal value of $\cot^{-1} x$ lie in

A. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C. $(0, \pi)$

D. $[0, \pi]$

Answer: C

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7. The principal values of $\sec^{-1} x$ lie in

A. $[0, \pi]$

B. $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

C. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

D. $\{\pi, 2\pi\} - \left\{\frac{3\pi}{2}\right\}$

Answer: B

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8. The value of $\tan^{-1}(\sqrt{3}) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) + \sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is

A. $-\frac{\pi}{12}$

B. $\frac{11\pi}{12}$

C. $\frac{5\pi}{4}$

D. $\frac{23\pi}{12}$

Answer: D

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9. The value of $\cot^{-1}(-\sqrt{3}) + \cos ec^{-1}(2) + \tan^{-1}(\sqrt{3})$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{5\pi}{6}$

D. $\frac{4\pi}{3}$

Answer: D

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10. The value of $2\cos^{-1}\left(-\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}(-1)$ is

A. $2\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. π

D. 2π

Answer: A

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11. Evaluate each of the following: $\sin^{-1}\left(\frac{\sin\pi}{6}\right)$ (ii) $\sin^{-1}\left(\frac{\sin(7\pi)}{6}\right)$ (iii) $\sin^{-1}\left(\frac{\sin(5\pi)}{6}\right)$

A. $-\frac{\pi}{6}$

B. $\frac{\pi}{6}$

C. $\frac{7\pi}{6}$

D. $-\frac{5\pi}{6}$

Answer: A

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12. Evaluate each of the following: $\sin^{-1}\left(\frac{\sin(2\pi)}{3}\right)$ (ii) $\cos^{-1}\left(\frac{\cos(7\pi)}{6}\right)$ (iii) $\tan^{-1}\left(\frac{\tan(3\pi)}{4}\right)$

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{4}$

C. $-\frac{\pi}{4}$

D. $-\frac{3\pi}{4}$

Answer: C

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13. $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$ is equal to

A. $\frac{\pi}{2}-\frac{3}{5}$

B. $\frac{3}{5}$

C. $-\frac{4}{5}$

D. $\frac{4}{5}$

Answer: D

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14. The numerical value of $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right.$ is equal to____

A. $\frac{5}{12}$

B. $-\frac{5}{12}$

C. $\frac{7}{17}$

D. $-\frac{7}{17}$

Answer: D

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15. Evaluate: $\left\{\frac{2\tan^{-1}1}{5}-\frac{\pi}{4}\right\}$ (ii) $\tan\left\{\frac{1}{2}\frac{\cos^{-1}\left(\sqrt{5}\right)}{3}\right\}$

A. $\frac{3-\sqrt{5}}{2}$

B. $\frac{3+\sqrt{5}}{2}$

C. $\frac{\sqrt{5}-3}{2}$

D. $\frac{-\sqrt{5}-3}{2}$

Answer: A

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16. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$, Then $x^{2013}+y^{2013}+z^{2013}-\frac{9}{x^{2013}+y^{2013}+z^{2013}}$ is equal to

A. 1

B. 0

C. -1

D. -3

Answer: B

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17. Prove the following : $\sin^{-1}\left(\frac{4}{5}\right) + 2\tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{2}$

A. $\tan^{-1}\left(\frac{3}{4}\right)$

B. $\tan^{-1}\left(\frac{4}{3}\right)$

C. $\tan^{-1}\left(\frac{3}{\sqrt{10}}\right)$

D. $\frac{\pi}{2}$

Answer: D

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18. $\frac{1}{2}\tan^{-1}\left(\frac{12}{5}\right)$ is equal to

A. $\tan^{-1}\left(\frac{3}{2}\right)$

B. $\tan^{-1}\left(\frac{2}{3}\right)$

C. $\tan^{-1}\left(\frac{3}{4}\right)$

D. $\tan^{-1}\left(\frac{7}{17}\right)$

Answer: B

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19. Value of $\sec^{-1}\left(\text{sec.}\ \frac{4\pi}{3}\right)$ is

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{4\pi}{3}$

D. $\frac{\pi}{3}$

Answer: B

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20. Find the value of the expression: $\sin\left(2\,\frac{\tan^{-1}1}{3}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)$

A. $\frac{14}{15}$

B. $\frac{3}{4}$

C. $\frac{2\sqrt{2}}{7}$

D. $\frac{2\sqrt{2}}{15}$

Answer: A

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21. Prove that : $\frac{\tan^{-1}1}{2}+\frac{\tan^{-1}1}{5}+\frac{\tan^{-1}1}{8}=\frac{\pi}{4}$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\tan^{-1}\left(\frac{12}{5}\right)$

D. $\tan^{-1}\left(\frac{4}{3}\right)$

Answer: A

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22. $\cos^{-1}(-x), |x| \leq 1$, is equal to

A. $-\cos^{-1} x$

B. $\cos^{-1} x$

C. $\frac{\pi}{2} - \cos^{-1} x$

D. $\frac{\pi}{2} + \sin^{-1} x$

Answer: D

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23. $\cos ec^{-1}(-x), x \in R - (-1, 1),$ is equal to

A. $\cos ec^{-1}x$

B. $-\sin^{-1}x$

C. $-\sin^{-1}\left(\frac{1}{x}\right)$

D. $\pi - \cos ec^{-1}x$

Answer: C

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24. $\cot^{-1}(-2)$ is equal to

A. $\pi - \tan^{-1}\left(\frac{1}{2}\right)$

B. $\tan^{-1}\left(\frac{1}{2}\right)$

C. $-\frac{\tan^{-1}1}{2}$

D. $\cot^{-1}2$

Answer: A

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25. If $\sin\{\cot^{-1}(x+1)\} = \cos(\tan^{-1}x)$, then find x

A. 1

B. $\frac{1}{2}$

C. 0

D. $-\frac{1}{2}$

Answer: D

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26. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - ..\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4}\right) = \frac{\pi}{2}$ for `0<|x|

A. -1

B. $\frac{-1}{2}$

C. $\frac{1}{2}$

D. 1

Answer: D

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27. if $\tan^{-1}\left(\frac{1}{x}\right)+\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)=\frac{\pi}{4}$ then x equals

A. $\frac{1}{\sqrt{5}}$

B. $\frac{4}{\sqrt{5}}$

C. 1

D. 3

Answer: D

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A. $\frac{x}{\sqrt{1=x^2}}$

B. x

C. $x\sqrt{1+x^2}$

D. $\sqrt{1+x^2}$

Answer: C

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29. If $x_1 = \cos^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ and $x_2 = \sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then

A. $x_1 < x_2$

B. $x_1 = x_2$

C. $x_1 > x_2$

D. Can't be determined

Answer: C

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30. If $\tan\left(\sin^{-1}\sqrt{1-x^2}\right) = \sin\left(\tan^{-1}2\right)$ then x is

A. $-\frac{\sqrt{5}}{3}$

B. $\frac{3}{\sqrt{10}}$

C. $\frac{\sqrt{5}}{3}$

D. Both (1) & (3)

Answer: D

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31. If $0 \leq x \leq 1 then \cos^{-1}\left(2x^2-1\right)$ equals

A. $2\cos^{-1}x$

B. $2\sin^{-1}x$

C. $\pi - 2\cos^{-1}x$

D. $\pi + 2\cos^{-1}x$

Answer: A

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32. Express in terms of : $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ to $\sin^{-1}x$ for $1 \geq x > \frac{1}{\sqrt{2}}$

A. $2\sin^{-1}x$

B. $2\cos^{-1}x$

C. $-2\sin^{-1}x$

D. $-2\cos^{-1}x$

Answer: B

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33. If $\sin^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{12}{x}\right)=\frac{\pi}{2}$, then x is equal to $\frac{7}{13}$ (b) $\frac{4}{3}$ (c) 13 (d) $\frac{13}{7}$

A. -13

B. 13

C. 15

D. 17

Answer: B

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34. $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), where$ -1

A. $\frac{1}{2}\cos^{-1}x^2$

B. $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$

C. $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$

D. $\frac{\pi}{2}-\frac{1}{2}\cos^{-1}x^2$

Answer: C

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35. If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\theta$, Prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\theta+\frac{y^2}{b^2}=\sin^2\theta.$

A. θ

B. $\sin^2\theta$

C. $\cos^2\theta$

D. 1

Answer: B

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36. $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)$ is equal to

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{5\pi}{12}$

Answer: B

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37. $\cos\left(\frac{\tan^{-1}3}{4}\right)+\cos\left(\tan^{-1}x\right)$ is equal to

A. $\frac{4}{5}+\frac{x}{\sqrt{1+x^2}}$

B. $\frac{3}{5}+\frac{1}{\sqrt{1+x^2}}$

C. $\frac{4}{5}+\frac{1}{\sqrt{1+x^2}}$

D. $\frac{3}{5}+\frac{x}{\sqrt{1+x^2}}$

Answer: C

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38. If $\cot^{-1} x + \tan^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{4}$ then x is

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 2

D. 3

Answer: D

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39. The value of $\cos^{-1}(-1) + \sin^{-1}(1)$ is

A. $-\frac{3\pi}{2}$

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$

Answer: D

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40. If $\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2}$ then $x =$

A. -5

B. 5

C. 25

D. -25

Answer: B

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41. The domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right)$ is

A. [-6,6]

B. $(-\infty, 2) \cup (2, 3)$

C. $(2, 3)$

D. $[-6, 2) \cup (2, 3)$

Answer: A

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42. The domain of definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real-valued x is $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

A. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

B. $\left[-\frac{1}{4}, \frac{1}{2}\right]$

C. $[-1, 1]$

D. $\left[-1, \frac{1}{2}\right]$

Answer: B

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43. $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ holds good for

A. All $x, y \in R$

B. $|x| < 1 |y| < 1$

C. $|x| > 1, |y| > 1$

D. $xy > -1$

Answer: D

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44. If $\sin^{-1} x + \sin^{-1}(1-x) + \cos^{-1} x = 0$, then x is

A. 0

B. 1

C. 2

D. No real value

Answer: D

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45. $\sin^{-1}(\sin x) = x$ if

A. $x \in R$

B. $x \in [-1, 1]$

C. $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. $x \in [-\pi, \pi]$

Answer: C

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Assignment Section B Objective Type Questions One Option Is Correct

1. The domain and range of $f(x) = \sin^1 x + \cos^{-1} x + \tan^{-1} x + \cot^{-1} x + \sec^{-1} x + \cos ec^{-1} x$ respectively are

A. $\{-1, 1\}, \frac{3\pi}{2}$

B. $\{-1, 1\}, \frac{\pi}{2}$

C. $(-1, 1), \frac{\pi}{2}$

D. $(-1, 1), 2\pi$

Answer: A

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2. The maximum and minimum values of $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$ respectively is

A. $\frac{3\pi}{4}, \frac{\pi}{2}$

B. $\frac{3\pi}{4}, \frac{\pi}{4}$

C. $\frac{\pi}{4}, \frac{-\pi}{4}$

D. $\pi, 0$

Answer: B

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3. If $x_1, x_2, x_3, x_4, x_5, x_6$ all are independent then the maximum and minimum values of $\left[\sin^{-1} x_1\right] + \left[\cos^{-1} x_2\right] + \left[\tan^{-1} x_3\right] + \left[\cot^{-1} x_4\right] + \left[\sec^{-1} x_5\right] + \left[\cos ec\right.$, where [] represents greatest integer function, respectively are

A. 9,3

B. 11,5

C. 12,-6

D. 12,-3

Answer: D

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4. Select the wrong option

A. $-1 \leq \sin^{-1} x \leq 1 \Rightarrow -\sin 1 \leq x \leq \sin 1$

B. $\frac{\pi}{3} \leq \cos^{-1} x \leq \frac{4\pi}{3} \Rightarrow -\frac{1}{2} \leq x \leq \frac{1}{2}$

C. $\frac{\pi}{4} \leq \cot^{-1} x \leq \frac{5\pi}{6} \Rightarrow -\sqrt{3} \leq x \leq 1$

D. $\sec^{-1} x \geq \frac{\pi}{4} \Rightarrow x \leq \sqrt{2}$

Answer: D

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5. If the number of solutions of $\sin^{-1} x + |x| = 1 \cos^{-1} x + |x| = 1, \tan^{-1} x + |x| = 1, \cot^{-1} x + |x| = 1$ are $n_1, n_2, n_3, n_4, n_5, n_6$ respectively, then the value of $n_1 + n_2 + n_3 + n_4 + n_5 + n_6$ is

A. 6

B. 7

C. 8

D. 9

Answer: A

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6. Let $x_i \in [-1, 1]$ for $i = 1, 2, 3, \ldots 24,$ such that $\sin^{-1} x_1 + \sin^{-1} x_2 + \ldots + \sin^{-1} x_{24} = 12\pi$ then the value of $x_1 + 2x_2 + 3x_3 + \ldots + 24x_{24}$ is

A. 276

B. 300

C. 325

D. 351

Answer: B

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7.

$$\sin^{-1}\left(-\left(\frac{1}{2}\right)\right)+\cos^{-1}\left(-\left(\frac{1}{2}\right)\right)+\cot^{-1}(-\sqrt{3})+\operatorname{cosec}^{-1}(\sqrt{2})+$$

equals

A. $\frac{9\pi}{4}$

B. $\frac{19\pi}{12}$

C. $\frac{3\pi}{2}$

D. $\frac{\pi}{2}$

Answer: B

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8. Let [.] represents the greatest integer function and $\left[\cos^{-1}\sin^{-1}\tan^{-1}x\right]=1$ then 'x' lies in the intervel.

A. $[\sin\tan\cos 2, \tan\sin\cos 1]$

B. $[\tan\sin\cos 2, \tan\sin\cos 1]$

C. $[\tan\sin\cos 1, \tan\sin\cos 2]$

D. $(\tan\sin\cos 2, \tan\sin\cos 1]$

Answer: D

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9. The value of $\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right)+\sin^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right)$ is (a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{3}$ (c) $\frac{10\pi}{3}$ (d) 0

A. 0

B. $\frac{\pi}{2}$

C. 10. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

Answer: A

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10. How many solutions does the equation $5\tan^{-1}x + 3\cot^{-1}x = 2\pi$ have ?

A. zero

B. Exactly one

C. Exactly two

D. Infinite

Answer: B

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11. Solve $\sin^{-1} x - \cos^{-1} x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

A. No solutions

B. Unique solution

C. Infinite number of solution

D. Two solutions

Answer: B

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12. about to only mathematics

A. -1

B. 1

C. 0

D. 2

Answer: A

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13. Let $\cos^{-1}\left(\frac{x}{2}\right) + \cos^{-1}\left(\frac{y}{3}\right) = \theta$ and denote by $f(x, y, \theta) = 0$ the rational integeral expression in x and y. Then for $\theta = \frac{\pi}{2}$ the locus represented by $f\left(x, y, \frac{\pi}{2}\right) = 0$ is (A) an ellipse (B) parabola (C) hyperbola (D) pair of lines

A. An ellipse

B. A parabola

C. A hyperbola

D. A pair of line

Answer: A

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14. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} =$

A. 0

B. 1

C. $\frac{1}{xyz}$

D. xyz

Answer: B

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15. The value of $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$ is :

A. 0

B. π

C. $-\pi$

D. $-2\tan^{-1}3$

Answer: B

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16. If a,b,c are real positive numbers and $\theta = \tan^{-1}\left[\frac{a(a+b+c)}{bc}\right]^{\frac{1}{2}} + \tan^{-1}\left[\frac{b(a+b+c)}{ca}\right]^{\frac{1}{2}} + \tan^{-1}\left[\frac{c(a+b}{ab}\right.$, then $\tan\theta$ equals

A. (a) 0

B. (b) 1

C. (c) $\frac{a+b+c}{abc}$

D. (d) 2

Answer: A

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17. Prove that: $\frac{\sin^{-1}4}{5}+\frac{\sin^{-1}5}{13}+\frac{\sin^{-1}(16)}{65}=\frac{\pi}{2}$

A. $\frac{3\pi}{2}$

B. $\frac{\pi}{2}$

C. $\pi-\sin^{-1}.\frac{3713}{4225}$

D. $\pi-\tan^{-1}.\frac{3713}{2016}$

Answer: B

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18. If $\alpha=\tan^{-1}\left(\frac{\sqrt{3}x}{2y-x}\right),\beta=\tan^{-1}\left(\frac{2x-y}{\sqrt{3}y}\right)$, then $\alpha-\beta=$ (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{3}$

A. $\frac{\pi}{6}$

B. $-\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $-\frac{\pi}{3}$

Answer: A

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19. Evaluate: $\left\{\frac{2\tan^{-1}1}{5}-\frac{\pi}{4}\right\}$ (ii) $\tan\left\{\frac{1}{2}\frac{\cos^{-1}\left(\sqrt{5}\right)}{3}\right\}$

A. $\frac{3\pm\sqrt{5}}{2}$

B. $\frac{3+\sqrt{5}}{2}$

C. $\frac{3-\sqrt{5}}{2}$

D. $\frac{1}{2}$

Answer: C

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20. If $-1\leq x\leq -\frac{1}{2}$, then $\sin^{-1}\left(3x-4x^3\right)$ equals

A. $3\sin^{-1}x$

B. $\pi - 3\sin^{-1}x$

C. $-\pi - 3\sin^{-1}x$

D. $\pi + 3\sin^{-1}x$

Answer: C

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21. The value of $\cos\left(2\cos^{-1}0.8\right)$ is

A. 0.48

B. 0.96

C. 0.6

D. 0.28

Answer: D

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22. Find the value of $4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right)$

A. π

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: D

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23. If $\cot^{-1}\left(\sqrt{\cos\alpha}\right) - \tan^{-1}\left(\sqrt{\cos\alpha}\right) = x$, then $\sin x$ is $\frac{\tan^2\alpha}{2}$ (b) $\frac{\cot^2\alpha}{2}$ (c) $\tan^2\alpha$ (d) $\frac{\cot\alpha}{2}$

A. $\tan^2.\frac{\alpha}{2}$

B. $\cot^2.\frac{\alpha}{2}$

C. $\tan\alpha$

D. $\cot.\frac{\alpha}{2}$

Answer: A

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24. If $\left|\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right| < \frac{\pi}{3}$, then

A. $\left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$

B. $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

C. $\left(0, \frac{1}{\sqrt{3}}\right)$

D. $[1, 2]$

Answer: B

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25. The set of values of x satisfying $\left|\sin^{-1} x\right| < \left|\cos^{-1} x\right|$, is

A. $\left[-1, \frac{1}{\sqrt{2}}\right]$

B. $\left[-1, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, 1\right]$

C. $\left(-1, \frac{1}{\sqrt{2}}\right)$

D. $\left[\frac{1}{\sqrt{2}}, 1\right]$ only

Answer: A

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26. Let $f(x) = \sec^{-1}(x-10) + \cos^{-1}(10-x)$. Then range of $f(x)$ is

A. $\left\{0, \frac{\pi}{2}, \pi\right\}$

B. $\left\{0, \frac{\pi}{2}\right\}$

C. $\left\{\frac{\pi}{2}\right\}$

D. $\{\pi\}$

Answer: D

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27. Let $f(x) = \cos ec^{-1}\left[1 + \sin^2 x\right], where[\cdot]$ denotes the greatest integer function, then the range of f

A. Is discrete and has two members

B. Is discrete and has four members

C. Is continuous

D. Is $\left(\frac{\pi}{2}, \frac{\pi}{6}\right)$

Answer: A

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28. The sum $\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{1}{2^n + 2^{1-n}}\right)$ equals

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer: B

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29. $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right), x\neq 0,$ is equation to

A. $\frac{1}{x}$

B. x

C. $\frac{2}{x}$

D. 2x

Answer: C

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30. The sum $\sum_{k=1}^{\infty} \cot^{-1}\left(2k^2\right)$ equals

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. $\frac{3\pi}{2}$

Answer: B

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31. The sum $\tan^{-1}.\frac{1}{1+x+x^2}+\tan^{-1}.\left(\frac{1}{3+3x+x^2}\right)+\tan^{-1}.\left(\frac{1}{7+5x+x^2}\right)+$ of first 100 terms of the series is

A. $\tan^{-1}\left(\frac{100}{1+x^2+100x}\right)$

B. $\tan^{-1}\left(\frac{2x-100}{1-x^2-100x}\right)$

C. $\tan^{-1}\left(\frac{100}{1+x^2-100x}\right)$

D. $\tan^{-1}\left(\frac{2x+100}{1+x^2-100x}\right)$

Answer: A

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32. Find the sum of infinite series

$$s=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\sin^{-1}\left(\frac{\sqrt{2}-1}{\sqrt{6}}\right)+\sin^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}\right)+\ldots+\sin^{-}$$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: B

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33. If $\cot^{-1}\left(\frac{n^2-10n+21\cdot 6}{\pi}\right) > \frac{\pi}{6}, n \in N$, then find the minimum value of n.

A. 4

B. 3

C. 5

D. 2

Answer: B

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34. Find the set of values of k for which $x^2 - kx + \sin^{-1}(\sin 4) > 0$ for all real x .

A. No values of k is possible

B. $[-4(\pi-4), 4(\pi-4)]$

C. $[0, 1]$

D. All real values of k

Answer: A

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35. If $p > q > 0 and pr < -1 < qr$, then find the value of $\tan^{-1}\left(\frac{p-q}{1+pq}\right) + \tan^{-1}\left(\frac{q-r}{1+qr}\right) + \tan^{-1}\left(\frac{r-p}{1+rp}\right)$.

A. 0

B. $-\pi$

C. π

D. $\frac{\pi}{2}$

Answer: C

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36. Number of solutions of the equation

$2\left(\sin^{-1}x\right)^2 - \sin^{-1}x - 6 = 0$ is

A. 2

B. 1

C. 0

D. 3

Answer: B

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Assignment Section C Objective Type Questions More Than One Options Are Correct

1. if $6\sin^{-1}\left(x^2 - 6x + 8.5\right) = \pi$ then

A. $x = 1$

B. $x = 2$

C. $x = 3$

D. $x = 4$

Answer: B::D

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2. Let $f(x) = \sin^{-1} x$ and $g(x) = \cos^{-1} x$, then which of the following statements are correct?

A. $f(x) > g(x), \quad \text{if} \quad x \in \left(\frac{1}{\sqrt{2}}, 1\right]$

B. $f(x) < g(x), \quad \text{if} \quad x \in \left[-1, \frac{1}{\sqrt{2}}\right)$

C. $f(x) > g(x), \quad \text{if} \quad x \in \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

D. $f(x) < g(x), \quad \text{if} \quad x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

Answer: A::B

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3. If the equation $\sin^{-1}\left(x^2+x+1\right)+\cos^{-1}(\lambda x+1)=\frac{\pi}{2}$ has exactly two solutions, then the value of λ is

A. -1

B. 0

C. 1

D. 2

Answer: B::C

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4. The solution of $\sin^{-1}|\sin x|=\sqrt{\sin^{-1}|\sin x|}$ is

A. $(n\pi-1)$

B. $n\pi$

C. $n\pi+1$

D. $2n\pi + 1$

Answer: A::B::C::D

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5. Which of the following is/are true?

A. $\tan\left|\tan^{-1} x\right| = |x|$

B. $\cot\left|\cot^{-1} x\right| = x$

C. $\tan^{-1}|\tan x| = |x|$

D. $\sin\left|\sin^{-1} x\right| = |x|$

Answer: A::B::D

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6. The solutions of the equation $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1} x\right)\right\}\right] = 0$ are

A. ± 1

B. $1 \pm \sqrt{2}$

C. $-1 \pm \sqrt{2}$

D. 2

Answer: A::B::C

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7. If the numerical value of $\tan\left(\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right)$ is $\left(\frac{a}{b}\right)$, where a, b are two positive integers and their H.C.F. is 1

A. $a + b = 23$

B. $a - b = 11$

C. $3b = a + 1$

D. $2a = 3b$

Answer: A::B::C

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8. Prove that $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{2x}{2-x^2}\right)$

A. 1

B. 2

C. 3

D. 4

Answer: A

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9. let $f(x)=e^{\cos^{-1}\sin\left(x+\frac{\pi}{3}\right)}$ then $f\left(\frac{8\pi}{9}\right)$ and $f\left(\frac{-7\pi}{4}\right)$

A. $f\left(\frac{8\pi}{9}\right)=e^{\frac{5\pi}{18}}$

B. $f\left(\frac{8\pi}{9}\right)=e^{\frac{13\pi}{18}}$

C. $f\left(-\frac{7\pi}{4}\right) = e^{\frac{\pi}{12}}$

D. $f\left(-\frac{7\pi}{9}\right) = e^{\frac{11\pi}{12}}$

Answer: B::C

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10. Value of x satisfying $\tan\left(\sec^{-1} x\right) = \sin\left(\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)$

A. $x = -\frac{3}{\sqrt{5}}$

B. $x = \frac{\sqrt{5}}{2}$

C. $x = \frac{3}{\sqrt{5}}$

D. $x = -\frac{\sqrt{5}}{2}$

Answer: A::C

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11. $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ equals

A. $\frac{\pi}{2} - \cos^{-1}.\frac{4}{5}$

B. $\tan^{-1}.\frac{1}{2}$

C. $\frac{\pi}{4} - \frac{1}{2}\cos^{-1}.\frac{4}{5}$

D. $\frac{\pi}{2} - \tan^{-1}.\frac{1}{2}$

Answer: B

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12. Let $f(x) = \tan^{-1}\left(\frac{1}{2}\tan 2x\right) + \tan^{-1}(\cot x) + \tan^{-1}(\cot^3 x)$ *then*

(1) $f\left(\frac{3\pi}{8}\right) = \pi$ (2) $f\left(\frac{\pi}{8}\right) = 0$ (3) $f\left(\frac{\pi}{8}\right) = \pi$ (4) $f\left(\frac{3\pi}{8}\right) = 0$

A. $f\left(\frac{3\pi}{8}\right) = \pi$

B. $f\left(\frac{\pi}{8}\right) = 0$

C. $f\left(\frac{\pi}{8}\right) = \pi$

D. $f\left(\frac{3\pi}{8}\right) = 0$

Answer: B::D

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13. Solution(s) of the equation

$\sin\left(-\frac{\pi}{3} + \tan^{-1} x + \cot^{-1} x\right) = \frac{1}{2}$ is/are

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. $\sqrt{3}$

D. $-\sqrt{3}$

Answer: A::B::C::D

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Assignment Section D Linked Comprehension Type Questions

1. Transformations of inverse trigonometric functions need to be handled with care. Consider the identity $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$ in its domain of definition. Suppose we set $\tan\theta = x$, we have $\sin 2\theta = \dfrac{2x}{1+x^2}$

Taking $\sin^{-1}$of both sides yields $2\theta = \sin^{-1}.\dfrac{2x}{1} + x^2 i.e., 2\tan^{-1}x = \sin^{-1}.\dfrac{2x}{1+x^2}$. But we will discover that the above identity is not valid for all x. Choose $x = \sqrt{3}, LHS = 2\tan^{-1}\sqrt{3} = 2\times\dfrac{\pi}{3} = \dfrac{2\pi}{3}, RHS = \sin.\dfrac{2\sqrt{3}}{1+3} = \sin^{-1}$. And so left hand and right hand side don't match. The reason is that we have disregarded the principal values of inverse functions. So it is well to remember that the iverse trigonometric formmulae have restrictions attached to the argument. When the values of x lie outside the interval of validity then the formula needs to be corrected.

Let $f(x) = \dfrac{\sin^{-1}(x)}{1+x^2}, g(x) = 2\tan^{-1}x$. Then the largest interval in R on which f and g both are agree

A. $[-1, 1]$

B. $(-\infty, -1]$

C. $[1, \infty)$

D. $(-\infty, 1)$

Answer: A

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2. Transformations of inverse trigonometric functions need to be handled with care. Consider the identity $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ in its domain of definition. Suppose we set $\tan\theta = x$, we have $\sin 2\theta = \frac{2x}{1+x^2}$

Taking $\sin^{-1}$of both sides yields $2\theta = \sin^{-1}.\frac{2x}{1} + x^2 i.e., 2\tan^{-1}x = \sin^{-1}.\frac{2x}{1+x^2}$. But we will discover that the above identity is not valid for all x. Choose $x = \sqrt{3}, LHS = 2\tan^{-1}\sqrt{3} = 2\times\frac{\pi}{3} = \frac{2\pi}{3}, RHS = \sin.\frac{2\sqrt{3}}{1+3} = \sin^{-1}$. And so left hand and right hand side don't match. The reason is that we have disregarded the principal values of inverse functions. So it is well to remember that the iverse trigonometric formmulae have restrictions attached to the argument. When the values of x lie outside the interval of validity then the formula needs to be corrected.

Let $f(x) = \frac{\sin^{-1}(x)}{1+x^2}, g(x) = 2\tan^{-1}x$. Then the largest interval in R on which f and g both are agree

A. $\pi + \sin^{-1}\left(\frac{4\pi}{4+\pi^2}\right)$

B. $\frac{\pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{4\pi}{4+\pi^2}\right)$

C. $\frac{\pi}{2} + \frac{1}{2}\sin^{-1}\left(\frac{4\pi}{4+\pi^2}\right)$

D. $-\pi + \sin^{-1}\left(\frac{4\pi}{4+\pi^2}\right)$

Answer: B

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3. Transformations of inverse trigonometric functions need to be handled with care. Consider the identity $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ in its domain of definition. Suppose we set $\tan\theta = x$, we have $\sin 2\theta = \frac{2x}{1+x^2}$

Taking $\sin^{-1}$of both sides yields $2\theta = \sin^{-1}.\frac{2x}{1} + x^2 i.e., 2\tan^{-1}x = \sin^{-1}.\frac{2x}{1+x^2}$. But we will discover that the above identity is not valid for all x. Choose

$x = \sqrt{3}, LHS = 2\tan^{-1}\sqrt{3} = 2 \times \frac{\pi}{3} = \frac{2\pi}{3}, RHS = \sin.\frac{2\sqrt{3}}{1+3} = \sin^{-1}$

. And so left hand and right hand side don't match. The reason is that we have disregarded the principal values of inverse functions. So it is well to remember that the iverse trigonometric formmulae have restrictions attached to the argument. When the values of x lie outside the interval of validity then the formula needs to be corrected.

Let $f(x) = \frac{\sin^{-1}(x)}{1+x^2}, g(x) = 2\tan^{-1}x$. Then the largest interval in R on which f and g both are agree

A. $2\tan^{-1}x = \sin^{-1}.\frac{2x}{1+x^2} - \frac{\pi}{2}$

B. $2\tan^{-1}x = -\pi - \sin^{-1}.\frac{2x}{1+x^2}$

C. $2\tan^{-1}x = \pi - \sin^{-1}.\frac{2x}{1+x^2}$

D. $2\tan^{-1}x = \frac{\pi}{2} + \sin^{-1}.\frac{2x}{1+x^2}$

Answer: B

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4. If the maximum possible value of $\left(\sin^{-1}x\right)^2+\left(\tan^{-1}y\right)^2+\pi\sin^{-1}x+\pi\tan^{-1}y+\frac{\pi^2}{2}$ is k, and [] represents the greatest integer function then the value of $\left[\frac{k}{2\pi^2}\right]$ is

A. 0

B. 1

C. 2

D. 3

Answer: A

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5. If the maximum value of $\left(\sec^{-1}x\right)^2+\left(\cos ec^{-1}x\right)^2$ approaches a, the minimum value of $\left(\tan^{-1}x\right)^3+\left(\cot^{-1}x\right)^3$ approaches b then $\left(a+\frac{b}{\pi}\right)$ is equal to

A. π^2

B. $\frac{41\pi^2}{32}$

C. $\frac{\pi^2}{32}$

D. $\frac{43\pi^2}{32}$

Answer: B

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6.

$\left(\cos^{-1}x+\sec^{-1}x+\tan^{-1}x+\cot^{-1}x\right)_{\max}+6\left(\left(\sin^{-1}x\right)^2+\pi\sin^{-1}x\right.$

equals

A. (a) $\frac{5\pi^2+3\pi}{2}$

B. (b) $\frac{\pi^2+3\pi}{2}$

C. (c) $\frac{5\pi-3\pi^2}{2}$

D. (d) $\frac{\pi+3\pi^3}{2}$

Answer: C

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7. Sometimes we are just concerned with finding integral solutions to equations. Consider the equation

$\tan^{-1}.\frac{1}{m}+\tan^{-1}.\frac{1}{n}=\tan^{-1}.\frac{1}{\lambda}$, where $m, n, \lambda \in N$

How many positive integral solutions (m, n) exist for the equation if $\lambda = 3$?

A. Two

B. Four

C. Six

D. Eight

Answer: B

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8. Sometimes we are just concerned with finding integral solutions to equations. Consider the equation

$\tan^{-1}\left(\frac{1}{m}\right)+\tan^{-1}\left(\frac{1}{n}\right)=\tan^{-1}\left(\frac{1}{\lambda}\right)$, where $m, n, \lambda \in N$

For $\lambda = 11$, an integral pair (m, n) satisfying the equation is:

A. (a) $(12, 72)$

B. (b) $(12, 133)$

C. (c) $(13, 74)$

D. (d) $(13, 136)$

Answer: B

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9. Sometimes we are just concerned with finding integral solutions to equations. Consider the equation

$\tan^{-1}.\frac{1}{m}+\tan^{-1}.\frac{1}{n}=\tan^{-1}.\frac{1}{\lambda}$, where $m, n, \lambda \in N$

If λ is such that $\lambda^2 = 1$ is a prime, then how many solutions (m,n) are there for the equation?

A. One

B. Two

C. Four

D. Infinite

Answer: B

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Assignment Section E Assertion Reason Type Questions

1. STATEMENT -1 : $\tan^{-1} x = \sin^{-1} y \Rightarrow y \in (-1, 1)$

and

STATEMENT -2 : $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

A. Statement -1 is True, Statement-2 is True, Statement -2 is a correct explanation for Statement -1

B. Statement -1 is True, Statement -2 is True, Statement -2 is NOT a correct explanation for Statement -1

C. Statement-1 is True, Statement -2 is False

D. Statement -1 is False, Statement -2 is True

Answer: A

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2. STATEMENT -1 : If $\left[\sin^{-1} x\right] > \left[\cos^{-1} x\right]$,where [] represents the greatest integer function, then $x \in [\sin 1, 1]$ is

and

STATEMENT -2 : $\cos^{-1}(\cos x) = x, x \in [-1, 1]$

A. Statement -1 is True, Statement-2 is True, Statement -2 is a correct explanation for Statement -2

B. Statement -1 is True, Statement -2 is True, Statement -2 is NOT a correct explanation for Statement -2

C. Statement-1 is True, Statement -2 is False

D. Statement -1 is False, Statement -2 is True

Answer: C

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3. STATEMENT -1 : The function $f:[-1,1] \to [0,\pi], f(x) = \cos^{-1} x$ is not one-one. and STATEMENT -2 :The function $f:(-\infty,\infty) \to [-1,1], f(x) = \cos x$ is not one-one.

A. Statement -1 is True, Statement-2 is True, Statement -2 is a correct explanation for Statement -3

B. Statement -1 is True, Statement -2 is True, Statement -2 is NOT a correct explanation for Statement -3

C. Statement-1 is True, Statement -2 is False

D. Statement -1 is False, Statement -2 is True

Answer: D

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4. STATEMENT -1 : The value of $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}, \forall x \in R - \{0\}$. and STATEMENT -2 : The value of $\tan^{-1}.\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1} x & x > 0 \\ -\pi + \cot^{-1} x & x < 0 \end{cases}$

A. Statement -1 is True, Statement-2 is True, Statement -2 is a correct explanation for Statement -4

B. Statement -1 is True, Statement -2 is True, Statement -2 is NOT a correct explanation for Statement -4

C. Statement-1 is True, Statement -2 is False

D. Statement -1 is False, Statement -2 is True

Answer: D

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5. STATEMENT-1 : The solution of $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = \frac{\pi}{2}$ is , $x = \pm \frac{1}{12}$. and STATEMENT - 2 As, $\sin^{-1} x$ is defined for $|x| \leq 1$.

A. Statement -1 is True, Statement-2 is True, Statement -2 is a correct explanation for Statement -5

B. Statement -1 is True, Statement -2 is True, Statement -2 is NOT a correct explanation for Statement -5

C. Statement-1 is True, Statement -2 is False

D. Statement -1 is False, Statement -2 is True

Answer: D

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Assignment Section F Matrix Match Type Question

1. Let

$t_1 = (\sin^{-1} x)^{\sin^{-1} x}, t_2 = (\sin^{-1} x)^{\cos^{-1} x}, t_3 = (\cos^{-1} x)^{\sin^{-1} x}, t_4 = (\cos$

,

Match the follwing items of Column I with Column II

	Column I		Column II
A.	$x \in (0, \cos 1)$	(p)	$t_1 > t_2 > t_4 > t_3$
B.	$x \in \left(\cos 1, \frac{1}{\sqrt{2}}\right)$	(q)	$t_4 > t_3 > t_1 > t_2$
C.	$x \in \left(\frac{1}{\sqrt{2}}, \sin 1\right)$	(r)	$t_2 > t_1 > t_4 > t_3$
D.	$x \in (\sin 1, 1)$	(s)	$t_3 > t_4 > t_1 > t_2$

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2. Which of the following affect the heat of reaction?

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3. Let (x,y) be such that $\sin^{-1}(ax) + \cos^{-1} y + \cos^{-1}(bxy) = \frac{\pi}{2}$

Column I	Column II
(A) If $a = 1, b = 0$, then (x, y)	(p) Lies on the circle $x^2 + y^2 = 1$
(B) If $a = 1, b = 1$, then (x, y)	(q) Lies on $(x^2 - 1)(y^2 - 1) = 0$
(C) If $a = 1, b = 2$, then (x, y)	(r) Lies on $y = x$
(D) If $a = 2, b = 2$, then (x, y)	(s) Lies on $(4x^2 - 1)(y^2 - 1) = 0$
	(t) Lies on $x + 2y = 0$

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4. Match the following

Column I	Column II
(A) The value of $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right\}$ is not equal to	(p) $-\frac{7}{17}$
(B) The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is not equal to	(q) $\frac{17}{6}$
(C) The maximum number of solution of $\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \pi$ in $\left(\frac{1}{2}, \frac{3}{2}\right)$ is not equal to	(r) $\frac{1}{2}$
(D) The value of x for which $\sin(\cot^{-1}(1 + x)) = \cos(\tan^{-1}x)$ is not equal to	(s) $-\frac{1}{2}$
	(t) 1

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Assignment Section G Integer Answer Type Questions

1. If $f(x) = \sin^{-1} x$ and $\lim_{x \to 1/2+} f\left(3x - 4x^3\right) = a - 3 \lim_{x \to 1/2+} f(x)$, then [a] is equal to {where [] denotes G.I.F}

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2. If $\cos^{-1}\left(4x^3 - 3x\right) = a + b\cos^{-1} x$ for $-1 < x < -\frac{1}{2}$, then [a + b+ 4] is equal to {where [] denotes G.I.F}`

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3. If $f(x) = \cos^{-1}\left(4x^3 - 3x\right)$ and $\lim_{x \to \frac{1}{2}+} f'(x) = a$ and $\lim_{x \to \frac{1}{2}-} f'(x) = b$ then $a + b + 3$ is equal to ____

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Assignment Section H Multiple True False Type Questions

1. STATEMENT-1 : If $\tan^2\left(\sin^{-1}x\right) > 1$ then $x \in \left(-1-\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}.1\right)$. STATEMENT-2 : The number of positive integral solution of $\frac{\tan^{-1}1}{y} + \cot^{-1}\left(\frac{1}{x}\right) = \cot^{-1}\left(\frac{1}{3}\right)$, where $\frac{x}{y} < 1$, is 2. STATEMENT -3 : If $\sin^{-1}x = -\cos^{-1}\sqrt{1-x^2}$ and $\sin^{-1}y = \cos^{-1}\sqrt{1-y^2}$, then the exact range of $\left(\tan^{-1}x + \tan 6 - 1y\right)$ is $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

A. T T F

B. F T F

C. T T T

D. F F T

Answer: C

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Assignment Section I Subjective Type Question

1. Let $\lambda = \sec^2\left(\tan^{-1} 2\right) + \cos ec^2\left(\cot^{-1} 3\right) + 2$ then the value of $17\lambda^2 + 7$ is .....

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2. The value of $\cot^{-1}(\cot(-10)) + \cos^{-1}(\cos 10)$ is given by $a + b\pi$, a, b being rational numbers, then (b-a) equals.....

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3. If M and m are the greatest and least value of the function $f(x) = \left(\cos^{-1} x\right)^2 + \left(\sin^{-1} x\right)^2$ then the value of $\left(\frac{M + 9m}{m}\right)^3$ is .....

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4. The value of

$$\sin^{-1}\left(\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}.\frac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right)\right)$$ is

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5. $ax+b\left(\sec\left(\tan^{-1}x\right)\right)=c$ and $ay+b\left(\sec\left(\tan^{-1}y\right)\right)=c$

The value of xy is

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6. A ball A of mass m falls on the surface of the earth from infinity. Another ball B of mass 2m falls on the earth from the height equal to six times the radius of the earth. Then ratio of velocities of A and B on reaching the earth is $\sqrt{x/y}$ where x and y are coprimes. Find x+y .

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Assignment Section J Ankash Challengers Questions

1. The total number of solutions of the equation $\sin^{-1}\left(\frac{3}{5}x\right)+\sin^{-1}\left(\frac{4}{5}x\right)=\sin^{-1}x$

A. 1

B. 0

C. 2

D. 3

Answer: D

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2. The value (s) of θ satisfying the equation $\theta=\tan^{-1}\left(2\tan^2\theta\right)-\frac{1}{2}\left(\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right)\right)$ is

A. (a) $n\pi+\frac{\pi}{4}$

B. (b) $n\pi + \tan^{-1}(-2)$

C. (c) $n\pi + \frac{\pi}{3}$

D. (d) $n\pi + \frac{\pi}{6}$

Answer: A::B

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3. Show that :

$$2\tan^{-1}\left\{\frac{\tan\alpha}{2}\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\} = \tan^{-1}\left(\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}\right).$$

A. $\tan^{-1}\left(\frac{\cos\alpha-\sin\beta}{\sin\alpha\cos\beta}\right)$

B. $\tan^{-1}\left(\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}\right)$

C. Both (1) & (2)

D. $\tan^{-1}\left(\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}\right)$

Answer: B

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4. The total number of ordered pairs of (x,y) satisfying the equation $13 + 12\left[\tan^{-1} x\right] = 24[Inx] + 8[e^x] + 6\left[\cos^{-1} y\right]$ is/are:

A. 0

B. 1

C. 2

D. Infinitely many

Answer: A

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