MATHS

BOOKS - SHRI BALAJI MATHS (ENGLISH)

ELLIPSE

Exercise 1 Single Choice Problems

1. If CF be the perpendicular from the centre C of the ellipse $\frac{x^2}{12}+\frac{y^2}{8}=1$, on the tangent at any point P and G is the point where the normal at P meets the major axis, then the value of $(CF \cdot PG)$ equals to :

A. 5

B. 6

C. 8

D. None of these

Answer: C

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2. The minimum length of intercept on any tangent to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ cut by the circle $x^2 + y^2 = 25$ is :

A. 8

B. 9

C. 2

D. 11

Answer: A

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3. Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line x+y=7, is minimum.

A. (2, 3)

B. (2, 1)

C. (1, 0)

D. None of these

Answer: B

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4. If lines $2x + 3y = 10$ and $2x - 3y = 10$ are tangents at the extremities of a latus rectum of an ellipse, whose centre is origin, then the length of the latus rectum is :

A. $\frac{110}{27}$

B. $\frac{98}{27}$

C. $\frac{100}{27}$

D. $\frac{120}{27}$

Answer: C

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5. Prove that the area bounded by the circle $x^2 + y^2 = a^2$ and the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to the area of another ellipse having semi-axis $a - b$ and b, $a > b$.

A. $a + b \text{ and } b$

B. $a - b \text{ and } a$

C. a and b

D. None of these

Answer: B

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6. If F_1 and F_2 are the feet of the perpendiculars from the foci $S_1 and S_2$ of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ on the tangent at any point P on the ellipse, then prove that $S_1F_1 + S_2F_2 \geq 8$.

A. $S_1F_1 + S_2F_2 \geq 2$

B. $S_1F_1 + S_2F_2 \geq 3$

C. $S_1F_1 + S_2F_2 \geq 6$

D. $S_1F_1 + S_2F_2 \geq 8$

Answer: D

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7. Consider the ellipse $\dfrac{x^2}{f(k^2+2k+5)} + \dfrac{y^2}{f(k+11)} = 1.$ If $f(x)$ is a positive decr4easing function, then the set of values of k for which the major axis is the x-axis is $(-3, 2)$. the set of values of k for which the major axis is the y-axis is $(-\infty, 2)$.

the set of values of k for which the major axis is the y-axis is $(-\infty, -3) \cup (2, \infty)$ the set of values of k for which the major axis is the y-axis is $(-3, -\infty,)$

A. $k \in (-7, -5)$

B. $k \in (-5, -3)$

C. $k \in (-3, 2)$

D. None of these

Answer: C

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8. If area of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ inscribed in a square of side length $5\sqrt{2}$ is A, then $\frac{A}{\pi}$ equals to :

A. 12

B. 10

C. 8

D. 11

Answer: A

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9. Any chord of the conic $x^2 + y^2 + xy = 1$ passing through origin is bisected at a point (p, q), then $(p + q + 12)$ equals to :

A. A) 13

B. B) 14

C. C) 11

D. D) 12

Answer: D

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10. Find the equation of pair of tangents drawn from point (4, 3) to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$. Also, find the angle between the tangents.

A. $\frac{3\sqrt{3}}{5\sqrt{17}}$

B. $\frac{\sqrt{43}}{10}$

C. $\frac{\sqrt{43}}{5}$

D. $\sqrt{\frac{3}{17}}$

Answer: C

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Exercise 2 Comprehension Type Problems

1. An ellipse hase semi-major of length 2 and semi-minor axis of length 1. It slides between the coordinates axes in the first quadrant while mantaining contact with both x-axis and y-axis. The locus of the centre of the ellipse is

A. $x^2 + y^2 = 3$

B. $x^2 + y^2 = 5$

C. $(x - 2)^2 + (y - 1)^2 = 5$

D. $(x - 2)^2 + (y - 1)^2 = 3$

Answer: B

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2. An ellipse hase semi-major of length 2 and semi-minor axis of length 1. It slides between the coordinates axes in the first quadrant while mantaining contact with both x-axis and y-axis. The locus of the centre of the ellipse is

A. $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} = 16$

B. $x^2 + y^2 + \frac{1}{x^2} - \frac{1}{y^2} = 2\sqrt{3} + 4$

C. $x^2 + y^2 - \frac{1}{x^2} - \frac{1}{y^2} = 2\sqrt{3} + 4$

D. $x^2 - y^2 + \frac{1}{x^2} - \frac{1}{y^2} = 2\sqrt{3} + 4$

Answer: A

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3. A coplanar beam of ligth emerging from a point source has the equation $\lambda x - y + 2(1 + a\lambda) - 0, \lambda \in R$. The rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having eqution $\mu x - y + 2(1 - \mu) = 0, \mu \in R$. Further, it is found that the foot of the perpendicular from the point (2,2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0$

The eccentricity of the ellipse of is equal to

A. $\frac{1}{3}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{2}{3}$

D. $\frac{1}{2}$

Answer: C

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4. A coplanar beam of ligth emerging from a point source has the equation $\lambda x - y + 2(1 + a\lambda) - 0, \lambda \in R$. The rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having eqution $\mu x-y+2(1-\mu)=0, \mu \in R$. Further, it is found that the foot of the perpendicular from the point (2,2) upon any tangent to the ellipse lies on the circle $x^2+y^2-4y-5=0$

The eccentricity of the ellipse of is equal to

A. $4\sqrt{5}$

B. $\sqrt{5}$

C. $3\sqrt{5}$

D. $2\sqrt{5}$

Answer: D

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5. A coplanar beam of ligth emerging from a point source has the equation $\lambda x - y + 2(1 + a\lambda) - 0, \lambda \in R$. The rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having eqution

$\mu x - y + 2(1-\mu) = 0, \mu \in R$. Further, it is found that the foot of the perpendicular from the point (2,2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0$

The eccentricity of the ellipse of is equal to

A. 6

B. 3

C. $\sqrt{5}$

D. $2\sqrt{5}$

Answer: A

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Exercise 3 Matching Type Problems

1.

	Column-I		Column-II
(A)	If the tangent to the ellipse $x^2+4y^2=16$ at the point $P(4\cos\phi, 2\sin\phi)$ is a normal to the circle $x^2+y^2-8x-4y=0$ then $\frac{\phi}{2}$ may be	(P)	0
(B)	The eccentric angle(s) of a point on the ellipse $x^2+3y^2=6$ at a distance 2 units from the centre of the ellipse is/are	(Q)	$\cos^{-1}\left(-\frac{2}{3}\right)$
(C)	The eccentric angle of point of intersection of the ellipse $x^2+4y^2=4$ and the parabola $x^2+1=y$ is	(R)	$\frac{\pi}{4}$
(D)	If the normal at the point $P(\sqrt{14}\cos\theta, \sqrt{5}\sin\theta)$ to the ellipse $\frac{x^2}{14}+\frac{y^2}{5}=1$ intersect it again at the point $Q(\sqrt{14}\cos 2\theta, \sqrt{5}\sin 2\theta)$, then θ is	(S)	$\frac{5\pi}{4}$
		(T)	$\frac{\pi}{2}$

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Exercise 4 Subjective Type Problems

1. For the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let O be the centre and S and S' be the foci. For any point P on the ellipse the value of $\frac{PS.\,PS'd^2}{9}$ (where d is the distance of O from the tangent at P) is equal to

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2. The number of distinct normal lines that can be drawn to the ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ from

the point $P(0, 6)$ is one (b) two (c) three (d) four

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