MATHS

BOOKS - IA MARON MATHS (HINGLISH)

DIFFERENTIATION OF FUNCTIONS

2 1 Definition Of The Derivatives

1. Find the increment $\triangle y$ and the ratio $\frac{\triangle y}{\triangle x}$ for the following function

(a)$y = \sqrt{x}$ at x=1 and $\triangle x = 0.0001$,

(b) $y = \frac{1}{x^2 + x - 6}$ at x=1 and $\triangle x = 0.2$

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2. find the derivatives of the following functions :

(a) y =cos ax, (b)$y = 5x^2 - 2x$

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3. Show that the following functions have no finite derivatives at the indicated points :

(a) $y = 5\sqrt{x^3}$ at the point x=0

(b) $y = 3\sqrt{x-1}$ at the point x=1

(c) $y = 3|x| + 1$ at the point x=0

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4. Find the average velocity of motion specified by the formula $s = \left(t^2 - 5t + 2\right)$ m from $t_1 = 5\,\text{sec}$ to t_2 =15 sec.

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5. find the derivatives of the following functions : (a) $y = x^3, (b) y = 1/x^2$

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6. Investigate the function y = |cosx| for differentiability at the points $x = \frac{\pi}{2} + n\pi$ (n an integer).

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2 2 Differentiation Of Explicit Functions

1. Find y ' , if

(a)

$$y = 5x^{2/3} - 3x^{5/2} + 2x^{-3} (b) y = \frac{a}{\left(3\sqrt{x}\right)^2} - \frac{b}{x^3\sqrt{x}}$$

(a,b constants)

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2. Find y ', if : (a)

$$y = 3\cos x + 2\sin x, (b) y = \frac{\sin x + \cos x}{\sin x - \cos x} (c) y = \left(x^2 + 1\right)$$

arc tan x, (d) $y = x^3 arc \sin x$

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3. Find the derivative of the given function and then compute the particular value of the derivative at the indicated value of the argument:

(a) $f(x) = 1 - \left(3\sqrt{x}\right)^2 + \frac{16}{x} atx = -8$

(b) $f(x) = \frac{\left(1 - 3\sqrt{x}\right)^2}{x} atx = 0.01$

(c) $f(t) = \frac{\cot t}{1 - \sin t}$ at t=$\frac{\pi}{6}$

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4. Find the derivation of the function

$$y = arc\sin\left(\frac{2x}{1 + x^2}\right)$$

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5. Show that the function y = $xe^{-x^2/2}$ satisfies the equation

$$x\left(\frac{dy}{dx}\right) = (1 - x^2)y$$

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6. Show that the function $y = xe^{-x}$ satisfies the equation xy'=(1-x)y

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7. Applying logrithmic differentiation find the derivatives of the following functions

$(a) y = (\cos x)^{\sin x}$

(b) $y = \left(\frac{3\sqrt{\sin 3x}}{1 - \sin 3x} \right)$

$(c) y = \frac{\sqrt{x-1}}{\left(3\sqrt{x+2}\right)^2 \left(\sqrt{x+3}\right)^3}$

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2 3 Successive Differentiation Of Explicit Function Leibnitz Formula

1. Compute the value of the nth derivative of the function $y = \frac{3x+2}{x^2 - 2x + 5}$ at the point x = 0.

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2. Using the Leibnitz formula give the derivatives of the indicated orders for the following functions

$(a) y = x^3 \sin x$, find y^{20}

(b) $y = e^{-x} \sin x$, find y''',

(c) $y = e^x \left(3x^2 - 4\right)$, find y^n

(d) $y = \left(1 - x^2\right) \cos x$, find y^{2n}

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2 4 Differentiation Of Inverse Implicit And Parametrically Represented Functions

1. Find $y''_{\times}$ if:

$(a) arc\tan y - y + c = 0$

(b) $e^{x} - e^{y} = y - x$

(c) $x + y = e^{x-y}$

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2. Find y 'x for the following implicit function

(a) y=x +arc tan y

(b) $x^2 + 5xy + y^2 - 2x + y - 6 = 0$ find y'' at the point

(1,1)

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2 5 Application Of The Derivative

1. Find the points on the curve $y = x^3 - 3x + 5$ at which the tangent line :

(a) is parallel to the straight line y = - 2x,

(b) is perpendicular to the straight line $y = \frac{x}{9}$,

(c) forms an angle of 45° with the positive direction of the x-axis.

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2. Find the angles at which the following lines intersect:

(a) the straight line y = 4 - x and the parabola

$y = 4 - \frac{x^2}{2}$

(b) the sinusoid y = sin x and the cosine curve y = cos x

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3. Prove that the segment of the tangent to the hyperbola $y = \frac{c}{x}$ which is contained between the coordinate axes is bisected at the point of tangency.

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4. Find the equations of the tangent and the normal to the following curves :

$(a) 4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at

the point (-2,3)

(b) $x^5 + y^5 - 2xy = 0$ at the point (1,1)

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5. $f(x) = 3x^5 - 15x^3 + 5x - 7$. Find out at which of the points x the rate of change of the function is minimal

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6. A point is in motion along a cubic parabola $12y = x^3$. Which of its coordinates changes faster ?

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7. A body of mass 6g is in rectilinear motion according to the law $s = -1 + (t+1) + (t+1)^3$ (s is in centimetres and t, in seconds). Find the kinetic enery of the body one second after it begins to move.

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8. If the velocity of a body moving in a straight line is proportional to the square root of the distance traversed, then it moves with

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9. A raft is pulled to the bank by means of a rope which is wound on a drum, at a rate of 3m/min. Determine the speed of the raft at the moment when it is 25m distant from the bank if the drum is situated on the bank 4m above water level.

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10. (a) Find the slope of the tangent to the cube parabola $y = x^3$ at the point $x = \frac{\sqrt{3}}{3}$

(b) Write the equations of the tangents to the curve $y = \frac{1}{1+x^2}$ at the 1 points of its intersection with the hyperbola $y = \frac{1}{x+1}$

(c) Write the equation of the normal to the parabola

$y = x^2 + 4x + 1$ perpendicular to the line joining the origin of coordinates with the vertex of the parabola.

(d) At what angle does the curve $y = e^x$ intersect the y-axis

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11. The velocity of a body in rectilinear motion is determined by the formula $v = 3t + t^2$. What acceleration will the body have 4s after the start ?

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12. The law of rectilinear motion of a body with a mass of 100 kg is $s = 2t^2 + 3t + 1$. Determine the kinetic energy $\frac{mv^2}{2}$ of the body 5s after the start

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13. If $s = ae^l + be^{-t}$ is the equation of motion of a particle, then its acceleration is equal to

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14. A body is thrown vertically with an initial velocity of a m/sec. What altitude will it reach in t seconds ? Find the velocity of the body. In how many seconds and at what

distance from the ground will the body reach the highest point ?

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2 6 Application Of The Derivative

1. All faces of a copper cube with 5 cm sides were uniformly ground down. As a result the weight of the cube was reduced by 0.96 g. Knowing the specific weight of copper (8) find the reduction the cube size, ie, the amount by which its side was reduced

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2. Find differentials of higher orders (x an independent variable) :

$(a) y = 4^{-x^2}$ find d^2y

(b) $y = \sqrt{\sin^2 x - 4}$ find d^2y

(c) $y = \sin^2 x$ find d^3y

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3. y = ln $\frac{1 - x^2}{1 + x^2}$ find d^2y if :

(a)x is an independent variable,

(b) x is a function of another variable. Consider the particular case when x = tan t

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2 7 Additional Problems

1. Given the functions :

$(a) f(x) = |x|$ and

(b) $\phi(x) = \left|x^3\right|$

Do derivaties for these functions exist at the point x=0

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2. Prove that the Chebyshev polynomials

satisfy the equation

$\left(1 - x^2\right)T''_n(x) - xT'_n(x) + n^2T_n(x) = 0$

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3. find derivatives of $y = 2x^2 - x^4$

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4. Determine the parameters a,b,c in the equation of the parabola $y = ax^2 + bx + c$ so that it becomes tangent to the straight line y = x at the point x = 1 and passes through the point (-1,0).

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5. Prove that the curves $y_1 = f(x)(f(x) > 0)$ and $y_2 = f(x)$ sin x, where f(x) is a differentiable function, are tangent to each other at the common points.

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6. Show that two cardioids $p = a(1 + \cos\phi)$ and $p = a(1 - \cos\phi)$ intersect at right angles

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7. How will the expression $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y$ be transformed (where y is a twice differentiable function of x) if we introduce a new independent variable t, putting x = cos t

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