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## MATHS

# BOOKS - SHRI BALAJI MATHS (ENGLISH) 

## VECTOR \& 3DIMENSIONAL GEOMETRY

## Exercise 1 Single Choice Problems

1. If $a x+b y+c z=p$, then minimum value of $x^{2}+y^{2}+z^{2}$ is
A. $\left(\frac{p}{a+b+c}\right)^{2}$
B. $\frac{p^{2}}{a^{2}+b^{2}+c^{2}}$
C. $\frac{a^{2}+b^{2}+c^{2}}{p^{2}}$
D. 0

Answer: B
2. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and the area of the triangle with adjacemnt sides parallel to $\vec{a}$ and $\vec{b}$ is 3 , then a.b is
A. $\sqrt{3}$
B. $2 \sqrt{3}$
C. $4 \sqrt{3}$
D. $\frac{\sqrt{3}}{2}$

## Answer: B

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3. Let $B_{1}, C_{1}$ and $D_{1}$ are points on $A B, A C$ and $A D$ of the parallelogram $A B C D$, such that $\overrightarrow{A B_{1}}=k_{1} \overrightarrow{A C,} \overrightarrow{A C_{1}}=k_{2} \overrightarrow{A C}$ and $\overrightarrow{A D_{1}}=k_{2} \overrightarrow{A D}$, where $k_{1}, k_{2}$ and $k_{3}$ are scalar.
A. $\lambda_{1}, \lambda_{3}$ and $\lambda_{2}$ are in AP
B. $\lambda_{1}, \lambda_{3}$ and $\lambda_{2}$ are in GP
C. $\lambda_{1}, \lambda_{3}$ and $\lambda_{2}$ are in HP
D. $\lambda_{1}+\lambda_{2}+\lambda_{3}=0$

## Answer: C

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4. Let $\vec{a}=2 \hat{i}+\hat{j}+\hat{k}$, and $\vec{b}=\hat{i}+\hat{j}$ if c is a vector such that $\vec{a} \cdot \vec{c}=|\vec{c}|,|\vec{c}-\vec{a}|=2 \sqrt{2} \quad$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{i} s 30^{\circ}$, then $|(\vec{a} \times \vec{b})| \times \vec{c} \mid$ is equal to
A. $\frac{2}{3}$
B. $\frac{3}{2}$
C. 2
D. 3

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5. If acute angle between the line $\vec{r}=\hat{i}+2 \hat{j}+\lambda(4 \hat{i}-3 \hat{k})$ and $x y$ plane is $\theta_{1}$ and acute angle between planes $x+2 y=0$ and $2 x+y=0$ is $\theta_{2}$, then $\left(\cos ^{2} \theta_{1}+\sin ^{2} \theta_{2}\right)$ equals to :
A. 1
B. $\frac{1}{4}$
C. $\frac{2}{3}$
D. $\frac{3}{4}$

## Answer: A

6. If $a, b, c, x, y, z$ are real and $a^{2}+b^{2}+c^{2}=25, x^{2}+y^{2}+z^{2}=36$ and $a x+b y+c z=30, \quad$ then $\frac{a+b+c}{x+y+z}$ is equal to :
A. 1
B. $\frac{6}{5}$
C. $\frac{5}{6}$
D. $\frac{3}{4}$

## Answer: C

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7. If $\vec{a}$ and $\vec{b}$ are non-zero, non-collinear vectors such that $|\vec{a}|=2, \vec{a} \cdot \vec{b}=1$ and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$. If $\vec{r}$ is any vector such that

$$
\vec{r} \cdot \vec{a}=2, \vec{r} \cdot \vec{b}=8,(\vec{r}+2 \vec{a}-10 \vec{b}) \cdot(\vec{a} \times \vec{b})=4 \sqrt{3} \text { and }
$$

satisfy to $\vec{r}+2 \vec{a}-10 \vec{b}=\lambda(\vec{a} \times \vec{b})$, then $\lambda$ is equal to : (a) $\frac{1}{2}$ (b)
2 (c) $\frac{1}{4}$ (d) none of these
A. $\frac{1}{2}$
B. 2
C. $\frac{1}{4}$
D. None of these

## Answer: D

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8. Let $\vec{a}=3 \hat{i}+2 \hat{j}+4 \hat{k}, \vec{b}=2(\hat{i}+\hat{k})$ and $\vec{c}=4 \hat{i}+2 \hat{j}+3 \hat{k}$.Sum of the values of $\alpha$ for which the equation $x \vec{a}+y \vec{b}+z \vec{c}=\alpha(x \hat{i}+y \hat{j}+z \hat{k})$ has non-trivial solution is:
A. -1
B. 4
C. 7
D. 8

## Answer: C

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9. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2 \hat{j}-\hat{k}$, then the value of
$\left\lvert\, \begin{array}{lll}\vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c}\end{array}\right.$
$\vec{b} \cdot \vec{a} \vec{b} \cdot \vec{b} \quad \vec{b} \cdot \vec{c}$ is equal to :
$\vec{c} \cdot \vec{a} \quad \vec{c} \cdot \vec{b} \quad \vec{c} \cdot \vec{c} \mid$
A. 2
B. 4
C. 16
D. 64

## Answer: C

10. $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}|=1,|\vec{b}|=4$ and $\vec{a}$. Vecb $=2 . I f$ vecc $=(2 \vec{a} \times \vec{b})-3 \vec{b}$ then find angle between $\vec{b}$ and $\vec{c}$.
A. $\frac{\pi}{6}$
B. $\frac{\pi}{3}$
C. $\frac{2 \pi}{3}$
D. $\frac{5 \pi}{3}$

## Answer: D

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11. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then the value of $|\vec{a}-2 \vec{b}|^{2}+|\vec{b}-2 \vec{c}|^{2}+|\vec{c}-2 \vec{a}|^{2}$ does not exceed to : (a) 9 (b) 12
(c) 18 (d) 21
A. 9
B. 12
C. 18
D. 21

## Answer: D

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12. Two adjacent sides OA and OB of a rectangle $O A C B$ are represented by $\quad \vec{a}$ and $\vec{b}$ respectively, where $o$ is origin. If $16|\vec{a} \times \vec{b}|=3(|\vec{a}|+|\vec{b}|)^{2}$ and $\theta$ is the angle between the diagonals OC and $A B$, then the value(s) of $\tan \left(\frac{\theta}{2}\right)$
A. $\frac{1}{\sqrt{2}}$
B. $\frac{1}{2}$
C. $\frac{1}{\sqrt{3}}$
D. $\frac{1}{3}$

## Answer: D

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13. If vectors $\overrightarrow{A B}=-3 \hat{i}+4 \hat{k}$ and $\overrightarrow{A C}=5 \hat{i}-2 \hat{j}+4 \hat{k}$ are the sides of a $\triangle A B C$, then the length of the median throught A is
A. $\sqrt{288}$
B. $\sqrt{72}$
C. $\sqrt{33}$
D. $\sqrt{18}$

## Answer: C

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14. 

If $\quad \vec{a}=2 \hat{i}+\lambda \hat{j}+3 \hat{k}, \vec{b}=3 \hat{i}+3 \hat{j}+5 \hat{k}, \vec{c}=\lambda \hat{i}+2 \hat{j}+2 \hat{k} \quad$ are linearly dependent vectors, then the number of possible values of $\lambda$ is :
i) 0
ii)1
iii)2
iv)More than 2
A. 0
B. 1
C. 2
D. More than 2

## Answer: C

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$[\vec{a}+\vec{b}-\vec{c} \vec{b}+\vec{c}-\vec{a} \vec{c}+\vec{a}-\vec{b}]$ is equal to
A. 0
B. $[\vec{a} \vec{b} \vec{c}]$
C. $2\left[\begin{array}{lll}\vec{a} & \vec{b} & \vec{c}\end{array}\right]$
D. $4[\vec{a} \vec{b} \quad \vec{c}]$

Answer: D

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16. If $\hat{a}$ and $\hat{b}$ are unit vectors then the vector defined as $\vec{V}=(\widehat{a} \times \hat{b}) \times(\widehat{a}+\hat{b})$ is collinear to the vector :
A. $\widehat{a}+\hat{b}$
B. $\hat{b}-\widehat{a}$
C. $2 \widehat{a}-\hat{b}$
D. $\widehat{a}+2 \hat{b}$

## Answer: B

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17. The sine of angle formed by the lateral face ADC and plane of the base $A B C$ of the terahedron $A B C D$, where $A=(3,-2,1), B=(3,1,5), C=(4,0,3)$ and $D=(1,0,0)$, is :
A. $\frac{2}{\sqrt{29}}$
B. $\frac{5}{\sqrt{29}}$
C. $\frac{3 \sqrt{3}}{\sqrt{29}}$
D. $\frac{-2}{\sqrt{29}}$

## Answer: B

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18. Let $\vec{a}_{r}=x_{r} \hat{i}+y_{r} \hat{j}+z_{r} \hat{k}, r=1,2,3$ three mutually prependicular unit vectors then the value of $\left|\begin{array}{lll}x_{1} & -x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \\ z_{1} & z_{2} & z_{3}\end{array}\right|$ is equal to
A. 0
B. $\pm 1$
C. $\pm 2$
D. $\pm 4$

## Answer: B

## D Watch Video Solution

19. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non coplanar vectors and $\vec{r}$ is any vector in space, then $(\overrightarrow{\times} \vec{b}),(\vec{r} \times \vec{c})+(\vec{b} \times \vec{c}) \times(\vec{r} \times \vec{a})+(\vec{c} \times \vec{a}) \times(\vec{r} \times$
(A) $[\vec{a} \vec{b} \vec{c}]$
(B) $2[\vec{a} \vec{b} \vec{c}] \vec{r}$
(C) $3[\vec{a} \vec{b} \vec{c}] \vec{r}$
(D) $4[\vec{a} \vec{b} \vec{c}] \vec{r}$
A. $[\vec{a} \vec{b} \vec{c}] \vec{r}$
B. $2[\vec{a} \vec{b} \vec{c}] \vec{r}$
C. $4[\vec{a} \vec{b} \vec{c}] \vec{r}$
D. $\overrightarrow{0}$

## Answer: B

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20. $E$ and $F$ are the interior points on the sides $B C$ and $C D$ of a parallelogram ABCD. Let $\overrightarrow{B E}=4 \overrightarrow{E C}$ and $\overrightarrow{C F}=4 \overrightarrow{F D}$. If the line EF meets the diagonal AC in G , then $\overrightarrow{A G}=\lambda \overrightarrow{A C}$, where $\lambda$ is equal to :
A. $\frac{1}{3}$
B. $\frac{21}{25}$
C. $\frac{7}{13}$
D. $\frac{21}{5}$
21. If $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{c}$ is such that $\vec{c}$ is such that $\vec{c}=\vec{c}=\vec{a} \times \vec{c}+\vec{b}$ then maximum value of $[\vec{a} \vec{b} \vec{c}]$ is
A. 1
B. $\frac{1}{2}$
C. 2
D. $\frac{3}{2}$

## Answer: B

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22. Conside the matrices $A=\left[\begin{array}{ccc}1 & 2 & 3 \\ 4 & 1 & 2 \\ 1 & -1 & 1\end{array}\right] \quad B=\left[\begin{array}{ccc}2 & 1 & 3 \\ 4 & 1 & -1 \\ 2 & 2 & 3\end{array}\right]$
$C=\left[\begin{array}{c}14 \\ 12 \\ 2\end{array}\right] \quad D=\left[\begin{array}{l}13 \\ 11 \\ 14\end{array}\right]$. Now $x=\left[\begin{array}{l}x \\ y \\ z\end{array}\right]$ is such that solutions of
equation $A X=C$ and $B X=D$ represent two points L and
respectively in 3 dimensional space. If $L^{\prime}$ and $M^{\prime}$ are hre reflections of L and $M$ in the plane $x+y+z=9$ then find coordinates of $L, M, L^{\prime}, M^{\prime}$
A. $(3,4,2)$
B. $(5,3,4)$
C. $(7,2,3)$
D. $(1,5,6)$

## Answer: A

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23. The value of $\alpha$ for which point $M(\alpha \hat{i}+2 \hat{j}+\hat{k})$, lie in the plane containing three points $A(\hat{i}+\hat{j}+\hat{k})$ and $C(3 \hat{i}-\hat{k})$ is:
A. 1
B. 5
C. $\frac{1}{2}$
D. $-\frac{1}{2}$

## Answer: B

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24. $Q$ is the image of point $P(1,-2,3)$ with respect to the plane $x-y+z=7$. The distance of Q from the origin is.
A. $\sqrt{\frac{70}{3}}$
B. $\frac{1}{2} \sqrt{\frac{70}{3}}$
C. $\sqrt{\frac{35}{3}}$
D. $\sqrt{\frac{15}{2}}$

## Answer: A

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25. $\widehat{a}, \hat{b}$ and $\widehat{a}-\hat{b}$ are unit vectors. The volume of the parallelopiped, formed with $\widehat{a}, \hat{b}$ and $\widehat{a} \times \hat{b}$ as coterminous edges is:
A. 1
B. $\frac{1}{4}$
C. $\frac{2}{3}$
D. $\frac{3}{4}$

## Answer: D

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26. A line passing through $P(3,7,1)$ and $R(2,5,7)$ meet the plane $3 x+2 y+11 z-9=0$ at Q . Then PQ is equal to :
A. $\frac{5 \sqrt{41}}{59}$
B. $\frac{\sqrt{41}}{59}$
C. $\frac{50 \sqrt{41}}{59}$
D. $\frac{25 \sqrt{41}}{59}$

## Answer: D

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27. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero non coplanar vectors and $\vec{p}, \vec{q}$ and $\vec{r}$ be three vectors given by $\vec{p}=\vec{a}+\vec{b}-2 \vec{c}, \vec{q}=3 \vec{a}-2 \vec{b}+\vec{c}$ and $\vec{r}=\vec{a}-4 v c b+2 \vec{c}$

If the volume of the parallelopiped determined by $\vec{a}, \vec{b}$ and $\vec{c}$ is $V_{1}$ and that of the parallelopiped determined by $\vec{a}, \vec{q}$ and $\vec{r}$ is $V_{2}$, then $V_{2}: V_{1}=$
A. 10
B. 15
C. 20
D. None of these

## Answer: B

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28. 

two
$x=a y+b, z=c y+d$ and $x=a^{\prime} y+b^{\prime}, z=c^{\prime} y+d^{\prime} \quad$ are pendicular to each other if
A. 1
B. 2
C. 3
D. 4

## Answer: A

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29. The perpendicular distance between the line $\vec{r}=2 \hat{i}-2 \hat{j}+3 \hat{k}+\lambda(\hat{i}-\hat{j}+4 \hat{k}) \quad$ and $\quad$ the plane $\vec{r} \cdot(\hat{i}+5 \hat{j}+\hat{k})=5$ is :
A. $\frac{10}{9}$
B. $\frac{10}{3 \sqrt{3}}$
C. $\frac{3}{10}$
D. $\frac{10}{3}$

## Answer: B

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30. If $(\vec{a} \times \vec{b}) \times \vec{c}=\vec{a} \times(\vec{b} \times \vec{c})$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a} \cdot \vec{b} \neq 0, \vec{b} \cdot \vec{c} \neq 0$, then $\vec{a}$ and $\vec{c}$ are :
A. Inclined at an angle of $\frac{\pi}{3}$
B. Inclined at an angle of $\frac{\pi}{6}$
C. Perpendicular
D. Parallel

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31. Let $\vec{r}$ be position vector of variable point in cartesian plane OXY such that $\vec{r} \cdot(\vec{r}+6 \hat{j})=7$ cuts the co-ordinate axes at four distinct points, then the area of the quadrilateral formed by joining these points is :
A. $4 \sqrt{7}$
B. $6 \sqrt{7}$
C. $7 \sqrt{7}$
D. $8 \sqrt{7}$

## Answer: D

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32. If $|\vec{a}|=2,|\vec{b}|=5$ and $\vec{a} \cdot \vec{b}=0$,
then
$\vec{a} \times(\vec{a} \times(\vec{a} \times(\vec{a} \times(\vec{a} \times(\vec{a} \times \vec{b})))))$ is equal to :
A. $64 \vec{a}$
B. $64 \vec{b}$
C. $-64 \vec{a}$
D. $-64 \vec{b}$

## Answer: D

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33. If O (origin) is a point inside the triangle $P Q R$ such that $\overrightarrow{O P}+k_{1} \overrightarrow{O Q}+k_{2} \overrightarrow{O R}=0$, where $k_{1}, k_{2}$ are constants such that $\frac{\operatorname{Area}(\Delta P Q R)}{\operatorname{Area}(\Delta O Q R)}=4$, then the value of $k_{1}+k_{2}$ is :
A. 2
B. 3
C. 4
D. 5

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34. Let $P Q$ and $Q R$ be diagonals of adjacent faces of a rectangular box, with its centre at 0 . If $\angle Q O R, \angle R O P$ and $\angle P O Q$ are $\theta, \phi$ and $\Psi$ respectively then the value of ' $\cos \theta+\cos \phi+\cos \Psi^{\prime}$ is :
A. -2
B. $-\sqrt{3}$
C. -1
D. 0

## Answer: C

35. 

$\vec{r}=a(\vec{m} \times \vec{n})+b(\vec{n} \times \vec{I})+c(\vec{I} \times \vec{m})$ and $[\vec{I} \vec{m} \vec{n}]=4, \quad$ find
A. $\frac{1}{4}$
B. $\frac{1}{2}$
C. 1
D. 2

## Answer: A

## D Watch Video Solution

36. The volume of tetrahedron, for which three co-terminous edges are
$\vec{a}-\vec{b}, \vec{b}+2 \vec{c}$ and $3 \vec{a}-\vec{c}$ is : (a) $6 k$ (b) $7 k$ (c) $30 k$ (d) $42 k$
A. 6 k
B. 7 k
C. 30k
D. 42 k

## Answer: D

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37. The equation of a plane passing through the line of intersection of the planes :
$x+2 y+z-10=0$ and $3 x+y-z=5$ and passing through the origin is :
A. $5 x+3 z=0$
B. $5 x-3 z=0$
C. $5 x+4 y+3 z=0$
D. $5 x-4 y+3 z=0$

## Answer: B

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38. Find the locus of a point whose distance from $x$-axis is equal the distance from the point $(1,-1,2)$ :
A. $y^{2}+2 x-2 y-4 z+6=0$
B. $x^{2}+2 x-2 y-4 z+6=0$
C. $x^{2}-2 x+2 y-4 z+6=0$
D. $z^{2}-2 x+2 y-4 z+6=0$

## Answer: C

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1. If equation of three lines are:
$\frac{x}{1}=\frac{y}{2}=\frac{z}{3}, \frac{x}{2}=\frac{y}{1}=\frac{z}{3}$ and $\frac{x-1}{1}=\frac{2-y}{1}=\frac{z-3}{0}$, then which of the following statement(s) is/are correct ?
A. Triangle formed by the line is equilateral
B. Triangle formed by the lines is isosceles
C. Equation of the plane containing the lines is $x+y=z$
D. Area of the triangle formed by the lines is $\frac{3 \sqrt{3}}{2}$

## Answer: B::C::D

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2. 

$$
\vec{a}=\hat{i}+6 \hat{j}+3 \hat{k}, \vec{b}=3 \hat{i}+2 \hat{j}+\hat{k} \text { and } \vec{c}=(\alpha+1) \hat{i}+(\beta-1) \hat{j}+\hat{k}
$$

are linearly dependent vectors and $|\vec{c}|=\sqrt{6}$, then the possible value(s)
of $(\alpha+\beta)$ can be : (a) 1 (b) 2 (c) 3 (d) 4
A. 1
B. 2
C. 3
D. 4

## Answer: A::C

## - Watch Video Solution

3. 

Consider
the
lines:
$L_{1}: \frac{x-2}{1}=\frac{y-1}{7}=\frac{z+2}{-5}, L_{2}: x-4=y+3=-z$ Then which of the following is/are correct ? (A) Point of intersection of $L_{1}$ and $L_{2} i s(1,-6,3)$
A. Point of intersection of $L_{1}$ and $L_{2}$ is $(1,-6,3)$
B. Equation of plane containing $L_{1}$ and $L_{2}$ is $x+2 y+3 z+2=0$
C. Acute angle between $L_{1}$ and $L_{2}$ is $\cot ^{-1}\left(\frac{13}{15}\right)$
D. Equation of plane containing $L_{1}$ and $L_{2}$ is $x+2 y+2 z+3=0$

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4. Let $\widehat{a}, \hat{b}$ and $\hat{c}$ be three unit vectors such that $\widehat{a}=\hat{b}+(\hat{b} \times \hat{c})$, then the possible values) of $|\widehat{a}+\hat{b}+\hat{c}|^{2}$ can be :
A. 1
B. 4
C. 16
D. 9

## Answer: A::D

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5. The values) of $\mu$ for which the straight lines

$$
\vec{r}=3 \hat{i}-2 \hat{j}-4 \hat{k}+\lambda_{1}(\hat{i}-\hat{j}+\mu \hat{k})
$$

$\vec{r}=5 \hat{i}-2 \hat{j}+\hat{k}+\lambda_{2}(\hat{i}+\mu \hat{j}+2 \hat{k})$ are coplanar is/are :
A. $\frac{5+\sqrt{33}}{4}$
B. $\frac{-5+\sqrt{33}}{4}$
C. $\frac{5-\sqrt{33}}{4}$
D. $\frac{-5-\sqrt{33}}{4}$

## Answer: A:C

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6. 

$\hat{i} \times[(\vec{a}-\hat{j}) \times \hat{i}] \times[(\vec{a}-\hat{k}) \times \hat{j}]+\vec{k} \times[(\vec{a}-\vec{i}) \times \hat{k}]=0$, then find vector $\vec{a}$.
A. $x+y=1$
B. $y+z=\frac{1}{2}$
C. $x+z=1$
D. None of these

Answer: A:C

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7. $\left[\begin{array}{lll}\vec{a} \times \vec{b} & \vec{c} \times \vec{d} & \vec{e} \times \vec{f}\end{array}\right]$ is equal to
A. $[\vec{a} \vec{b} \vec{d}][\vec{c} \vec{e} \vec{f}]-[\vec{a} \vec{b} \vec{c}][\vec{d} \vec{e} \vec{f}]$
B. $[\vec{a} \vec{b} \vec{e}][\vec{f} \vec{c} \vec{d}]-[\vec{a} \vec{b} \vec{f}][\vec{e} \vec{c} \vec{d}]$
c. $[\vec{c} \vec{d} \vec{a}][\vec{b} \vec{e} \vec{f}]-[\vec{c} \vec{d} \vec{b}][\vec{a} \vec{e} \vec{f}]$
D. $[\vec{b} \vec{c} \vec{d}][\vec{a} \vec{e} \vec{f}]-[\vec{b} \vec{c} \vec{f}][\vec{a} \vec{e} \vec{d}]$

## Answer: A::B::C

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8. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D respectively in three dimensionalspace no three of $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ are collinear and satisfy the relation $3 \vec{a}-2 \vec{b}+\vec{c}-2 \vec{d}=0$, then
A. A, B, C and D are coplanar
B. The line joining the points $B$ and $D$ divides the line joining the point

A and C in the ratio of $2: 1$
C. The line joining the points $A$ and $C$ divides the line joining the points $B$ and $D$ in the ratio of $1: 1$
D. The four vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are linearly dependent.

## Answer: A::C::D

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9. If OAB is a tetrahedron with edges and $\hat{p}, \hat{q}, \hat{r}$ are unit vectors along bisectors of
$\overrightarrow{O A}, \overrightarrow{O B}: \overrightarrow{O B}, \overrightarrow{O C}: \overrightarrow{O C}, \overrightarrow{O A}$
$\widehat{a}=\frac{\overrightarrow{O A}}{|\overrightarrow{O A}|}, \vec{b}=\frac{\overrightarrow{O B}}{|\overrightarrow{O B}|}, \vec{c}=\frac{\overrightarrow{O C}}{|\overrightarrow{O C}|}$, then :
A. $\frac{[\widehat{a} \hat{b} \hat{c}]}{[\hat{p} \hat{q} \hat{r}]}=\frac{3 \sqrt{3}}{2}$
B. $\left.\frac{\left[\begin{array}{lll}\widehat{a}+\hat{b} & \hat{b}+\hat{c} & \hat{c}+\hat{a}\end{array}\right]}{[\hat{p}+\hat{q}} \begin{array}{lll}\hat{q}+\hat{r} & \hat{r}+\hat{p}\end{array}\right]=\frac{3 \sqrt{3}}{4}$
C. $\frac{\left[\begin{array}{lll}\widehat{a}+\hat{b} & \hat{b}+\hat{c} & \hat{c}+\widehat{a}\end{array}\right]}{[\hat{p} \hat{q} \hat{r}]}=\frac{3 \sqrt{3}}{2}$
D. $\frac{[\widehat{a} \hat{b} \hat{c}]}{\left[\begin{array}{lll}\hat{p}+\hat{q} & \hat{q}+\hat{r} & \hat{r}+\hat{p}]\end{array}=\frac{3 \sqrt{3}}{4}\right.}$

## Answer: A:D

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10. $\vec{a}$ and $\vec{c}$ are unit vectors and $|\vec{b}|=4$ the angle between $\vec{a}$ and $\vec{b} i s \cos ^{-1}(1 / 4)$ and $\vec{b}-2 \vec{c}=\lambda \vec{a}$ the value of $\lambda$ is
A. 2
B. 3
C. -3
D. 4

## Answer: C::D

## - Watch Video Solution

11. Consider the lines $x=y=z$ and line
$2 x+y+z-1=0=3 x+y+2 z-2$, then
A. The shortest distance between the two lines is $\frac{1}{\sqrt{2}}$
B. The shortest distance between the two lines is $\sqrt{2}$
C. Plane containing the line $L_{2}$ and parallel to line $L_{1}$ is

$$
z-x+1=0
$$

D. Perpendicular distance of origin from plane containing line $L_{2}$ and parallel to line $L_{1}$ is $\frac{1}{\sqrt{2}}$
12. Let $\vec{r}=(\vec{a} \times \vec{b}) \sin x+(\vec{b} \times \vec{c}) \cos y+2(\vec{c} \times \vec{a})$, where $\vec{a}, \vec{b}, \vec{c}$ are non-zero and non-coplanar vectors. If $\vec{r}$ is orthogonal to $\vec{a}+\vec{b}+\vec{c}$, then find the minimum value of $\frac{4}{\pi^{2}}\left(x^{2}+y^{2}\right)$.
A. $\pi^{2}$
B. $\frac{5 \pi^{2}}{4}$
C. $\frac{35 \pi^{2}}{4}$
D. $\frac{37 \pi^{2}}{4}$

## Answer: B::D

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13. If $(\vec{a} \times \vec{b}) \times(\vec{c} \times \vec{d})=h \vec{a}+k \vec{b}=r \vec{c}+s \vec{d}$, where $\vec{a}, \vec{b}$ are non-collinear and $\vec{c}, \vec{d}$ are also non-collinear then :
A. $h=[\vec{b} \vec{c} \vec{d}]$
B. $k=[\vec{a} \vec{c} \vec{d}]$
C. $r=\left[\begin{array}{lll}\vec{a} & \vec{b} & \vec{d}\end{array}\right]$
D. $s=-\left[\begin{array}{lll}\vec{a} & \vec{b} & \vec{c}\end{array}\right]$

## Answer: B::C::D

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14. 

Let
a be
a real
number
and $\vec{\alpha}=\hat{i}+2 \hat{j}, \vec{\beta}=2 \hat{i}+a \hat{j}+10 \hat{k}, \vec{\gamma}=12 \hat{i}+20 \hat{j}+a \hat{k}$ be three vectors, then $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ are linearly independent for :
A. $a>0$
B. $a<0$
C. $a=0$
D. No value of a

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15. The volume of a right triangular prism $\mathrm{ABC} A_{1} B_{1} C_{1}$ is equal to 3 cubic unit. Then the co-ordinates of the vertex $A_{1}$, if the co-ordinates of the base vertices of the prism are $A(1,0,1), B(2,0,0)$ and $C(0,1,0)$, are
A. $(2,2,2)$
B. $(0,2,0)$
C. $(0,-2,2)$
D. $(0,-2,0)$

## Answer: A: D

16. If $\vec{a}=x \hat{i}+y \hat{j}+z \hat{k}, \vec{b}=y \hat{i}+z \hat{j}+x \hat{k}$ and $\vec{c}=z \hat{i}+x \hat{j}+y \hat{k}$, then $\vec{a} \times(\vec{b} \times \vec{c})$ is
(a)parallel to $(y-z) \hat{i}+(z-x) \hat{j}+(x-y) \hat{k} \quad$ (b)orthogonal to $\hat{i}+\hat{j}+\hat{k} \quad$ (c)orthogonal to $\quad(y+z) \hat{i}+(z+x) \hat{j}+(x+y) \hat{k}$
(d)orthogonal to $x \hat{i}+y \hat{j}+z \hat{k}$
A. Parallel to $(y-z) \hat{i}+(z-x) \hat{j}+(x-y) \hat{k}$
B. Orthogonal to $\hat{i}+\hat{j}+\hat{k}$
C. Orthogonal to $(y+z) \hat{i}+(z+x) \hat{j}+(x+y) \hat{k}$,
D. Orthogonal to $x \hat{i}+y \hat{j}+z \hat{k}$

## Answer: A::B::C::D

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17. If a line has a vector equation, $\vec{r}=2 \hat{i}+6 \hat{j}+\lambda(\hat{i}-3 \hat{j})$ then which of the following statements holds good?
A. the line is parallel to $2 \hat{i}+6 \hat{j}$
B. the line passes through the point $3 \hat{i}+3 \hat{j}$
C. the line passes through the point $\hat{i}+9 \hat{j}$
D. the line is parallel to xy plane

## Answer: B::C::D

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18. Let $M, N, P$ and $Q$ be the mid points of the edges $A B, C D, A C$ and $B D$ respectively of the tetrahedron $A B C D$. Further, $M N$ is perpendicular to both $A B$ and $C D$ and $P Q$ is perpendicular to both $A C$ and $B D$. Then which of the following is/are correct:
A. $A B=C D$
B. $B C=D A$
C. $A C=B D$
D. $A N=B N$

## D Watch Video Solution

19. The solution vectors $\vec{r}$ of the equation $\vec{r} \times \hat{i}=\hat{j}+\hat{k}$ and $\vec{r} \times \hat{j}=\hat{k}+\hat{j}$ represent two straight lines which are :
A. Intersecting
B. Non coplanar
C. Coplanar
D. Non intersecting

## Answer: B::D

20. The lines with vector equations are, $\vec{r}_{1}=3 \hat{i}+6 \hat{j}+\lambda(-4 \hat{i}+3 \hat{j}+2 \hat{k})$ and $\vec{r}_{2}=-2 \hat{i}+7 \hat{j}+\mu(-4 \hat{i}+$ are such that :
A. they are coplanar
B. they do not intersect
C. they are skew
D. the angle between then is $\tan ^{-1}(3 / 7)$

## Answer: B::C::D

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## Exercise 3 Comprehension Type Problems

1. The vertices of $\Delta A B C$ are $\mathrm{A}(2,0,0), \mathrm{B}(0,1,0), \mathrm{C}(0,0,2)$. Its orthocentre is H and circumcentre is $\mathrm{S} . \mathrm{P}$ is a point equidistant from $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and the
origin 0.
Q. The z -coordinate of H is :
A. 1
B. $1 / 2$
C. $1 / 6$
D. $1 / 3$

## Answer: D

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2. The vertices of $\Delta A B C$ are $\mathrm{A}(2,0,0), \mathrm{B}(0,1,0), \mathrm{C}(0,0,2)$. Its orthocentre is H and circumcentre is $\mathrm{S} . \mathrm{P}$ is a point equidistant from $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and the origin 0.
Q. The y-coordinate of $S$ is :
A. $5 / 6$
B. $1 / 3$
C. $1 / 6$
D. $1 / 2$

## Answer: C

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3. The vertices of $\Delta A B C$ are $(2,0,0), \mathrm{B}(0,1,0), \mathrm{C}(0,0,2)$. Its orthocentre is $H$ and circumcentre is $S . P$ is a point equidistant from $A, B, C$ and the origin 0 .
Q. PA is equal to :
A. 1
B. $\sqrt{2}$
C. $\sqrt{\frac{3}{2}}$
D. $\frac{3}{2}$

## Answer: D

4. Consider a plane $\pi: \vec{r} \cdot \vec{n}=d$ (where $\vec{n}$ is not a unti vector). There are two points $A(\vec{a})$ and $B(\vec{b})$ lying on the same side of the plane. Q. If foot of perpendicular from A and B to the plane $\pi$ are P and Q respectively, then length of $P Q$ be :
A. $\frac{|(\vec{b}-\vec{a}) \cdot \vec{n}|}{|\vec{n}|}$
B. $|(\vec{b}-\vec{a}) \cdot \vec{n}|$
c. $\frac{|(\vec{b}-\vec{a}) \times \vec{n}|}{|\vec{n}|}$
D. $|(\vec{b}-\vec{a}) \times \vec{n}|$

## Answer: C

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5. Consider a plane $\pi: \vec{r} \cdot \vec{n}=d$ (where $\vec{n}$ is not a unti vector). There are two points $A(\vec{a})$ and $B(\vec{b})$ lying on the same side of the plane. Q. Reflection of $A(\vec{a})$ in the plane $\pi$ has the position vector:
A. $\vec{a}+\frac{2}{(\vec{n})^{2}}(d-\vec{a} \cdot \vec{n}) \vec{n}$
B. $\vec{a}-\frac{1}{(\vec{n})^{2}}(d-\vec{a} \cdot \vec{n}) \vec{n}$
C. $\vec{a}+\frac{2}{(\vec{n})^{2}}(d+\vec{a} \cdot \vec{n}) \vec{n}$
D. $\vec{a}+\frac{2}{(\vec{n})^{2}} \vec{n}$

## Answer: A

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6. Consider a plane $\prod: \vec{r} \cdot(2 \hat{i}+\hat{j}-\hat{k})=5$, a line $L_{1}: \vec{r}=(3 \hat{i}-\hat{j}+2 \hat{k})+k(2 \hat{i}-3 \hat{j}-\hat{k}) \quad$ and $\quad$ a point $A(3,-4,1) \cdot L_{2}$ is a line passing through A intersecting $L_{1}$ and parallel

## to plane $\prod$.

Then equation of $L_{2}$ is :
A. (a) $\vec{r}=(1+\lambda) \hat{i}+(2-3 \lambda) \hat{j}+(1-\lambda) \hat{k}: \lambda \in R$
B. (b) $\vec{r}=(3+\lambda) \hat{i}-(4-2 \lambda) \hat{j}+(1+3 \lambda) \hat{k}, \lambda \in R$
C. (c) $\vec{r}=(3+\lambda) \hat{i}-(4+3 \lambda) \hat{j}+(1-\lambda) \hat{k}, \lambda \in R$
D. (d) None of the above

## Answer: C

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7. Consider a plane $\prod: \vec{r} \cdot(2 \hat{i}+\hat{j}-\hat{k})=5$, a line $L_{1}: \vec{r}=(3 \hat{i}-\hat{j}+2 \hat{k})+k(2 \hat{i}-3 \hat{j}-\hat{k}) \quad$ and $\quad$ a point $A(3,-4,1) \cdot L_{2}$ is a line passing through A intersecting $L_{1}$ and parallel to plane $\prod$.

Then equation of plane containing $L_{1}$ and $L_{2}$ is:
A. (a) parallel to yz-plane
B. (b) parallel to $x$-axis
C. (c) parallel to $y$-axis
D. (d) passing through origin

## Answer: B

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8. Consider a plane $\prod: \vec{r} \cdot(2 \hat{i}+\hat{j}-\hat{k})=5$, a line $L_{1}: \vec{r}=(3 \hat{i}-\hat{j}+2 \hat{k})+k(2 \hat{i}-3 \hat{j}-\hat{k}) \quad$ and $\quad$ a point $A(3,-4,1) \cdot L_{2}$ is a line passing through A intersecting $L_{1}$ and parallel to plane $\prod$.
Q. Line $L_{1}$ intersects plane $\prod$ at $Q$ and $x y$-plane at $R$ the volume of tetrahedron OAQR is :
(where ' O ' is origin)
A. (a) 0
B. (b) $\frac{14}{3}$
C. (c) $\frac{3}{7}$
D. (d) $\frac{7}{3}$

## Answer: D

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9. Consider three planes :
$2 x+p y+6 z=8, x+2 y+q z=5$ and $x+y+3 z=4$
Q. Three planes intersect at a point if :
A. $p=2, q \neq 3$
B. $p \neq 2, q \neq 3$
C. $p \neq 2, q=3$
D. $p=2, q=3$

## Answer: B

10. Consider three planes:
$2 x+p y+6 z=8, x+2 y+q z=5$ and $x+y+3 z=4$
Q. Three planes do not have any common point of intersection if :
A. $p=2, q \neq 3$
B. $p \neq 2, q \neq 3$
C. $p \neq 2, q=3$
D. $p=2, q=3$

## Answer: C

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11. Consider a tetrahedron $D-A B C$ with position vectors if its angular points as
$A(1,1,1), B(1,2,3), C(1,1,2)$
and centre of tetrahedron $\left(\frac{3}{2}, \frac{3}{4}, 2\right)$.
Q. Shortest distance between the skew lines AB and CD :
A. $\frac{1}{2}$
B. $\frac{1}{3}$
C. $\frac{1}{4}$
D. $\frac{1}{5}$

## Answer: B

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12. Consider a tetrahedron $D-A B C$ with position vectors if its angular points as
$A(1,1,1), B(1,2,3), C(1,1,2)$
and centre of tetrahedron $\left(\frac{3}{2}, \frac{3}{4}, 2\right)$.
Q . If N be the foot of the perpendicular from point D on the plane face
$A B C$ then the position vector of N are :
A. $(-1,1,2)$
B. $(1,-1,2)$
C. $(1,1,-2)$
D. $(-1,-1,2)$

## Answer: B

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## Exercise 4 Matching Type Problems

## Column-l

(A) Lines $\frac{x-1}{-2}=\frac{y+2}{3}=\frac{z}{-1}$ and
$\overrightarrow{\mathbf{r}}=(\vec{i}-j+\hat{k})+t(\hat{i}+j+k)$ are
(B) Lines $\frac{x+5}{1}=\frac{y-3}{7}=\frac{z+3}{3}$ and
$x-y+2 z-4=0=2 x+y-3 z+5$ are
(C) Lines $(x=t-3, y=-2 t+1, z=-3 x-2)$ and $\overrightarrow{\mathbf{r}}=(t+1) \dot{t}+(2 t+3) \hat{j}+(t-9) k$ are
(D) Lines $\overrightarrow{\mathbf{r}}=(\hat{i}+3 j-k)+t(2 \dot{i}-j-k)$ and $\overrightarrow{\mathbf{r}}=(-i-2 j+5 \hat{k})+s\left(i-2 j+\frac{3}{4} k\right)$ are
1.

## Column-II

(P) Intersecting
(Q) Perpendicular
(R) Parallel
(S) Skew
(T) Coincident


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## Exercise 5 Subjective Type Problems

1. A rod $A B$ of length $2 L$ and mass $m$ is lying on a horizontal frictionless
surface. A particle of same mass $m$ travelling along the surface hits the rod at distance $\frac{L}{2}$ from COM with a velocity $v_{0}$ in a direction perpendicular to rod and sticks to it.


Distance of point $P$ on rod from $B$ which is at rest immediately afte collision is

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2. If $\hat{a}, \hat{b}$ and $\hat{c}$ are non-coplanar inti vectors such that $\left[\begin{array}{lll}\widehat{a} \hat{b} \hat{c}\end{array}\right]=\left[\begin{array}{lll}\hat{b} \times \hat{c} & \hat{c} \times \widehat{a} & \widehat{a} \times \hat{b}\end{array}\right]$, then find the projection of $\hat{b}+\hat{c}$ on $\widehat{a} \times \hat{b}$.
3. Let $O A B C$ be a tetrahedron whose edges are of unit length. If $\vec{O} A=\vec{a}, \overrightarrow{O B}=\vec{b}$, and $\overrightarrow{O C}=\alpha(\vec{a}+\vec{b})+\beta(\vec{a} \times \vec{b})$, then $(\alpha \beta)^{2}=\frac{p}{q}$ (where $\mathrm{p} \& \mathrm{q}$ are relatively prime to each other), then the value of $\left[\frac{q}{2 p}\right]$ is

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4. If $A$ is the matrix $\left[\begin{array}{cc}1 & -3 \\ -1 & 1\end{array}\right]$, then
$A-\frac{1}{3} A^{2}+\frac{1}{9} A^{3} \ldots \ldots \ldots \ldots+\left(-\frac{1}{3}\right)^{n} A^{n+1}+\ldots \ldots \ldots \infty=\frac{3}{13}\left[\begin{array}{ll}1 & a \\ b & 1\end{array}\right.$.
. Find $\left|\frac{a}{b}\right|$.

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5. A sequence of $2 \times 2$ matrices $\left\{M_{n}\right\}$ is defined as follows

$$
M_{n}=\left[\begin{array}{cc}
\frac{1}{(2 n+1)!} & \frac{1}{(2 n+2)!} \\
\sum_{k=0}^{n} \frac{(2 n+2)!}{(2 k+2)!} & \sum_{k=0}^{n} \frac{(2 n+1)!}{(2 k+1)!}
\end{array}\right]
$$

$\lim _{n \rightarrow \infty} \operatorname{det} .\left(M_{n}\right)=\lambda-e^{-1}$. Find $\lambda$.

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6. Let $|\vec{a}|=1,|\vec{b}|=1$ and $|\vec{a}+\vec{b}|=\sqrt{3}$. If $\vec{c}$ be a vector such that $\vec{c}=\vec{a}+2 \vec{b}-3(\vec{a} \times \vec{b})$ and $p=|(\vec{a} \times \vec{b}) \times \vec{c}|$, then find [ $p^{2}$ ].(where [ ] represents greatest integer function).

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7. Let $\vec{r}=(\vec{a} \times \vec{b}) \sin x+(\vec{b} \times \vec{c}) \cos y+2(\vec{c} \times \vec{a})$, where $\vec{a}, \vec{b}, \vec{c}$ are non-zero and non-coplanar vectors. If $\vec{r}$ is orthogonal to $\vec{a}+\vec{b}+\vec{c}$, then find the minimum value of $\frac{4}{\pi^{2}}\left(x^{2}+y^{2}\right)$.

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8. The plane denoted by $P_{1}: 4 x+7 y+4 z+81=0$ is rotated through a right angle about its line of intersection with plane
$P_{2}: 5 x+3 y+10 z=25$. If the plane in its new position be denoted by P, and the distance of this plane from the origin is $d$, then the value of $\left[\frac{k}{2}\right]$ (where[.] represents greatest integer less than or equal to $k$ ) is....

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9. $A B C D$ is a regular tetrahedron, $A$ is the origin and $B$ lies on $x$-axis. $A B C$ lies in the xy-plane $|\overrightarrow{A B}|=2$ Under these conditions, the number of possible tetrahedrons is:

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10. A, B, C, D are four points in the space and satisfy $|\overrightarrow{A B}|=3,|\overrightarrow{B C}|=7,|\overrightarrow{C D}|=11$ and $|\overrightarrow{D A}|=9$. Then find the value of $\overrightarrow{A C} \cdot \overrightarrow{B D}$.

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11. Let OABC be a regular tetrahedron of edge length unity. Its volume be V and $6 V=\sqrt{\frac{p}{q}}$ where p and q are relatively prime. The find the value of $(p+q)$.

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12. If $\vec{a}$ and $\vec{b}$ are non zero, non collinear vectors and $\vec{a}_{1}=\lambda \vec{a}+3 \vec{b}, \vec{b}_{1}=2 \vec{a}+\lambda \vec{b}, \vec{c}_{1}=\vec{a}+\vec{b}$. Find the sum of all possible real values of $\lambda$ so that points $A_{1}, B_{1}, C_{1}$ whose position vectors are $\vec{a}_{1}, \vec{b}_{1}, \vec{c}_{1}$ respectively are collinear is equal to.

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13. Let $P$ and $Q$ are two points on the curve $y=\log _{\frac{1}{2}}(x-0.5)+\log _{2} \sqrt{4 x^{2}-4 x+1}$ and P is also on the circle $x^{2}+y^{2}=10$. Q lies inside the given circle such that its abscissa is an integer.
Q. $O P \cdot O Q$, o being the origin is

## (D) Watch Video Solution

14. Let $P$ and $Q$ are two points on the curve $y=\log _{\frac{1}{2}}(x-0.5)+\log _{2} \sqrt{4 x^{2}-4 x+1}$ and P is also on the circle $x^{2}+y^{2}=10 . \mathrm{Q}$ lies inside the given circle such that its abscissa is an integer.

Q . The coordinates of P are given by

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15. If $a, b, c, l, m, n \in R-\{0\} \quad$ such that
$a l+b m+c n=0, b l+c m+a n=0, c l+a m+b n=0$. If $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are distinct and $f(x)=a x^{3}+b x^{2}+c x+2$. Find $\mathrm{f}(1)$.

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16. Let $\vec{u}$ and $\vec{v}$ be unit vectors such that $\vec{u} \times \vec{v}+\vec{u}=\vec{w}$ and $\vec{w} \times \vec{u}=\vec{v}$. Find the value of $[\vec{u} \vec{v} \vec{w}]$.
