MATHS

BOOKS - SHRI BALAJI MATHS (ENGLISH)

DIFFERENTIAL EQUATIONS

Exercise Single Choice Problems

1. $\frac{dy}{dx}\left(\frac{1+\cos x}{y}\right) = -\sin x$ and $f\left(\frac{\pi}{2}\right) = -1$, then $f(0)$ is:

A. -2

B. -1

C. -3

D. -4

Answer: A

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2. If $y=\frac{2}{\sqrt{a^2-b^2}}\left\{\tan^{-1}\left(\frac{a-b}{a+b}\right)\frac{\tan x}{2}\right\}$, then show that $\frac{d^2y}{dx^2}=\frac{b\sin x}{(a+b\cos x)^2}$

A. $\frac{d^3y}{dx^3}-9\frac{d^2y}{dx^2}+23\frac{dy}{dx}+15y=0$

B. $\frac{d^3y}{dx^3}-9\frac{d^2y}{dx^2}-23\frac{dy}{dx}+15y=0$

C. $\frac{d^3y}{dx^3}+9\frac{d^2y}{dx^2}-23\frac{dy}{dx}+15y=0$

D. $\frac{d^3y}{dx^3}+9\frac{d^2y}{dx^2}+23\frac{dy}{dx}-15y=0$

Answer: A

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3. If $y=y(x)$ and it follows the relation $e^{xy^2}+y\cos(x^2)=5$ then $y'(0)$ is equal to

A. 4

B. -16

C. -4

D. 16

Answer: B

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4. If $(x^2+y^2)dy = xydx$ and y(1)=1 and $y(x_o) = e$, then $x_o =$

A. $\sqrt{3}e$

B. $\sqrt{e^2 - \frac{1}{2}}$

C. $\sqrt{\frac{e^2-1}{2}}$

D. $\sqrt{e^2 + \frac{1}{2}}$

Answer: A

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5. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{y}$ determines a family of circles with

A. Variable radil and fixed centre at $(0, 1)$

B. Variable radii and fixed centre at $(0, \ -1)$

C. Fixed radius 1 and variable centres along y-axis

D. Fixed radius 1 and variable centres along x-axis

Answer: C

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6. Interval contained in the domain of definition of non-zero solution of the differential equation $(x - 3)^2 y' + y = 0$ is:

A. $R - \{3\}$

B. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

C. $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$

D. $(-\pi, \pi)$

Answer: A

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7. A function $y = f(x)$ satisfies $(x+1)f'(x) - 2(x^2+x)f(x) = \frac{e^{x^2}}{x+1}, Aax > -1$. If $f(0) = 5$, then f(x) is

A. $\left(\frac{3x+5}{x+1}\right). e^{x^2}$

B. $\left(\frac{6x+5}{x+1}\right). e^{x^2}$

C. $\left(\frac{6x+5}{(x+1)^2}\right). e^{x^2}$

D. $\left(\frac{5x+6x}{x+1}\right). e^{x^2}$

Answer: B

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8. The solution of the differential equation

$2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$, given $y(1) = \sqrt{\frac{\pi}{2}}$, is

A. $\sin(x^2y^2) - 1 = 0$

B. $\cos\left(\frac{\pi}{2} + x^2y^2\right) + x = 0$

C. $\sin(x^2y^2) = e^{x-1}$

D. $\sin(x^2y^2) = e^{(x-1)}$

Answer: C

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9. If $y = e^{(\alpha+1)x}$ be solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$, then α is:

A. 0

B. 1

C. -1

D. 2

Answer: B

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10. The order and degree of the differential equation $\left(\frac{dy}{dx}\right)^{1/3} - 4\frac{d^2y}{dx^2} - 7x = 0$ are α and β, then the value of $(\alpha + \beta)$ is:

A. 3

B. 4

C. 2

D. 5

Answer: D

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11. General solution of differential equation of $f(x)\frac{dy}{dx} = f^2(x) + yf(x) + f'(x)y$ is:

(c being arbitary constant.)

A. $y = f(x) + ce^x$

B. $y = -f(x) + ce^x$

C. $y = -f(x) + ce^x f(x)$

D. $y = cf(x) + e^x$

Answer: C

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12. The order and degree respectively of the differential equation of all tangent lines to parabola $x^2 = 2y$ is:

A. $1, 2$

B. $2, 1$

C. $1, 1$

D. $1, 3$

Answer: A

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13. The general solution of the differential equation $\frac{dy}{dx} + x(x+y) = x(x+y)^3 - 1$ is:

A. $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^4}\right| = x^2 + C$

B. $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right| = x^2 + C$

C. $2\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right| = x^2 + C$

D. $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right| = x + C$

Answer: B

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14. The general solution of $\frac{dy}{dx}=2y\tan x+\tan^2 x$ is:

A. A) $y\cos^2 x=\frac{x}{2}+\frac{\sin 2x}{4}+C$

B. B) $y\sec^2 x=\frac{x}{2}-\frac{\sin 2x}{4}+C$

C. C) $y\cos^2 x=\frac{x}{2}-\frac{\cos 2x}{4}+C$

D. D) $y\cos^2 x=\frac{x}{2}-\frac{\sin 2x}{4}+C$

Answer: A

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15. The solution of differential equation $\frac{d^2y}{dx^2}=\frac{dy}{dx}, y(0)=3$ and $y'(0)=2$:

A. is a periodic function

B. approaches to zero as $x\to-\infty$

C. has an asymptote parallel to x-axis

D. has an asymptote parallel to y-axis

Answer: C

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16. The solution of the differential equation $(x^2+1)\frac{d^2y}{dx^2}=2x\frac{dy}{dx}$ under the conditions y(0)=1 and y '(0) =3,` is

A. $y=x^2+3x+1$

B. $y=x^3+3x+1$

C. $y-x^4+3x+1$

D. $y=3\tan^{-1}x+x^2+1$

Answer: B

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17. The differential equation of the family of curves $cy^2=2x+c$ (where c is an arbitrary constant.) is:

A. $\frac{xdy}{dx} = 1$

B. $\left(\frac{dy}{dx}\right) = \frac{2xly}{dx} + 1$

C. $y^2 = 2xy\frac{dy}{dx} + 1$

D. $y^2 = \frac{2ydy}{dx} + 1$

Answer: C

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18. The solution of the equation $\frac{dy}{dx} + \frac{1}{x}\tan y = \frac{1}{x^2}\tan y \sin$ y is:

A. $2y = \sin y(1 - 2cx^2)$

B. $2x = \cot y(1 + 2cx^2)$

C. $2x = \sin y(1 + 2cx^2)$

D. $2x \sin y = 1 - 2cx^2$

Answer: C

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19. Solution of the differential equation $xdy - \sqrt{x^2 + y^2}dx = 0$ is :

A. A) $y - \sqrt{x^2 + y^2} = cx^2$

B. B) $y + \sqrt{x^2 + y^2} = cx$

C. C) $x - \sqrt{x^2 + y^2} = cx^2$

D. D) $y + \sqrt{x^2 + y^2} = cx^2$

Answer: D

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20. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $f(x) =$

A. $\frac{1}{4x} + \frac{3x^2}{4}$

B. $\frac{3}{4x} + \frac{x^2}{4}$

C. $\frac{2}{3}x^2 + \frac{1}{3x}$

D. $\frac{1}{4x^3} + \frac{3x}{4}$

Answer: C

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21. The population p(t) at time t of a certain mouse species satisfies the differential equation $d\frac{p(t)}{dt} = 0.5p(t) - 450$ If $p(0) = 850$, then the time at which the population becomes zero is ....

(1) 2 In 18 (2) In 9 (3) $\frac{1}{2}$ In 18 (4) In 18

A. $\frac{1}{2}\ln 18$

B. $\ln 18$

C. $2\ln 18$

D. $\ln 9$

Answer: C

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22. The solution of the differential equation $\sin 2y \frac{dy}{dx} + 2\tan x \cos^2 y = 2\sec x \cos^3 y$ is: (where C is arbitary constant)

A. $\cos y \sec x = \tan x + C$

B. $\sec y \cos x = \tan x + C$

C. $\sec y \sec x = \tan x + C$

D. $\tan y \sec x = \sec x + C$

Answer: C

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23. The solution of the differential equation $\frac{dy}{dx} = (4x + y + 1)^2$, is

A. $4x + y + 1 = 2\tan(2x + y + C)$

B. $4x + y + 1 = 2\tan(x + 2y + C)$

C. $4x + y + 1 = 2\tan(2y + c)$

D. $4x + y + 1 = 2\tan(2x + c)$

Answer: D

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24. If a curve is such that line joining origin to any point $P(x, y)$ on the curve and the line parallel to y-axis through P are equally inclined to tangent to curve at P, then the differential equation of the curve is:

A. $x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} = x$

B. $x\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx} = x$

C. $y\left(\frac{dy}{dx}\right)^2 - 2x\frac{dy}{dx} = x$

D. $y\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} = x$

Answer: A

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25. If $y = f(x)$ satisfy the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2, \ f(1) = 1,$ then value of $f(3)$ equals:

A. 7

B. 5

C. 9

D. 27

Answer: A

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26. Let $y = f(x)$ and $\frac{x}{y}\frac{dy}{dx} = \frac{3x^2 - y}{2y - x^2}, \ f(1) = 1$ then the possible value of $\frac{1}{3}f(3)$ equals :

A. 9

B. 4

C. 3

D. 2

Answer: C

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Exercise One Or More Than One Answer Is Are Correct

1. Let y=f(x) be a real valued function satisfying $x\frac{dy}{dx}=x^2+y-2$, f(1)=1 then f(3) equal

A. 8

B. 7

C. 5

D. 2

Answer: C

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2. about to only mathematics

A. $xy = \sin x + c\cos x$

B. $xy\sec x = \tan x + c$

C. $xy + \sin x + c\cos x = 0$

D. None of these

Answer: A::B

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3. If a function satisfies $(x-y)f(x+y) - (x+y)f(x-y) = 2(x^2y - y^3)\ \forall x, y \in R$ and $f(1) =$

, then

A. f (x) must be polynomial function

B. $f(3) = 12$

C. $f(0) = 0$

D. $f(3) = 13$

Answer: A::B::C

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4. A function y=f (x) satisfies the differential equation $f(x)\sin 2x - \cos x + (1 + \sin^2 x)f'(x) = 0$ with initial condition $y(0) = 0$. The value of $f\left(\frac{\pi}{6}\right)$ is equal to (A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{2}{5}$

A. $\frac{1}{5}$

B. $\frac{3}{5}$

C. $\frac{4}{5}$

D. $\frac{2}{5}$

Answer: D

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5. Solution of the differential equation $\left(2+2x^2\sqrt{y}\right)ydx+\left(x^2\sqrt{y}+2\right)xdy=0$ is/are:

A. $xy\left(x^2\sqrt{y}+5\right)=c$

B. $xy\left(x^2\sqrt{y}+3\right)=c$

C. $xy\left(y^2\sqrt{x}+3\right)=c$

D. $xy\left(y^2\sqrt{x}+5\right)=c$

Answer: B

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6. If y (x) satisfies the differential equation $\frac{dy}{dx}=\sin 2x+3y\cot x$ and $y\left(\frac{\pi}{2}\right)=2$ then which of the following stattement (s) is/are correct ?

A. $y\left(\frac{\pi}{6}\right)=0$

B. $y'\left(\frac{\pi}{3}\right)=\frac{9-3\sqrt{2}}{2}$

C. $y(x)$ increases in the interval

D. $\int_{-\pi/2}^{\pi/2} y(x)dx = x$

Answer: A::C

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Exercise Comprehension Type Problem

1. A differentiable function y = g(x) satisfies $\int_0^x (x-t+1)g(t)dt = x^4+x^2$ for all $x \geq 0$ then y=g(x) satisfies the differential equation

A. $\frac{dy}{dx}-y=12x^2+2$

B. $\frac{dy}{dx}+2y=12x^2+2$

C. $\frac{dy}{dx}+y=12x^2+2$

D. $\frac{dy}{dx}+y=12x+2$

Answer: C

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2. A differentiable function $y = g(x)$ satisfies $\int_0^x (x - t + 1)g(t)dt = x^4 + x^2, \forall x \geq 0$. y =g (x)` satisfies the differential equation :

A. 0

B. 1

C. e^2

D. Data insufficient

Answer: A

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3. Suppose f and g are differentiabel functions such that $xg(f(x))f'(g(x))g'(x) = f(g(x))g'(f(x))f'(x)\,\forall x \in R$ and f is positive $\forall n \in R.$ Also $\int_0^x f(g(t))dt = \frac{1}{2}\left(1 - e^{-2x}\right)\forall x \in R, g(f(0)) = 1$ and $h(x) = \frac{g(f(x))}{f(g(x))}$

The graph of $y = h(x)$ is symmetric with respect to line:

A. $x = -1$

B. $x = 0$

C. $x = 1$

D. $x = 2$

Answer: C

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4. Let f and g be differentiable functions such that: $xg(f(x))f'(g(x))g'(x) = f(g(x))g'(f(x))f'(x)\,\forall x \in R$ Also, $f(x) > 0$ and $g(x) > 0\,\forall x \in R$ $\int_0^x f(g(t))dt = 1 - \frac{e^{-2x}}{2}, \forall x \in R$

and $g(f(0)) = 1, h(x) = \frac{g(f(x))}{f(g(x))} \forall x \in R$ Now answer the question:

$f(g(0)) + g(f(0)) =$ (A) 1 (B) 2 (C) 3 (D) 4

A. 1

B. 2

C. 3

D. 4

Answer: B

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5. Suppose f and g are differentiabel functions such that $xg(f(x))f'(g(x))g'(x) = f(g(x))g'(f(x))f'(x) \forall x \in R$ and f is positive $\forall n \in R.$ Also $\int_0^x f(g(t))dt = \frac{1}{2}(1 - e^{-2x}) \forall x \in R, g(f(0)) = 1$ and $h(x) = \frac{g(f(x))}{f(g(x))}$

The graph of $y = h(x)$ is symmetric with respect to line:

A. 1

B. $e^{1/3}$

C. e

D. e^2

Answer: C

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6. Given a function ' g' which has a derivative $g'(x)$ for every real x and satisfies $g'(0) = 2$ and $g(x+y) = e^y g(x) + e^y g(y)$ for all x and y then: Find g(x).

A. $x(2 + xe^x)$

B. $e(e^x + 1)$

C. $2xe^x$

D. $x + \ln(x+1)$

Answer: C

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7. Given a function ' g' which has a derivative $g'(x)$ for every real x and satisfies $g'(0) = 2$ and $g(x + y) = e^y g(x) + e^y g(y)$ for all x and y then: Find g(x).

A. R

B. $\left[-\frac{2}{e}, \infty\right)$

C. $\left[-\frac{1}{e}, \infty\right)$

D. $[0, \infty)$

Answer: B

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8. Given a function ' g' which has a derivative $g'(x)$ for every real x and satisfies $g'(0) = 2$ and $g(x + y) = e^y g(x) + e^y g(y)$ for all x and y then: Find g(x).

A. 0

B. 1

C. 2

D. Does not exist

Answer: A

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Exercise Matching Type Problems

1.

	Column-I (Differential equation)		Column-II Solution (Integral curves)
(A)	$y - x\frac{dy}{dx} = y^2 + \frac{dy}{dx}$	(P)	$y = A_1x^2 + A_2x + A_3$
(B)	$(2x - 10y^3)\frac{dy}{dx} + y = 0$	(Q)	$x^2y^2 + 1 = cy$
(C)	$\left(\frac{dy}{dx}\right)\left(\frac{d^3y}{dx^3}\right) - 3\left(\frac{d^2y}{dx^2}\right)^2 = 0$	(R)	$(x+1)(1-y) = cy$
(D)	$(x^2y^2 - 1)dy + 2xy^3dx = 0$	(S)	$x = A_1y^2 + A_2y + A_3$
		(T)	$xy^2 = 2y^5 + c$

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2.

	Column-I		Column-II
(A)	Solution of differential equation $[3x^2y+2xy-e^x(1+x)]dx+(x^3+x^2)dy=0$ is :	(P)	$y^2(x^2+1+ce^{x^2})=1$
(B)	Solution of differential equation $ydx-xdy-3xy^2e^{x^2}dx=0$ is :	(Q)	$(x^2+x^3)y-xe^x=c$
(C)	Solution of differential equation $\frac{dy}{dx}=xy(x^2y^2-1)$ is :	(R)	$\frac{x}{y}-\frac{3}{2}e^{x^2}=c$
(D)	Solution of differential equation $\frac{dy}{dx}(x^2y^3+xy)=1$ is :	(S)	$\frac{1}{x}=2-y^2+ce^{-y^2/2}$
		(T)	$\frac{2}{x}=1-y^2+ce^{-y/2}$
	(where c is arbitrary constant).		

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Exercise Subjective Type Problems

1. Find the value of |a| for which the area of triangle included between the coordinate axes and any tangent to the curve $x^a y = \lambda^a$ is constant (where λ is constnat.),

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2. Let $y = f(x)$ satisfies the differential equation $xy(1+y)dx = dy. Iff(0) = 1$ and $f(2) = \frac{e^2}{k - e^2}$, then find the value of k.

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3. If $y^2 = 3\cos^2 x + 2\sin^2 x$, then the value of $y^4 + y^3 \frac{d^2y}{dx^2}$ is

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4. Let $f(x)$ be a diffentiable function in $[-1, \infty)$ and $f(0) = 1$ such that $\lim_{t \to x+1} \frac{t^2 f(x+1) - (x+1)^2 f(t)}{f(t) - f(x+1)} = 1.$ Find the value of $\lim_{x \to 1} \frac{\ln(f(x)) - \ln 2}{x - 1}$.

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5. Let y = $(a\sin x+(b+c)\cos x)e^{x+d}$, where a,b,c and d are parameters represent a family of curves, then differential equation for the given family of curves is given by $y'' - \alpha y' + \beta y = 0$, then $\alpha + \beta =$

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6. Let $y = f(x)$ satisfies the differential equation $xy(1+y)dx = dy. Iff(0) = 1$ and $f(2) = \frac{e^2}{k-e^2}$, then find the value of k.

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