



MATHS

BOOKS - SHRI BALAJI MATHS (ENGLISH)

INDEFINITE AND DEFINITE INTEGRATION

Exercise Single Choice Problems

1. $\int a^x \left(\ln x + \ln a \cdot \ln \left(\frac{x}{e} \right)^x \right) dx =$

A. $a^x \ln \left(\frac{e}{x} \right)^{2x} + C$

B. $a^x \ln \left(\frac{x}{e} \right)^x + C$

C. $a^x + \ln \left(\frac{x}{e} \right)^x + C$

D. None of these

Answer: B



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2. The value of :

$$\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \frac{1}{\sqrt{n}\sqrt{n+3}} + \dots + \frac{1}{\sqrt{n}\sqrt{2n}} \right)$$

is:

- A. $\sqrt{2} - 1$
- B. $2(\sqrt{2} - 1)$
- C. $\sqrt{2} + 1$
- D. $2(\sqrt{2} + 1)$

Answer: B



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3. $\int \frac{\sin x}{\sin(x - \alpha)} dx = Ax + B \log(\sin(x - \alpha)) + C$ then find out (A, B)

- A. $(\sin \alpha, \cos \alpha)$

B. $(\cos \alpha, \sin \alpha)$

C. $(-\sin \alpha, \cos \alpha)$

D. $(-\cos \alpha, \sin \alpha)$

Answer: B



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4. The value of the integral $\int_0^2 \frac{\log(x^2 + 2)}{(x + 2)^2} dx$ is:

A. $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} + \frac{5}{12} \log 2 - \frac{1}{4} \log 3$

B. $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} - \frac{5}{12} \log 2 - \frac{1}{4} \log 3$

C. $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} + \frac{5}{12} \log 2 + \frac{1}{4} \log 3$

D. $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} - \frac{5}{12} \log 2 + \frac{1}{4} \log 3$

Answer: D



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5. If $I_1 = \int_0^1 \frac{1+x^8}{1+x^4} dx$ and $I_2 = \int_0^1 \frac{1+x^9}{1+x^2} dx$, then:

A. $I_1 > 1, I_2 < 1$

B. $I_1 < 1, I_2 > 1$

C. $1 < I_1 < I_2$

D. $I_2 < I_1 < 1$

Answer: D



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6. Let $f: (0, 1) \rightarrow (0, 1)$ be a differentiable function such that $f(x) \neq 0$

for all $x \in (0, 1)$ and $f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$. Suppose for all x ,

$$\lim_{x \rightarrow x} \frac{\int_0^1 \sqrt{1(f(s))^2} dx \int_0^x \sqrt{1(f(s))^2} ds}{f(t) - f(x)} = f(x)$$

Then, the value of $f\left(\frac{1}{4}\right)$ belongs to

A. $\left\{ \frac{\sqrt{7}}{4}, \frac{\sqrt{15}}{4} \right\}$

B. $\left\{ \frac{\sqrt{7}}{3}, \frac{\sqrt{15}}{3} \right\}$

C. $\left\{ \frac{\sqrt{7}}{2}, \frac{\sqrt{15}}{2} \right\}$

D. $\{ \sqrt{7}, \sqrt{15} \}$

Answer: A

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7. If $f(\theta) = \frac{4}{3}(1 - \cos^6 \theta - \sin^6 \theta)$, then

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left[\sqrt{f\left(\frac{1}{n}\right)} + \sqrt{f\left(\frac{2}{n}\right)} + \sqrt{f\left(\frac{n}{n}\right)} \right] =$$

A. $\frac{1 - \cos 1}{2}$

B. $1 - \cos 2$

C. $\frac{\sin 2}{2}$

D. $\frac{1 - \cos 2}{2}$

Answer: D

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8. The value of $\int_0^1 \frac{(x^6 - x^3)}{(2x^3 + 1)^3} dx$ is equal to :

A. $-\frac{1}{6}$

B. $-\frac{1}{12}$

C. $-\frac{1}{18}$

D. $-\frac{1}{36}$

Answer: D



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9. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that

$$f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt.$$

The value of $\int_0^1 f(x) dx$ is

A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{7}{12}$

D. $\frac{5}{12}$

Answer: D



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10. If $f'(x) = f(x) + \int_0^1 f(x) dx$ and given $f(0) = 1$, then $\int f(x) dx$ is equal to :

A. $\frac{2}{3-e}e^x + \left(\frac{3-e}{1-e}\right)x + C$

B. $\frac{2}{3-e}e^x + \left(\frac{1-e}{3-e}\right)x + C$

C. $\frac{3}{2-e}e^x + \left(\frac{1+e}{3+e}\right)x + C$

D. $\frac{2}{2-e}e^x + \left(\frac{1-e}{3+e}\right)x + C$

Answer: B



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11. For $x \in \mathbb{R}$, and a continuous function f let

$$I_1 = \int_{\sin^2 t}^{1 + \cos^2 t} x f\{x(2 - x)\} dx \text{ and } I_2 = \int_{\sin^2 t}^{1 + \cos^2 t} f\{x(2 - x)\} dx.$$

Then $\frac{I_1}{I_2}$ is

A. I_2

B. $\frac{1}{2}I_2$

C. $2I_2$

D. $3I_2$

Answer: A



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12. If the integral $\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln|\sin x - 2 \cos x| + k$ then a is equal to (1) -1 (2) -2 (3) 1 (4) 2

A. 1

B. 2

C. -1

D. -2

Answer: B



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13. $\int \frac{(2 + \sqrt{x}) dx}{(x + 1 + \sqrt{x})^2}$ is equal to:

A. $\frac{x}{x + \sqrt{x} + 1} + C$

B. $\frac{2x}{x + \sqrt{x} + 1} + C$

C. $\frac{-2x}{x + \sqrt{x} + 1} + C$

D. $\frac{-x}{x + \sqrt{x} + 1} + C$

Answer: B



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14. Evaluate $\int \frac{\left(\sqrt[3]{x + \sqrt{2 - x^2}}\right) \left(\sqrt[6]{1 - x\sqrt{2 - x^2}}\right) dx}{\sqrt[3]{1 - x^2}}, x \in (0, 1):$

A. $2^{\frac{1}{6}}x + C$

B. $2^{\frac{1}{12}}x + C$

C. $2^{\frac{1}{3}}x + C$

D. None of these

Answer: A



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15. $\int \frac{dx}{\sqrt{1 - \tan^2 x}} = \frac{1}{\lambda} \sin^{-1}(\lambda \sin x) + C$, then $\lambda =$

A. $\sqrt{2}$

B. $\sqrt{3}$

C. 2

D. $\sqrt{5}$

Answer: A



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16. $\int \frac{dx}{\sqrt[3]{x^{5/2}(x+1)^{7/2}}}$ is equal to:

A. $-\left(\frac{x-1}{x}\right)^{1/6} + C$

B. $6\left(\frac{x+1}{x}\right)^{-1/6} + C$

C. $\left(\frac{x}{x+1}\right)^{5/6} + C$

D. $-\left(\frac{x}{x+1}\right)^{5/6} + C$

Answer: B



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17. If $I_n = \int (\sin x)^n dx$, $n \in N$, then $5I_4 - 6I_6$ is equal to

A. $\sin x \cdot (\cos x)^5 + C$

B. $\sin 2x \cos 2x + C$

C. $\frac{\sin 2x}{8} [1 + \cos^2 2x - 2 \cos 2x] + C$

D. $\frac{\sin 2x}{8} [1 + \cos^2 2x + 2 \cos 2x] + C$

Answer: C



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18. Evaluate: $\int \frac{x^2}{(a + bx)^2} dx$

A. $\frac{1}{b^3} \left(a + bx - a \ln|a + bx| - \frac{a^3}{a + bx} \right) + C$

B. $\frac{1}{b^3} \left(a + bx - 2a \ln|a + bx| - \frac{a^3}{a + bx} \right) + C$

C. $\frac{1}{b^3} \left(a + bx + 2n \ln|a + bx| \frac{a^2}{a + bx} \right) + C$

D. $\frac{1}{b^3} \left(a + bx + 2n \ln|a - bx| \frac{a^2}{a + bx} \right) + C$

Answer: B



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19. $\int \frac{8x^{43} + 13x^{38}}{(x^{13} + x^5 + 1)^4} dx =$

A. $\frac{x^{39}}{3(x^{13} - x^5 + 1)^3} + C$

B. $\frac{x^{39}}{(x^{13} - x^5 + 1)^3} + C$

C. $\frac{x^{39}}{5(x^{13} - x^5 + 1)^5} + C$

D. None of these

Answer: A



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20. $\int \left(\frac{\cos 6x + 6 \cos 4x + 15 \cos 2x + 10}{10 \cos^2 x + 5 \cos x \cos 3x + \cos x \cos 5x} \right) dx = f(x) + C$, then

$f(10)$ is equal to:

A. 20

B. 10

C. $2 \sin 10$

D. $2 \cos 10$

Answer: A

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21. $\int (1 + x - x^{-1})e^{x+x^{-1}} dx =$

A. $(x + 1)e^{x+x^{-1}} + C$

B. $(x - 1)e^{x+x^{-1}} + C$

C. $-xe^{x+x^{-1}} + C$

D. $xe^{x+x^{-1}} + C$

Answer: D

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22. $\int e^x \left(\frac{2 \tan x}{1 + \tan x} + \cot^2 \left(x + \frac{\pi}{4} \right) \right) dx$ is equal to $c e^x \tan \left(\frac{\pi}{4} - x \right) + c e^x \tan \left(x - \frac{\pi}{4} \right) + c e^x \tan \left(\frac{3\pi}{4} - x \right) + c$ (d) none

of these

A. 0

B. 1

C. -1

D. 2

Answer: B

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23. The value of

$\int e^{(x \sin x + \cos x)} \cdot \left(\frac{x^4 \cos^3 x - x \sin x + \cos x}{x^2 \cos^2 x} \right) dx$, is equal to

A. $e^{x \sin x + \cos x} \left(x - \frac{1}{\cos x} \right) C$

B. $e^{x \sin x + \cos x} \left(x - \frac{1}{x \cos x} \right) C$

$$C. e^{x \sin x + \cos x} \left(1 - \frac{1}{x \cos x} \right) C$$

$$D. e^{x \sin x + \cos x} \left(1 - \frac{1}{\cos x} \right) C$$

Answer: B



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24. $\int \frac{1 + x + \sqrt{x + x^2}}{(\sqrt{x} + \sqrt{1 + x}) dx}$ is equal to

A. $\frac{1}{3} (2^{1/2} - 1)$

B. $\frac{2}{3} (2^{1/2} - 1)$

C. $\frac{2}{3} (2^{2/3} - 1)$

D. $\frac{1}{3} (2^{3/2} - 1)$

Answer: C



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25. $\int x^{x^2+1}(2 \ln x + 1) dx$

A. $x^{2x} + C$

B. $x^2 \ln x + C$

C. $x^{(x^x)} + C$

D. $(x^3)^x + C$

Answer: D



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26. If $\int \frac{\cos e^{c^2 x} - 2010}{\cos^{2010} x} dx = \frac{f(x)}{(g(x))^{2010}} + C$, where $f\left(\frac{\pi}{4}\right) = 1$,

then the number of solution of the equation $\frac{f(x)}{g(x)} = \{x\}$ in $[0, 2\pi]$

is/are: (where $\{ \}$ represents fractional part function)

A. 0

B. 1

C. 2

D. 3

Answer: A

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27. $\int x^x \left((\ln x)^2 + \ln x + \frac{1}{x} \right) dx$ is equal to:

A. $x^2 \left((\ln x)^2 - \frac{1}{x} + C \right)$

B. $x^x (\ln x - x) + C$

C. $x^x \frac{(\ln x)^2}{2} + C$

D. $x^x \ln x + C$

Answer: D

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28. $\int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dx$ is equal to (a) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + C$ (b) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C$ (c) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + C$ (d) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$

A. $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + C$

B. $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C$

C. $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C$

D. $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$

Answer: D

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29. Evaluate: $\int \left(\frac{\log x - 1}{1 + (\log x)^2} \right)^2 dx$

A. $\frac{x}{x^2 + 1} + C$

B. $\frac{\ln x}{(\ln x)^2 + 1} + C$

$$C. \frac{x}{1 + (\ln x)^2} + C$$

$$D. e^x \left(\frac{x}{x^2 + 1} \right) + C$$

Answer: C



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30. $I = \int \frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}} = k^4 \sqrt{\frac{x-1}{x+2}} + C$, then 'k' is equal to:

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{4}{3}$

Answer: D



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31. If $\int \frac{1 - x^7}{x(1 + x^7)} dx = a \log_e |x| + b \log_e |x^7 + 1| + c$, then

A. $2P - 7Q = 0$

B. $2P + 7Q = 0$

C. $7P + 2Q = 0$

D. $7P - 2Q = 1$

Answer: B



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32. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2 \sin^2 x \cos^2 x} dx =$

A. $\sin 2x + C$

B. $\frac{\sin 2x}{2} + C$

C. $\frac{-\sin 2x}{2} + C$

D. $-2 \sin 2x + C$

Answer: C

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33. $\int \frac{(\sin 2x)^{1/3} d(\sqrt[3]{\tan x})}{\sin^{2/3} x + \cos^{2/3} x}$ is equal to

A. $\frac{1}{2^{2/3}} \ln(1 + \tan^{1/3} x) + C$

B. $\ln(1 + \tan^{2/3} x) + C$

C. $2^{1/3} \ln(1 + \tan^{2/3} x) + C$

D. $\frac{1}{2^{2/3}} \ln(1 + \tan^{2/3} x) + C$

Answer: D

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34. $\int \sqrt{\frac{(2012)^{2x}}{1 - (2012)^{2x}}} (2012)^{\sin^{-1} (2012)^x} dx =$

A. $(\log_{2012} e)^2 (2012)^{\sin^{-1} (2012)^x} + C$

B. $(\log_{2012} e)^2 (2012)^{x - \sin^{-1}(2012)^x} + C$

C. $(\log_{2012} e)^2 (2012)^{\sin^{-1}(2012)^x} + C$

D. $\frac{(2012)^{\sin^{-1}(2012)^x}}{(\log_{2012} e)^2} + C$

Answer: C

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35. $\int \frac{x + 2}{(x^2 + 3x + 3)\sqrt{x + 1}} dx$ is equal to

A. $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3(x + 1)}} \right) + C$

B. $\frac{2}{\sqrt{3}} \tan^{-1} \left(\sqrt{\frac{x}{3(x + 1)}} \right) + C$

C. $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x}{3(x + 1)} \right) + C$

D. $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}(x + 1)} \right) + C$

Answer: A

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36. $\int \frac{f(x) \cdot g'(x) - f'(x)g(x)}{f(x) \cdot g(x)} \{\log g(x) - \log f(x)\} dx$

A. $\log \left(\frac{g(x)}{f(x)} \right) + C$

B. $\frac{1}{2} \left(\frac{g(x)}{f(x)} \right)^2 + C$

C. $\frac{1}{2} \left(\log \left(\frac{g(x)}{f(x)} \right) \right)^2 + C$

D. $\log \left(\left(\frac{g(x)}{f(x)} \right)^2 \right) + C$

Answer: C



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37. $\int \left(\int e^x \left(\ln x + \frac{2}{x} - \frac{1}{x^2} \right) dx \right) dx =$

A. $e^x \ln x + C_1 x + C_2$

B. $e^x \ln x + \frac{1}{x} + C_1 x + C_2$

C. $\frac{\ln x}{x} + C_1 x + C_2$

D. None of these

Answer: A



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38. Maximum value of the function $f(x) = \pi^2 \int_0^1 t \sin(x + \pi t) dt$ over all real number x :

A. $\sqrt{\pi^2 + 1}$

B. $\sqrt{\pi^2 + 2}$

C. $\sqrt{\pi^2 + 3}$

D. $\sqrt{\pi^2 + 4}$

Answer: D



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39. Let 'f' is a function, continuous on $[0, 1]$ such that $f(x) \leq \sqrt{5} \forall x \in \left[0, \frac{1}{2}\right]$ and $f(x) \leq \frac{2}{x} \forall x \in \left[\frac{1}{2}, 1\right]$ then smallest 'a'

for which $\int_0^1 f(x)dx \leq a$ holds for all 'f' is

A. $\sqrt{5}$

B. $\frac{\sqrt{5}}{2} + 2\ln 2$

C. $2 + \ln\left(\frac{\sqrt{5}}{2}\right)$

D. $2 + 2\ln\left(\frac{\sqrt{5}}{2}\right)$

Answer: D



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40. Let $I_n = \int_0^{\pi/2} (\sin x + \cos x)^n dx$ ($n \geq 2$). Then the value of n .

$I_n - 2(n-1)I_{n-1}$ is

A. 0

B. $2e^2$

C. e^2

D. 1

Answer: B



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41. Let a function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x + \sin x$. The value

of $\int_0^{2\pi} f^{-1}(x) dx$ will be:

A. $2\pi^2$

B. $2\pi^2 - 2$

C. $2x^2 + 2$

D. π^2

Answer: A



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42. The value of the definite integral

$\int_{-1}^1 e^{-x^4} \left(2 + \ln(x + \sqrt{x^2 + 1}) + 5x^3 - 8x^4 \right) dx$ is equal to:

A. $4e$

B. $\frac{4}{e}$

C. $2e$

D. $\frac{2}{e}$

Answer: B



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43. $\int_{-10}^0 \frac{\left| \frac{2\{x\}}{3x - [x]} \right|}{\frac{2[x]}{3x - [x]}} dx$ is equal to (where $[*]$ denotes greatest integer function.)

A. $\frac{28}{3}$

B. $\frac{1}{3}$

C. 0

D. None of these

Answer: A



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44. If $f(x) = \frac{x}{1 + (\ln x)(\ln x) \dots \infty} \forall x \in [1, \infty)$ then $\int_1^{2e} f(x) dx$

equals:

A. $\frac{e^2 - 1}{2}$

B. $\frac{e^2 + 1}{2}$

C. $\frac{e^2 - 2e}{2}$

D. None of these

Answer: A



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45. $\int_0^4 \frac{(y^2 - 4y + 5) \sin(y - 2) dy}{[2y^2 - 8y + 1]}$ is equal to

A. 0

B. 2

C. -2

D. None of these

Answer: A



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46. Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right), x > 0.$ If

$\int_1^4 \frac{3}{x} e^{\sin x} dx = F(k) - F(1),$ then one of the possible values of $k,$

is: 15 (b) 16 (c) 63 (d) 64

A. 15

B. 16

C. 63

D. 64

Answer: D



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47. Let $f: R^+ \rightarrow R$ be a differentiable function with $f(1) = 3$ and satisfying :

$$\int_1^{xy} f(t)dt = y \int_1^x f(t)dt + x \int_1^y f(t)dt \forall x, y \in R^+, \text{ then } f(e) =$$

A. 3

B. 4

C. 1

D. None of these

Answer: D



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48. Calculate the reciprocal of the limit $\lim_{x \rightarrow \infty} \int_0^x x e^{t^2 - x^2} dt$

A. 0

B. 1

C. 2

D. 3

Answer: C



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49.

Let

$$L = \lim_{n \rightarrow \infty} \left(\frac{(2.1 + n)}{1^2 + n.1 + n^2} + \frac{(2.2 + n)}{2^2 + n.2 + n^2} + \frac{(2.3 + n)}{3^2 + n.3 + n^2} + \dots + \right)$$

then value of e^L is:

A. 2

B. 3

C. 4

D. $\frac{3}{2}$

Answer: B

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50. The value of the definite integral $\int_0^2 \left(\sqrt{1+x^3} + \sqrt[3]{x^2+2x} \right) dx$ is :

A. 4

B. 5

C. 6

D. 7

Answer: C

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51. The value of the definite integral $\int_0^{\infty} \frac{\ln x}{x^2+4} dx$ is:

A. $\frac{\pi \ln 3}{2}$

B. $\frac{\pi \ln 2}{3}$

C. $\frac{\pi \ln 2}{4}$

D. $\frac{\pi \ln 4}{3}$

Answer: C



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52. The value of the definite integral

$$\int_0^{10} \left((x - 5) + (x - 5)^2 + (c - 5)^3 \right) dx \text{ is:}$$

A. $\frac{125}{3}$

B. $\frac{250}{3}$

C. $\frac{125}{6}$

D. $\frac{250}{4}$

Answer: B



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53. The value of definite integral $\int_0^{\infty} \frac{dx}{(1+x^9)(1+x^2)}$ equal to:

A. $\frac{\pi}{16}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

Answer: C



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54. The value of the definite integral $\int_0^{\frac{\pi}{2}} \left(\frac{1 + \sin 3x}{1 + 2 \sin x} \right) dx$ equals to

A. $\frac{\pi}{2}$

B. 1

C. $\frac{1}{2}$

D. $\frac{\pi}{4}$

Answer: B



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55. The value of $\lim_{x \rightarrow \infty} \frac{\int_0^x (\tan^{-1} x)^2}{\sqrt{x^2 + 1}} dx$

A. $\frac{\pi^2}{16}$

B. $\frac{\pi^2}{4}$

C. $\frac{\pi^2}{2}$

D. None of these

Answer: B



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56.

If

$$\int_0^1 \left(\sum_{r=1}^{2013} \frac{x}{x^2 + r^2} \prod_{r=1}^{2013} (x^2 + r^2) \right) dx = \frac{1}{2} \left[\left(\prod_{r=1}^{2013} (1 + r^2) \right) - k^2 \right]$$

then k=

A. 2013

B. 2013!

C. 2013^2 D. 2013^{2013} **Answer: B**[Watch Video Solution](#)

57. Let $f(x) = 2x - \tan^{-1} x - \ln(x + \sqrt{1 + x^2})$; $x \in \mathbb{R}$, then

A. strictly increases $\forall x \in \mathbb{R}$ B. strictly increases only in $(0, \infty)$

C. strictly decreases $\forall x \in R$

D. strictly decreases in $(0, \infty)$ and strictly increases in $(-\infty, 0)$

Answer: A



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58. The value of the definite integral

$$\int_0^{\frac{\pi}{2}} \frac{dx}{\tan x + \cot x + \cos x + \sec x} \text{ is}$$

A. $1 - \frac{\pi}{4}$

B. $\frac{\pi}{4} + 1$

C. $\pi + \frac{1}{4}$

D. None of these

Answer: A



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59. The value of the definite integral $\int_3^7 \frac{\cos x^2}{\cos(x)^2 + \cos(10-x)^2} dx$ is:

A. 2

B. 1

C. $\frac{1}{2}$

D. None of these

Answer: A



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60. The value of the integral $\int_{e^{-1}}^{e^2} \left| \frac{\log_e x}{x} \right| dx$ is

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. $\frac{3}{2}$

D. 5

Answer: B



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61. The value of $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\int_2^{\cos e^{c^2 x}} t g(t) dt}{x^2 - \frac{\pi^2}{16}}$ is:

A. $\frac{2}{\pi} g(2)$

B. $-\frac{4}{\pi} g(2)$

C. $-\frac{16}{\pi} g(2)$

D. $-\frac{16}{\pi} g(2)$

Answer: C



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62. The value of $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n-k}{n^2} \cos\left(\frac{4k}{n}\right)$ equals to

A. $\frac{1}{4} \sin 4 + \frac{1}{16} \cos 4 - \frac{1}{16}$

$$B. \frac{1}{4} \sin 4 - \frac{1}{16} \cos 4 + \frac{1}{16}$$

$$C. \frac{1}{16} (1 - \sin 4)$$

$$D. \frac{1}{16} (1 - \cos 4)$$

Answer: D



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63. For each positive integer n , define a function f_n on $[0, 1]$ as follows:

$$f_n(x) = \begin{cases} 0 & \text{if } x=0 \\ \sin \frac{\pi}{2n} & \text{if } 0 < x \leq \frac{1}{n} \\ \sin \frac{2\pi}{2n} & \text{if } \frac{1}{n} < x \leq \frac{2}{n} \\ \sin \frac{3\pi}{2n} & \text{if } \frac{2}{n} < x \leq \frac{3}{n} \\ \sin \frac{n\pi}{2n} & \text{if } \frac{n-1}{n} < x \leq 1 \end{cases}$$

Then the value of $\lim_{x \rightarrow \infty} \int_0^1 f_n(x) dx$ is:

A. π

B. $\frac{\pi}{2}$

C. $\frac{1}{\pi}$

D. $\frac{2}{\pi}$

Answer: D

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64. $\int_0^{n+1} \min \{|x - 1|, |x - 2|, |x - 3|, \dots, |x - n|\} dx$ equals to (where n be a positive integer)

A. $\frac{(n + 1)}{4}$

B. $\frac{(n + 2)}{4}$

C. $\frac{(n + 3)}{4}$

D. $\frac{(n + 4)}{4}$

Answer: A

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65. For positive integers $K = 1, 2, 3, \dots, n$ Let S_k denotes the area of ΔOAB_k (where O is the origin) such that $\angle AOB_k = \frac{K\pi}{2n}$, $OA = 1$

and $OB_K = K$. The value of the $\lim_{n \rightarrow \infty} \frac{1}{n^2} \cdot \sum_{K=1}^n S_K$ is

A. $\frac{2}{\pi^2}$

B. $(4) / (\pi^2)$

C. $\frac{8}{\pi^2}$

D. $\frac{1}{2\pi^2}$

Answer: D



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66. If $A = \int_0^1 \prod_{r=1}^{2014} (r - x) dx$ and $B = \int_0^1 \prod_{r=0}^{2013} (r + x) dx$, then:

A. $A = 2B$

B. $2A = B$

C. $A + B = 0$

D. $A = B$

Answer: D

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67. If $f(x) = \left[\frac{x}{120} + \frac{x^3}{30} \right]$ defined in $[0, 3]$ then $\int_0^1 ((f(x)) + 2) dx =$
(where $[.]$ denotes greatest integer function)

A. 0

B. 1

C. 2

D. 4

Answer: C

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68. If $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}$, $g(x) = \int_0^{\cos x} (1 + \sin t)^2 dt$, then the value of $f' \left(\frac{\pi}{2} \right)$ is equal to:

A. 1

B. -1

C. 0

D. $\frac{1}{2}$

Answer: D



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69. Let $f(x) = \frac{1}{x^2} \int_4^x (4t^2 - 2f'(t)) dt$, find $9f'(4)$

A. 16

B. 4

C. 8

D. 32

Answer: B



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70. Evaluate $\lim_{x \rightarrow \infty} \left(\frac{1^2}{n^3 + 1^3} + \frac{2^2}{n^3 + 2^3} + \frac{3^2}{n^3 + 3^3} + \dots + \frac{4}{9n} \right)$.

A. $\frac{1}{3} \ln 3$

B. $\frac{\ln 9}{3}$

C. $\frac{\ln 4}{3}$

D. $\frac{\ln 6}{3}$

Answer: A



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71. The value of $\int_0^{2\pi} \cos^{-1} \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) dx$ is:

A. π^2

B. $\frac{\pi^2}{2}$

C. $2\pi^2$

D. π^3

Answer: D



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72. Given a function 'g' continuous everywhere such that

$$\int_0^1 g(t) dt = 2 \text{ and } g(1) = 5. \text{ If } f(x) = \frac{1}{2} \int_0^x (x-t)^2 g(t) dt, \text{ then the}$$

value of $f''(1) - f'(1)$ is:

A. 0

B. 1

C. 2

D. 10/7

Answer: B



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73. If $\int_0^x \frac{x^3 \cos^4 x \sin^2 x}{\pi^2 - 3\pi x + 3x^2} dx = \lambda \int_0^{\pi/2} \sin^2 x dx$, then the value of λ is:

A. $\frac{\pi}{12}$

B. $\frac{\pi}{(8)}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: A



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74. $\int_0^{\sqrt{3}} \left(\frac{1}{2} \frac{d}{dx} \left(\tan^{-1} \frac{2x}{1-x^2} \right) \right) dx$ equals to:

A. $\frac{\pi}{3}$

B. $-\frac{\pi}{6}$

C. $\frac{\pi}{2}$

D. None of these

Answer: B

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75. Let $y = \{x\}^{[x]}$ then the value of $\int_0^3 y dx$ equals to (where $\{.\}$ and $[.]$ denote fractional part and integerpart function respectively)

A. 1

B. $\frac{11}{6}$

C. 3

D. $\frac{5}{6}$

Answer: C

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76. $\int_0^1 \frac{\tan^{-1} x}{x} dx =$

A. $\int_0^{\pi/4} \frac{\sin x}{x} dx$

B. $\int_0^{\pi/2} \frac{x}{\sin x} dx$

C. $\frac{1}{2} \int_0^{\pi/2} \frac{x}{\sin x} dx$

D. $\frac{1}{2} \int_0^{\pi/2} \frac{x}{\sin x} dx$

Answer: C

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77. The value of $\int_0^{4/\pi} \left(3x^2 \sin \frac{1}{x} - x \cos \frac{1}{x} \right) dx$ is:

A. $\frac{8\sqrt{2}}{\pi^3}$

B. $\frac{24\sqrt{2}}{\pi^3}$

C. $\frac{32\sqrt{2}}{\pi^3}$

D. None of these

Answer: C

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78. Number of values of x satisfying the equation

$$\int_{-1}^x \left(8t^2 + \frac{28}{3}t + 4 \right) dt = \frac{\left(\frac{3}{2}\right)x + 1}{\log_{x+1} \sqrt{x+1}}, \text{ is}$$

A. 0

B. 1

C. 2

D. 3

Answer: B



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79.

Evaluate

:

$$\lim_{n \rightarrow \infty} \frac{1^4 + 2^4 + 3^4 + \dots + n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + \dots + n^3}{n^5}$$

A. $\frac{1}{30}$

B. zero

C. $\frac{1}{4}$

D. $\frac{1}{5}$

Answer: D



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80. The value of $\lim_{x \rightarrow 0^+} \frac{\int_1^{\cos x} (\cos^{-1} t) dt}{2x - \sin 2x}$ is equal to:

A. 0

B. -1

C. $\frac{2}{3}$

D. $-\frac{1}{4}$

Answer: D



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81. Consider a parabola $y = \frac{x^2}{4}$ and the point $F(0, 1)$.

Let $A_1(x_1, y_1), A_2(x_2, y_2), A_3(x_3, y_3), \dots, A_n(x_n, y_n)$ are 'n' points on the parabola such $x_k > 0$ and $\angle OFA_k = \frac{k\pi}{2n}$ ($k = 1, 2, 3, \dots, n$).

Then the value of $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n FA_k$, is equal to :

A. $\frac{2}{\pi}$

B. $\frac{4}{\pi}$

C. $\frac{8}{\pi}$

D. None of these

Answer: B



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82. The minimum value of $f(x) = \int_0^4 e^{|x-t|} dt$ where $x \in [0, 3]$ is:

A. A) $2e^2 - 1$

B. B) $e^4 - 1$

C. C) $e(e^2 - 1)$

D. D) $e^2 - 1$

Answer: C



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83. If $\int_0^{\infty} \frac{\cos x}{x} dx = \frac{\pi}{2}$, then $\int_0^{\infty} \frac{\cos^3 x}{x} dx$ is equals to:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. π

D. $\frac{3\pi}{2}$

Answer: A



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84. $\int \sqrt{1 + \sin x} \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right) dx$ is:

A. A) $\frac{1 + \sin x}{2} + C$

B. B) $(1 + \sin x)^2$

C. C) $\frac{1}{\sqrt{1 + \sin x}} + C$

D. D) $\sin x + C$

Answer: D



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85. If $I_n = \int_0^\pi \frac{\sin(2nx)}{\sin 2x} dx$, then the value of $I_{n+\frac{1}{2}}$ is equal to ($n \in I$):

A. 1: $\frac{n\pi}{2}$

B. 2: π

C. 3: $\frac{\pi}{2}$

D. 4: 0

Answer: D



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86. The value of function $f(x) = 1 + x + \int_1^x (\ln^2 t + 2 \ln t) dt$ where $f'(x)$ vanishes is:

A. $\frac{1}{e}$

B. 0

C. $\frac{2}{e}$

D. $1 + \frac{2}{e}$

Answer: D



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87. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that

$$f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt.$$

The value of $\int_0^1 f(x) dx$ is

A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{7}{12}$

D. $\frac{5}{12}$

Answer: D



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88. The value of the definite integral $\int_{-(\pi/2)}^{\pi/2} \frac{\cos^2 x}{1 + 5^x}$ equal to:

A. $\frac{3n}{4}$

B. π

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: D

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$$89. \int \left(\frac{x^2 - x + 1}{x^2 + 1} \right) e^{\cot^{-1}(x)} dx = f(x) \cdot e^{\cot^{-1}(x)} + C$$

where C is constant of integration. Then f(x) is equal to:

A. $-x$

B. $\sqrt{1-x}$

C. x

D. $\sqrt{1+x}$

Answer: C

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$$90. \lim_{x \rightarrow \infty} \frac{1}{n^3} \left(\sqrt{n^2 + 1} + 2\sqrt{n^2 + 2^2} + \dots + n\sqrt{(n^2 + n^2)} \right) = :$$

A. $\frac{3\sqrt{2} - 1}{2}$

B. $\frac{2\sqrt{2} - 1}{3}$

C. $\frac{3\sqrt{3} - 1}{3}$

D. $\frac{4\sqrt{2} - 1}{2}$

Answer: B

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91. $\int \frac{(x^3 - 1)}{(x^4 + 1)(x + 1)} dx$ is

A. $\frac{1}{4}\ln(1 + x^4) + \frac{1}{3}\ln(1 + x^3) + c$

B. $\frac{1}{4}\ln(1 + x^4) - \frac{1}{3}\ln(1 + x^3) + c$

C. $\frac{1}{4}\ln(1 + x^4) - \ln(1 + x) + c$

D. $\frac{1}{4}\ln(1 + x^4) + \ln(1 + x) + c$

Answer: C

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92. The value of $\lim_{x \rightarrow 0^+} \frac{\int_1^{\cos x} (\cos^{-1} t) dt}{2x - \sin(2x)}$ is equal to

A. 0

B. -1

C. $\frac{2}{3}$

D. $\frac{-1}{4}$

Answer: D

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93. Let $f(x) = \lim_{n \rightarrow \infty} \frac{\cos x}{1 + (\tan^{-1} x)^n}$. Then the value of $\int_0^{\infty} f(x) dx$ is

equal to

A. $\tan(\sin 1)$

B. $\sin(\tan 1)$

C. 0

D. $\sin\left(\frac{\tan 1}{2}\right)$

Answer: B



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94. The value of $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{k}{n^2 + n + 2k} \right) =$

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. 1

Answer: C



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95. The value of $\lim_{y \rightarrow 1^+} \frac{\int_1^y |t - 1| dt}{\tan(y - 1)}$ is:

A. 0

B. 1

C. 2

D. does not exist

Answer: A



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96. Given that

$$\int \frac{dx}{(1+x^2)^n} = \frac{x}{2(n-1)(1+x^2)^{n-1}} + \frac{(2n-3)}{2(n-1)} \int \frac{dx}{(1+x^2)^{n-1}}.$$
 Find

the value of $\int_0^1 \frac{dx}{(1+x^2)^4}$, (you may or may not use reduction formula given)

A. 1: $\frac{11}{48} + \frac{5\pi}{64}$

B. 2: $\frac{11}{48} + \frac{5\pi}{32}$

C. 3: $\frac{1}{24} + \frac{5\pi}{64}$

D. 4: $\frac{1}{96} + \frac{5\pi}{32}$

Answer: A



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97. Find the value of $\int_0^{\pi/4} (\sin x)^4 dx$:

A. $\frac{3\pi}{16}$

B. $\frac{3\pi}{32} - \frac{1}{4}$

C. $\frac{3\pi}{32} - \frac{3}{4}$

D. $\frac{3\pi}{16} - \frac{7}{8}$

Answer: B



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98. $\int \frac{\cos 9x + \cos 6x}{2 \cos 5x - 1} dx = A \sin 4x + B \sin + C$, then $A + B$ is equal to:

(Where C is constant of integration)

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. 2

D. $\frac{5}{4}$

Answer: D



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99. $\int \frac{dx}{x^{2014} + x} = \frac{1}{p} \ln \left(\frac{x^q}{1 + x^r} \right) + C$ where $p, q, r \in N$ then the value of $(p + q + r)$ equals (Where C is constant of integration)

A. 6039

B. 6048

C. 6047

D. 6021

Answer: A

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100. If $\int_0^1 e^{-x^2} dx = a$, then $\int_0^1 x^2 e^{-x^2} dx$ is equal to

A. 1: $\frac{1}{2e}(e - 1)$

B. 2: $\frac{1}{2e}(ea + 1)$

C. 3: $\frac{1}{e}(e - 1)$

D. 4: $\frac{1}{e}(e + 1)$

Answer: A

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101. $f(x)$ is a continuous function for all real values of x and satisfies

$$\int_n^{n+1} f(x) dx = \frac{n^2}{2} \quad \forall n \in I. \text{ Then } \int_{-3}^5 f(|x|) dx \text{ is equal to } \frac{19}{2} \text{ (b) } \frac{35}{2}$$

(c) $\frac{17}{2}$ (d) none of these

A. $\frac{19}{2}$

B. $\frac{35}{2}$

C. $\frac{17}{2}$

D. $\frac{37}{2}$

Answer: B



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102. If $\int \frac{dx}{x^4(1+x^3)^2} = a \ln \left| \frac{1+x^3}{x^3} \right| + \frac{b}{x^3} + \frac{c}{1+x^3} + d$. then (where d is arbitrary constant)

A. 1: $a = \frac{1}{3}, b = \frac{1}{3}, c = \frac{1}{3}$

B. 2: $a = \frac{2}{3}, b = -\frac{1}{3}, c = \frac{1}{3}$

$$\text{C. 3: } a = \frac{2}{3}, b = -\frac{1}{3}, c = -\frac{1}{3}$$

$$\text{D. 4: } a = \frac{2}{3}b = \frac{1}{3}, c = -\frac{1}{3}$$

Answer: C



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103. The value of :

$$\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \frac{1}{\sqrt{n}\sqrt{n+3}} + \dots + \frac{1}{\sqrt{n}\sqrt{2n}} \right)$$

is:

A. 2

B. 4

C. $2(\sqrt{2} - 1)$

D. $2\sqrt{2} - 1$

Answer: A



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104. Let $f(x) = \int_z^2 \frac{dy}{\sqrt{1+y^3}}$. The value of the integral $\int_0^2 x f(x) dx$ is equal to:

A. 1

B. $\frac{1}{3}$

C. $\frac{4}{3}$

D. $\frac{2}{3}$

Answer: D



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105. The value of the definite integral $\int_0^{\pi/3} \ln(1 + \sqrt{3} \tan x) dx$ equals

A. $\frac{\pi}{3} \ln 2$

B. $\frac{\pi}{3}$

C. $\frac{\pi^2}{6} \ln 2$

D. $\frac{\pi}{2} \ln 2$

Answer: A



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106. about to only mathematics

A. 100 a

B. a

C. 0

D. 0.4

Answer: B



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107. The value of $\int_0^1 \lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{x^{k+2} 2^k}{k!} dx$ is: (a) $e^2 - 1$ (b) 2 (c) $\frac{e^2 - 1}{2}$
(d) $\frac{e^2 - 1}{4}$

A. $e^2 - 1$

B. 2

C. $\frac{e^2 - 1}{2}$

D. $\frac{e^2 - 1}{4}$

Answer: D



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108. Evaluate: $\int \frac{x^5}{\sqrt{1+x^3}} dx$

A. $\frac{1}{15} (1+x^2)^{5/2} - \frac{1}{9} (1+x^3)^{3/2} + xc$

B. $\frac{2}{15} (1+x^3)^{5/2} - \frac{1}{9} (1+x^3)^{3/2} + c$

C. $\frac{2}{15} (1+x^2)^{5/2} - \frac{2}{9} (1+x^3)^{3/2} + c$

$$D. \frac{1}{15}(1+x^3)^{5/2} - \frac{2}{9}(1+x^3)^{3/2} + c$$

Answer: C



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109. If $f(x) = \int_0^x \frac{\sin t}{t} dt$, which of the following is true?

A. $f(0) > f(1.1)$

B. $f(0) < f(1.1) > f(2.1)$

C. $f(0) < f(1.1) < f(2.1) > f(3.1)$

D. $f(0) < f(1.1) < f(2.1) < f(3.1) > f(4.1)$

Answer: D



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110. Evaluate: $\int \frac{x^3 + 3x^2 + x + 9}{(x^2 + 1)(x^2 + 3)} dx$.

A. $\ln|x^2 + 3| + 3 \tan^{-1} x + c$

B. $\frac{1}{2} \ln|x^2 + 3| + \tan^{-1} x + c$

C. $\frac{1}{2} \ln|x^2 + 3| + 3 \tan^{-1} x + c$

D. $\ln|x^2 + 3| - \tan^{-1} x + c$

Answer: C

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111. $\int \frac{\sqrt{\sec^5 x}}{\sqrt{\sin^3 x}} dx$ equals to:

A. $(\tan x)^{3/2} - \sqrt{\tan x} + C$

B. $2 \left(\frac{1}{3} (\tan x)^{3/2} - \frac{1}{\sqrt{\tan x}} \right) + C$

C. $\frac{1}{3} (\tan x)^{3/2} - \sqrt{\tan x} + C$

D. $\sqrt{\sin x} + \sqrt{\cos x} + C$

Answer: B

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112. $\lim_{x \rightarrow 0} \frac{e^{\sin(x)} - 1}{x}$ equals to :

A. 1

B. 2

C. e

D. does not exist

Answer: A



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113. If $A = \int_0^{\pi} \frac{\sin x}{x^2} dx$, then $\int_0^{\pi/2} \frac{\cos 2x}{x} dx$ is equal to:

A. $1 - A$

B. $\frac{3}{2} - A$

C. $A - 1$

D. $1 + A$

Answer: C



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Exercise One Or More Than One Answer Is Are Correct

1. $\int \frac{dx}{(1 + \sqrt{x})^8} = -\frac{1}{3(1 + \sqrt{x})^{k_1}} + \frac{2}{7(1 + \sqrt{x})^{k_2}} + C$, then:

A. $k_1 = 5$

B. $k_1 = 6$

C. $k_2 = 7$

D. $k_2 = 8$

Answer: B::C



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2. If $\int_{-\alpha}^{\alpha} \left(e^x + \cos x \ln(x + \sqrt{1+x^2}) \right) dx > \frac{3}{2}$, then possible value of α can be:

A. 1

B. 2

C. 3

D. 4

Answer: A::B::C::D



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3. If $\int \sqrt{\frac{x}{a^3 - x^3}} dx = \frac{d}{b} \sin^{-1} \left(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}} \right) + C$ (where b & d are coprime integer) then $b + d$ equals to.

A. $A = \frac{2}{3}$

B. $B = a^{3/2}$

C. $A = \frac{1}{3}$

$$D. B = a^{1/2}$$

Answer: A::B



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4. Let $\int x \sin x \sec^3 x dx = \frac{1}{2}(x \cdot f(x) - g(x)) + k$, then

A. $f(x) \notin (-1, 1)$

B. $g(x) = \sin x$ has 6 solution for $x \in [-\pi, 2\pi]$

C. $g'(x) = f(x), \forall x \in R$

D. $f(x)g(x)$ has no solution

Answer: A::C::D



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5.

If

$$\int (\sin 3\theta + \sin \theta) \cos \theta e^{\sin \theta} d\theta = (A \sin^3 \theta + B \cos^2 \theta + C \sin \theta + D \cos \theta + I)$$

then:

A. $A = -4$

B. $B = -12$

C. $C = -20$

D. None of these

Answer: A::B::C



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6. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}} dx = A \sin^{-1} \left(\frac{x^{3/2}}{B} \right) + C$, where

C is any arbitrary constant, then:

A. $A = \frac{2}{3}$

B. $B = a^{3/2}$

$$C. A = \frac{1}{3}$$

$$D. B = a^{1/2}$$

Answer: A::B

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7. If $f(\theta) = \lim_{x \rightarrow \infty} \sum_{r=0}^{n\theta} \frac{2r}{n\sqrt{(3\theta n - 2r)(n\theta + 2r)}}$ then:

A. $f(1) = \frac{\pi}{6}$

B. $f(\theta) = \frac{\theta}{2} \int_0^\theta \frac{dx}{\sqrt{\theta^2 - \left(x - \frac{\theta}{2}\right)^2}}$

C. $f(\theta)$ is constant function

D. $y = f(\theta)$ is invertible

Answer: A::B::D

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8. If $f(x + y) = f(x)f(y)$ for all x, y and

$f(0) \neq 0$, and $F(x) = \frac{f(x)}{1 + (f(x))^2}$ then:

A. $\int_{-2010}^{2011} F(x) dx = \int_0^{2011} F(x) dx$

B. $\int_{-2010}^{2011} F(x) dx = \int_0^{2011} F(x) dx = \int_0^{2011} F(x) dx$

C. $\int_{-2010}^{2011} F(x) dx = 0$

D. $\int_{-2010}^{2010} (2F(-x) - F(x)) dx = e \int_0^{2010} F(x) dx$

Answer: B::D



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9. Let $J = \int_{-1}^2 \left(\cot^{-1} \frac{1}{x} + \cot^{-1} x \right) dx$, $K = \int_{-2\pi}^{7\pi} \frac{\sin x}{|\sin x|} dx$. Then

which of the following alternative (s) is/are correct ?

A. $2J + 3K = 8\pi$

B. $4J^2 + K^2 = 26\pi^2$

C. $2J = K = 3\pi$

$$D. \frac{J}{K} = \frac{2}{5}$$

Answer: A::B



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10. Which of the following function (s) is/are even ?

$$A. f(x) = \int_0^x \ln(t + \sqrt{1+t^2}) dt$$

$$B. g(x) = \int_0^x \frac{(2^t + 1)t}{2^t - 1} dt$$

$$C. h(x) = \int_0^x (\sqrt{1+t+t^2} - \sqrt{1-t+t^2}) dt$$

$$D. l(x) = \int_0^x \ln\left(\frac{1-t}{1+t}\right) dt$$

Answer: A::B::C::D



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11. Let $\lim_{x \rightarrow \infty} \sqrt{\frac{x - \cos^2 x}{x + \sin x}}$ **and** $I_2 = \lim_{h \rightarrow 0^+} \int_{-1}^1 h \frac{dx}{h^2 + x^2}$. **Then**

A. Both l_1 and l_2 are less than $22/7$

B. One of the two limits is rational and other irrational

C. $l_2 > l_1$

D. l_2 is greater than 3 times of l_1

Answer: A::B::C::D

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12. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}} dx = A \sin^{-1} \left(\frac{x^{3/2}}{B} \right) + C$,

where C is any arbitrary constant, then:

A. $A = \frac{2}{3}$

B. $B = a^{3/2}$

C. $A = \frac{1}{3}$

D. $B = a^{1/2}$

Answer: A::B



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13. If $\int \frac{dx}{1 - \sin^4 x} = a \tan x + b \tan^{-1}(c \tan x) + D$, then:

A. $a = \frac{1}{2}$

B. $b = \sqrt{2}$

C. $c = \sqrt{2}$

D. $b = \frac{1}{2\sqrt{2}}$

Answer: A:C



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14. The value of definite integral: $\int_{-2014}^{2014} \frac{dx}{1 + \sin^{2015} x + \sqrt{1 + \sin^{4030} x}}$

equals :

A. 0

B. 2014

C. $(2014)^2$

D. 4028

Answer: B



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15. Let $L = \lim_{n \rightarrow \infty} \int_a^{\infty} \frac{ndx}{1 + n^2x^2}$, where $a \in R$, then L can be

A. π

B. $\frac{\pi}{2}$

C. 0

D. $\frac{\pi}{3}$

Answer: A::B::C



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16. Let $I = \int_0^1 \sqrt{\frac{1 + \sqrt{x}}{1 - \sqrt{x}}} dx$ and $J = \int_0^1 \sqrt{\frac{1 - \sqrt{x}}{1 + \sqrt{x}}} dx$ then correct

statement (s) is/are:

A. $I + J = 2$

B. $I - J = \pi$

C. $I = \frac{2 + \pi}{2}$

D. $J = \frac{4 - \pi}{2}$

Answer: B::C



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Exercise Comprehension Type Problems

1. Let $f(x) = \int x^2 \cos^2 x (2x + 6 \tan x - 2x \tan^2 x) dx$ and $f(x)$ passes through the point $(\pi, 0)$

If $f: R - (2n + 1)\frac{\pi}{2} \rightarrow R$ then $f(x)$ be a :

- A. even function
- B. odd function
- C. neither even nor odd
- D. even as well as odd both

Answer: A

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2. Let $f(x) = \int x^2 \cos^2 x (2x + 6 \tan x - 2x \tan^2 x) dx$ and $f(x)$ passes through the point $(\pi, 0)$

The number of solution (s) of the equation $f(x) = x^3$ in $[0, 2\pi]$ be:

- A. 0
- B. 3
- C. 2
- D. None of these

Answer: B



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3. Let $f(x)$ be a non constant twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1-x)$ and $f'\left(\frac{1}{4}\right) = 0$. Then

A. 2

B. 3

C. 4

D. 5

Answer: C



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4. Let $f(x)$ be a non constant twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1-x)$ and $f'\left(\frac{1}{4}\right) = 0$. Then

A. 1

B. π

C. 2

D. 0

Answer: D



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5. Let $f(x)$ be a non constant twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1 - x)$ and $f'\left(\frac{1}{4}\right) = 0$. Then

A. $\int_0^2 f'(t)e^{\cos \pi t} dt$

B. 1

C. 2

D. π

Answer: A



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6. Consider the function $f(x)$ and $g(x)$, both defined from $R \rightarrow R$

$$f(x) = \frac{x^3}{2} + 1 - x \int_0^x g(t)dt \text{ and } g(x) = x - \int_0^1 f(t)dt, \text{ then}$$

The number of points of intersection of $f(x)$ and $g(x)$ is/are:

A. 0

B. 1

C. $\frac{3}{2}$

D. does not exist

Answer: B



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7. Consider the function $f(x)$ and $g(x)$, both defined from $R \rightarrow R$

$$f(x) = \frac{x^3}{2} + 1 - x \int_0^x g(t)dt \text{ and } g(x) = x - \int_0^1 f(t)dt, \text{ then}$$

The number of points of intersection of $f(x)$ and $g(x)$ is/are:

A. 0

B. 1

C. 2

D. 3

Answer: A



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8. Consider the function $f(x)$ and $g(x)$, both defined from $R \rightarrow R$

$$f(x) = \frac{x^3}{2} + 1 - x \int_0^x g(t)dt \text{ and } g(x) = x - \int_0^1 f(t)dt, \text{ then}$$

The number of points of intersection of $f(x)$ and $g(x)$ is/are:

A. $\frac{9}{4}$

B. $\frac{9}{2}$

C. $\frac{9}{8}$

D. None of these

Answer: C



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9. Let $f(x)$ be function defined on $[0, 1]$ such that $f(1) = 0$ and for any $a \in (0, 1]$, $\int_0^a f(x)dx - \int_a^1 f(x)dx = 2f(a) + 3a + b$ where b is constant.

$b =$

A. $\frac{3}{2e} - 3$

B. $\frac{3}{2e} - \frac{3}{2}$

C. $\frac{3}{2e} + 3$

D. $\frac{3}{2e} + \frac{3}{2}$

Answer: A



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10. Let $f(x)$ be function defined on $[0, 1]$ such that $f(1) = 0$ and for any $a \in (0, 1]$, $\int_0^a f(x)dx - \int_a^1 f(x)dx = 2f(a) + 3a + b$ where b is constant.

The length of the subtangent of the curve $y = f(x)$ at $x = 1/2$ is:

A. $\sqrt{e} - 1$

B. $\frac{\sqrt{e} - 1}{2}$

C. $\sqrt{e} + 1$

D. $\frac{\sqrt{e} + 1}{2}$

Answer: A

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11. Let $f: R \rightarrow \left[\frac{3}{4}, \infty \right)$ be a surjective quadratic function with line of symmetry $2x - 1 = 0$ and $f(1) = 1$

If $g(x) = \frac{f(x) + f(-x)}{2}$ then $\int \frac{dx}{\sqrt{g(e^x) - 2}}$ is equal to:

A. $\sec^{-1}(e^{-x}) + C$

B. $\sec^{-1}(e^x) + C$

C. $\sin^{-1}(e^{-x}) + C$

D. $\sin^{-1}(e^x) + C$

Answer: B

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12. Let $f: R \rightarrow \left[\frac{3}{4}, \infty \right)$ be a surjective quadratic function with line of symmetry $2x - 1 = 0$ and $f(1) = 1$

$$\int \frac{e^x}{f(e^x)} dx$$

A. $\cot^{-1}\left(\frac{2x^2 - 1}{\sqrt{3}}\right) + C$

B. $\frac{2}{\sqrt{3}} \cot^{-1}\left(\frac{2x^2 - 1}{\sqrt{3}}\right) + C$

C. $\tan^{-1}\left(\frac{2x^2 - 1}{\sqrt{3}}\right) + C$

D. $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2 - 1}{\sqrt{3}}\right) + C$

Answer: D



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13. Let $g(x) = x^C e^{Cx}$ and $f(x) = \int_0^x te^{2t}(1+3t^2)^{1/2} dt$. If

$L = \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}$ is non-zero finite number then :

The value of L . Is :

A. 7

B. $\frac{3}{2}$

C. 2

D. 3

Answer: C



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14. Let $g(x) = x^C e^{Cx}$ and $f(x) = \int_0^x te^{2t}(1+3t^2)^{1/2} dt$. If

$L = \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}$ is non-zero finite number then :

The value of L . Is :

A. $\frac{2}{7}$

B. $\frac{1}{2}$

C. $\frac{\sqrt{3}}{4}$

D. $\frac{\sqrt{3}}{2}$

Answer: D



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Exercise Matching Type Problems

Column-I		Column-II	
(A)	$\int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx =$	(P)	$\frac{\pi}{6}$
(B)	$\int_0^{41\pi} \cos x dx =$	(Q)	$20 + \frac{1}{\sqrt{2}}$
(C)	$\int_{-1/2}^{1/2} \left([x] + \ln\left(\frac{1+x}{1-x}\right) \right) dx =$ where $[\]$ greatest integer function	(R)	$\ln 4 - \ln 3$
(D)	$\int_0^{\pi/2} \frac{2\sqrt{\cos \theta}}{3(\sqrt{\sin \theta} + \sqrt{\cos \theta})} d\theta =$	(S)	$-\frac{1}{2}$

1.



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Column-I		Column-II	
(A)	If quadratic equation $3x^2 + ax + 1 = 0$ and $2x^2 + bx + 1 = 0$ have a common root then value of $5ab - 2a^2 - 3b^2 =$	(P)	6
(B)	Number of solution of $x^4 - 2x^2 \sin^2 \frac{\pi x}{2} + 1 = 0$ is/are	(Q)	1
(C)	Number of points of discontinuity $y = \frac{1}{u^2 + u - 2}$ where $u = \frac{1}{x-1}$ is/are.	(R)	2
(D)	$\int \frac{dx}{\sqrt[3]{x^{5/2}(1+x)^{7/2}}} = A \left(\frac{x+1}{x} \right)^{-1/A} + C$ (Where C is integration constant), then A =	(S)	3

2.



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(A)	$\int_0^{1.5} [x^2] dx$	(P)	$-\pi$
(B)	$\int_0^4 \{\sqrt{x}\} dx$	(Q)	$4(\sqrt{2}-1)$
where $\{x\}$ denotes the fractional part of x			
(C)	$\int_0^{2\pi} [\sin x + \cos x] dx$	(R)	$\frac{7}{3}$
(D)	$\int_0^{\pi} \sin x - \cos x dx$	(S)	$2-\sqrt{2}$

3.



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Exercise Subjective Type Problems

1. $\int \frac{x + (\cos^{-1} 3x)^2}{\sqrt{1-9x^2}} dx = \frac{1}{k_1} \left(\sqrt{1-9x^2} + (\cos^{-1} 3x)^{k_2} \right) + c$, then

$k_1^2 + k_2^2 =$ (where C is an arbitrary constnat.)



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2. If $\int_0^{\infty} \frac{x^3}{(a^2+x^2)} dx = \frac{1}{ka^6}$, then find the value of $\frac{k}{8}$.



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3. Let $f(x) = x \cos x$, $x \in \left[\frac{3\pi}{2}, 2\pi \right]$ and $g(x)$ be its inverse. If $\int_0^{2\pi} g(x) dx = a\pi^2 + \beta\pi + \gamma$, where α, β and $\gamma \in R$, then find the value of $2(\alpha + \beta + \gamma)$.



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4. If

$$\int (x^6 + x^4 + x^2) \sqrt{2x^4 + 3x^2 + 6} dx = \frac{(ax^6 + \beta x^4 + \gamma x^2)^{3/2}}{18} + C$$

where C is constnat, then find the value of $(\beta + \gamma - \alpha)$.



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5. If the value of the definite integral

$$\int_{-1}^1 \cos^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right) \cdot \left(\cot^{-1} \frac{x}{\sqrt{1-(x^2)^{|x|}}} \right) dx = \frac{\pi^2 (\sqrt{a} - \sqrt{c})}{\sqrt{c}}$$

where $a, b, c \in N$ in their lowest form, then find the value of $(a + b + c)$.

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6. The value of

$$\int \frac{\tan x}{\tan^2 x + \tan x + 1} dx = x - \frac{2}{\sqrt{A}} \tan^{-1} \left(\frac{2 \tan x + 1}{\sqrt{A}} \right) + C$$
 Then

the value of A is:

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7. Let $\int_0^1 \frac{4x^3(1 + (x^4)^{2010})}{(1 + x^4)^{2012}} dx = \frac{\lambda}{\mu}$

where λ and μ are relatively prime positive integers. Find unit digit of μ .

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8. Let $\int_1^{\sqrt{5}} (x^{2x^2+1} + \ln(x^{2x^{2x^2+1}})) dx = N$. Find the value of $(N - 6)$.



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9. If $\int \frac{dx}{\cos^3 x - \sin^3} = A \tan^{-1}(f(x)) + b \ln \left| \frac{\sqrt{2} + f(x)}{\sqrt{2} - f(x)} \right| + C$ where $f(x) = \sin x + \cos x$ find the value of $(12A + 9\sqrt{2}V) - 3$.



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10. Find the value of $|a|$ for which the area of triangle included between the coordinate axes and any tangent to the curve $xy^a = \lambda^{a+1}$ is constant (where λ is constant.)



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11. Let $I = \int_0^\pi x^6(\pi - x)^8 dx$, then $\frac{\pi^{15}}{({}^{15}C_9)I} =$



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12. If $I = \int_0^{100} (\sqrt{x}) dx$, then the value of $\frac{9I}{155}$ is:

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13. Let $I_n = \int_0^\pi \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)} dx$ where $n \in N \cup \{0\}$. If

$l_1^2 + l_2^2 + l_3^2 + \dots + l_{20}^2 = m\pi^2$, then find the largest prime factor of m .

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14. If M be the maximum value of $72 \int_0^y \sqrt{x^4 + (y - y^2)^2} dx$ for $y \in [0, 1]$, then find $\frac{M}{6}$.

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15. Find the number points where $f(\theta) = \int_{-1}^1 \frac{\sin \theta dx}{1 - 2x \cos \theta + x^2}$ is discontinuous where $\theta \in [0, 2\pi]$.

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16. $\lim_{n \rightarrow \infty} \left[\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{2n}} + \frac{1}{\sqrt{3n}} + \dots + \frac{1}{n} \right]$

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17. The maximum value of $\int_{-\pi/2}^{2\pi/2} \sin x \cdot f(x) dx$, subject to the condition $|f(x)| \leq 5$ is M, then $M/10$ is equal to :

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18. Given a function g , continuous everywhere such that $g(1) = 5$ and $\int_0^1 g(t) dt = 2$. If $f(x) = \frac{1}{2} \int_0^x (x-t)^2 g(t) dt$, then

find the value of $f'(1) + f''(1)$.

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19. If $f(n) = \frac{1}{\pi} \int_0^{\pi/2} \frac{\sin^2(n\theta)d\theta}{\sin^2 \theta}$, $n \in N$, then evaluate $\frac{f(15) + f(3)}{f(12) - f(10)}$.

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20. Let $f(2 - x) = f(2 + x)$ and $f(4 - x) = f(4 + x)$. Function $f(x)$ satisfies $\int_0^2 f(x)dx = 5$. if $\int_0^{50} f(x)dx = l$. Find $[\sqrt{l} - 2]$. (where $[\cdot]$ denotes greatest integer function.)

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21. Let $l_n = \int_{-1}^1 |x| \left(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^{2n}}{2n} \right) dx$ if $\lim_{x \rightarrow \infty} l_n$ can

be expressed as rational $\frac{p}{q}$ in this lowest form, then find the value of

$$\frac{pq(p+q)}{10}$$

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22. Let $\lim_{x \rightarrow \infty} n^{\frac{1}{2}} \left(1 + \frac{1}{n}\right) \cdot (1^1 \cdot 2^2 \cdot 3^3 \dots n^n)^{\frac{1}{n^2}} = e^{\frac{-p}{q}}$

where p and q are relative prime positive integers. Find the value of $|p+q|$.

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23. If $\int_a^b |\sin x| dx = 8$ and $\int_0^{a+b} |\cos x| dx = 9$ then the value of $\frac{1}{\sqrt{2\pi}} \left| \int_a^b x \sin x dx \right|$ is _____.

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24. If $f(x), g(x), h(x)$ and $\phi(x)$ are polynomial in x , $\left(\int_1^x f(x)h(x) dx \right) \left(\int_1^x g(x)\phi(x) dx \right) - \left(\int_1^x g(x)h(x) dx \right)$ is

divisible by $(x - 1)^\lambda$. Find maximum value of λ .

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25. If $\int_0^2 (3x^2 - 3x + 1) \cos(x^3 - 3x^2 + 4x - 2) dx = a \sin(b)$, where a and b are positive integers. Find the value of $\frac{a + b}{2}$.

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26. let $f(x) = \int_0^x e^{x-y} f'(y) dy - (x^2 - x + 1)e^x$

Find the number of roots of the equation $f(x) = 0$.

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27. For a positive integer n , let $I_n = \int_{-\pi}^{\pi} \left(\frac{\pi}{2} - |x|\right) \cos nx dx$

Find the value of $[I_1 + I_3 + I_4]$ (where $[.]$ denotes greatest integer function).

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