MATHS

BOOKS - SHRI BALAJI MATHS (ENGLISH)

SOLUTION OF TRIANGLES

Exercise 1 Single Choice Problems

1. In a ΔABC if $9(a^2+b^2)=17c^2$ then the value of the $\dfrac{\cot A+\cot B}{\cot C}$ is

A. $\dfrac{13}{4}$

B. $\dfrac{7}{4}$

C. $\dfrac{5}{4}$

D. $\dfrac{9}{4}$

Answer: D

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2. Let H be the orthocentre of triangle ABC. Then angle subtended by side BC at the centre of incircle of ΔCHB is

A. $\frac{A}{2}+\frac{\pi}{2}$

B. $\frac{B+C}{2}+\frac{\pi}{2}$

C. $\frac{B-C}{2}+\frac{\pi}{2}$

D. $\frac{B+C}{2}+\frac{\pi}{4}$

Answer: B

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3. Circum radius of a ΔABC is 3 units, let O be the circum and H be the orthocentre then the value of $\frac{1}{64}\left(AH^2+BC^2\right)\left(BH^2+AC^2\right)\left(CH^2+AB^2\right)$ equals :

A. 3^4

B. 9^3

C. 27^6

D. 81^4

Answer: B

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4. The angles A, B and C of a triangle ABC are in arithmetic progression. If $2b^2 = 3c^2$ then the angle A is :

A. 15°

B. 60°

C. 75°

D. 90°

Answer: C

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5. In a triangle ABC if $\tan.\frac{A}{2}\tan.\frac{B}{2}=\frac{1}{3}$ and ab = 4, then the value of c can be

A. 1

B. 2

C. 4

D. 6

Answer: B

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6. In a triangle ABC the expression $a\cos B\cos C+b\cos C\cos A+c\cos A\cos B$ equals to :

A. $\frac{rs}{R}$

B. $\frac{r}{sR}$

C. $\frac{R}{rs}$

D. $\frac{Rs}{r}$

Answer: A

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7. The set of all real numbers a such that $a^2 + 2a, 2a + 3, and a^2 + 3a + 8$ are the sides of a triangle is_____

A. $(0, \infty)$

B. $(5, 8)$

C. $\left(-\frac{11}{3}, \infty\right)$

D. $(5, \infty)$

Answer: D

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8. In triangle ABC, $\angle B = \frac{\pi}{3}, and \angle C = \frac{\pi}{4}$. Let D divided BC internally in the ratio $1:3$. Then $\frac{\sin \angle BAD}{\sin \angle CAD}$ equals (a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{\frac{2}{3}}$

A. $\frac{1}{\sqrt{6}}$

B. $\frac{1}{3}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{\sqrt{2}}{3}$

Answer: A

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9. Let AD, BE, CF be the lengths of internal bisectors of angles A, B, C respectively of triangle ABC. Then the harmonic mean of $AD \sec \frac{A}{2}, BE \sec \frac{B}{2}, CF \sec \frac{C}{2}$ is equal to :

A. Harmonic mean of sides of ΔABC

B. Geometric mean of sides of ΔABC

C. Arithmetic mean of sides of ΔABC

D. Sum of reciprocals of the sides of ΔABC

Answer: A

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10. In $\Delta ABC, \text{ if } 2b = a + c \text{ and } A - C = 90^\circ, \text{ then sin B equal}$

All symbols used have usual meaning in $\Delta ABC.]$

A. $\frac{\sqrt{7}}{5}$

B. $\frac{\sqrt{5}}{8}$

C. $\frac{\sqrt{7}}{4}$

D. $\frac{\sqrt{5}}{3}$

Answer: C

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11. In a triangle ABC, if $2a\cos\left(\frac{B-C}{2}\right)=b+c$, then secA is equal to :

A. $\frac{2}{\sqrt{3}}$

B. $\sqrt{2}$

C. 2

D. 3

Answer: C

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12. In a triangle ABC if $BC=1$ and $AC=2$, then what is the maximum possible value of angle A?

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: A

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13. $\Delta I_1 I_2 I_3$ is an excentral triangle of an equilateral triangle ΔABC such that $I_1 I_2 = 4$ unit, if ΔDEF is pedal triangle of ΔABC, then $\frac{Ar(\Delta I_1 I_2 I_3)}{Ar(\Delta DEF)} =$

A. 16

B. 4

C. 2

D. 1

Answer: A

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14. Let ABC be a triangle with $\angle BAC = 2\pi/3$ and $AB = x$ such that (AB) (AC) = 1. If x varies, then find the longest possible length of the angle bisector AD

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{\sqrt{2}}{3}$

Answer: B

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15. In an equilateral ΔABC, (where symbols used have usual meanings), then r, R and r_1 form :

A. an A.P.

B. a G.P.

C. an H.P.

D. none of these

Answer: A

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16. If in a triangle ABC, $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, then

A. A.P.

B. G.P.

C. H.P.

D. none of these

Answer: A

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17. In ΔABC, $\tan A = 2$, $\tan B = \frac{3}{2}$ and $c = \sqrt{65}$, then circumradius of the triangle is : (a) 65 (b) $\frac{65}{7}$ (c) $\frac{65}{14}$ (d) none of these

A. 65

B. $\frac{65}{7}$

C. $\frac{65}{14}$

D. none of these

Answer: C

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18. In a triangle ABC, if the sides a,b,c, are roots of $x^3 - 11x^2 + 38x - 40 = 0$, then find the value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$

A. $\frac{61}{144}$

B. $\frac{61}{72}$

C. $\frac{169}{144}$

D. $\frac{59}{144}$

Answer: A

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19. In $\Delta ABC, \quad \text{if} \quad \angle C = 90^{\circ}, \quad \text{then} \quad \frac{a+c}{b} + \frac{b+c}{a}$ is equal to :

A. $\frac{c}{r}$

B. $\frac{1}{2Rr}$

C. 2

D. $\frac{R}{r}$

Answer: A

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20. In a $\Delta ABC, \quad \text{if} \quad a^2 \sin B = b^2 + c^2$, then :

A. $\angle A$ is obtuse

B. $\angle A$ is acute

C. $\angle B$ is abtuse

D. $\angle A$ is right angle

Answer: A

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21. If R and R' are the circumradii of triangles ABC and OBC, where O is the orthocenter of triangle ABC, then :

A. $R' = \frac{R}{2}$

B. $R' = 2R$

C. $R' = R$

D. $R' = 3R$

Answer: C

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22. The acute angle of a rhombus whose side is the geometric mean of its diagonals is

A. 15°

B. 20°

C. 30°

D. 60°

Answer: C

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23. In a ΔABC right angled at A, a line is drawn through A to meet BC at D dividing BC in $2:1$. If $\tan(\angle ADC) = 3$ then $\angle BAD$ is : (a) 30° (b) 45°

(c) 60° (d) 75°

A. 30°

B. 45°

C. 60°

D. 75°

Answer: B

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24. A circle is cirumscribed in an equilateral triangle of side $'l'$. The area of any square inscribed in the circle is :

A. $\frac{4}{3}l^2$

B. $\frac{2}{3}l^2$

C. $\frac{1}{3}l^2$

D. l^2

Answer: B

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25. if the sides of a triangle are in the ratio $2:\sqrt{6}:\sqrt{3}+1$, then the largest angle of the trangle will be (1) 60 (3) 72 (2) 75 (4) 90

A. 60°

B. 72°

C. 75°

D. 90°

Answer: C

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26. In a triangle ABC if a, b, c are in A.P. and $C-A=120^{\circ}$, then $\frac{s}{r}=$ (where notations have their usual meaning)

A. $\sqrt{15}$

B. $2\sqrt{15}$

C. $3\sqrt{15}$

D. $6\sqrt{15}$

Answer: C

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27. If in $\Delta ABC, a = 5, b = 4 \text{ and } \cos(A - B) = \frac{31}{32}$, then side c is

A. $\sqrt{6}$

B. $6\sqrt{6}$

C. 6

D. $(216)^{1/4}$

Answer: C

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28. If semiperimeter of a triangle is 15, then the value of $(b+c)\cos(B+C)+(c+a)\cos(C+A)+(a+b)\cos(A+B)$ is equal to :

(where symbols used have usual meanings)

A. -60

B. -15

C. -30

D. can not be determined

Answer: C

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29. Let triangle ABC be an isosceles with AB=AC. Suppose that the angle bisector of its angle B meets the side AC at a point D and that $BC=BD+AD$. Measure of the angle A in degrees, is :

A. 80

B. 100

C. 110

D. 130

Answer: B

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30. In triangle ABC if $A:B:C=1:2:4$, then $\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)=\lambda a^2b^2c^2$, where $\lambda=$

(where notations have their usual meaning)

A. 1

B. 2

C. 4

D. 9

Answer: A

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31. In a triangle ABC with altitude AD, $\angle BAC = 45^\circ$, $DB = 3$ and $CD = 2$. The area of the triangle ABC is :

A. 6

B. 15

C. $15/4$

D. 12

Answer: B

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32. A triangle has base 10 cm long and the base angles of 50° and 70°. If the perimeter of the triangle is $x + y\cos z^\circ$ where $z \in (0, 90)$ then the value of $x + y + z$ equals :

A. 40

B.

C.

D.

Answer:

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33. Let H be the orthocentre of triangle ABC. Then angle subtended by side BC at the centre of incircle of ΔCHB is

A. $\frac{A}{2} + \frac{\pi}{2}$

B. $\frac{B+C}{2} + \frac{\pi}{2}$

C. $\frac{B-C}{2} + \frac{\pi}{2}$

D. $\frac{B+C}{2} + \frac{\pi}{4}$

Answer: B

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34. Triangel ABC is right angles at A. The points P and Q are on hypotenuse BC such that BP = PQ = QC. If AP = 3 and AQ = 4, then length BC is equal to

A. $\sqrt{27}$

B. $\sqrt{36}$

C. $\sqrt{45}$

D. $\sqrt{54}$

Answer: C

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35. In a ΔABC, if b =(sqrt3-1) a and angle C=30^(@),` then the value of (A-B) is equal to (All symbols used have usual meaning in the triangel.)

A. 15°

B. 45°

C. 75°

D. 105°

Answer: D

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36. Through the centroid of an equilateral triangle, a line parallel to the base is drawn. On this line, an arbitrary point P is taken inside the triangle. Let h denote the perpendicular distance of P from the base of the triangle. Let h_1 and h_2 be the perpendicular distance of P from the other two sides of the triangle . Then :

A. $h = \frac{h_1 + h_2}{2}$

B. $h = \sqrt{h_1 h_2}$

C. $h = \frac{2h_1 h_2}{h_1 + h_2}$

D. $h = \frac{(h_1 + h_2)\sqrt{3}}{4}$

Answer: A

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37. The angles A, B and C of a triangle ABC are in arithmetic progression. AB=6 and BC=7. Then AC is :

A. $\sqrt{41}$

B. $\sqrt{39}$

C. $\sqrt{42}$

D. $\sqrt{43}$

Answer: D

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38. In ΔABC, if $A - B = 120^\circ$ and $R = 8r$, then the value of $\frac{1+\cos C}{1-\cos C}$ equals :

(All symbols used hav their usual meaning in a triangle)

A. 12

B. 15

C. 21

D. 31

Answer: B

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39. The lengths of the sides CB and CA of a triangle ABC are given by a and b and the angle C is $\frac{2\pi}{3}$. The line CD bisects the angle C and meets AB at D. Then the length of CD is : (a) $\frac{1}{a+b}$ (b) $\frac{a^2+b^2}{a+b}$ (c) $\frac{ab}{2(a+b)}$ (d) $\frac{ab}{a+b}$

A. $\frac{1}{a+b}$

B. $\frac{a^2+b^2}{a+b}$

C. $\frac{ab}{2(a+b)}$

D. $\frac{ab}{a+b}$

Answer: D

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40. In ΔABC, angle A is 120°, $BC + CA = 20, \text{ and } AB + BC = 21$

Find the length of the side BC

A. 13

B. 15

C. 17

D. 19

Answer: A

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41. A triangle has sides 6,7, and 8. The line through its incenter parallel to the shortest side is drawn to meet the other two sides at P and Q. Then find the length of the segment PQ.

A. $\frac{12}{5}$

B. $\frac{15}{4}$

C. $\frac{30}{7}$

D. $\frac{33}{9}$

Answer: C

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42. The perimeter of a ΔABC is $48cm$ and one side is $20cm$. Then remaining sides of ΔABC must be greater than : (a) $8cm$ (b) $9cm$ (c) $12cm$ (d) $4cm$

A. 8 cm

B. 9 cm

C. 12 cm

D. 4 cm

Answer: D

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43.

In an equilateral ΔABC, (where symbols used have usual meanings), then r, R and r_1 form :

i) an A.P.

ii)a G.P.

iii) an H.P.

iv) neither an A.P., G.P. nor H.P.

A. an A.P.

B. a G.P.

C. an H.P.

D. neither an A.P., G.P. nor H.P.

Answer: A

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44. Prove that $\dfrac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}=\sin^2$

A. $\cos^2 A$

B. $\sin^2 A$

C. $\cos A \cos B \cos C$

D. $\sin A \sin B \sin C$

Answer: B

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45. Circumradius of an isosceles ΔABC with $\angle A = \angle B$ is 4 times its in radius, then cosA is root of the equation :

A. $x^2 - x - 8 = 0$

B. $8x^2 - 8x + 1 = 0$

C. $x^2 - x - 4 = 0$

D. $4x^2 - 4x + 1 = 0$

Answer: B

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46. A is the orthocentre of ΔABC and D is reflection point of A w.r.t. perpendicualr bisector of BC, then orthocenter of ΔDBC is :

A. D

B. C

C. B

D. A

Answer: A

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47. Let a, b, c be positive and not all equal. Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.

A. ≥ 0

B. > 0

C. ≤ -1

D. < 0

Answer: D

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48. In triangle ABC if $A:B:C = 1:2:4$, then $(a^2 - b^2)(b^2 - c^2)(c^2 - a^2) = \lambda a^2 b^2 c^2$, where $\lambda =$

(where notations have their usual meaning)

A. 1

B. 2

C. 3

D. $\frac{1}{3}$

Answer: A

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49. In any triangle, the minimum value of $r_1 r_2 r_3 / r^3$ is equal to

A. 1

B. 3

C. 8

D. 27

Answer: D

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50. In a triangle ABC, side BC = 3, AC = 4 and AB = 5. The value of $\sin A + \sin 2B + \sin 3C$ is equal to :

A. $\frac{24}{25}$

B. $\frac{14}{25}$

C. $\frac{64}{25}$

D. None

Answer: B

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51. In any triangle ABC, the value of $\frac{r_1 + r_2}{1 + \cos C}$ is equal to (where notation have their usual meaning) :

A. 2R

B. 2r

C. R

D. $\frac{2R^2}{r}$

Answer: A

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52. In a triangle ABC, medians AD and BE are deawn. IF $AD = 4, \angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the trianlge

ABC is-

A. $\frac{8}{3\sqrt{3}}$

B. $\frac{16}{3\sqrt{3}}$

C. $\frac{32}{3\sqrt{3}}$

D. $\frac{64}{3\sqrt{3}}$

Answer: C

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53. The sides of a triangle are $\sin\alpha, \cos\alpha, \sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$ then the greatest angle of the triangle is :

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: C

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54. Let ABC be a right with $\angle BAC = \frac{\pi}{2}$, then $\left(\frac{r^2}{2R^2} + \frac{r}{R}\right)$ is equal to :

(where symbols used have usual meaning in a striangle)

A. sinB sinC

B. tanB tanC

C. secB secC

D. cotB cotC

Answer: A

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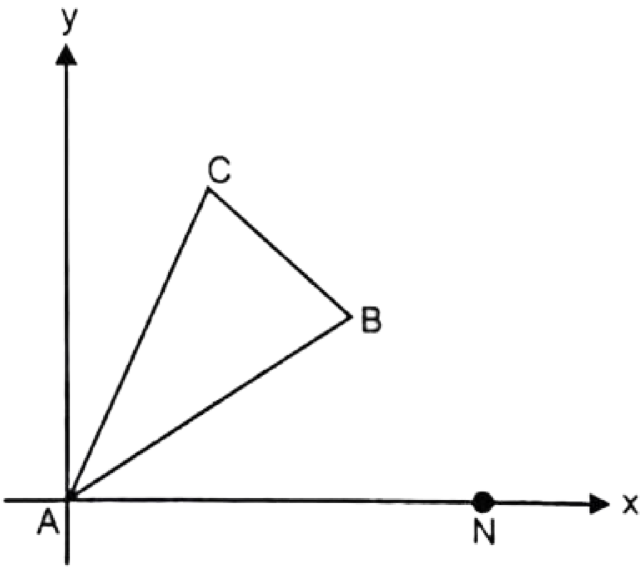
55. Find the radius of the circle escribed to the triangle ABC (Shown in the figure below) on the side BC if $\angle NAB = 30^\circ, \angle BAC = 30^\circ, AB = AC = 5.$



A. $\dfrac{\left(10\sqrt{2}+5\sqrt{3}-5\right)\left(2-\sqrt{3}\right)}{2\sqrt{2}}$

B. $\dfrac{\left(10\sqrt{2}+5\sqrt{3}+5\right)}{2\sqrt{2}}(2-\sqrt{3})$

C. $\dfrac{\left(10\sqrt{2}+5\sqrt{3}-5\right)}{2\sqrt{2}}(2+\sqrt{3})$

D. $\dfrac{\left(10\sqrt{2}+5\sqrt{2}+1\right)}{2\sqrt{3}}(\sqrt{3}-1)$

Answer: A

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56. In a ΔABC, with usual notations, if $b > c$ then distance between foot of median and foot of altitude both drawn from vertex A on BC is :

A. $\dfrac{a^2 - b^2}{2c}$

B. $\dfrac{b^2 - c^2}{2a}$

C. $\dfrac{b^2 + c^2 - a^2}{2a}$

D. $\dfrac{b^2 + c^2 - a^2}{2c}$

Answer: B

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57. In a triangle ABC the expression $a \cos B \cos C + b \cos C \cos A + c \cos A \cos B$ equals to :

A. $\frac{rs}{R}$

B. $\frac{r}{sR}$

C. $\frac{R}{rs}$

D. $\frac{Rs}{r}$

Answer: A

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58. A circle of area 20 sq. units is centered at the point O. Suppose ΔABC is inscribed in that circle and has area 8 sq. units. The central angles α, β and γ are as shown in the figure. The value of $(\sin\alpha + \sin\beta + \sin\gamma)$ is equal to

A. $\frac{4\pi}{5}$

B. $\frac{3\pi}{4}$

C. $\frac{2\pi}{5}$

D. $\frac{\pi}{4}$

Answer: A

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Exercise 2 One Or More Than One Answer Is Are Correct

1. If r_1, r_2, r_3 are radii of the escribed circles of a triangle ABC and r it the radius of its incircle, then the root(s) of the equation $x^2 - r(r_1r_2 + r_2r_3 + r_3r_1)x + (r_1r_2r_3 - 1) = 0$ is/are :

A. r_1

B. $r_2 + r_3$

C. 1

D. $r_1r_2r_3 - 1$

Answer: C::D

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2. In $\Delta ABC, \angle A = 60^\circ, \angle B = 90^\circ, \angle C = 30^\circ$. Let H be its orthocentre, then :

(where symbols used have usual meanings)

A. $AH = c$

B. $CH = a$

C. $AH = a$

D. $BH = 0$

Answer: A::B::D

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3. In an equilateral triangle, if inradius is a rational number then

A. circumradius is always rational

B. exradii are always rational

C. area is always ir-rational

D. perimeter is always rational

Answer: A::B::C

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4. Let A, B, C be angles of a triangle ABC and let $D=\frac{5\pi+A}{32}, E=\frac{5\pi+B}{32}, F=\frac{5\pi+C}{32}$, then :

(where $D, E, F\neq\frac{n\pi}{2}, n\in I, I$ denote set of integers)

A. $\cot D\cot E+\cot E\cot F+\cot D\cot F=1$

B. $\cot D+\cot E+\cot F=\cot D\cot E\cot F$

C. $\tan D\tan E+\tan E\tan F+\tan F\tan D=1$

D. $\tan D+\tan E+\tan F=\tan D\tan E\tan F$

Answer: B::C

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5. In a triangle ABC, if a=4, b=8 and $\angle C = 60^\circ$, then :

(where symbols used have usual meanings)

A. $c = 6$

B. $c = 4\sqrt{3}$

C. $\angle A = 30^\circ$

D. $\angle B = 90^\circ$

Answer: B::C::D

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6. If in a triangle $\frac{r}{r_1} = \frac{r_2}{r_3}$, then

A. $a^2 + b^2 + c^2 = 8R^2$

B. $\sin^2 A + \sin^2 B + \sin^2 C = 2$

C. $a^2 + b^2 = c^2$

D. $\Delta = s(s+c)$

Answer: A::B::C

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7. ABC is a triangle whose circumcentre, incentre and orthocentre are O, I and H respectively which lie inside the triangle, then :

A. $\angle BOC = A$

B. $\angle BIC = \frac{\pi}{2} + \frac{A}{2}$

C. $\angle BHC = \pi - A$

D. $\angle BHC = \pi - \frac{A}{2}$

Answer: B::C

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8. In a triangle ABC, tanA and tanB satisfy the inequality $\sqrt{3}x^2 - 4x + \sqrt{3} < 0$, then which of the following must be correct ? (where symbols used have usual meanings)

A. $a^2 + b^2 - ab < c^2$

B. $a^2 + b^2 > c^2$

C. $a^2 + b^2 + ab > c^2$

D. $a^2 + b^2 < c^2$

Answer: A::C

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9. If in $\Delta ABC, \angle C = \frac{\pi}{8}, a = \sqrt{2} \text{ and } b = \sqrt{2 + \sqrt{2}}$ then find the measure of angle A (in degree).

A. 45°

B. 135°

C. 30°

D. 150°

Answer: A

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10. In triangle ABC, $a=3, b=4, c=2$. Point D and E trisect the side BC. If $\angle DAE=\theta$, then $\cot^2\theta$ is divisible by :

A. 2

B. 3

C. 5

D. 7

Answer: B::C

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11. In a triangle ABC, 3sinA + 4cosB = 6 and 4sinB + 3cosA = 1. Find the measure of angle C.

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: B

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12. If the line joining the incentre to the centroid of a triangle ABC is parallel to the side BC. Which of the following are correct ?

A. $2b = a + c$

B. $2a = b + c$

C. $\cot\frac{A}{2}\cot\frac{C}{2} = 3$

D. $\cot\frac{B}{2}\cot\frac{C}{2}=3$

Answer: B::D

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13. In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P, then the length of the third side can be (a) $5-\sqrt{6}$ (b) $3\sqrt{3}$ (c) 5 (d) $5+\sqrt{6}$

A. $5-\sqrt{6}$

B. $5+\sqrt{6}$

C. $6-\sqrt{5}$

D. $6+\sqrt{5}$

Answer: A::B

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14. If area of $\Delta ABC(\Delta)$ and angle C are given and if c opposite to given angle is minimum, then

A. $a = \sqrt{\dfrac{2\Delta}{\sin C}}$

B. $b = \left(\dfrac{2\Delta}{\sin C}\right)$

C. $a = \dfrac{4\Delta}{\sin C}$

D. $b = \dfrac{4\Delta}{\sin^2 C}$

Answer: A::B

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15. In a triangle ABC , if $\tan A = 2\sin 2C$ and $3\cos A = 2\sin B \sin C$, then C=

A. $\dfrac{\pi}{8}$

B. $\dfrac{\pi}{6}$

C. $\dfrac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: C::D

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Exercise 3 Comprehension Type Problems

1. Internal bisectors of ΔABC meet the circumcircle at point D, E, and F

Area of ΔDEF is

A. $2R^2 \cos^2\left(\frac{A}{2}\right)\cos^2\left(\frac{B}{2}\right)\cos^2\left(\frac{C}{2}\right)$

B. $2R^2 \sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$

C. $2R^2 \sin^2\left(\frac{A}{2}\right)\sin^2\left(\frac{B}{2}\right)\sin^2\left(\frac{C}{2}\right)$

D. $2R^2 \cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)$

Answer: D

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2. Internal angle bisecotors of ΔABC meets its circum circle at D, E and F where symbols have usual meaning.

Q. The ratio of area of triangle ABC and triangle DEF is :

A. ≥ 1

B. ≤ 1

C. $\geq 1/2$

D. $\leq 1/2$

Answer: B

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3. Let triangle ABC is right triangle right angled at C such that $A < B$ and $r = 8, R = 41$.

Q. Area of ΔABC is :

A. 720

B. 1440

C. 360

D. 480

Answer: A

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4. Let triangle ABC is right triangle right angled at C such that $A < B$ and $r = 8, R = 41$.

Q. $\tan\frac{A}{2} =$

A. $\frac{1}{18}$

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. $\frac{1}{9}$

Answer: D

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5. Let the incircle of ΔABC touches the sides BC, CA, AB at A_1, B_1, C_1 respectively. The incircle of $\Delta A_1B_1C_1$ touches its sides of B_1C_1, C_1A_1 and A_1B_1 at A_2, B_2, C_2 respectively and so on.

Q. $\lim_{n \to \infty} \angle A_n =$

A. 0

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: D

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6. Let the incircle of ΔABC touches the sides BC, CA, AB at A_1, B_1, C_1 respectively. The incircle of $\Delta A_1B_1C_1$ touches its sides of

B_1C_1, C_1A_1 and A_1B_1 at A_2, B_2, C_2 respectively and so on.

Q. In $\Delta A_4B_4C_4$, the value of $\angle A_4$ is:

A. $\frac{3\pi + A}{6}$

B. $\frac{3\pi - A}{8}$

C. $\frac{5\pi - A}{16}$

D. $\frac{5\pi + A}{16}$

Answer: D

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7. Let ABC be a given triangle. Points D and E are on sides AB and AC respectively and point F is on line segment DE. Let $\frac{AD}{AB} = x, \frac{AE}{AC} = y, \frac{DF}{DE} = z$. Let area of $\Delta BDF = \Delta_1$, Area of $\Delta CEF = \Delta_2$ and area of $\Delta ABC = \Delta$.

Q. $\frac{\Delta_2}{\Delta}$ is equal to :

A. $(1 - x)y(1 - z)$

B. $(1-x)(1-y)z$

C. $x(1-y)(1-z)$

D. $(1-x)yz$

Answer: C

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8.

a, b, c are the length of sides BC, CA, AB respectively of ΔABC satisfying $\log\left(1+\frac{c}{a}\right)+\log a-\log b=\log 2$. a, b, c are in :

i) A.P.

ii) G.P.

iii) H.P.

iv) none

A. A.P.

B. G.P.

C. H.P.

D. None

Answer: A

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9. a, b, c are the length of sides BC, CA, AB respectively of ΔABC satisfying $\log\left(1+\frac{c}{a}\right)+\log a-\log b=\log 2.$

Also the quadratic equation $a\left(1-x^2\right)+2bx+c\left(1+x^2\right)=0$ has two equal roots.

. Measure of angle C is :

A. 30°

B. 45°

C. 60°

D. 90°

Answer: D

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10. a, b, c ar the length of sides BC, CA, AB respectively of ΔABC satisfying $\log\left(a+\frac{c}{a}\right)+\log a-\log b=\log 2.$

Also the quadratic equation $a\left(1-x^2\right)+2bx+c\left(1+x^2\right)=0$ has two equal roots.

Q. The value of $(\sin A+\sin B+\sin C)$ is equal to :

A. $\frac{5}{2}$

B. $\frac{12}{5}$

C. $\frac{8}{3}$

D. 2

Answer: B

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11. Let ABC be a triangle inscribed in a circle and let $l_a = \frac{m_a}{M_a}, l_b = \frac{m_b}{M_b}, l_c = \frac{m_c}{M_c}$ where m_a, m_b, m_c are the lengths of the angle bisectors of angles A, B and C respectively , internal to the triangle and M_a, M_b and M_c are the lengths of these internal angle bisectors extended until they meet the circumcircle.

Q. l_a equals :

A. $\frac{\sin A}{\sin\left(B + \frac{A}{2}\right)}$

B. $\frac{\sin B \sin C}{\sin^2\left(\frac{B+C}{2}\right)}$

C. $\frac{\sin B \sin C}{\sin^2\left(B + \frac{A}{2}\right)}$

D. $\frac{\sin B + \sin C}{\sin^2\left(B + \frac{A}{2}\right)}$

Answer: C

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12. Let ABC be a triangle inscribed in a circle and let $l_a = \frac{m_a}{M_a}, l_b = \frac{m_b}{M_b}, l_c = \frac{m_c}{M_c}$ where m_a, m_b, m_c are the lengths of the angle bisectors of angles A, B and C respectively , internal to the triangle and M_a, M_b and M_c are the lengths of these internal angle bisectors extended until they meet the circumcircle.

Q. The maximum value of the product $(l_a l_b l_c) \times \cos^2\left(\frac{B-C}{2}\right) \times \frac{\cos^2(C-A)}{2}\Bigg) \times \cos^2\left(\frac{A-B}{2}\right)$ is equal to :

A. $\frac{1}{8}$

B. $\frac{1}{64}$

C. $\frac{27}{64}$

D. $\frac{27}{32}$

Answer: C

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13. Let ABC be a triangle inscribed in a circle and let $l_a = \frac{m_a}{M_a}, l_b = \frac{m_b}{M_b}, l_c = \frac{m_c}{M_c}$ where m_a, m_b, m_c are the lengths of the angle bisectors of angles A, B and C respectively , internal to the triangle and M_a, M_b and M_c are the lengths of these internal angle bisectors extended until they meet the circumcircle.

Q. The minimum value of the expression $\frac{l_a}{\sin^2 A} + \frac{l_b}{\sin^2 B} + \frac{l_c}{\sin^2 C}$ is :

A. 2

B. 3

C. 4

D. none of these

Answer: B

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Exercise 4 Matching Type Problems

1.

	Column-I		Column-II
(A)	Find the sum of the series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{12}+\ldots\ldots\infty$, where the terms are the reciprocals of the positive integers whose only prime factors are two's and three's	(P)	7
(B)	The length of the sides of ΔABC are a, b and c and A is the angle opposite to side a. If $b^2+c^2=a^2+54$ and $bc=\frac{a^3}{\cos A}$ then the value of $\left(\frac{b^2+c^2}{9}\right)$, is	(Q)	10
(C)	The equations of perpendicular bisectors of two sides AB and AC of a triangle ABC are $x+y+1=0$ and $x-y+1=0$ respectively. If circumradius of ΔABC is 2 units and the locus of vertex A is $x^2+y^2+gx+c=0$, then (g^2+c^2), is equal to	(R)	13
(D)	Number of solutions of the equation $\cos\theta\sin\theta+6(\cos\theta-\sin\theta)+6=0$ in $[0, 30]$, is equal to	(S)	3

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2. In ΔABC, If $r_1=21, r_2=24, r_3=28$, then

	Column-I		Column-II
(A)	$a=$	(P)	8
(B)	$b=$	(Q)	12
(C)	$s=$	(R)	26
(D)	$r=$	(S)	28
		(T)	42

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Exercise 5 Subjective Type Problems

1. If the median AD of triangle ABC makes an angle $\frac{\pi}{4}$ with the side BC, then find the value of $|\cot B - \cot C|\cdot$

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2. In parallelogram ABCD, the bisector of angle A meets DC at P and AB= 2AD. Prove that:

BP bisects angle B

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3. In a ΔABC, inscribed circle with centre I touches side AB, AC and BC at D, E, F respectively . Let area of quadrilateral ADIE is 5 units and area of quadrilteral BFID is 10 units. Find the value of $\frac{\cos\left(\frac{C}{2}\right)}{\sin\left(\frac{A-B}{2}\right)}$.

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4. If Δ be area of incircle of a triangle ABC and $\Delta_1, \Delta_2, \Delta_3$ be the area of excircles then find the least value of $\frac{\Delta_1\Delta_2\Delta_3}{729\Delta^3}$

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5. In ΔABC, P is an interior point such that $\angle PAB = 10^\circ, \angle PBA = 20^\circ, \angle PCA = 30^\circ, \angle PAC = 40^\circ$ then ΔABC is

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6. In an acute angled triangle ABC, $\angle A = 20^\circ$, let DEF be the feet of altitudes through A, B, C respectively and H is the orthocentre of ΔABC. Find $\frac{AH}{AD} + \frac{BH}{BE} + \frac{CH}{CF}$.

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7. If the quadratic equation $ax^2 + bx + c = 0$ has equal roots where a, b, c denotes the lengths of the sides opposite to vertex A, B and C of the ΔABC respectively then find the number of integers in the range of $\frac{\sin A}{\sin C} + \frac{\sin C}{\sin A}$

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8. If in the triangle ABC, $\tan\frac{A}{2}, \tan\frac{B}{2}$ and $\tan\frac{C}{2}$ are in harmonic progression then the least value of $\cot^2\frac{B}{2}$ is equal to :

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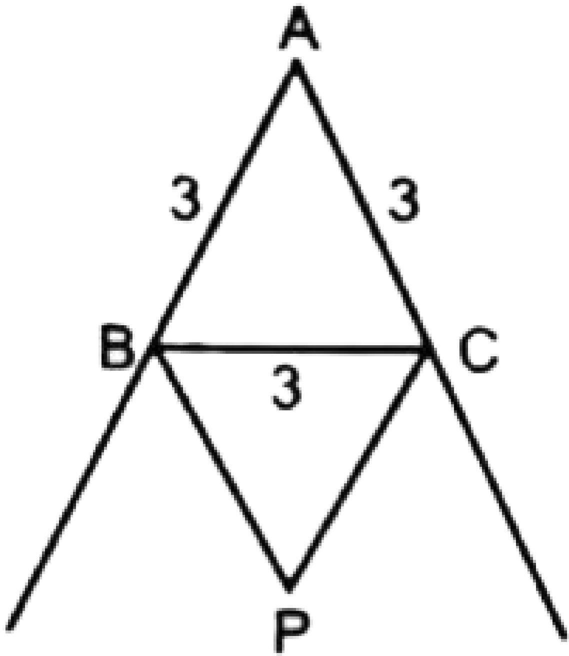
9. Sides AB and AC in an equilateral triangle ABC with side length 3 is extended to form two rays from point A as shown in the figure. Point P is chosen outside the triangle ABC and between the two rays such that $\angle ABP + \angle BCP = 180^\circ$. If the maximum length of CP is M, then $M^2/2$

is equal to :



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10.

Let a, b, c be sides of a triangle ABC and Δ denotes its area .

If $a = 2, \Delta = \sqrt{3}$ and $a \cos C + \sqrt{3} a \sin C - b - c = 0$, then find the value of $(b + c)$.

(symbols used have usual meaning in ΔABC).

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11. Circumradius of ΔABC is 3 cm and its area is $6cm^2$. If DEF is the triangle formed by feet of the perpendicular drawn from A,B and C on the sides BC, CA and AB, respectively, then the perimeter of ΔDEF (in cm) is

_____

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