



MATHS

BOOKS - SHRI BALAJI MATHS (ENGLISH)

CONTINUITY, DIFFERENTIABILITY AND DIFFERENTIATION

Exercise Single Choice Problems

1. Let 'f' be a differentiable real valued function satisfying

$$f(x + 2y) = f(x) + f(2y) + 6xy(x + 2y) \quad \forall x, y \in \mathbb{R}. \quad \text{Then}$$

$f'(0), f'(1), f'(2), \dots$ are in

A. AP

B. GP

C. HP

D. None of these

Answer: A



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2. Find the number of points of non-differentiability for

$$f(x) = \max \{ ||x| - 1|, 1/2 \}.$$

A. 4

B. 3

C. 2

D. 5

Answer: D



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3. Number of points of discontinuity of $f(x) = \left\{ \frac{x}{5} \right\} + \left[\frac{x}{2} \right]$ in $x \in [0, 100]$ is/are (where $[-]$ denotes greatest integer function and $\{ \}$

denotes fractional part function)

A. 50

B. 51

C. 52

D. 61

Answer: A

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4. If $f(x)$ has isolated point of discontinuity at $x = a$ such that $f(x)$ is continuous at $x = a$ then :

A. $\lim_{x \rightarrow a} f(x)$ doesn't exist

B. $\lim_{x \rightarrow a} f(x) + f(a) = 0$

C. $f(a) = 0$

D. None of these

Answer: B



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5. If $f(x)$ is a thrice differentiable function such that

$$\lim_{x \rightarrow 0} \frac{f(4x) - 3f(3x) + 3f(2x) - f(x)}{x^3} = 12 \text{ then the value of } f'''(0)$$

equals to :

A. 0

B. 1

C. 12

D. None of these

Answer: C



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6.

$$y = \frac{1}{(1 + \tan \theta)^{\sin \theta - \cos \theta} + (\cot \theta)^{\cos \theta - \cot \theta}} + \frac{1}{(1 + \tan \theta)^{\cos \theta - \sin \theta} + (\cot \theta)^{\sin \theta - \cot \theta}}$$

then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$ is

A. 0

B. 1

C. $\sqrt{3}$

D. None of these

Answer: A



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7. Let $f'(x) = \sin(x^2)$ and $y = f(x^2 + 1)$ then $\frac{dy}{dx}$ at $x = 1$ is:

A. $2 \sin 2$

B. $2 \cos 2$

C. $2 \sin 4$

D. $\cos 2$

Answer: C



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8. If $f(x) = |\sin x - |\cos x||$, then the value of $f'(x)$ at $x = \frac{7\pi}{6}$ is

A. $\frac{\sqrt{3} + 1}{2}$

B. $\frac{1 - \sqrt{3}}{2}$

C. $\frac{\sqrt{3} - 1}{2}$

D. $\frac{-1 - \sqrt{3}}{2}$

Answer: C



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9. If $(\sin x)(\cos y) = \frac{1}{2}$, then $\frac{d^2y}{dx^2}$ at $(\frac{\pi}{4}, \frac{\pi}{4})$ is -4 (b) -2 (c) -6 (d) 0

A. -4

B. -2

C. -6

D. 0

Answer: A



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10. f is a differentiable function such that

$$x = f(t^2), y = f(t^3) \text{ and } f(1) \neq 0 \text{ if } \left(\frac{d^2y}{(dx^2)} \right)_{t=1} =$$

A. $\frac{3}{4} \left(\frac{f''(1) + f(1)}{(f'(1))^2} \right)$

B. $\frac{3}{4} \left(\frac{f(1) \cdot f'(1) - f''(1)}{f(f'(1))^2} \right)$

C. $\frac{4}{3} \frac{f'(1)}{(f'(1))^2}$

D. $\frac{4}{3} \left(\frac{f'(1)f''(1) - f''(1)}{(f'(1))^2} \right)$

Answer: A



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11. let $f(x) = \begin{cases} ax + 1 & \text{if } x \leq 1 \\ 3 & \text{if } x = 1 \\ bx^2 + 1 & \text{if } x > 1 \end{cases}$ if $f(x)$ is continuous at $x = 1$

then value of $a - b$ is (A) 0 (B) 1 (C) 2 (D) 4

A. 0

B. 1

C. 2

D. 4

Answer: A



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12.

If

$$y = 1 + \frac{\alpha}{\left(\frac{1}{x} - \alpha\right)} + \frac{\beta/x}{\left(\frac{1}{x} - \alpha\right)\left(\frac{1}{x} - \beta\right)} + \frac{\gamma/x^2}{\left(\frac{1}{x} - \alpha\right)\left(\frac{1}{x} - \beta\right)\left(\frac{1}{x} - \gamma\right)},$$

find $\frac{dy}{dx}$

A. $y\left(\frac{\alpha}{1/x - \alpha} + \frac{\beta}{1/x - \beta} + \frac{\gamma}{1/x - \gamma}\right)$

B. $\frac{y}{x}\left(\frac{\alpha}{1/x - \alpha} + \frac{\beta}{1/x - \beta} + \frac{\gamma}{1/x - \gamma}\right)$

C. $y\left(\frac{\alpha}{1/x - \alpha} + \frac{\beta}{1/x - \beta} + \frac{\gamma}{1/x - \gamma}\right)$

D. $y\left(\frac{\alpha/x}{1/x - \alpha} + \frac{\beta/x}{1/x - \beta} + \frac{\gamma/x}{1/x - \gamma}\right)$

Answer: B**Watch Video Solution**

13. If $f(x) = \sqrt{\frac{1 + \sin^{-1} x}{1 - \tan^{-1} x}}$, then $f(0)$ is equal to :

A. 4

B. 3

C. 2

D. 1

Answer: D



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14. Let $f(x) = \begin{cases} \sin^2 x, & x \text{ is rational} \\ -\sin^2 x, & x \text{ is irrational} \end{cases}$, then set of points, where f

(x) is continuous, is:

A. $\left\{ (2n + 1) \frac{\pi}{2} \in I \right\}$

B. a null set

C. $\{n\pi, n \in I\}$

D. set of all rational numbers

Answer: C



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15. The number of values of x in $(0, 2\pi)$ where the fraction $f(x) = \frac{\tan x + \cot x}{2} - \left| \frac{\tan x - \cot x}{2} \right|$ is continuous but non-derivable :

A. 3

B. 4

C. 0

D. 1

Answer: B



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16. If $f(x) = |x - 1|$ and $g(x) = f(f(f(x)))$, then $g'(x)$ is equal to:

A. 1 for $x > 2$

B. 1 for $2 < x < 3$

C. -1 for $2 < x < 3$

D. -1 for $x > 3$

Answer: C



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17. If $f(x)$ is continuous function $\forall x \in R$ and the range of $f(x)$ is $(2, \sqrt{26})$ and $g(x) = \left[\frac{f(x)}{c} \right]$ is continuous $\forall x \in R$, then find the least positive integral value of c , where $[.]$ denotes the greatest integer function.

A. 3

B. 5

C. 6

D. 7

Answer: C



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18. If $y = x + e^x$, then $\left(\frac{d^2y}{dy^2}\right)_{x=\ln 2}$ is :

A. $-\frac{1}{9}$

B. $-\frac{2}{27}$

C. $\frac{2}{27}$

D. $\frac{1}{9}$

Answer: B



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19. Let $f(x) = x^3 + 4x^2 + 6x$ and $g(x)$ be inverse then the value of $g'(-4)$:

A. -2

B. 2

C. $\frac{1}{2}$

D. None of these

Answer: C



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20. If $f(x) = 2 + |x| - |x - 1| - |x + 1|$, then $f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) + f'\left(\frac{5}{2}\right)$ is equal to:

A. 1

B. -1

C. 2

D. -2

Answer: D



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21. If $f(x) = \cos(x^2 - 4[x])$, $0 < x < 1$, (whre $[.]$ denotes greatest integer function) then $f' \left(\frac{\sqrt{\pi}}{2} \right)$ is equal to:

A. $-\sqrt{\frac{\pi}{2}}$

B. $\sqrt{\frac{\pi}{2}}$

C. 0

D. $\sqrt{\frac{\pi}{4}}$

Answer: A

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22. Let $g(x)$ be then inverse of $f(x)$ such that $f'(x) = \frac{1}{1+x^5}$, then

$\frac{d^2(g(x))}{dx^2}$ is equal to:

A. (a) $\frac{1}{1+(g(x))^5}$

B. (b) $\frac{g'(x)}{1+(g(x))^5}$

C. (c) $5(g(x))^4(1 + g(x))^5$

D. (d) $1 + (g(x))^5$

Answer: C

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23. Let $f(x) = \begin{cases} \min(x, x^2) & x \geq 0 \\ \max(2x, x - 1) & x < 0 \end{cases}$, then which of the following

is not true ?

A. $f(x)$ is not differentiable at $x = 0$

B. $f(x)$ is not differentiable are exactly two points

C. $f(x)$ is continuous everywhere

D. $f(x)$ is strictly increasing $\forall x \in \mathbb{R}$

Answer: B

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24. if $f(x) = \lim_{x \rightarrow \infty} \left(\prod_{i=1}^n \cos\left(\frac{x}{2^i}\right) \right)$ then $f'(x)$ is equal to:

A. $\frac{\sin x}{x}$

B. $\frac{x}{\sin x}$

C. $\frac{x \cos x - \sin x}{x^2}$

D. $\frac{\sin x - x \cos x}{\sin^2 x}$

Answer: C



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25. If $f(x) = \begin{cases} \frac{1 - \tan x}{4x - \pi}, & x \neq \frac{\pi}{4} \\ k, & x = \frac{\pi}{4} \end{cases}$ is continuous at $x = \frac{\pi}{4}$ then the value

of k is

A. 1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -1

Answer: C



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26. Let $f(x) = \begin{cases} \left(e^{\frac{1}{x^2}} \sin \frac{1}{x}, & x \neq 0 \right) \\ (\lambda, & x = 0) \end{cases}$, then $f'(0)$

A. 1

B. -1

C. 0

D. Does not exist

Answer: C



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27. Let f be a differentiable function satisfying $f'(x) = 2f(x) + 10 \forall x \in R$ and $f(0) = 0$, then the number of real roots of the equation $f(x) + 5 \sec^2 x = 0$ in $(0, 2\pi)$ is:

- A. 0
- B. 1
- C. 2
- D. 3

Answer: A



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28. If $f(x) = \begin{cases} \frac{\sin \{ \cos x \}}{x - \frac{\pi}{2}} & x \neq \frac{\pi}{2} \\ 1 & x = \frac{\pi}{2} \end{cases}$, where $\{k\}$ represents the fractional

part of k , then:

- A. $f(x)$ is continuous at $x = \frac{\pi}{2}$

B. $\lim_{x \rightarrow \frac{\pi}{2}} f(x)$ does not exist

C. $\lim_{x \rightarrow \frac{\pi}{2}} f(x)$ exists, but f is not continuous at $x = \frac{\pi}{2}$

D. $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = 1$

Answer: B



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29. Let $f(x)$ be a polynomial in x . Then the second derivative of

$f(e^x)$ is $f^{e^x} \dot{e}^x + f'(e^x)$ $f^{e^x} \dot{e}^{2x} + f'(e^x) \dot{e}^{2x}$ $f^{e^x} e^{2x}$ (d)

$f^{e^x} \dot{e}^{2x} + f'(e^x) \dot{e}^x$

A. $f''(e^x)e^x + f(e^x)$

B. $f''(e^x)e^{2x} + f'(e^x)e^{2n}$

C. $f''(e^x)e^x + f'(e^x)e^{2x}$

D. $f''(e^x)e^{2x} + e^x f'(e^x)$

Answer: D



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30. If $e^{f(x)} = \log x$ and $g(x)$ is the inverse function of $f(x)$, then $g'(x)$ is

A. $e^x + x$

B. $e^{e^x} e^{e^x}$

C. $e^{e^x + x}$

D. e^{e^x}

Answer: C



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31. If $y = f(x)$ is differentiable $\forall x \in R$, then

A. $y = |f(x)|$ is differentiable $\forall x \in R$

B. $y = f^2(x)$ is not-differentiable for atleast one x

C. $y = f(x)|f(x)|$ is non-differentiable for atleast one x

D. $y = |f(x)|^3$ is differentiable $\forall x \in \mathbb{R}$

Answer: D



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32. If $f(x) = (x - 1)^4(x - 2)^3(x - 3)^2$ then the value of $f'(1) + f''(2) + f''(3)$ is:

A. 0

B. 1

C. 2

D. 6

Answer: A



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33. Indicate all correct alternatives: if $f(x) = \frac{x}{2} - 1$, then on the interval $[0, \pi]$:

- A. $\tan(f(x))$ and $\frac{1}{f(x)}$ are both continuous
- B. $\tan(f(x))$ and $\frac{1}{f(x)}$ are both discontinuous
- C. $\tan(f(x))$ and $f^{-1}(x)$ are both continuous
- D. $\tan f(x)$ is continuous but $f^{-1}(x)$ is not

Answer: C



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34. Let $f(x) = \begin{cases} \frac{\left(\frac{1}{e^{x-2}} - 3\right)}{\frac{1}{3^{x-2}} + 1} & x > 2 \\ \frac{b \sin \{-x\}}{\{-x\}} & x < 2 \\ c & x = 2 \end{cases}$, where $\{.\}$ denotes fraction part

function, is continuous at $x = 2$, then $b + c =$

A. 0

B. 1

C. 2

D. 4

Answer: A



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35. Let $f(x) = \frac{e^{\tan x} - e^x + \ln(\sec x + \tan x) - x}{\tan x - x}$ be a continuous

function at $x=0$. The value $f(0)$ equals

A. a) $\frac{1}{2}$

B. b) $\frac{2}{3}$

C. c) $\frac{3}{2}$

D. d) 2

Answer: C



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36. Let $f(x) = \begin{cases} (1+ax)^{1/x} & x < 0 \\ \frac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1} & x > 0 \end{cases}$, is continuous at $x = 0$, then

$3(e^a + b + c)$ is equal to:

A. 3

B. 6

C. 7

D. 8

Answer: D

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37. $\sqrt{x+y} + \sqrt{y-x} = c$, then $\frac{d^2y}{dx^2}$ equals

A. $\frac{2}{c}$

B. $-\frac{2}{c^2}$

C. $\frac{2}{c^2}$

D. *No \neq of these*

Answer: C



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38. If $f(x) = x^2 + x^4 + \log x$ and g is the inverse of f , then $g'(2)$ is:

A. 8

B. $\frac{1}{8}$

C. 2

D. $\frac{1}{4}$

Answer: B



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39. The number of points at which the function,

$$f(x) = \begin{cases} \min \{|x|, x^2\} & \text{if } x \in (-\infty, 1) \\ \min(2x - 1, x^2) & \text{otherwise} \end{cases} \text{ is not}$$

differentiable is:

A. 0

B. 1

C. 2

D. 3

Answer: B



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40. If $f(x)$ is a function such that

$$f(x) + f''(x) = 0 \text{ and } g(x) = (f(x))^2 + (f'(x))^2 \text{ and } g(3) = 8,$$

then $g(8) =$

A. 0

B. 3

C. 5

D. 8

Answer: D



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41. Let f is twice differentiable on \mathbb{R} such that $f(0) = 1$, $f'(0) = 0$ and $f''(0) = -1$, then for

$$a \in \mathbb{R}, \lim_{x \rightarrow \infty} \left(f\left(\frac{a}{\sqrt{x}}\right) \right)^x =$$

A. e^{-e^2}

B. $e^{\frac{a^2}{4}}$

C. $e^{-\frac{a^2}{2}}$

D. e^{-2a^2}

Answer: C



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42. Let $f_1(x) = e^x$ and $f_{n+1}(x) = e^{f_n(x)}$ for any $n \geq 1, n \in N$. Then for any fixed n , the value of $\frac{d}{dx} f^{(n)}(x)$ equals:

A. $f_n(x)$

B. $f_n(x)f_{n-1}(x) \dots f_2(x)f_1(x)$

C. $f_n(x)f_{n-1}(x)$

D. $f_n(x)f_{n-1}(x) \dots f_2(x)f_1(x)e^x$

Answer: B



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43. If $y = \tan^{-1} \left(\frac{x^{1/3} - a^{1/3}}{1 + x^{1/3}a^{1/3}} \right)$, $x > 0, a > 0$, then $\frac{dy}{dx}$ is:

A. $\frac{1}{x^{2/3}(1 + x^{2/3})}$

$$\text{B. } \frac{3}{x^{2/3}(1 + x^{2/3})}$$

$$\text{C. } \frac{1}{3x^{2/3}(1 + x^{2/3})}$$

$$\text{D. } \frac{1}{3x^{1/3}(1 + x^{2/3})}$$

Answer: C

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44. The value of $k + f(0)$ so that $f(x) = \begin{cases} \frac{\sin(4k-1)x}{3x}, & x < 0 \\ \frac{\tan(4k+1)x}{5x}, & 0 < x < \frac{\pi}{2} \\ 1, & x = 0 \end{cases}$

can be made continuous at $x = 0$ is:

A. 1

B. 2

C. $\frac{5}{4}$

D. 0

Answer: B



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45. If $y = \tan^{-1}\left(\frac{x}{1 + \sqrt{1 - x^2}}\right)$, $|x| \leq 1$, then $\frac{dy}{dx}$ at $\left(\frac{1}{2}\right)$ is:

A. (a) $\frac{1}{\sqrt{3}}$

B. (b) 3

C. (c) $\frac{\sqrt{3}}{2}$

D. (d) $\frac{2}{\sqrt{3}}$

Answer: A



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46. Let $f(x) = \frac{e^x x \cos x - x \log_e(1 + x) - x}{x^2}$, $x \neq 0$. If $f(x)$ is continuous at $x = 0$, then $f(0)$ is equal to

A. 0

B. 1

C. -1

D. 2

Answer: A



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47. A function $f(x) = \max(\sin x, \cos x, 1 - \cos x)$ is non-derivable for n values of $x \in [0, 2\pi]$. Then the value of n is:

A. 2

B. 1

C. 3

D. 4

Answer: C



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48. Let g be the inverse function of a differentiable function f and $G(x) = \frac{1}{g(x)}$. If $f(4) = 2$ and $f'(4) = \frac{1}{16}$, then the value of $(G'(2))^2$ equals to:

- A. 1
- B. 4
- C. 16
- D. 64

Answer: A



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49. If $f(x) = \max \left(x^4, x^2, \frac{1}{81} \right) \forall x \in [0, \infty)$, then the sum of square of reciprocal of all the values of x where $f(x)$ is non-differentiable, is equal to:

A. 1

B. 81

C. 82

D. $\frac{82}{81}$

Answer: C



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50. If $f(x)$ is derivable at $x = 2$ such that $f(2) = 2$ and $f'(2) = 4$, then the value of $\lim_{h \rightarrow 0} \frac{1}{h^2} (\ln(f(2 + h^2)) - \ln(f(2 - h^2)))$ is equal to:

A. 1

B. 2

C. 3

D. 4

Answer: D



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51. Let $f(x) = (x^2 - 3x + 2)|(x^3 - 6x^2 + 11x - 6)| + \left| \sin\left(x + \frac{\pi}{4}\right) \right|$

Number of points at which the function $f(x)$ is non-differentiable in $[0, 2\pi]$, is

A. 5

B. 4

C. 3

D. 2

Answer: C



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52. Let f be a real - valued function defined on \mathbb{R} (the set of real numbers) such that $f(x) = \sin^{-1}(\sin x) + \cos^{-1}(\cos x)$

The value of $f(10)$ is equal to

A. 28

B. 24

C. 32

D. 18

Answer: C



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53. If $f(x) = \frac{(x+1)^7 \sqrt{1+x^2}}{(x^2-x+1)^6}$, then the value of $f'(0)$ is equal to:

A. 10

B. 11

C. 13

D. 15

Answer: C



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54. Statement.1 : The function $f(x) = \lim_{x \rightarrow \infty} \frac{\log_e(1+x) - x^{2n} \sin(2x)}{1+x^{2n}}$

is discontinuous at $x = 1$

Statement.2: $L. H. L. = R. H. L. \neq f(1)$.

A. a) Statement:1 is true , Statement:2 is true and Statement:2 is

correct explanation for Statement-1

B. b) Statement-1 is true, Statement-2 is true and Statement-2 is not

the correct explanation for Statement-1

C. c) Statement-1 is true, Statement-2 is false

D. d) Statement-1 is false, Statement-2 is true

Answer: C



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55. If $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1 - x & \text{if } x \text{ is irrational} \end{cases}$, then number of points for $x \in R$, where $y = f(f(x))$ discontinuous is:

A. (a) 0

B. (b) 1

C. (c) 2

D. (d) Infinitely many

Answer: A



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56. Number of points where $f(x) = \begin{cases} \max(|x^2 - x - 2|, x^2 - 3x) & x \geq 0 \\ \max(\ln(-x), e^x) & x < 0 \end{cases}$ is non-

differentiable will be:

A. 1

B. 2

C. 3

D. None of these

Answer: C



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57. If the function

$f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ and $g(x) = f^{-1}(x)$, then the

reciprocal of $g' \left(\frac{-7}{6} \right)$ is _____

A. $\frac{1}{5}$

B. $-\frac{1}{5}$

C. $\frac{6}{7}$

D. $-\frac{6}{7}$

Answer: A



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58. Find k , if possible, so that

$$f(x) = \begin{cases} \frac{\ln(2 - \cos 2x)}{\ln^2(1 + \sin 3x)} & x < 0 \\ k & x = 0 \text{ is continuous at } x = 0. \\ \frac{e^{\sin 2x} - 1}{\ln(1 + \tan 9x)} & x > 0 \end{cases}$$

A. $\frac{2}{3}$

B. $\frac{1}{9}$

C. $\frac{2}{9}$

D. not possible

Answer: C



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59. A function is represented parametrically by the equations

$$x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t} \text{ Then the value of } \left| \frac{dy}{dx} - x \left(\frac{dy}{dx} \right)^3 \right| \text{ is } \underline{\hspace{2cm}}$$

A. 2

B. 0

C. -1

D. -2

Answer: C



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60. If $y^2 = 1 + 2\sqrt{2} \cos 2x$, then:

$$\frac{d^2y}{dx^2} = y(py^2 + 1) \text{ then the value of } (p + q) \text{ equals to:}$$

A. 7

B. 8

C. 9

D. 10

Answer: D



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61. If $y^{-2} = 1 + 2\sqrt{2} \cos 2x$, then :

$\frac{d^2y}{dx^2} = y(py^2 + 1)(qy^2 - 1)$ then the value of $(p + q)$ equals to:

A. 7

B. 10

C. 9

D. 15

Answer: B



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62. Let $f: R \rightarrow R$ is not identically zero, differentiable function and satisfy the equals $f(xy) = f(x)f(y)$ and $f(x+z) = f(x) + f(z)$, then $f(5) =$

A. 3

B. 5

C. 10

D. 15

Answer: B



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63. If $y = (x + \sqrt{1+x^2})^n$ then $(1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx}$

A. n^2y

B. y^{-n^2}

C. $-y$

D. $2x^2y$

Answer: A



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64. Let $g(x) = f\left(x - \sqrt{1 - x^2}\right)$ and $f'(x) = 1 - x^2$ then $g'(x)$ equal to:

A. $1 - x^2$

B. $\sqrt{1 - x^2}$

C. $2x\left(x + \sqrt{1 - x^2}\right)$

D. $2x\left(x - \sqrt{1 - x^2}\right)$

Answer: C



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65. Let $f(x) = \lim_{x \rightarrow \infty} \frac{\log_2(2+x) - x^{2x} \sin x}{1+x^{2n}}$ then:

A. a) $f(x)$ is continuous at $x = 1$

B. b) $\lim_{x \rightarrow 1} f(x) = \log_2 3$

C. c) $\lim_{x \rightarrow 1^+} f(x) = -\sin 1$

D. d) $\lim_{x \rightarrow 1^+} f(x)$ does not exist

Answer: C



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66. If $f(x+y) = f(x) \times f(y)$ for all $x, y \in \mathbb{R}$ and $f(5) = 2, f'(0) = 3$, then $f'(5) =$

A. 3

B. 1

C. -6

D. 6

Answer: C



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67. Let $f(x) = \lim_{x \rightarrow \infty} \frac{\log_e(2+x) - x^{2x} \sin x}{1+x^{2n}}$ then:

A. a) $f(x)$ continuous at $x = 1$

B. b) $\lim_{x \rightarrow 1^-} f(x) = \log_e 3$

C. c) $\lim_{x \rightarrow 1^+} f(x) = -\sin 1$

D. d) $\lim_{x \rightarrow 1^-} f(x)$ does not exist

Answer: C



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68. If $f(x) = \begin{cases} \frac{x - e^x + 1 - (1 - \cos 2x)}{x^2} & x \neq 0 \\ x^2 & x = 0 \end{cases}$ is continuous at $x = 0$ then

which of the following statement is false ?

A. a) $k = \frac{-5}{2}$

B. b) $\{k\} = \frac{1}{2}$

C. c) $[k] = -2$

D. d) $[k]\{k\} = \frac{-3}{2}$

Answer: C



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69. Let $f(x) = ||x^2 - 10x + 21| - p|$, then the exhaustive set of values of for which $f(x)$ has exactly 6 points of non-derivability, is:

A. $(4, \infty)$

B. $(0, 4)$

C. $[0, 4]$

D. $(-4, 4)$

Answer: B

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70. If $f(x) = \sqrt{\frac{1 + \sin^{-1} x}{1 - \tan^{-1} x}}$, then $f(0)$ is equal to :

A. 4

B. 3

C. 2

D. 1

Answer: D

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71. For $t \in (0, 1)$, Let $x = \sqrt{2^{\sin^{-1}(t)}}$ and $y = \sqrt{2^{\cos^{-1}t}}$, then $1 + \left(\frac{dy}{dx}\right)^2$ equals :

A. $\frac{x^2}{y^2}$

B. $\frac{y^2}{x^2}$

C. $\frac{x^2 + y^2}{y^2}$

D. $\frac{x^2 + y^2}{x^2}$

Answer: D



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72. Let $f(x) = -1 + |x - 2|$ and $g(x) = 1 - |x|$ then set of all possible value (s) of for which $(f \circ g)(x)$ is discontinuous is:

A. $\{0, 1, 2\}$

B. $\{0, 2\}$

C. $\{0\}$

D. an empty set

Answer: D



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73. If $f(x) = [x]\tan(\pi x)$ then $f'(k')$ is equal to ($k \in$ and $[.]$ denotes greatest integer function):

A. $(k - 1)\pi(-1)^k$

B. $k\pi$

C. $k\pi(-1)^{k+1}$

D. $(k - 1)\pi(-1)^{k-1}$

Answer: B



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74. If $f(x) = \begin{cases} \frac{ae^{\sin x} + be^{-\sin x} - c}{x^2} & x \neq 0 \\ 2 & x = 0 \end{cases}$ is continuous at $x = 0$, then:

A. $a = b = c$

B. $a = 2b = 3c$

C. $a = b = 2c$

D. $2a = 2b = c$

Answer: D



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75. If $\tan x \cot y = \sec \alpha$ where α is constant and

$\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then $\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ equal to:

A. 0

B. 1

C. 2

D. 3

Answer: A



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76. If $y = (x - 3)(x - 2)(x - 1)x(x + 1)(x + 2)(x + 3)$, then

$\frac{d^2y}{dx^2}$ at $x = 1$ is:

A. (a) - 101

B. (b) - 48

C. (c) 56

D. (d) 190

Answer: C



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77. Let $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, If $f(x)$ is discontinuous at $x = 0$, show that $f(x)$ is discontinuous at all x .

- A. all natural numbers only
- B. all integers only
- C. all rational numbers only
- D. all real numbers

Answer: D



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78. If

$$f(x) = 3x^9 - 2x^4 + 2x^3 - 3x^2 + x + \cos x + 5 \text{ and } g(x) = f^{-1}(x),$$

then the value of $g'(6)$ equals:

- A. 1
- B. $\frac{1}{2}$

C. 2

D. 3

Answer: A



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79. about to only mathematics

A. $\frac{d'f'' - f'g''}{(g')^2}$

B. $\frac{g'f'' - f'g''}{(g')^3}$

C. $\frac{fg'' - g'f''}{(g')^3}$

D. None of these

Answer: B



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80.

Let

$$f(x) = \begin{cases} x + 1 & x < 0 \\ (x - 1) & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x + 1 & x < 0 \\ (x - 1)^2 & x \geq 0 \end{cases} \text{ then}$$

the number of points where $g(f(x))$ is not differentiable.

A. 0

B. 1

C. 2

D. None of these

Answer: C



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81.

$f(x) = [\sin x] + [\cos x]$, $x \in [0, 2\pi]$, where $[.]$ denotes the greatest integer function. Total number of point where $f(x)$ is non-differentiable is equal to

i) 2

ii) 3

iii) 4

iv) 5

A. 2

B. 3

C. 4

D. 5

Answer: D



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82. Let $f(x) = \cos x$, $g(x) = \begin{cases} \min \{f(t) : 0 \leq t \leq x\}, & x \in [0, \pi] \\ (\sin x) - 1, & x > \pi \end{cases}$

Then

A. $g(x)$ is discontinuous at $x = \pi$

B. $g(x)$ is continuous for $x \in [0, \infty)$

C. $g(x)$ is differentiable at $x = \pi$

D. $g(x)$ is differentiable for $x \in [0, \infty)$

Answer: B



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83. If $f(x) = (4 + x)^n$, $n \in \mathbb{N}$ and $f^r(0)$ represents the r^{th} derivative of $f(x)$ at $x = 0$, then the value of $\sum_{r=0}^{\infty} \frac{(f^r(0))}{r!}$ is equal to

A. 2^n

B. 3^n

C. 5^n

D. 4^n

Answer: C



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84. Let $f(x) = \begin{cases} \frac{x}{1+|x|}, & |x| \geq 1 \\ \frac{x}{1-|x|}, & |x| < 1 \end{cases}$, then domain of $f'(x)$ is:

- A. $(-\infty, \infty)$
- B. $(-\infty, \infty) - \{-1, 0, 1\}$
- C. $(-\infty, \infty) - \{-1, 1\}$
- D. $(-\infty, \infty) - \{0\}$

Answer: C

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85. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ and $g(x) = f^{-1}(x)$, then the value of $g'\left(\frac{-7}{6}\right)$ equals :

- A. $\frac{1}{5}$
- B. $-\frac{1}{5}$

C. $\frac{6}{7}$

D. $-\frac{6}{7}$

Answer: A



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86. The number of points at which the function $f(x) = (x - |x|)^2(1 - x + |x|)^2$ is not differentiable in the interval $(-3, 4)$ is ___

A. Zero

B. One

C. Two

D. Three

Answer: A



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87. If $f(x) = \sqrt{\frac{1 + \sin^{-1} x}{1 - \tan^{-1} x}}$, then $f'(0)$ is equal to:

- A. 4
- B. 3
- C. 2
- D. 1

Answer: D



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88. If

$f(x) = [(e^{x-1}), , , 0 \leq x \leq 1), (x + 1 - \{x\}, ', 1 < x < 3)$ and $g(x) = x$

such that $f(x)g(x)$ is continuous $[0, 3]$ then the ordered pair (a,b) is

(where $\{.\}$ denotes fractional part function):

- A. $(2, 3)$

B. (1, 2)

C. (3, 2)

D. (2, 2)

Answer: C



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89. Use the following table and the fact that $f(x)$ is invertible and differentiable everywhere to find $(f^{-1}(3))'$:

x	$f(x)$	$f'(x)$
3	1	7
6	2	10
9	3	5

A. 0

B. $\frac{1}{5}$

C. $\frac{1}{10}$

D. $\frac{1}{7}$

Answer: B



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90. Let $f(x) = (1 - x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$, and let $g(x) = \int \left(\frac{2(t-1)}{t+1} - \ln t \right) f(t) dt$ for $t \in [1, x]$ for all $x \in (1, \infty)$. Which of the following is true?

A. (a) 0

B. (b) 1

C. (c) 2

D. (d) 3

Answer: C



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Exercise One Or More Than One Answer Is Are Correct

1. If $f(x) = \tan^{-1}(\operatorname{sgn}(n^2 - \lambda x + 1))$ has exactly one point of discontinuity, then the value of λ can be:

A. 1

B. -1

C. 2

D. -2

Answer: C::D



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$$2. f(x) = \begin{cases} 2(x + 1) & x \leq -1 \\ \sqrt{1 - x^2} & -1 < x < 1, \text{ then:} \\ ||x| - 1| - 1 & x \geq 1 \end{cases}$$

A. $f(x)$ is non-differentiable at exactly three points

B. $f(x)$ is continuous in $(-\infty, 1]$

C. $f(x)$ is differentiable in $(-\infty, -1)$

D. $f(x)$ is finite type of discontinuity at $x = 1$, but continuous at

$$x = -1$$

Answer: A::C::D



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3. Let $f(x) = \begin{cases} \frac{x(3e^{1/x} + 4)}{2 - e^{1/x}} & (x \neq 0) \\ 0 & x = 0 \end{cases}$ $x \neq \frac{1}{\ln 2}$ which of the

following statement (s) is/are correct ?

A. $f(x)$ is continuous at $x = 0$

B. $f(x)$ is non-dervable at $x = 0$

C. $f'(0^+) = -3$

D. $f'(0^-)$ does not exist

Answer: A::B::C



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4. Let $|f(x)| \leq \sin^2 x$, $\forall x \in R$, then

A. $f(x)$ is continuous at $x = 0$

B. $f(x)$ is differentiable at $x = 0$

C. $f(x)$ is continuous but not differentiable at $x = 0$

D. $f(0) = 0$

Answer: A::B::D



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$$5. \text{ Let } f(x) = \begin{cases} \frac{a(1 - x \sin x) + b \cos x + 5}{x^2} & x < 0 \\ \left(1 + \left(\frac{dx + dx^3}{dx^2}\right)\right)^{\frac{1}{x}} & x > 0 \end{cases}$$

If f is continuous at $x = 0$ then correct statement (s) is/are:

A. $a + c = -1$

B. $b + x = -4$

C. $a + b = -5$

D. $c + d = an$ irrational number

Answer: A::B::C::D



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6. If $f(x) = |||x| - 2| + p|$ have more than 3 points non-derivability then the value of p can be :

A. 0

B. -1

C. -2

D. 2

Answer: B::C



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7. Identify the options having correct statement:

A. $f(x) = \sqrt[3]{x^2|x|} - |x|$ is nowhere non-differentiable

B. $\lim_{x \rightarrow \infty} ((x + 5)\tan^{-1}(x + 1)) - ((x + 1)\tan^{-1}(x + 1)) = 2\pi$

C. $f(x) = \sin\left(\ln\left(x + \sqrt{x^2 + 1}\right)\right)$ is an odd function

D. $f(x) = \frac{4 - x^2}{4x - x^3}$ is discontinuous at exactly one point

Answer: A::B::C



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8. A twice differentiable function $f(x)$ is defined for all real numbers and satisfies the following conditions

$f(0) = 2$; $f'(0) = -5$ and $f''(0) = 3$. The function $g(x)$ is defined by

$g(x) = e^{ax} + f(x) \forall x \in \mathbb{R}$, where 'a' is any constant. If

$g'(0) + g''(0) = 0$. Find the value(s) of 'a'

A. 1

B. -1

C. 2

D. -2

Answer: A::D



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9. If $f(x) = |\sin x|$, then

A. differentiable everywhere

B. not differentiable at $x = n\pi, n \in I$

C. not differentiable at $x = 0$

D. continuous at $x = 0$

Answer: A::D



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10. Let $[.]$ represent the greatest integer function and $f(x) = [\tan^2 x]$

then :

- A. $\lim_{x \rightarrow 0} f(x)$ does not exist
- B. $f(x)$ is continuous at $x = 0$
- C. $f(x)$ is not differentiable at $x = 0$
- D. $f'(0) = 0$

Answer: B::D



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11. Let f be a differentiable function satisfying $f'(x) = f'(-x) \forall x \in R$. Then which of the following is correct regarding f :

- A. (a) If $f(1) = f(2)$, then $f(-1) = f(-2)$
- B. (b) $\frac{1}{2}f(x) + \frac{1}{2}f(y) = f\left(\frac{1}{2}(x + y)\right)$ for all real values of x, y

C. (c) Let $f(x)$ be an even function, then $f(x) = 0 \forall x \in R$

D. (d) $f(x) + f(-x) = 2f(0) \forall x \in R$

Answer: A::D



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12. Let $f: R \rightarrow R$ be a function, such that $|f(x)| \leq x^{4n}, n \in N \forall n \in R$

then $f(x)$ is:

A. discontinuous at $x = 0$

B. continuous at $x = 0$

C. non-differentiable at $x = 0$

D. differentiable at $x = 0$

Answer: B::D



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13. Let $f(x) = [x]$ and $g(x) = 0$ when x is an integer and $g(x) = x^2$ when x is not an integer ($[]$ is the greatest integer function) then:

- A. $\lim_{x \rightarrow 1} g(x)$ exists, but $g(x)$ is not continuous at $x = 1$
- B. $\lim_{x \rightarrow 1} f(x)$ does not exist
- C. $g \circ g$ is continuous for all x
- D. $f \circ g$ is continuous for all x

Answer: A::B



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14. Let the function f be defined by $f(x) = \begin{cases} p + qx + x^2, & x < 2 \\ 2px + 3qx^2, & x \geq 2 \end{cases}$,

Then:

- A. $f(x)$ is continuous in \mathbb{R} if $3p + 10q = 4$
- B. $f(x)$ is differentiable in \mathbb{R} if $p = q = \frac{4}{13}$
- C. If $p = -2, q = 1$, then $f(x)$ is continuous in \mathbb{R}

D. $f(x)$ is differentiable in \mathbb{R} if $1p + 11q = 4$

Answer: A::B::C



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15. Let $f(x) = |2x - 9| + |2x + 9|$. Which of the following are true ?

A. $f(x)$ is not differentiable at $x = \frac{9}{2}$

B. $f(x)$ is not differentiable at $x = \frac{-9}{2}$

C. $f(x)$ is differentiable at $x = 0$

D. $f(x)$ is differentiable at $x = \frac{-9}{2}, 0, \frac{9}{2}$

Answer: A::B::C



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16. Let $f(x) = \max(x, x^2x^3) \in -2 \leq x \leq 2$. Then:

A. $f(x)$ is continuous in $-2 \leq x \leq 2$

B. $f(x)$ is not differentiable at $x = 1$

C. $f(-1) + f\left(\frac{3}{2}\right) = \frac{35}{8}$

D. $f(-1)f'\left(\frac{3}{2}\right) = \frac{-35}{4}$

Answer: A::B::C

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17. Let $f(x)$ be a differentiable function satisfying $f(y)f\left(\frac{x}{y}\right) = f(x) \forall x, y \in \mathbb{R}, y \neq 0$ and $f(1) \neq 0, f'(1) = 3$ then

A. $\text{sgn}(f(x))$ is non-differentiable at exactly one point

B. $\lim_{x \rightarrow 0} \frac{x^2(\cos x - 1)}{f(x)} = 0$

C. $f(x) = x$ has 3 solutions

D. $f(f(x)) - f^3(x) = 0$ has infinitely many solutions

Answer: A::B::C::D



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18. Let $f(x) = (x^2 - 3x + 2)(x^2 + 3x + 2)$ and α, β, γ satisfy $\alpha < \beta < \gamma$ are the roots of $f'(x) = 0$ then which of the following is/are correct ([.] denots greatest integer function) ?

A. $[\alpha] = -2$

B. $[\beta] = -1$

C. $[\beta] = 0$

D. $[\alpha] = 1$

Answer: A:C



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19. Let $f: R \rightarrow R$ is given by

$$f(x) = \begin{cases} p + qx + x^2 & x < 2 \\ 2px + 3qx^2 & x \geq 2 \end{cases} \text{ then:}$$

A. $f(x)$ is continuous in \mathbb{R} if $3p + 10q = 4$

B. $f(x)$ is differentiable in \mathbb{R} is $p = q = \frac{4}{13}$

C. If $p = -2, q = 1$, then $f(x)$ is continuous in \mathbb{R}

D. $f(x)$ is differentiable in \mathbb{R} is $2p + 11q = 4$

Answer: A::B::C



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20. Let $y = e^{x \sin(x^3)} + (\tan x)^x$. Find $\frac{dy}{dx}$.

A.

$$e^{x \sin(x^3)} [3x^3 \cos(x^3) + \sin(x^3)] + (\tan x)^x [\ln \tan x + 2x \sec^2 2x]$$

B.

$$e^{x \sin(x^3)} [x^3 \cos(x^3) + \sin(x^3)] + (\tan x)^x [\ln \tan x + 2x \sec^2 2x]$$

C. $e^{x \sin(x^3)} [x^3 \sin(x^3) + \sin(x^3)] + (\tan x)^x [\ln \tan x + 2 \sec^2 2x]$

D. $e^{x \sin(x^3)} [3x^3 \cos(x^3) + \sin(x^3)] + (\tan x)^x \left[\ln \tan x + \frac{x \sec^2}{\tan x} \right]$

Answer: A::D



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21.

Let

$$f(x) = x + (1-x)x^2 + [1-x](1-x^2) + \dots + (1-x)\dots\dots(1-x^r$$

then:

$$\text{A. } f(x) = - \prod_{r=1}^n (1 - n^r)$$

$$\text{B. } f(x) = 1 - \prod_{r=1}^n (1 - x^r)$$

$$\text{C. } f(x) = (1 - f(x)) \left(\prod_{r=1}^n \frac{rx^{r-1}}{(1-x^r)} \right)$$

$$\text{D. } f'(x) = f(x) \left(\left(\prod_{r=1}^n \frac{rx^{r-4}}{(1-x^r)} \right) \right)$$

Answer: B::C



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22. Let $f(x) = \begin{cases} x^2 + a & 0 \leq x < 1 \\ 2x + b & 1 \leq x \leq 2 \end{cases}$ and $g(x) = \begin{cases} 3x + b & 0 \leq x < 1 \\ x^3 & 1 \leq x \leq 2 \end{cases}$

If derivative of $f(x)$ w.r.t. $g(x)$ at $x = 1$ exists and is equal to λ , then which of the following is/are correct?

A. $a + b = -3$

B. $a - b = 1$

C. $\frac{ab}{\lambda} = 3$

D. $\frac{-b}{\lambda} = 3$

Answer: A::B::C::D

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23. If $f(x) = \begin{cases} \frac{\sin [x^2] \pi}{x^2 - 3x - 18} + ax^2 + b, & \text{for } 0 \leq x \leq 1 \\ 2 \cos \pi x + \tan^{-1} x, & \text{for } 1 < x \leq 2 \end{cases}$ differentiable

function in $[0, 2]$, where $[\cdot]$ denotes the greatest integer function, then

A. $a = \frac{1}{3}$

$$B. a = \frac{1}{6}$$

$$C. b = \frac{\pi}{4} - \frac{13}{6}$$

$$D. b = \frac{\pi}{4} - \frac{7}{3}$$

Answer: B::C



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24. Let $f(x) \begin{cases} 1 + x, & 0 \leq x \leq 2 \\ 3 - x, & 2 < x \leq 3 \end{cases}$:

$g(x) = f(f(x))$:

Column-I		Column-II	
(A) If domain of $g(x)$ is $[a, b]$ then $b - a$ is	(P)		1
(B) If range of $g(x)$ is $[c, d]$ then $c + d$ is	(Q)		2
(C) $f(f(f(2))) + f(f(f(3)))$, is	(R)		3
(D) $m =$ maximum value of $g(x)$ then $2m - 2$ is :	(S)		4

A. $x = 1$

B. $x = 2$

C. $x = \frac{5}{2}$

D. $x = 3$

Answer: A::B



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25.

Let

$$f(x) = (x + 1)(x + 2)(x + 3) \dots (x + 100) \text{ and } g(x) = f(x)f''(x) - f'(x)f''(x)$$

Let n be the numbers of real roots of $g(x) = 0$, then:

- A. $n < 2$
- B. $n > 2$
- C. $n < 100$
- D. $n > 100$

Answer: A::C



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26. Let $f(x) = \begin{cases} |x| - 3 & x < 1 \\ |x - 2| + a & x \geq 1 \end{cases}$
 $g(x) = \begin{cases} 2 - |x| & x < 1 \\ \text{Sgn}(x) - b & x \geq 1 \end{cases}$ If $h(x)=f(x)$ is discontinuous at exactly

one point, then which of the following are correct?

A. $a = -3, b = 0$

B. $a = -3, b = -1$

C. $a = 2, b = 1$

D. $a = 0, b = 1$

Answer: A::B::C::D

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27. Let $f(x)$ be a continuous function in $[-1, 1]$ such that

$$f(x) = \begin{cases} \frac{\ln(ax^2 + bx + c)}{x^2} & -1 \leq x < 0 \\ 1 & x = 0 \\ \frac{\sin(e^{x^2} - 1)}{x^2} & 0 < x \leq 1 \end{cases}$$

Then which of the following

is/are correct

A. $a + b + c = 0$

B. $b = a + c$

C. $c = 1 + b$

D. $b^2 + c^2 = 1$

Answer: C::D



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28. $f(x)$ is differentiable function satisfying the relationship

$$f^2(x) + f^2(y) + 2(xy - 1) = f^2(x + y) \forall x, y \in R$$

Also $f(x) > 0 \forall x \in R$ and $f(\sqrt{2}) = 2$. Then which of the following statement (s) is/are correct about $f(x)$?

A. $[f(3)] = 3$ ([.] denotes greatest integer function)

B. $f(\sqrt{7}) = 3$

C. $f(x)$ is even

D. $f'(0) = 0$

Answer: A::B::C::D



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29.

The function $f(x) = \left[\sqrt{1 - \sqrt{1 - x^2}} \right]$, (where $[.]$ denotes greatest integer function):

i) has domain $[-1,1]$

ii) is discontinuous at two points in its domain

iii) is discontinuous at $x=0$

iv) is discontinuous at $x = 1$

A. has domain $[-1, 1]$

B. is discontinuous at two points in its domain

C. is discontinuous at $x = 0$

D. is discontinuous at $x = 1$

Answer: A::B::D

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30. A function $f(x)$ satisfies the relation

$$f(x + y) = f(x) + f(y) + xy(x + y), \forall x, y \in \mathbb{R}. \text{ If } f'(0) = -1, \text{ then}$$

- A. $f(x)$ is a polynomial function
- B. $f(x)$ is an exponential function
- C. $f(x)$ is twice differentiable for all $x \in \mathbb{R}$
- D. $f'(3) = 8$

Answer: A::C::D

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31. The points of discontinuities of $f(x) = \left[\frac{6x}{\pi} \right] \cos \left[\frac{3x}{\pi} \right] \sin \left[\frac{\pi}{6}, \pi \right]$ is/are:(where $[.]$ denotes greatest integer function)

- A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. π

Answer: B::C::D



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32. Check the continuity of $f(x) = \begin{cases} \frac{x^2}{2} & \text{if } 0 \leq x \leq 1 \\ 2x^2 - 3x + \frac{3}{2} & \text{if } 1 < x \leq 2 \end{cases}$ at $x = 1$

A. $f(x), f'(x)$ are continuous

B. $f'(x)$ is continuous, $f''(x)$ is not continuous

C. $f''(x)$ is continuous

D. $f''(x)$ is non differentiable

Answer: A::B::D



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33. If $x = \phi(t)$, $y = \Psi(t)$, then $\frac{d^2y}{dx^2}$ is equal to

A. $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^2}$

B. $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^3}$

C. $\frac{\psi''}{\phi'} - \frac{\psi'\phi''}{(\phi')^2}$

D. $\frac{\psi''}{(\phi')^2} - \frac{\psi'\phi''}{(\phi')^3}$

Answer: B::D



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34. $f(x) = [x]$ and $g(x) = \begin{cases} 0, & x \in I \\ x^2, & \notin I \end{cases}$ where $[.]$ denotes the greatest

integer function. Then

A. $g \circ f$ is continuous for all x

B. $g \circ f$ is not continuous for all x

C. fog is continuous everywhere

D. fog is not continuous everywhere

Answer: A



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35. Let $f: \mathbb{R}^+ \rightarrow \mathbb{R}$ defined as $f(x) = e^x + \ln x$ and $g = f^{-1}$ then correct statement (s) is/are:

A. $g''(e) = \frac{1 - e}{(1 + e)^3}$

B. $g''(e) = \frac{e - 1}{(1 + e)^3}$

C. $g'(e) = e + 1$

D. $g'(e) = \frac{1}{e + 1}$

Answer: A:D



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36. Let $f(x) = \begin{cases} \frac{3x-x^2}{2} & x < 2 \\ [x-1] & 2 \leq x < 3 \\ x^2 - 8x + 17 & x \geq 3 \end{cases}$ then which of the

following hold(s) good ?

- A. $\lim_{x \rightarrow 2} f(x) = 1$
- B. $f(x)$ is differentiable at $x = 2$
- C. $f(x)$ is continuous at $x = 2$
- D. $f(x)$ is discontinuous at $x = 3$

Answer: A::C::D



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Exercise Comprehension Type Problems

1. Let $f(x) = \lim_{n \rightarrow \infty} n^2 \tan\left(\ln\left(\sec \frac{x}{n}\right)\right)$ and $g(x) = \min(f(x), \{x\})$

(where $\{.\}$ denotes fractional part function)

Left derivative of $\phi(x) = e^{\sqrt{2f(x)}} at $x = 0$ is:$

A. 0

B. 1

C. -1

D. Does not exist

Answer: C



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2. Let $f(x) = \lim_{n \rightarrow \infty} n^2 \tan\left(\ln\left(\sec \frac{x}{n}\right)\right)$ and $g(x) = \min(f(x), \{x\})$

(where $\{ \cdot \}$ denotes fractional part function)

Left derivative of $\phi(x) = e^{\sqrt{2f(x)}} \text{ at } x = 0$ is:

A. 2

B. 1

C. 0

D. 3

Answer: A



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3.

$$f(x) = x^2 + xg'(1) + g''(2) \text{ and } g(x) = f(1)x^2 + xf'(x) + f'(x).$$

The value of $g(0)$ is

A. 0

B. 1

C. 2

D. 3

Answer: D



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4. Let $f(x)$ and $g(x)$ be two differentiable functions, defined as:

$$f(x) = x^2 + xg'(1) + g''(2) \text{ and } g(x) = f(1)x^2 + xf'(x) + f''(x).$$

The number of integers in the domain of the function

$$F(x) = \sqrt{-\frac{f(x)}{g(x)}} + \sqrt{3-x} \text{ is:}$$

- A. (a) 0
- B. (b) 1
- C. (c) 2
- D. (d) Infinite

Answer: C



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5. Define: $f(x) = |x^2 - 4x + 3| \ln x + 2(x - 2)^{1/3}, x > 0$

$$h(x) = \begin{cases} x - 1, & x \in Q \\ x^2 - x - 2, & x \notin Q \end{cases}$$

$f(x)$ is non-differentiable at..... points and the sum of corresponding x value (s) is

A. 3, 6

B. 2, 3

C. 2, 4

D. 2, 5

Answer: D



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6. Define: $f(x) = |x^2 - 4x + 3| \ln x + 2(x - 2)^{1/3}, x > 0$

$$h(x) = \begin{cases} x - 1, & x \in Q \\ x^2 - x - 2, & x \notin Q \end{cases}$$

$h(x)$ is discontinuous at $x = \dots\dots$

A. $1 + \sqrt{2}$

B. $\tan \frac{3\pi}{8}$

C. $\tan \frac{7\pi}{8}$

D. $\sqrt{2} - 1$

Answer: D



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7. Consider a function defined in $[-2, 2]$

$$f(x) = \begin{cases} \{x\} & -2 \leq x < -1 \\ |\operatorname{sgn} x| & -1 \leq x \leq 1 \\ \{-x\} & 1 < x \leq 2 \end{cases}, \text{ where } \{.\} \text{ denotes the fractional}$$

part function.

The total number of points of discontinuity of $f(x)$ for $x \in [-2, 2]$ is:

A. 0

B. 1

C. 2

D. 4

Answer: B



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8. Let a function $f(x)$ be defined in $[-2, 2]$ as

$$f(x) = \begin{cases} \{x\}, & -2 \leq x < -1 \\ |\operatorname{sgn} x|, & -1 \leq x \leq 1 \\ \{-x\}, & 1 < x \leq 2 \end{cases} \quad \text{where } \{x\} \text{ and } \operatorname{sgn} x \text{ denote}$$

fractional part and signum functions, respectively. Then find the area bounded by the graph of $f(x)$ and the x-axis.

A. 0

B. 1

C. 2

D. 3

Answer: D



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9. Consider a function $f(x)$ in $[0, 2\pi]$ defined as :

$$f(x) = \begin{cases} [\sin x] + [\cos x] & 0 \leq x \leq \pi \\ [\sin x] - [\cos x] & \pi < x \leq 2\pi \end{cases}$$

where $[.]$ denotes greatest integer function then.

$\lim_{x \rightarrow \left(\frac{3\pi}{2}\right)^+}$, $f(x)$ equals

A. 2

B. 3

C. 4

D. 5

Answer: B



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10. Consider a function $f(x)$ in $[0, 2\pi]$ defined as :

$$f(x) = \begin{cases} [\sin x] + [\cos x] & 0 \leq x \leq \pi \\ [\sin x] - [\cos x] & \pi < x \leq 2\pi \end{cases}$$

where $[\cdot]$ denotes greatest integer function then.

$$\lim_{x \rightarrow \left(\frac{3\pi}{2}\right)^+}, f(x) \text{ equals}$$

A. 0

B. 1

C. -1

D. 2

Answer: C



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11. Let $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1) & 2 \leq x \leq 3 \end{cases}$ where $[x]$ = greatest integer less

than or equal to x , then:

The number of values of x for $x \in [0, 3]$ where $f(x)$ is discontinuous is:

A. 0

B. 1

C. 2

D. 3

Answer: C



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12. Let $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x - 1) & 2 \leq x \leq 3 \end{cases}$ where $[x]$ = greatest integer less

than or equal to x , then:

The number of values of x for $x \in [0, 3]$ where $f(x)$ is non-differentiable

is :

A. 0

B. 1

C. 2

D. 3

Answer: D



13. Let $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x - 1) & 2 \leq x \leq 3 \end{cases}$ where $[x]$ = greatest integer less than or equal to x , then:

The number of integers in the range of $y = f(x)$ is:

- A. 3
- B. 4
- C. 5
- D. 6

Answer: A

14. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a continuous and differentiable function such that $f(x + y) = f(x) \cdot f(y) \forall x, y$, $f(x) \neq 0$ and $f(0) = 1$ and $f'(0) = 2$.

Let $f(xy) = g(x) \cdot G(y) \forall x, y$ and $g'(1) = 2$. $g(1) \neq 0$

Identify the correct option:

A. $f(2) = e^4$

B. $f(2) = 2e^2$

C. $(1) < 4$

D. $f(3) > 729$

Answer: A



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15. Let $f: R \rightarrow R$ be a continuous and differentiable function such that

$$f(x+y) = f(x) \cdot F(y) \forall x, y, f(x) \neq 0 \text{ and } f(0) = 1 \text{ and } f'(0) = 2.$$

Let $f(xy) = g(x) \cdot G(y) \forall x, y$ and $g'(1) = 2$. $g(1) \neq 0$

Identify the correct option:

A. $g(2) = 2$

B. $g(3) = 3$

C. $g(3) = 9$

D. $g(3) = 6$

Answer: C



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16. Let $f: R \rightarrow R$ be a continuous and differentiable function such that $f(x+y) = f(x) \cdot f(y) \forall x, y$, $f(x) \neq 0$ and $f(0) = 1$ and $f'(0) = 2$.

Let $g(xy) = g(x) \cdot g(y) \forall x, y$ and $g'(1) = 2$. $g(1) \neq 0$

The number of values of x , where $f(x)g(x) = 0$:

A. 0

B. 1

C. 2

D. 3

Answer: B



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17. Let $f(x) = \frac{\cos^2 x}{1 + \cos x + \cos^2 x}$ and $g(x) = \lambda \tan x + 1(1 - \lambda)\sin x - x$, where $\lambda \in \mathbb{R}$ and $x \in [0, \pi/2]$.

$g'(x)$ equals

A. $\frac{(1 - \cos x)(f(x) - \lambda)}{\cos x}$

B. $\frac{(1 - \cos x)(\lambda - f(x))}{\cos x}$

C. $\frac{(1 - \cos x)(\lambda - f(x))}{f(x)}$

D. $\frac{(1 - \cos x)(\lambda - f(x))}{(f(x))^2}$

Answer: C



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18.

Let

$$f(x) = \frac{\cos^2 x}{1 + \cos x + \cos^2 x} \text{ and } g(x) = \lambda \tan x + (1 - \lambda)\sin x - x,$$

where $\lambda \in R$ and $x \in [0, \pi/2)$.

The values of ' λ ' such that $g'(x) \geq 0$ for any $x \in [0, \pi/2)$:

A. (a) $[1, \infty)$

B. (b) $[0, \infty)$

C. (c) $\left[\frac{1}{2}, \infty\right)$

D. (d) $\left[\frac{1}{3}, \infty\right)$

Answer: D



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19. Population evolve to maximize their reproductive potential in the habitat in which they live. The strategy shown by pacific solmon fish for it's survivalness is :

(A) Producing few individuals of large size

(B) Breed only once in their life time

(C) Produce many individuals of large size

Question Type: Single Correct Type

1 A, B, C

2 A,B

3 Only B

4 B, C

A. 1

B. 2

C. 4

D. Non-existent

Answer: B



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20. Let f and g be two differentiable functions such that:

$$f(x) = g'(1)\sin x + (g''(2) - 1)x$$

$$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f'\left(-\frac{\pi}{2}\right)$$

The number of solution (s) of the equation $f(x) = g(x)$ is/are :

A. 1

B. 2

C. 3

D. infinite

Answer: B

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21. Let f and g be two differentiable functions such that:

$$f(x) = g'(1)\sin x + (g''(2) - 1)x$$

$$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f'\left(-\frac{\pi}{2}\right)$$

If $\int \frac{g(\cos x)}{f(x) - x} dx = \cos x + \ln(h(x)) + C$ where C is constant and

$h\left(\frac{\pi}{2}\right) = 1$ then $\left| h\left(\frac{2\pi}{3}\right) \right|$ is:

A. $3\sqrt{2}$

B. $2\sqrt{3}$

C. $\sqrt{3}$

D. $\frac{1}{\sqrt{3}}$

Answer: B



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22. Let f and g be two differentiable functions such that:

$$f(x) = g'(1)\sin x + (g''(2) - 1)x$$

$$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f'\left(-\frac{\pi}{2}\right)$$

If $\phi(x) = f^{-1}(x)$ then $\phi'\left(\frac{\pi}{2} + 1\right)$ equals to :

A. $\frac{\pi}{2} + 1$

B. $\frac{\pi}{2}$

C. 1

D. 0

Answer: C



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23. Suppose a function $f(x)$ satisfies the following conditions

$$f(x + y) = \frac{f(x) + f(y)}{f + f(x)f(y)}, \forall x, y \in R \text{ and } f'(0) = 1$$

Also $-1 < f(x) < 1, \forall x \in R$

$f(x)$ increases in the complete interval :

A. $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$

B. $(-\infty, \infty)$

C. $(-\infty, 1) \cup (-1, 0)$

D. $(0, 1) \cup (1, \infty)$

Answer: B



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24. Suppose a function $f(x)$ satisfies the following conditions

$$f(x + y) = \frac{f(x) + f(y)}{f + f(x)f(y)}, \forall x, y \in R \text{ and } f'(0) = 1$$

Also $-1 < f(x) < 1, \forall x \in R$

The value of the limit $\lim_{x \rightarrow \infty} (f(x))^x$ is:

A. 0

B. 1

C. e

D. e^2

Answer: B



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25. Let $f(x)$ be a polynomial satisfying $\lim_{x \rightarrow \infty} \frac{x^4 f(x)}{x^8 + 1} = 3$

$f(2) = 5, f(3) = 10, f(-1) = 2, f(-6) = 37$

The value of $\lim_{x \rightarrow -6} \frac{f(x) - x^2 - 1}{3(x + 6)}$ equals to:



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26. Let $f(x)$ be a polynomial satisfying $\lim_{x \rightarrow \infty} \frac{x^4 f(x)}{x^8 + 1} = 3$

$f(2) = 5, f(3) = 10, f(-1) = 2, f(-6) = 37$

The number of points of discontinuity of

$$f(x) = \frac{1}{x^2 + 1 - f(x)} \text{ in } \left[\frac{-15}{2}, \frac{5}{2} \right] \text{ equals:}$$

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27. Consider $f(x) = x^{\ln x}$, and $g(x) = e^2 x$. Let α and β be two values of x satisfying $f(x) = g(x)$ ($\alpha < \beta$)

$$\lim_{x \rightarrow \beta} \frac{f(x) - c\beta}{g(x) - \beta^2} = l \text{ then the value of } x - l \text{ equals to:}$$

A. $4 - e^2$

B. $e \% (2) - 4$

C. $4 - e$

D. $e - 4$

Answer: B

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28. Consider $f(x) = x^{\ln x}$, and $g(x) = e^2 x$. Let α and β be two values of x satisfying $f(x) = g(x)$ ($\alpha < \beta$)

If $h(x) = \frac{f(x)}{g(x)}$ then $h'(\alpha)$ equals to:

- A. (a) e
- B. (b) $-e$
- C. (c) $3e$
- D. (d) $-3e$

Answer: D



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29. Let $f_n x + f_n(y) = \frac{x^n + y^n}{x^n y^n} \forall x, y \in R - \{0\}$. where $n \in N$ and $g(x) = \max \left\{ f_2(x), f_3(x), \frac{1}{2} \right\} \forall x \in R - \{0\}$

The number of values of x for which $g(x)$ is non-differentiable ($x \in R - \{0\}$):

A. 1

B. 2

C. $\sqrt{2}$

D. 4

Answer: B



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30. Let $f_n x + f_n(y) = \frac{x^n + y^n}{x^n y^n} \forall x, y \in R - \{0\}$. where $n \in N$ and $g(x) = \max \left\{ f_2(x), f_3(x), \frac{1}{2} \right\} \forall x \in R - \{0\}$

The number of values of x for which $g(x)$ is non-differentiable ($x \in R - \{0\}$):

A. 2

B. 4

C. 5

D. 1

Answer: A



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Exercise Matching Type Problems

1. Let a function $f(x) = [x]\{x\} - |x|$ where $[\cdot]$, $\{ \cdot \}$ are greatest integer and fractional part respectively then match the following List-I with List-II.

Column-I		Column-II	
(A)	$f(x)$ is continuous at x equal to	(P)	3
(B)	$\frac{4}{3} \int_2^3 f(x) dx$ is equal to	(Q)	1

(C)	If $g(x) = x - 1$ and if $f(x) = g(x)$ where $x \in (-3, \infty)$, then number of solutions	(R)	4
(D)	If $l = \lim_{x \rightarrow 4^+} f(x)$, then $-l$ is equal to	(S)	2



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Column-I		Column-II	
(A)	$\lim_{x \rightarrow \infty} \frac{x^2 + 2x - 1}{2x^2 - 3x - 2} =$	(P)	$\frac{1}{2}$
(B)	$\lim_{x \rightarrow 0} \frac{\log \sec x/2 \cos x}{\log \sec x \cos \frac{x}{2}} =$	(Q)	2
(C)	Let $f(x) = \max.(\cos x, x, 2x - 1)$ where $x \geq 0$ then number of points of non-differentiability of $f(x)$ is	(R)	5
(D)	If $f(x) = [2 + 3 \sin x]$, $0 < x < \pi$ then number of points at which the function is discontinuous, is	(S)	16

2.

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3. The function $f(x) = ax(x - 1) + b$ $x < 1$
 $= x - 1$ $1 \leq x \leq 3$
 $= px^2 + qx + 2$ $x > 3$

if (i) $f(x)$ is continuous for all x

(ii) $f'(1)$ does not exist

(iii) $f'(x)$ is continuous at $x = 3$, then

Column-I		Column-II	
(A)	a cannot has value	(P)	$1/3$
(B)	b has value	(Q)	0
(C)	p has value	(R)	-1
(D)	q has value	(S)	1

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Exercise Subjective Type Problems

1. Let $f(x) = \begin{cases} ax(x-1) + b & x < 1 \\ x + 2 & 1 \leq x \leq 3 \\ px^2 + qx + 2 & x > 3 \end{cases}$ is continuous $\forall x \in \mathbb{R}$

except $x = 1$ but $|f(x)|$ is differentiable everywhere and $f'(x)$ is continuous at $x = 3$ and $|a + p + q| = k$, then $k =$

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2. If $y = \sin(8 \sin^{-1} x)$ then $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = -ky$, where $k =$

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3. If $y^2 = 4ax$, then $\frac{d^2y}{dx^2} = \frac{ka^2}{y^3}$, where $k^2 =$

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4. The number of values of x , $x \in [-2, 3]$ where $f(x) = [x^2]\sin(\pi x)$ is discontinuous is (where $[.]$ denotes greatest integer function)

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5. If $f(x)$ is continuous and differentiable in $[-3, 9]$ and $f'(x) \in [-2, 8] \forall x \in (-3, 9)$. Let N be the number of divisors of the greatest possible value of $f(9) - f(-3)$, then find the sum of digits of N .

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6. In $f(x) = [(\cos x^2, x < 0), (\sin x^3 - |x^3 - 1|, x \geq 0)]$ then find the number of points where $g(x) = f(|x|)$ is non-differentiable.

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7.

Consider

$$f(x) = x^2 + ax + 3 \text{ and } g(x) = x + b \text{ and } F(x) = \lim_{n \rightarrow \infty} \frac{f(x) + x^{2n}g(x)}{1 + x^{2n}}$$

If $F(x)$ is continuous at $x=-1$, then

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8. Let $f(x) = \begin{cases} 2 - x, & -3 \leq x \leq 0 \\ x - 2, & 0 < x < 4 \end{cases}$ Then $f^{-1}(x)$ is discontinuous at $x=$

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9. If $f(x) + 2f(1 - x) = x^2 + 2 \forall x \in R$ and $f(x)$ is a differentiable function, then the value of $f'(8)$ is

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10. Let $f(x) = \text{signum}(x)$ and $g(x) = x(x^2 - 10x + 21)$, then the number of points of discontinuity of $f[g(x)]$ is

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11. If $\frac{d^2}{dx^2} \left(\frac{\sin^4 x + \sin^2 x + 1}{\sin^2 x + \sin x + 1} \right) = a \sin^2 x + b \sin x + c$ then the value of $b + c - a$ is

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12. $f(x) = a \cos(\pi x) + b$, $f' \left(\frac{1}{2} \right) = \pi$ and $\int_{1/2}^{3/2} f(x) dx = \frac{2}{\pi} + 1$, then find the value of $-\frac{12}{\pi} \left(\frac{\sin^{-1} a}{3} + \cos^{-1} b \right)$.

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13.

Let

$\alpha(x) = f(x) - f(2x)$ and $\beta(x) = f(x) - f(4x)$ and $\alpha'(1) = 5\alpha'(2) = 7$

then find the value of $\beta'(1) - 10$

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14. Let $f(x) = -4e^{\frac{1-x}{2}} + \frac{x^3}{3} + \frac{x^2}{2} + x + 1$ and g be inverse function of f and $h(x) = \frac{a + bx^{3/2}}{x^{5/4}}$, $h'(5) = 0$, then $\frac{a^2}{5b^2g'(\frac{-7}{6})} =$

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15. If $y = e^{2\sin^{-1}x}$ then $\left| \frac{(x^2 - 1)y'' + xy'}{y} \right|$ is equal to

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16. Let f be continuous function on $[0, \infty)$ such that $\lim_{x \rightarrow \infty} \left(f(x) + \int_0^x f(t)(dt) \right)$ exists. Find $\lim_{x \rightarrow \infty} f(x)$.

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17. Let $f(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$ and let $g(x) = f^{-1}(x)$. Find $g''''(0)$.



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18. In $f(x) = [(\cos x^2, , x < 0), (\sin x^3 - |x^3 - 1|, , x \geq 0)]$ then find the number of points where $g(x) = f(|x|)$ is non-differentiable.

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19. Let $f: R \rightarrow R$ be a differentiable function satisfying:

$$f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x} \forall x, y \in R^+ \text{ also } f(1) = 0, f'(1) = 1$$

find $\lim_{x \rightarrow e} \left[\frac{1}{f(x)} \right]$ (where $[.]$ denotes greatest integer function).

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20. For the curve $\sin x + \sin y = 1$ lying in first quadrant. If $\lim_{x \rightarrow 0} x^\alpha \frac{d^2y}{dx^2}$ exists and non-zero than $2\alpha =$

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21. Let $f(x) = x \tan^{-1}(x^2) + x^4$ Let $f^k(x)$ denotes k^{th} derivative of $f(x)$ w.r.t. x , $k \in N$. If $f^{2m}(0) \neq 0$, m belongs to N , then $m =$

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22. If $x = \cos \theta$ and $y = \sin^3 \theta$, then $\left| \frac{y d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right|_{at \theta} = \frac{\pi}{2}$ is:

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23. The value of x , $x \in (2, \infty)$ where $f(x) = \sqrt{x \sqrt{8x - 16}} + \sqrt{x - \sqrt{8x - 16}}$ is not differentiable is:

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24. The number of non differentiability of point of function $f(x) = \min \left([x], \{x\}, \left| x - \frac{3}{2} \right| \right)$ for $x \in (0, 2)$, where $[.]$ and $\{.\}$

denote greatest integer function and fractional part function respectively.



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