MATHS

BOOKS - SHRI BALAJI MATHS (ENGLISH)

AREA UNDER CURVES

Axercise Single Choice Problems

1. The area enclosed by the curve

$[x+3y]=[x-2]$ where $x \in [3, 4]$ is :

(where[.] denotes greatest integer function)

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. 1

Answer: B

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2. The area of region(s) enclosed by the curve $y = x^2$ and $y = \sqrt{|x|}$ is

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{16}{3}$

Answer: B

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3. Find the area enclosed by the figure described by the equation $x^4 + 1 = 2x^2 + y^2$.

A. 2

B. $\frac{16}{3}$

C. $\frac{8}{3}$

D. $\frac{4}{3}$

Answer: C

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4. The area defined by $|y| \le e^{|x|} - \frac{1}{2}$ in cartesian co-ordinate system, is :

A. $(2 - 2\ln 2)$

B. $(4 - \ln 2)$

C. $(2 - \ln 2)$

D. $(2 - 2\ln 2)$

Answer: D

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5. For each positive integer $n > a, A_n$ represents the area of the region restricted to the following two inequalities : $\frac{x^2}{n^2} + y^2$ and $x^2 + \frac{y^2}{n^2} < 1$. Find $\lim_{n \to \infty} A_n$.

A. 4

B. 1

C. 2

D. 3

Answer: A

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6. Find the ratio in which the area bounded by the curves $y^2 = 12x and x^2 = 12y$ is divided by the line $x = 3$.

A. $7:15$

B. $15:49$

C. $1:3$

D. $17:49$

Answer: B

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7. The value of positive real parameter 'a' such that area of region blunded by parabolas $y = x - ax^2, ay = x^2$ attains its maximum value is equal to :

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{3}$

D. 1

Answer: D

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8. For $0 < r < 1,$ let n_r dennotes the line that is normal to the curve $y = x^r$ at the point $(1, 1)$ Let S_r denotes the region in the first quadrant bounded by the curve $y = x^r$,the x-axis and the line n_r' Then the value of r the minimizes the area of S_r is :

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{2} - 1$

C. $\frac{\sqrt{2} - 1}{2}$

D. $\sqrt{2} - \frac{1}{2}$

Answer: B

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9. The area bounded by $|x| = 1 - y^2$ and $|x| + |y| = 1$ is:

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. 1

Answer: C

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10. Point A lies on the curve $y = e^{x^2}$ and has the coordinate $\left(x, e^{-x^2}\right)$ where $x > 0$. Point B has the coordinates $(x, 0)$. If $'O'$ is the origin, then the maximum area of the ΔAOB is

A. $\frac{1}{\sqrt{8e}}$

B. $\frac{1}{\sqrt{4e}}$

C. $\frac{1}{\sqrt{2e}}$

D. $\frac{1}{\sqrt{e}}$

Answer: A

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11. If the area enclosed between the curves $y = ax^2 and x = ay^2 (a > 0)$ is 1 square unit, then find the value of a.

A. $\frac{1}{\sqrt{3}}$

B. $\frac{1}{2}$

C. 1

D. $\frac{1}{3}$

Answer: D

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12. Let $f(x) = x^3 - 3x^2 + 3x + 1$ and g be the inverse of it , then area bounded by the curve $y = g(x)$ wirth x-axis between $x = 1$ to $x = 2$ is (in square units):

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{3}{4}$

D. 1

Answer: B

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13. Area bounded by $x^2y^2 + y^4 - x^2 - 5y^2 + 4 = 0$ is equal to :

A. $\frac{4\pi}{2} + \sqrt{2}$

B. $\frac{4\pi}{3} - \sqrt{2}$

C. $\frac{4\pi}{3} - \sqrt{2}$

D. none of these

Answer: C

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14. Let $f(x): R^+ \to R^+$ is an invertible function such that $f'(x) > 0 and f(x) > 0 \forall x \in [1, 5]$. If $f(1) = 1$ and $f(5) = 5$ and area under the curve $y = f(x)$ on x-axis from $x = 1 \to x = 5 is 8$ sq. units, then area bounded by $y = f^{-1}(x)$ on x-axis from $x = 1 \to x = 5$ is

A. 12

B. 16

C. 18

D. 20

Answer: B

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15. A circel centered at origin and having radius π units is divided by the curve $y = \sin x$ in two parts. Then area of the upper part equals to :

A. $\frac{\pi^2}{2}$

B. $\frac{\pi^3}{4}$

C. $\frac{\pi^3}{2}$

D. $\frac{\pi^3}{8}$

Answer: C

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16. The area of the loop formed by $y^2 = x(1 - x^3)$ dx is:

A. $\int_0^1 \sqrt{x - x^4} dx$

B. $2\int_0^1 \sqrt{x - x^4}\,dx$

C. $\int_{-1}^1 \sqrt{x - x^4}\,dx$

D. $4\int_0^{1/2} \sqrt{x - x^4}\,dx$

Answer: B

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17. If $f(x) = \min\left[x^2, \sin\frac{x}{2}, (x - 2\pi)^2\right]$, the area bounded by the curve $y = f(x)$, x-axis, $x = 0$ and $x = 2\pi$ is given by

Note: x_1 is the point of intersection of the curves x^2 and $\sin\frac{x}{2}$, x_2 is the point of intersection of the curves $\sin\frac{x}{2}$ and $(x - 2\pi)^2)$

A.

$$\int_0^{x1}\left(\sin\frac{x}{2}\right)dx + \int_{x_1}^{\pi} x^2 dx + \int_{\pi}^{x_2}(x - 2\pi)^2 dx + \int_{x_2}^{2\pi}\left(\sin\frac{x}{2}\right)dx$$

B. $\int_0^{x1} x^2 dx + \int_{x_1}^{x3}\left(\sin\frac{x}{2}\right)dx + \int_{x_2}^{2\pi}(x - 2\pi)^2 dx$, where $x_1 \in \left(0, \frac{\pi}{3}\right)$ and $x_2 \in \left(\frac{5\pi}{3}, 2\pi\right)$

C. $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2} \sin\left(\frac{x}{2}\right) dx + \int_{x_2}^{2\pi} (x - 2\pi)^2 dx,$ where $x_1 \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ and $x_2 \in \left(\frac{3\pi}{2}, 2\pi\right)$

D. $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2} \sin\left(\frac{x}{2}\right) dx + \int_{x_2}^{2\pi} (x - 2\pi)^2 dx,$ where $x_1 \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $x_2 \in (\pi, 2\pi)$

Answer: B

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18. The area enclosed between the curves $|x| + |y| \geq 2$ and $y^2 = 4\left(1 - \frac{x^2}{9}\right)$ is :

A. $(6\pi - 4)$ sq. units

B. $(6\pi - 8)$ se. units

C. $(3\pi - 4)$ se. units

D. $(3\pi - 2)$ sq. units

Answer: B

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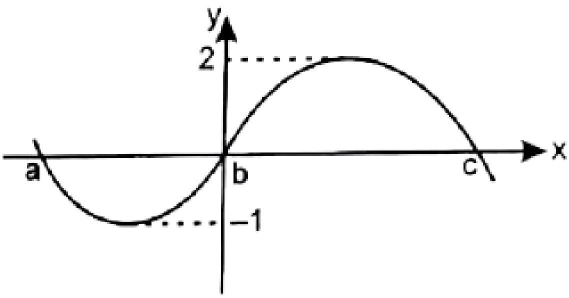
Axercise One Or More Than One Answer Is Are Correct

1. Let f (x) be a polynomial function of degree 3 where $a < b < c$ and $f(a) = f(b) = f(c)$. If the graph of f (x) is as shown, which of the following statements are INCORRECT ? (Where $c > |a|$)



A. (a) $\int_a^c f(x)dx = \int_b^c f(x)dx + \int_c^b f(x)dx$

B. (b) $\int_a^c f(x)dx < a$

C. (c) $\int_a^b f(x)dx < \int_c^b f(x)dx$

D. (d) $\frac{1}{b-a}\int_a^b f(x)dx > \frac{1}{c-b}\int_b^c f(x)dx$

Answer: B::C::D

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2. $T_n = \sum_{r=2n}^{3n-1} \frac{r}{r^2+n^2}, S_n = \sum_{r=2n+1}^{3n} \frac{r}{r^2+n^2},$ then $\forall n \in \{1, 2, 3...\}$:

A. $T_n > \frac{1}{2}\ln 2$

B. $S_n < \frac{1}{2}\ln 2$

C. $T_n < \frac{1}{2}\ln 2$

D. $S_n > \frac{1}{2}\ln 2$

Answer: A::D

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3. If a curve $y = a\sqrt{x} + bx$ passes through point $(1, 2)$ and the area bounded by curve, line $x = 4$ and x-axis is 8, then : (a) $a = 3$ (b) $b = 3$ (c) $a = -1$ (d) $b = -1$

A. $a = \frac{15}{4}$

B. $b = 3$

C. $a = -1$

D. $b = -\frac{7}{4}$

Answer: A::D

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Axercise Comprehension Type Problems

1. Let $f: A \to B f(x) = \frac{x+a}{bx^2 + cx + 2}$, where A represent domain set and B represent range set of function $f(x)$ a,b,c

$\in R, f(-1)=0$ and $y=1$ is an asymptote of $y=f(x)$ and $y=g(x)$ is the inverse of $f(x)$.

g (0) is equal to :

A. -1

B. -3

C. $-\frac{5}{2}$

D. $-\frac{3}{2}$

Answer: A

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2. Let $f: A \to B f(x)=\frac{x+a}{bx^2+cx+2}$, where A represent domain set and B represent range set of function $f(x)$ a,b,c $\in R, f(-1)=0$ and $y=1$ is an asymptote of $y=f(x)$ and $y=g(x)$ is the inverse of $f(x)$.

g (0) is equal to :

A. $2\sqrt{5} + \ln\left(\frac{3-\sqrt{5}}{5+\sqrt{5}}\right)$

B. $2\sqrt{5} + 2\ln\left(\frac{3+\sqrt{5}}{3-\sqrt{5}}\right)$

C. $3\sqrt{5} + 4\ln\left(\frac{3-\sqrt{5}}{3+\sqrt{5}}\right)$

D. $3\sqrt{5} + 2\ln\left(\frac{3-\sqrt{5}}{3+\sqrt{5}}\right)$

Answer: D

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3. Let $f: A \to B f(x) = \frac{x+a}{bx^2 + cx + 2}$, where A represent domain set and B represent range set of function $f(x)$ a,b,c $\in R, f(-1) = 0$ and $y = 1$ is an asymptote of $y = f(x)$ and $y = g(x)$ is the inverse of $f(x)$.

Area of region enclosed by asymptotes of curves $y = f(x)$ and $y = g(x)$ is:

A. 4

B. 9

C. 12

D. 25

Answer: B

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4. Consider the area $S_0, S_1, S_2 \ldots.$ bounded by the x-axis and half-waves of the curve $y = e^{-x} \sin x, \quad \text{where} \quad x \geq 0.$

The value of S_0 is

A. $\frac{1}{2}(1 + e^x)$

B. $\frac{1}{2}\left(1 + e^{-\pi}\right)$

C. $\frac{1}{2}\left(1 - e^{-\pi}\right)$

D. $\frac{1}{2}(e^{\pi} - 1)$

Answer: B

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5. For $j = 0, 1, 2...n$ let S_j be the area of region bounded by the x-axis and the curve $ye^x = \sin x$ for $j\pi \le x \le (j+1)\pi$

The ratio $\dfrac{S_{2009}}{S_{2010}}$ equals :

A. e^{-x}

B. e^{π}

C. $\dfrac{1}{2}e^x$

D. $2e^{\pi}$

Answer: B

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6. For $j = 0, 1, 2...n$ let S_j be the area of region bounded by the x-axis and the curve $ye^x = \sin x$ for $j\pi \le x \le (j+1)\pi$

The value of $\sum_{j=0}^{\infty} S_j$ equals to :

A. $\dfrac{e^x(1+e^x)}{2(e^\pi-1)}$

B. $\dfrac{1+e^\pi}{2(e^\pi-1)}$

C. $\dfrac{1+e^\pi}{e^\pi-1}$

D. $\dfrac{e^\pi(1+e^\pi)}{(e^\pi-1)}$

Answer: B

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Axercise Matching Type Problems

1.

	Column-I		Column-II
(A)	Area of region formed by points (x, y) satisfying $[x]^2 = [y]^2$ for $0 \le x \le 4$ is equal to (where [ ] denotes greatest integer function)	(P)	48
(B)	The area of region formed by points (x, y) satisfying $x + y \le 6$, $x^2 + y^2 \le 6y$ and $y^2 \le 8x$ is $\frac{k\pi - 2}{12}$, then $k =$	(Q)	27
(C)	The area in the first quardant bounded by the curve $y = \sin x$ and the line $\frac{2y-1}{\sqrt{2}-1} = \frac{2}{\pi}(6x - \pi)$ is $\left[\frac{\sqrt{3}-\sqrt{2}}{2} - \frac{(\sqrt{2}+1)\pi}{k}\right]$, then $k =$	(R)	7
(D)	If the area bounded by the graph of $y = xe^{-ax}$ $(a > 0)$ and the abscissa axis is $\frac{1}{9}$ then the value of 'a' is equal to	(S)	4
		(T)	3

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Axercise Subjective Type Problems

1. Find function f(x) which satisfy the relation $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}\ \forall x, y \in R, y \ne 0, f(y) \ne 0$ and $f'(1) = 2$

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2. Let f (x) be a function which satisfy the equation $f(xy) = f(x) + f(y)$ for all $x > 0, y > 0$ such that $f'(1) = 2$. Let A be the area of the region bounded by the curves $y = f(x), y = |x^3 - 6x^2 + 11x - 6|$ and $x = 0$, then find value of $\frac{28}{17}A$.

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3. Find the area enclosed by circle $x^2 + y^2 = 4$, parabola $y = x^2 + x + 1$, the curve $y = \left[\frac{\sin^2 x}{4} + \frac{\cos x}{4}\right]$ and X-axis (where,[.] is the greatest integer function.

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4. Let the function $f: [-4, 4] \to [-1, 1]$ be defined implicitly by the equation $x + 5y - y^5 = 0$ If the area of triangle formed by tangent and normal to $f(x) at x = 0$ and the line $y = 5$ is A, find $\frac{A}{13}$.

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5. Area of the region bounded by $[x]^2 = [y]^2, \quad \text{if} \quad x \in [1, 5]$, where [ ] denotes the greatest integer function is:

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6. Consider $y = x^2$ and $f(x)$where f (x), is a differentiable function satisfying

$f(x+1) + f(z-1) = f(x+z)\,\forall x, z \in R$ and $f(0) = 0, f'(0) = 4.$

If area bounded by curve $y = x^2$ and $y = f(x)$ is $\Delta,$ find the value of $\left(\frac{3}{16}\Delta\right).$

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7. The least integer which is greater than or equal to the area of region in $x - y$ plane satisfying $x^6 - x^2 + y^2 \leq 0$ is:

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8. The set of points (x,y) in the plane satisfying $x^{2/5}+|y|=1$ form a curve enclosing a region of area $\frac{p}{q}$ square units, when p and q are relatively prime positive intergers. Find $p-q$.

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