



## MATHS

### BOOKS - SHRI BALAJI MATHS (ENGLISH)

#### DETERMINANTS

##### Exercise 1 Single Choice Problems

1. The value of

$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma$  is \_\_\_\_ .

A. 1

B.  $\frac{1}{2}$

C.  $\frac{3}{8}$

D.  $\frac{9}{4}$

**Answer: A**



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2. Let the following system of equations

$$kx + y + z = 1$$

$$x + ky + z = k$$

$$x + y + kz = k^2$$

has no solution . Find  $|k|$ .

A. 0

B. 1

C. 2

D. 3

**Answer: C**



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3. If  $a, b,$  and  $c$  are all different and if

$$\begin{vmatrix} a & a^2 & 1 + a^3 \\ b & b^2 & 1 + b^3 \\ c & c^2 & 1 + c^3 \end{vmatrix} = 0 \text{ Prove that } abc = -1.$$

A. 2

B.  $-1$

C. 1

D. 0

**Answer: B**



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4. If the system of linear equations

$$x + 2ay + az = 0$$

$$x + 3by + bz = 0$$

$$x + 4cy + cz = 0$$

has a non zero solutions, then  $a, b, c$  are in

A. are in A.P.

B. are in G.P.

C. are in H.P.

D. satisfy  $a+2b+3c=0$

**Answer: C**



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5. if the number of quadratic polynomials  $ax^2 + 2bx + c$  which satisfy the following conditions :

(i)  $a, b, c$  are distinct

(ii)  $a, b, c \in \{1, 2, 3, \dots, 2001, 2002\}$

(iii)  $x+1$  divides  $ax^2 + 2bx + c$  is equal to  $1000\lambda$ , then find the value of  $\lambda$ .

A. 2002

B. 2001

C. 2003

D. 2004

**Answer: A**



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6. If the system of equations  $2x+ay+6z=8$ ,  $x+2y+z=5$ ,  $2x+ay+3z=4$  has a unique solution then 'a' cannot be equal to :

A. 2

B. 3

C. 4

D. 5

**Answer: C**



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7. If one root of the equation  $\begin{vmatrix} 7 & 6 & x^2 - 13 \\ 2 & x^2 - 13 & 2 \\ x^2 - 13 & 3 & 7 \end{vmatrix} = 0$  is  $x=2$  the sum of all other five roots is

- A.  $-2$
- B.  $0$
- C.  $2\sqrt{3}$
- D.  $\sqrt{15}$

**Answer: A**

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8. The system of equations

$$kx + (k + 1)y + (k - 1)z = 0$$

$$(k + 1)x + ky + (k + 2)z = 0$$

$$(k - 1)x + (k + 2)y + kz = 0$$

has a nontrivial solution for :

- A. Exactly three real value of  $k$

B. Exactly two real values of  $k$

C. Exactly one real value of  $k$

D. Infinite number of values of  $k$

**Answer: C**



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9. if  $a_1, a_2, \dots, a_n, \dots$  form a G.P. and  $a_1 > 0$ , for all  $I \geq 1$

$$\begin{vmatrix} \log a_n, & \log a_n + \log a_{n+2}, & \log a_{n+2} \\ \log a_{n+3}, & \log a_{n+3} + \log a_{n+5}, & \log a_{n+5} \\ \log a_{n+6}, & \log a_{n+6} + \log a_{n+8}, & \log a_{n+8} \end{vmatrix}$$

A. 0

B.  $\left( \sum_{i=1}^{n^2+n} a_i \right)$

C. 1

D. 2

**Answer: A**



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10. if  $D_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$  and  $D_2 = \begin{vmatrix} a_1 + 2a_2 + 3a_3 & 2a_3 & 5a_2 \\ b_1 + 2b_2 + 3b_3 & 2b_3 & 5b_2 \\ c_1 + 2c_2 + 3c_3 & 2c_3 & 5c_2 \end{vmatrix}$  then  $\frac{D_2}{D_1}$  is equal to :

A. 10

B. -10

C. 20

D. -20

Answer: B



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11. If  $A = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ ,  $B = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$ , then

A.  $\Delta_1 = \Delta_2$



B.  $\Delta_1 = 2\Delta_2$

C.  $\Delta_1 + \Delta_2 = 0$

D.  $\Delta_1 + 2\Delta_2 = 0$

**Answer: C**



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12. The value of the determinant  $\begin{vmatrix} 1 & 0 & -1 \\ a & 1 & 1-a \\ b & a & 1+a-b \end{vmatrix}$  depends on :

A. only a

B. only b

C. neither a nor b

D. both a and b

**Answer: C**



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13. Sum of solution of the equation  $\begin{vmatrix} 1 & 2 & x \\ 2 & 3 & x^2 \\ 3 & 5 & 2 \end{vmatrix} = 10$  is :

A. 1

B. -1

C. 2

D. 4

**Answer: B**



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14. if  $D = \begin{vmatrix} x + d & x + e & x + f \\ x + d + 1 & x + e + 1 & x + f + 1 \\ x + a & x + b & x + c \end{vmatrix}$  then D does not depend

on :

A. a

B. e

C. d

D. x

**Answer: D**



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15. Value of 
$$\begin{vmatrix} x - y - z & 2x & 2x \\ 2y & y - z - x & 2y \\ 2z & 2z & z - x - y \end{vmatrix}$$

A.  $xyz(x + y + z)^2$

B.  $(x + y + z)(x + y + z)^2$

C.  $(x + y + z)^3$

D.  $(x + y + z)^2$

**Answer: C**



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16. A rectangle ABCD is inscribed in a circle. Let PQ be the diameter of the circle parallel the side AB. If  $\angle BPC = 30^\circ$ , then the ratio of the area of rectangle to the area of circle is

A.  $\frac{\sqrt{3}}{x}$

B.  $\frac{\sqrt{3}}{2x}$

C.  $\frac{3}{\pi}$

D.  $\frac{\sqrt{3}}{9\pi}$

Answer: A



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17. Let  $ab = 1$ ,  $\Delta = \begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix}$  then the

minimum value of  $\Delta$  is :

A. 3

B. 9

C. 27

D. 81

**Answer: C**



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18. The determinant

$$\begin{vmatrix} 2 & a + b + c + d & ab + cd \\ a + b + c + d & 2(a + b)(c + d) & ab(c + d) + cd(a + b) \\ ab + cd & ab(c + d) + cd(a + b) & 2abcd \end{vmatrix} = 0$$

for

A.  $a+b+c+d=0$

B.  $ab+cd=0$

C.  $ab(c+d)+cd(a+b)=0$

D. any  $a, b, c, d$

**Answer: D**



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19. Let  $\det A = \begin{vmatrix} l & m & n \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$  and if

$$(l - m)^2 + (p - q)^2 = 9, (m - n)^2 + (q - r)^2 = 16, (n - l)^2 + (r - p)^2 =$$

, then the value  $(\det.A)^2$  equals :

A. 36

B. 100

C. 144

D. 160

**Answer: C**



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20. A solution set of the equations  $x + 2y + z = 1$ ,  $x + 3y + 4z = k$ ,

$x + 5y + 10z = k^2$  is

A. 0

B. 4

C. 2

D. 3

**Answer: C**



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21. If  $\left| \begin{matrix} (x+1) & (x+1)^2 & (x+1)^3 \\ (x+2) & (x+2)^2 & (x+2)^3 \\ (x+3) & (x+3)^2 & (x+3)^3 \end{matrix} \right|$  is expressed as a polynomial in  $x$ ,

then the term independent of  $x$  is :

A. 0

B. 2

C. 12

D. 16

**Answer: C**



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**22.** If  $A, B, C$  are the angles of triangle  $ABC$ , then the minimum value of

$$\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} \text{ is equal to :}$$

A. 0

B.  $-1$

C. 1

D.  $-2$

**Answer: D**



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23. If the system of linear equations

$$x + 2ay + az = 0$$

$$x + 3by + bz = 0$$

$$x + 4cy + cz = 0$$

has a non-zero solution, then a, b, c

A. A.P

B. G.P

C. H.P

D. None of these

**Answer: C**



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24. If a, b and c are the roots of the equation  $x^3 + 2x^2 + 1 = 0$ , find

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$$

A. 8

B. -8

C. 0

D. 2

**Answer: A**



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25. The system of homogeneous equation

$$\lambda x + (\lambda + 1)y + (\lambda - 1)z = 0, (\lambda + 1)x + \lambda y + (\lambda + 2)z = 0, (\lambda - 1)x +$$

has non-trivial solution for :

A. 1.exactly three real value of  $\lambda$

B. 2.exactly two real values of  $\lambda$

C. 3.exactly three real value of  $\lambda$

D. 4.infinitely many real value of  $\lambda$

Answer: C

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26. If one root of the equation 
$$\begin{vmatrix} 7 & 6 & x^2 - 13 \\ 2 & x^2 - 13 & 2 \\ x^2 - 13 & 3 & 7 \end{vmatrix} = 0$$
 is  $x=2$  the

sum of all other five roots is

- A.  $-2$
- B.  $0$
- C.  $2\sqrt{5}$
- D.  $\sqrt{15}$

Answer: A

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Exercise 2 One Or More Than One Answer Is Are Correct

1.  $\Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a + b & a + b \\ 0 & 1 & 2a + 3b \end{vmatrix}$  is divisible by  $a + b$  b.  $a + 2b$  c.  $2a + 3b$  d.  $a^2$

A.  $(2a+b)$  is a factor of  $f(a,b)$

B.  $(a+2b)$  is a factor of  $f(a,b)$

C.  $(a+b)$  is a factor of  $f(a,b)$

D.  $a$  is factor of  $f(a,b)$

**Answer: B::C::D**



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2.

If  $\begin{vmatrix} 1 + \cos^2 \theta & \sin^2 \theta & 2\sqrt{3} \tan \theta \\ \cos^2 \theta & 1 + \sin^2 \theta & 2\sqrt{3} \tan \theta \\ \cos^2 \theta & \sin^2 \theta & 1 + 2\sqrt{3} \tan \theta \end{vmatrix} = 0$  then  $\theta$  may be :

i)  $\pi/6$

ii)  $5\pi/6$

iii)  $7\pi/6$

iv)  $11\pi/6$

A.  $\frac{\pi}{6}$

B.  $\frac{5\pi}{6}$

C.  $\frac{7\pi}{6}$

D.  $\frac{11\pi}{6}$

**Answer: B::D**



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**3.**

Let  $\Delta = \begin{vmatrix} a & a+d & a+3d \\ a+d & a+2d & a \\ a+2d & a & a+d \end{vmatrix}$  then :

i)  $\Delta$  depends on a

ii)  $\Delta$  depends on d

iii)  $\Delta$  is independent of a,d

iv)  $\Delta = 0$

A.  $\Delta$  depends on a

B.  $\Delta$  depends on d

C.  $\Delta$  is independent of a,d

D.  $\Delta = 0$

**Answer: A:B**

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4. The value(s) of  $\lambda$  for which the system of equations

$$(1 - \lambda)x + 3y - 4z = 0$$

$$x - (3 + \lambda)y + 5z = 0$$

$$3x + y - \lambda z = 0$$

possesses non-trivial solutions .

A.  $-1$

B.  $0$

C.  $1$

D. 2

**Answer: A::B**



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5. Let  $D(x) = \begin{vmatrix} x^2 + 4x - 3 & 2x + 4 & 13 \\ 2x^2 + 5x - 9 & 4x + 5 & 26 \\ 8x^2 - 16x + 1 & 16x - 6 & 104 \end{vmatrix} = \alpha x^3 + \beta x^2 + \gamma x + \delta$

then :

A.  $\alpha + \beta = 0$

B.  $\beta + \gamma = 0$

C.  $\alpha + \beta + \gamma + \delta = 0$

D.  $\alpha + \beta + \gamma = 0$

**Answer: A::B::D**



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6. if the system of equations

$$ax + y + 2z = 0$$

$$x + 2y + z = b$$

$$2x + y + az = 0$$

has no solution then  $(a+b)$  can be equals to :

A. A)  $-1$

B. B)  $2$

C. C)  $3$

D. D)  $4$

**Answer: A::B::C::D**



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**Exercise 3 Comprehension Type Problems**



1. Consider the system of equations

$$2x + \lambda y + 6z = 8$$

$$x + 2y + \mu z = 5$$

$$x + y + 3z = 4$$

The system of equations has :

No solution if :

A. 1)  $\lambda = 2, \mu = 3$

B. 2)  $\lambda \neq 2, \mu = 3$

C. 3)  $\lambda \neq 2, \mu \neq 3$

D. 4)  $\lambda = 2, \mu \in R$

**Answer: B**



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2. Consider the system of equations

$$2x + \lambda y + 6z = 8$$

$$x + 2y + \mu z = 5$$

$$x + y + 3z = 4$$

The system of equations has : Exactly one solution if :

A.  $\lambda = 2, \mu = 3$

B.  $\lambda \neq 2, \mu = 3$

C.  $\lambda \neq 2, \mu \neq 3$

D.  $\lambda = 2, \mu \in R$

**Answer: A**



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**3. Consider the system of equations**

$$2x + \lambda y + 6z = 8$$

$$x + 2y + \mu z = 5$$

$$x + y + 3z = 4$$

The system of equations has :

Infinitely many solutions if :

A.  $\lambda = 2, \mu = 3$

B.  $\lambda \neq 2, \mu = 3$

$$C. \lambda \neq 2, \mu \neq 3$$

$$D. \lambda = 2, \mu \in R$$

**Answer: A**

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## Exercise 4 Subjective Type Problems

1. The greatest value of  $n$  for which the determinant

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ {}^n C_1 & {}^{n+3} C_1 & {}^{n+6} C_1 \\ {}^n C_2 & {}^{n+3} C_2 & {}^{n+6} C_2 \end{vmatrix} \text{ is divisible by } 3^n, \text{ is}$$

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2. Find the value of  $\lambda$  for which

$$\begin{vmatrix} 2a_1 + b_1 & 2a_2 + b_2 & 2a_3 + b_3 \\ 2b_1 + c_1 & 2b_2 + c_2 & 2b_3 + c_3 \\ 2c_1 + a_1 & 2c_2 + a_2 & 2c_3 + a_3 \end{vmatrix} = \lambda \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

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3. Find the co-efficient of  $x$  in the expansion of the determinant

$$\begin{vmatrix} (1+x)^2 & (1+x)^4 & (1+x)^6 \\ (1+x)^3 & (1+x)^6 & (1+x)^9 \\ (1+x)^4 & (1+x)^8 & (1+x)^{12} \end{vmatrix}$$



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4. If  $x, y, z \in \mathbb{R}$  and 
$$\begin{vmatrix} x & y^2 & z^3 \\ x^4 & y^5 & z^6 \\ x^7 & y^8 & z^9 \end{vmatrix} = 2$$

then find the value of

$$\begin{vmatrix} y^5 z^6 (z^3 - y^3) & x^4 z^6 (x^3 - z^3) & x^4 y^5 (y^3 - x^3) \\ y^2 z^3 (y^6 - z^6) & x z^3 (z^6 - x^6) & x y^2 (x^6 - y^6) \\ y^2 z^3 (z^3 - y^3) & x z^3 (x^3 - z^3) & x y^2 (y^3 - x^3) \end{vmatrix}$$



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5. if the system of equations :

$$2x + 3y - z = 0$$

$$3x + 2y + kz = 0$$

$$4x + y + z = 0$$

have a set of non-zero integral solutions then , find the smallest positive value of  $z$ .

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6. Find  $a \in R$  for which the system of equations  $2ax-2y+3z=0$  ,  $x+ay + 2z=0$  and  $2x+az=0$  also have a non-trivial solution.

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7. If three non-zero distinct real numbers form an arithmetic progression and the squares of these numbers taken in the same order constitute a geometric progression. Find the sum of all possible common ratios of the geometric progression.

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8.

Let  $\Delta_1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ ,  $\Delta_2 = \begin{vmatrix} 6a_1 & 2a_2 & 2a_3 \\ 3b_1 & b_2 & b_3 \\ 12c_1 & 4c_2 & 4c_3 \end{vmatrix}$  and

$$\Delta_3 = \begin{vmatrix} 3a_1 + b_1 & 3a_2 + b_2 & 3a_3 + b_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix}$$

then  $\Delta_3 - \Delta_2 = k\Delta_1$ , find  $k$ .



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9. The minimum value of determinant

$$\Delta = \begin{vmatrix} 1 & \cos \theta & 1 \\ -\cos \theta & 1 & \cos \theta \\ -1 & -\cos \theta & 2 \end{vmatrix} \quad \forall \theta \in R \text{ is :}$$



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10. For a unique value of  $\mu$  &  $\lambda$ , the system of equations given by

$$x + y + z = 6$$

$$x + 2y + 3z = 14$$

$$2x + 5y + \lambda z = \mu$$

has infinitely many solutions, then  $\frac{\mu - \lambda}{4}$  is equal to



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11. Let  $\lim_{n \rightarrow \infty} n \sin (2\pi e \lfloor n \rfloor) = k \pi$ , where  $n \in \mathbb{N}$ . Find  $k$  :



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12. If the system of linear equations

$$(\cos \theta)x + (\sin \theta)y + \cos \theta = 0$$

$$(\sin \theta)x + (\cos \theta)y + \sin \theta = 0$$

$$(\cos \theta)x + (\sin \theta)y - \cos \theta = 0$$

is consistent , then the number of possible values of  $\theta$ ,  $\theta \in [0, 2\pi]$  is :



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