MATHS

BOOKS - SHRI BALAJI MATHS (ENGLISH)

PARABOLA

Exercise 1 Single Choice Problems

1. If BC is a latus rectum of parabola $y^2 = 4ax$ and A is the vertex, then the minimum length

of the projection of BC on a tangent drawn in the portion BAC is

A. 2

B. 4

C. $2\sqrt{3}$

D. $2\sqrt{2}$

Answer: D

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2. A normal is drawn to the parabola $y^2 = 9x$ at the point P(4, 6). A circle is described on SP as diameter, where S is the focus. The length of the intercept made by the circle on the normal at point P is :

A. $\frac{17}{4}$

B. $\frac{15}{4}$

C. 4

D. 5

Answer: B

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3. about to only mathematics

A. 70

B. 71

C. 80

D. 75

Answer: D

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4. Find the length of normal chord which subtends an angle of 90^0 at the vertex of the parabola $y^2 = 4x$.

A. $6\sqrt{3}$

B. $7\sqrt{2}$

C. $8\sqrt{2}$

D. $9\sqrt{2}$

Answer: A

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5. If $b\,and\,c$ are lengths of the segments of any focal chord of the parabola $y^2 = 4ax$, then write the length of its latus rectum.

A. $\frac{bc}{b+c}$

B. $\frac{2bc}{b+c}$

C. $\frac{b+c}{2}$

D. $\sqrt{bc}$

Answer: B

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6. The length of the shortest path that begins at the point $(-1, 1)$, touches the x-axis and then ends at a point on the parabola $(x-y)^2 = 2(x+y-4)$, is :

A. $3\sqrt{2}$

B. 5

C. $4\sqrt{10}$

D. 13

Answer: A

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7. The normal to the parabola $y^2 = 4ax$ at three points P,Q and R meet at A. If S is the focus, then prove that $SP \cdot SR = aSA^2$.

A. 2^3

B. $a^2(SO')$

C. $a(SO')^2$

D. None of these

Answer: C

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8. A and B are two points on the parabola $y^2 = 4ax$ with vertex O. if OA is perpendicular to OB and they have lengths r_1 and r_2 respectively, then the valye of $\frac{r_1^{4/3} r_2^{4/3}}{r_1^{2/3} + r_2^{2/3}}$ is

A. $16a^2$

B. a^2

C. $4a$

D. None of these

Answer: A

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9. Length of the shortest chord of the parabola $y^2 = 4x + 8$, which belongs to the family of lines $(1 + \lambda)y + (\lambda - 1)x + 2(1 - \lambda) = 0$ is

A. 6

B. 5

C. 8

D. 2

Answer: C

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10. If locus of mid point of any normal chord of the parabola : $y^2 = 4x$ is $x - a = \frac{b}{y^2} + \frac{y^2}{c}$, where $a, b, c \in N$, then $(a + b + c)$ equals to :

A. 5

B. 8

C. 10

D. None of these

Answer: B

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11. Let tangents at P and Q to curve $y^2 - 4x - 2y + 5 = 0$ intersect at T. If S(2, 1) is a point such that $(SP)(SQ) = 16$, then the length ST is equal to :

A. 3

B. 4

C. 5

D. None of these

Answer: B

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12. Abscissa of two points P and Q on parabola $y^2 = 8x$ are roots of equation $x^2 - 17x + 11 = 0$. Let Tangents at P and Q

meet at point T, then distance of T from the focus of parabola is :

A. 7

B. 6

C. 5

D. 4

Answer: A

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13. If $Ax + By = 1$ is a normal to the curve $ay = x^2$, then :

A. $4A^2(1 - aB) = aB^3$

B. $4A^2(2 + aB) = aB^3$

C. $4A^2(1 + aB) + aB^3 = 0$

D. $2A^2(2 - aB) = aB^3$

Answer: D

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14. The equation of a curve which passes through the point (3, 1), such the segment of any tangent between the point of tangency and the x-axis is bisected at its point of intersection with y-axis, is :

A. $x = 3y^2$

B. $x^2 = 9y$

C. $x = y^2 + 2$

D. $2x = 3y^2 + 3$

Answer: A

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15. The parabola $y = 4 - x^2$ has vertex P. It intersects x-axis at A and B. If the parabola is translated from its initial position to a new position by moving its vertex along the line $y = x + 4$, so that it intersects x-axis at B and C, then abscissa of C will be :

A. 3

B. 4

C. 5

D. 8

Answer: D

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16. A focal chord for parabola $y^2 = 8(x + 2)$ is inclined at an angle of 60° with positive x-axis and intersects the parabola at P and Q. Let perpendicular bisector of the chord PQ intersects the x-axis at R, then the distance of R from focus is :

A. $\frac{8}{3}$

B. $\frac{16\sqrt{3}}{3}$

C. $\frac{16}{3}$

D. $8\sqrt{3}$

Answer: C

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17. The chord of contact of a point $A(x_A, y_A)$ of $y^2 = 4x$ passes through (3, 1) and point A lies on $x^2 + y^2 = 5^2$. Then :

A. $5x_A^2 + 24x_A + 11 = 0$

B. $13x_A^2 + 8x_A - 21 = 0$

C. $5x_A^2 + 24x_A + 61 = 0$

D. $13x_A^2 + 21x_A - 31 = 0$

Answer: A

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Exercise 2 One Or More Than One Answer Is Are Correct

1. PQ is a double ordinate of the parabola $y^2 = 4ax$. If the normal at P intersect the line passing through Q and parallel to axis of x at G, then locus of G is a parabola with -

A. vertex at (4a, 0)

B. focus at (5a, 0)

C. directrix as the line $x - 3a = 0$

D. length of latus rectum equal to 4a

Answer: A::B::C::D

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Exercise 3 Comprehension Type Problems

1. Consider the following lines :

$L_1 : x - y - 1 = 0$

$L_2 : x + y - 5 = 0$

$L_3 : y - 4 = 0$

Let L_1 is axis to a parabola, L_2 is tangent at the vertex to this parabola and L_3 is another tangent to this parabola at some point P.

Let 'C' be the circle circumscribing the triangle

formed by tangent and normal at point P and axis of parabola. The tangent and normals at normals at the extremities of latus rectum of this parabola forms a quadrilateral ABCD.

Q. The equation of the circle 'C' is :

A. $x^2 + y^2 - 2x - 31 = 0$

B. $x^2 + y^2 - 2y - 31 = 0$

C. $x^2 + y^2 - 2x - 2y - 31 = 0$

D. $x^2 + y^2 + 2x + 2y = 31$

Answer: A

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2. Consider the following lines :

$L_1 : x - y - 1 = 0$

$L_2 : x + y - 5 = 0$

$L_3 : y - 4 = 0$

Let L_1 is axis to a parabola, L_2 is tangent at the vertex to this parabola and L_3 is another tangent to this parabola at some point P.

Let 'C' be the circle circumscribing the triangle formed by tangent and normal at point P and axis of parabola. The tangent and normals at

normals at the extremities of latus rectum of this parabola forms a quadrilateral ABCD.

Q. The given parabola is equal to which of the following parabola ?

A. $y^2 = 16\sqrt{2}x$

B. $x^2 = -4\sqrt{2}y$

C. $y^2 = -\sqrt{2}x$

D. $y^2 = 8\sqrt{2}x$

Answer: D

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3. Consider the following lines :

$L_1 : x - y - 1 = 0$

$L_2 : x + y - 5 = 0$

$L_3 : y - 4 = 0$

Let L_1 is axis to a parabola, L_2 is tangent at the vertex to this parabola and L_3 is another tangent to this parabola at some point P.

Let 'C' be the circle circumscribing the triangle formed by tangent and normal at point P and axis of parabola. The tangent and normals at normals at the extremities of latus rectum of

this parabola forms a quadrilateral ABCD.

Q. The equation of the circle 'C' is :

A. 16

B. 8

C. 64

D. 32

Answer: C

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Exercise 4 Matching Type Problems

1.

	Column-I		Column-II
(A)	The equation of tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ which cuts off equal intercepts on axes is $x-y=a$ where $\lvert a\rvert$ equal to	(P)	$\sqrt{2}$
(B)	The normal $y=mx-2am-am^2$ to the parabola $y^2=4ax$ subtends a right angle at the vertex if $\lvert m\rvert$ equal to	(Q)	$\sqrt{3}$
(C)	The equation of the common tangent to parabola $y^2=4x$ and $x^2=4y$ is $x+y+\frac{k}{\sqrt{3}}=0$, then k is equal to	(R)	$\sqrt{8}$
(D)	An equation of common tangent to parabola $y^2=8x$ and the hyperbola $3x^2-y^2=3$ is $4x-2y+\frac{k}{\sqrt{2}}=0$, then k is equal to	(S)	$\sqrt{41}$
		(T)	2

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2.

	Column-I		Column-II
(A)	Area of ΔPQR is equal to	(P)	2
(B)	Radius of circumcircle of ΔPQR is equal to	(Q)	$\frac{5}{2}$
(C)	Distance of the vertex from the centroid of ΔPQR is equal to	(R)	$\frac{3}{2}$
(D)	Distance of the centroid from the circumcentre of ΔPQR is equal to	(S)	$\frac{2}{3}$
		(T)	$\frac{11}{6}$

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Exercise 5 Subjective Type Problems

1. Points A and B lie on the parabola $y = 2x^2 + 4x - 2$, such that origin is the mid-point of the linesegment AB. If l be the length of the line segment AB, then find the unit digit of l^2.

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2. For the parabola $y = -x^2$, let $a < 0$ and $b > 0$, $P(a, -a^2)$ and $Q(b, -b^2)$. Let M be the mid-point of PQ and R be the point of intersection of the vertical line through M, with the parabola. If the ratio of the area of the region bounded by the parabola and the line segment PQ to the area of the triangle PQR be $\frac{\lambda}{\mu}$, where λ and μ are relatively prime positive integers, then find the value of $(\lambda + \mu)$:

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3. The chord AC of the parabola $y^2 = 4ax$ subtends an angle of 90° at points B and D on the parabola. If points A, B, C and D are represented by $\left(at_i^2, 2at_i\right), i = 1, 2, 3, 4$ respectively, then find the value of $\left|\frac{t_2 + t_4}{t_1 + t_3}\right|$.

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