MATHS

BOOKS - SHRI BALAJI MATHS (ENGLISH)

BIONMIAL THEOREM

Exercise 1 Single Problems

1. Let $N = 2^{1224} - 1, \alpha = 2^{153} + 2^{77} + 1$ and $\beta = 2^{408} - 2^{204} + 1$. Then which of the following statement is correct ?

A. A) α divides N but β does not

B. B) β divides N but α does not

C. C) α and β both divide N

D. D) neither α nor β divides N

Answer: C

Watch Video Solution

2. If $\left(1+2x+x^2\right)^n = \sum_{r=0}^{2n} a_r x^r$, then $a_r =$

A. 0

B. ${}^nC_r$

C. a_r

D. 1

Answer: A

Watch Video Solution

3. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same, if α equals $-\frac{5}{3}$ b. $\frac{10}{3}$ c. $-\frac{3}{10}$ d. $\frac{3}{5}$

A. $-\frac{5}{3}$

B. $\frac{3}{5}$

C. $-\frac{3}{10}$

D. $\frac{10}{3}$

Answer: C

Watch Video Solution

4. If $(1+x)^{2010} = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_{2010} x^{2010}$ then the sum of series $C_2 + C_5 + C_8 + \ldots\ldots\ldots + C_{2009}$ equals to :

A. $\frac{1}{2}\left(2^{2010} - 1\right)$

B. $\frac{1}{3}\left(2^{2010} - 1\right)$

C. $\frac{1}{2}\left(2^{2009} - 1\right)$

D. $\frac{1}{3}\left(2^{2009} - 1\right)$

Answer: B

Watch Video Solution

5. Let $\alpha_n = \left(2+\sqrt{3}\right)^n$. Find $\lim_{n \to \infty} \left(\alpha_n - [\alpha_n]\right)$([.] denotes greateset integer function)

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Answer: A

Watch Video Solution

6. The number $N = {}^{20}C_7 - {}^{20}C_8 + {}^{20}C_9 - {}^{20}C_{10} + \ldots\ldots - {}^{20}C_{20}$ is not divisible by :

A. A) 3

B. B) 7

C. C) 11

D. D) 19

Answer: C

Watch Video Solution

7. The value of the expression $\log_2\left(1+\frac{1}{2}\sum_{k=1}^{11} {}^{12}C_k\right)$:

A. 11

B. 12

C. 13

D. 14

Answer: A

Watch Video Solution

8. The constant term in the expansion of $\left(x+\frac{1}{x^3}\right)^{12}$ is :

A. 26

B. 169

C. 260

D. 220

Answer: D

Watch Video Solution

9. If $\frac{3}{4!}+\frac{4}{5!}+\frac{5}{6!}+\ldots.+50term=\frac{1}{3!}-\frac{1}{(k+3)!}$, then sum of coefficients in the expansion $(1+2x_1+3x_2+\ldots.+100x_{100})^k$ is:

A. (a) $(5050)^{49}$

B. (b) $(5050)^{51}$

C. (c) $(5050)^{52}$

D. (d) $(5050)^{50}$

Answer: D

Watch Video Solution

10. Statement-1: The remainder when $(128)^{(128)^{128}}$ is divided by 7 is 3. because Statement-2: $(128)^{128}$ when divided by 3 leaves the remainder 1.

A. Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

B. Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.

C. Statement-1 is true, statement-2 is false.

D. Statement-1 is false, statement-2 is true.

Answer: D

Watch Video Solution

11. If $n > 3$, then $xyz^nC_0 - (x-1)(y-1)(z-1)^nC_1 + (x-2)(y-2)(z-2)^nC_2 - (x-3)(y-3)(z-3)^nC_3 + \ldots.. + (-1)^n(x-n)(y-n)(z-n)^nC_n$ equals :

A. xyz

B. $x+y+z$

C. $xy+yz+zx$

D. 0

Answer: D

Watch Video Solution

12. If $\alpha_1, \alpha_2, \ldots\ldots.., \alpha_n$ are the n, n^{th} roots of unity, $\alpha_r = e\dfrac{i2(r-1)\pi}{n}, r = 1, 2, \ldots n$ then ${}^nC_1\alpha_1 + {}^nC_2\alpha_2 + \ldots.. + {}^nC_n\alpha_n$ is equal to :

A. (a) $\left(1+\frac{\alpha_2}{\alpha_1}\right)^n - 1$

B. (b) $\frac{\alpha_1}{2}\left[(1+\alpha_1)^n - 1\right]$

C. (c) $\frac{\alpha_1 + \alpha_{n-1} - 1}{2}$

D. (d) $(\alpha_1 + \alpha_{n-1})^n - 1$

Answer: A

Watch Video Solution

13. The remainder when $2^{30} \cdot 3^{20}$ is divided by 7 is :

A. 1

B. 2

C. 4

D. 6

Answer: B

Watch Video Solution

14. ${}^{26}C_0 + {}^{26}C_1 + {}^{26}C_2 + \ldots.. + {}^{26}C_{13}$ is equal to :

A. $2^{25} - \frac{1}{2} \cdot {}^{26}C_{13}$

B. $2^{25} + \frac{1}{2} \cdot {}^{26}C_{13}$

C. 2^{13}

D. $2^{26} + \frac{1}{2} \cdot {}^{26}C_{13}$

Answer: B

Watch Video Solution

15. If a_r is the coefficient of x^r in the expansion of $(1 + x + x^2)^n (n \in N)$. Then the value of $(a_1 + 4a_4 + 7a_7 + 10a_{10} + \ldots\ldots.)$ is equal to :

A. a) 3^{n-1}

B. b) 2^n

C. c) $\frac{1}{3} \cdot 2^n$

D. d) $n \cdot 3^{n-1}$

Answer: D

Watch Video Solution

16. Let $\binom{n}{k}$ represents the combination of $'n'$ things taken $'k'$ at a time, then the value of the sum $\binom{99}{97} + \binom{98}{96} + \binom{97}{95} + \dots\dots\dots + \binom{3}{1} + \binom{2}{0}$ equals-

A. $\binom{99}{97}$

B. $\binom{100}{98}$

C. $\binom{99}{98}$

D. $\binom{100}{97}$

Answer: D

Watch Video Solution

17. The last digit of $9! + 3^{9966}$ is :

A. 1

B. 3

C. 7

D. 9

Answer: D

Watch Video Solution

18. Let x be the 7^{th} term from the beginning and y be the 7^{th} term from the end in the expansion of $\left(3^{1/3} + \frac{1}{4^{1/3}}\right)^n$. If $y = 12x$ then the value of n is :

A. 9

B. 8

C. 10

D. 11

Answer: A

Watch Video Solution

19. $^{10}(C_0)^2 - {}^{10}(C_1)^2 + {}^{10}(C_2)^2 - \ldots\ldots - ({}^{10}C_9)^2 + ({}^{10}C_{10})^2 =$

A. $10!$

B. $({}^{10}C_5)^2$

C. $-{}^{10}C_5$

D. ${}^{10}C_5$

Answer: C

Watch Video Solution

20. Find the ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(x^2+\frac{2}{x}\right)^{15}$.

A. $1:4$

B. $1:32$

C. $7:64$

D. $7:16$

Answer: B

Watch Video Solution

21. In the expansion of $(1+x)^2(1+y)^3(1+z)^4(1+w)^5$, the sum of the coefficient of the terms of degree 12 is :

A. 61

B. 71

C. 81

D. 91

Answer: D

Watch Video Solution

22. If $\sum_{r=0}^{n}\left(\frac{r^3+2r^2+3r+2}{r+1}\right)^{n}C_r=\frac{2^4+2^3+2^2-2}{3}$

A. 2

B. 2^2

C. 2^3

D. 2^4

Answer: A

Watch Video Solution

Exercise 2 One Or More Than One Answer Is Are Correct

1. The number $N = {}^{20}C_7 - {}^{20}C_8 + {}^{20}C_9 - {}^{20}C_{10} + \ldots.. - {}^{20}C_{20}$ is divisible by :

A. 3

B. 4

C. 7

D. 19

Answer: A::B::C::D

Watch Video Solution

2. If $(1 + x + x^2 + x^3)^{100} = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{300} x^{300}$, then

A. $a_1 = 100$

B. $a_0 + a_1 + a_2 + \ldots.. + a_{300}$ is divisible by 1024

C. coefficients equidistant from beginning and end are equal

D. $a_0 + a_2 + a_4 + \ldots\ldots.. + a_{300} = a_1 + a_3 + a_5 + \ldots\ldots + a_{299}$

Answer: A::B::C::D

Watch Video Solution

3. $\sum_{r=0}^{4}(-1)^{r16}C_r$ is divisible by :

A. 5

B. 7

C. 11

D. 13

Answer: A::B::D

Watch Video Solution

4. Arrange the expansion of $\left(x^{1/2}+\frac{1}{2x^{1/4}}\right)$ in decreasing powers of x. Suppose the coefficient of the first three terms form an arithemetic progression. Then the number of terms in the expression having integer powers of x is -

A. 0

B. 2

C. 4

D. 8

Answer: A::C::D

Watch Video Solution

5. Let $\left(1+x^2\right)^2(1+x)^n=\sum_{k=0}^{n+4} a_k x^k$. If a_1, a_2 and a_3 are in arithmetic progression, then the possible value/values of n is/are a. 5 b. 4 c. 3 d. 2

A. 6

B. 4

C. 3

D. 2

Answer: B::C::D

Watch Video Solution

6. $\sum_{m=1}^{n}\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}{}^{n}C_{m}\cdot{}^{m}C_{p}\cdot{}^{p}C_{k}\right)\right)=$

A. is less than 500 if n = 3

B. is greater than 600 if n = 3

C. is less than 5000 if n = 4

D. is greater than 4000 if n = 4

Answer: C::D

Watch Video Solution

7. If ${}^{100}C_6 + 4.\ {}^{100}C_7 + 6.\ {}^{100}C_8 + 4.\ {}^{100}C_9 + {}^{100}C_{10}$ has the value equal to ${}^{x}C_y$, then the possible value (s) of $x + y$ can be :

A. 112

B. 114

C. 196

D. 198

Answer: B::D

Watch Video Solution

8. If the co-efficient of x^{2r} is greater than half of the co-efficient of x^{2r+1} in the expansion of $(1 + x)^{15}$, then the possible value of 'r' equal to :

A. 5

B. 6

C. 7

D. 8

Answer: A::B::C

Watch Video Solution

9. Let $f(x) = 1 + x^{111} + x^{222} + x^{333} \ldots\ldots\ldots\ldots\ldots + x^{999}$ then $f(x)$ is divisible by

A. $x + 1$

B. x

C. $x - 1$

D. $1 + x^{222} + x^{444} + x^{666} + x^{888}$

Answer: D

Watch Video Solution

Exercise 3 Matching Type Problems

1.

	Column-I		Column-II
(A)	If ${}^{n-1}C_r = (k^2-3)\,{}^nC_{r+1}$ and $k \in R^+$, then least value of $5[k]$ is (where [·] represents greatest integer function)	(P)	10
(B)	$\sum_{i=0}^{m} {}^{20}C_i\, {}^{40}C_{m-i}$, where ${}^nC_r = 0$ if $r > n$, is maximum when $\frac{m}{5}$ is	(Q)	5
(C)	Number of non-negative integral solutions of inequation $x+y+z \le 4$ is	(R)	35
(D)	Let $A = \{1, 2, 3, 4, 5\}$, $f : A \to A$, The number of onto functions such that $f(x) = x$ for atleast 3 distinct $x \in A$, is not a multiple of	(S)	6
		(T)	12

Watch Video Solution

2.

	Column-I		Column-II
(A)	If the sum of first 84 terms of the series $\frac{4+\sqrt{3}}{1+\sqrt{3}} + \frac{8+\sqrt{15}}{\sqrt{3}+\sqrt{5}} + \frac{12+\sqrt{35}}{\sqrt{5}+\sqrt{7}} + \ldots$ is 549k, then k is equal to	(P)	3
(B)	If $x, y \in R$, $x^2+y^2-6x+8y+24=0$, the greatest value of $\frac{16}{5}\cos^2\left(\sqrt{x^2+y^2}\right) - \frac{24}{5}\sin\left(\sqrt{x^2+y^2}\right)$ is	(Q)	2
(C)	If $(\sqrt{3}+1)^6 + (\sqrt{3}-1)^6 = 416$, if $xyz = [(\sqrt{3}+1)^6]$, $x, y, z \in N$, (where [·] denotes the greatest integer function), then the number of ordered triplets (x, y, z) is	(R)	5
(D)	If $(1+x)(1+x^2)(1+x^4)\ldots(1+x^{128}) = \sum_{r=0}^{n} x^r$, then $\frac{n}{85}$ is equal to	(S)	9

Watch Video Solution

Exercise 4 Subjective Type Problems

1. The sum of series $3 \cdot {}^{2007}C_0 - 8 \cdot {}^{2007}C_1 + 13 \cdot {}^{2007}C_2 - 18 \cdot {}^{2007}C_3 + \ldots\ldots$ upto 2008 terms is K, then K is :

Watch Video Solution

2. In the polynomial function $f(x) = (x-1)(x^2-2)(x^3-3)\ldots\ldots.(x^{11}-11)$ the coefficient of x^{60} is :

Watch Video Solution

3. If $3^{101} - 2^{100}$ is divided by 11, the remainder is

Watch Video Solution

4. Find the hundred's digit in the co-efficient of x^{17} in the expansion of $\left(1+x^5+x^7\right)^{20}$.

Watch Video Solution

5. Let $x=\left(3\sqrt{6}+7\right)^{89}$. If {x} denotes the fractional part of 'x' then find the remainder when $x\{x\}+(x\{x\})^2+(x\{x\})^3$ is divided by 31.

Watch Video Solution

6. Let $n \in N, S_n = \sum_{r=0}^{3n} \hat{}\,(3n)C_r$ and $T_n = \sum_{r=0}^{n} \hat{}\,(3n)C_{3r}$, then $|S_n - 3T_n|$ equals

Watch Video Solution

7. Find the sum of possible real values of x for which the sixth term of $\left(3^{\log_3 \sqrt{9^{|x-2|}}} + 7^{\frac{1}{5}\log_7\left(3^{|x-2|-9}\right)}\right)^7$ equals 567.

Watch Video Solution

8. Let q be a positive with $q \leq 50.$

If the sum

$^{98}C_{30} + 2 \quad ^{97}C_{30} + 3. \quad ^{96}C_{30} + \ldots\ldots + 68. \quad ^{31}C_{30} + 69. \quad ^{30}C_{30} = ^{100}$

Find the sum of the digits of q.

Watch Video Solution

9. The remainder when $\left(\sum_{k=1}^{5} {}^{20}C_{2k-1}\right)^{6}$ is divided by 11, is :

Watch Video Solution

10. Let $a = 3^{\frac{1}{223}} + 1$ and for all $\geq 3, let f(n) =^n C_0 \dot{a}^{n-1} -^n C_1 \dot{a}^{n-2} +^n C_2 \dot{a}^{n-3} - + (-1)^{n-1} \hat{} nC_{n-}$

f(2007)+f(2008)=3^k w h e r ek in N ,$thenthevalueofk$` is.

Watch Video Solution

11. In the polynomial $(x-1)(x^2-2)(x^3-3)\ldots(x^{11}-11)$, the coefficient of x^{60} is :

Watch Video Solution

12. Let the sum of all divisior of the form $2^p \cdot 3^q$ (with p, q positive integers) of the number $19^{88}-1$ be λ. Find the unit digit of λ.

Watch Video Solution

13. Find the sum of possible real values of x for which the sixth term of $\left(3^{\log_3\sqrt{9^{|x-2|}}}+7^{\frac{1}{5}\log_7\left(3^{|x-2|-9}\right)}\right)^7$ equals 567.

Watch Video Solution

14. Let $1+\sum_{r=1}^{10}\left(3^r.{}^{10}C_r+r.{}^{10}C_r\right)=2^{10}\left(\alpha.4^5+\beta\right)$ where $\alpha, \beta \in N$ and $f(x)=x^2-2x-k^2+1.$ If α, β lies between the roots of $f(x)=0$, then find the smallest positive integral value of k.

Watch Video Solution

15. If $S_n={}^nC_0{}^nC_1+{}^nC_1{}^nC_2+\ldots+{}^nC_{n-1}{}^nC_n$ and if $\dfrac{S_{n+1}}{S_n}=\dfrac{15}{4}$, find n.

Watch Video Solution