



MATHS

BOOKS - VK JAISWAL ENGLISH

BIONMIAL THEOREM

Exercise 1 Single Problems

1. Let $N = 2^{1224} - 1$, $\alpha = 2^{153} + 2^{77} + 1$ and $\beta = 2^{408} - 2^{204} + 1$. Then which of the following statement is correct ?

- A. α divides N but β does not
- B. β divides N but α does not
- C. α and β both divide N
- D. neither α nor β divides N

Answer: C



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2. If $(1 + 2x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $a_r =$

A. 0

B. ${}^n C_r$

C. a_r

D. 1

Answer: A



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3. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same, if α equals (a) $-\frac{5}{3}$ (b) $\frac{10}{3}$ (c) $-\frac{3}{10}$ (d) $\frac{3}{5}$

A. $-\frac{5}{3}$

B. $\frac{3}{5}$

C. $-\frac{3}{10}$

D. $\frac{10}{3}$

Answer: C



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4. If $(1 + x)^{2010} = C_0 + C_1x + C_2x^2 + \dots + C_{2010}x^{2010}$ then the sum of series $C_2 + C_5 + C_8 + \dots + C_{2009}$ equals to :

A. $\frac{1}{2}(2^{2010} - 1)$

B. $\frac{1}{3}(2^{2010} - 1)$

C. $\frac{1}{2}(2^{2009} - 1)$

D. $\frac{1}{3}(2^{2009} - 1)$

Answer: B



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5. Let $\alpha_n = (2 + \sqrt{3})^n$. Find $\lim_{n \rightarrow \infty} (\alpha_n - [\alpha_n])$ ([.] denotes greatest integer function)

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Answer: A

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6. The number $N = {}^{20}C_7 - {}^{20}C_8 + {}^{20}C_9 - {}^{20}C_{10} + \dots - {}^{20}C_{20}$ is not divisible by :

A. 3

B. 7

C. 11

D. 19

Answer: C



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7. The value of the expression $\log_2 \left(1 + \frac{1}{2} \sum_{k=1}^{11} {}^{12}C_k \right)$:

A. 11

B. 12

C. 13

D. 14

Answer: A



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8. The constant term in the expansion of $\left(x + \frac{1}{x^3}\right)^{12}$ is :

- A. 26
- B. 169
- C. 260
- D. 220

Answer: D



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9. If $\frac{3}{4!} + \frac{4}{5!} + \frac{5}{6!} + \dots + 50\text{term} = \frac{1}{3!} - \frac{1}{(k+3)!}$, then sum of coefficients in the expansion $(1 + 2x_1 + 3x_2 + \dots + 100x_{100})^k$ is:

- A. $(5050)^{49}$
- B. $(5050)^{51}$
- C. $(5050)^{52}$

D. $(5050)^{50}$

Answer: D

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10. If $\alpha_1, \alpha_2, \dots, \alpha_n$ are the n, n^{th} roots of unity,

$\alpha_r = e^{\frac{i2(r-1)\pi}{n}}, r = 1, 2, \dots, n$ then

${}^nC_1\alpha_1 + {}^nC_2\alpha_2 + \dots + {}^nC_n\alpha_n$ is equal to :

A. $\left(1 + \frac{\alpha_2}{\alpha_1}\right)^n - 1$

B. $\frac{\alpha_1}{2} [(1 + \alpha_1)^n - 1]$

C. $\frac{\alpha_1 + \alpha_{n-1} - 1}{2}$

D. $(\alpha_1 + \alpha_{n-1})^n - 1$

Answer: A

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11. The remainder when $2^{30} \cdot 3^{20}$ is divided by 7 is :

A. 1

B. 2

C. 4

D. 6

Answer: B



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12. ${}^{26}C_0 + {}^{26}C_1 + {}^{26}C_2 + \dots + {}^{26}C_{13}$ is equal to :

A. $2^{25} - \frac{1}{2} \cdot {}^{26}C_{13}$

B. $2^{25} + \frac{1}{2} \cdot {}^{26}C_{13}$

C. 2^{13}

D. $2^{26} + \frac{1}{2} \cdot {}^{26}C_{13}$

Answer: B

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13. If a_r is the coefficient of x^r in the expansion of $(1 + x + x^2)^n$ ($n \in \mathbb{N}$)

. Then the value of $(a_1 + 4a_4 + 7a_7 + 10a_{10} + \dots)$ is equal to :

A. 3^{n-1}

B. 2^n

C. $\frac{1}{3} \cdot 2^n$

D. $n \cdot 3^{n-1}$

Answer: D

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14. Let $\binom{n}{k}$ represents the combination of 'n' things taken 'k' at a time,
then the value of the sum

$\binom{99}{97} + \binom{98}{96} + \binom{97}{95} + \dots + \binom{3}{1} + \binom{2}{0}$ equals:

A. $\binom{99}{97}$

B. $\binom{100}{98}$

C. $\binom{99}{98}$

D. $\binom{100}{97}$

Answer: D



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15. The last digit of $91 + 3^{9966}$ is :

A. 1

B. 3

C. 7

D. 9

Answer: D



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16. Let x be the 7^{th} term from the beginning and y be the 7^{th} term from the end in the expansion of $\left(3^{1/3} + \frac{1}{4^{1/3}}\right)^n$. If $y = 12x$ then the value of n is :

- A. 9
- B. 8
- C. 10
- D. 11

Answer: A



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17. The expression $({}^{10}C_0)^2 - ({}^{10}C_1)^2 + ({}^{10}C_2)^2 - \dots + ({}^{10}C_8)^2 - ({}^{10}C_9)^2 + ({}^{10}C_{10})^2$ equals :

- A. $10!$
- B. $({}^{10}C_5)^2$
- C. $-{}^{10}C_5$
- D. ${}^{10}C_5$

Answer: C



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18. Find the ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(x^2 + \frac{2}{x}\right)^{15}$.

- A. 1 : 4
- B. 1 : 32

C. 7: 64

D. 7: 16

Answer: B



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19. In the expansion of $(1 + x)^2(1 + y)^3(1 + z)^4(1 + w)^5$, the sum of the coefficient of the terms of degree 12 is :

A. 61

B. 71

C. 81

D. 91

Answer: D



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20. If $\sum_{r=0}^n \left(\frac{r^3 + 2r^2 + 3r + 2}{(r + 1)^2} \right) {}^n C_r = \frac{2^4 + 2^3 + 2^2 - 2}{3}$

then the value of n is :

A. 2

B. 2^2

C. 2^3

D. 2^4

Answer: A



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Exercise 2 One Or More Than One Answer Is Are Correct

1. The number $N = {}^{20}C_7 - {}^{20}C_8 + {}^{20}C_9 - {}^{20}C_{10} + \dots - {}^{20}C_{20}$ is not divisible by :

A. 3

B. 4

C. 7

D. 19

Answer: A::B::C::D



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2. If $(1 + x + x^2 + x^3)^{100} = a_0 + a_1x + a_2x^2 + \dots + a_{300}x^{300}$

then which of the following statement(s) is/are correct ?

A. $a_1 = 100$

B. $a_0 + a_1 + a_2 + \dots + a_{300}$ is divisible by 1024

C. coefficients equidistant from beginning and end are equal

D. $a_0 + a_2 + a_4 + \dots + a_{300} = a_1 + a_3 + a_5 + \dots + a_{299}$

Answer: A::B::C::D



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3. $\sum_{r=0}^4 (-1)^{r16} C_r$ is divisible by :

A. 5

B. 7

C. 11

D. 13

Answer: A:D



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4. The expansion of $\left(\sqrt{x} + \frac{1}{2\sqrt[4]{x}}\right)^n$ is arranged in decreasing powers of

x. If coefficient of first three terms from an A.P. then in expansion, the integral powers of x are :

A. 0

B. 2

C. 4

D. 8

Answer: A::C::D



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5. Let $(1 + x^2)^2(1 + x)^n = \sum_{k=0}^{n+4} a_k x^k$. If a_1 , a_2 and a_3 are in arithmetic

progression, then the possible value/values of n is/are a. 5 b. 4 c. 3 d. 2

A. 6

B. 4

C. 3

D. 2

Answer: B::C::D



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6. Evaluate $\sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \binom{n}{i} \binom{n}{j} \binom{n}{k}$, where $\binom{n}{r} = {}^n C_r$: (a) is less than 500 if $n = 3$ (b) is greater than 600 if $n = 3$ (c) is less than 5000 if $n = 4$ (d) is greater than 4000 if $n = 4$

A. is less than 500 if $n = 3$

B. is greater than 600 if $n = 3$

C. is less than 5000 if $n = 4$

D. is greater than 4000 if $n = 4$

Answer: C::D

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7. If ${}^{100}C_6 + 4 \cdot {}^{100}C_7 + 6 \cdot {}^{100}C_8 + 4 \cdot {}^{100}C_9 + {}^{100}C_{10}$ has the value equal to ${}^x C_y$, then the possible value (s) of $x + y$ can be :

A. 112

B. 114

C. 196

D. 198

Answer: B::D



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8. If the co-efficient of x^{2r} is greater than half of the co-efficient of x^{2r+1} in the expansion of $(1 + x)^{15}$, then the possible value of 'r' equal to :

A. 5

B. 6

C. 7

D. 8

Answer: A::B::C



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9. Let $f(x) = 1 + x^{111} + x^{222} + x^{333} \dots \dots \dots + x^{999}$ then $f(x)$ is divisible by

A. $x + 1$

B. x

C. $x - 1$

D. $1 + x^{222} + x^{444} + x^{666} + x^{888}$

Answer: A::D



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Exercise 3 Matching Type Problems

	Column-I	Column-II
(A)	If ${}^{n-1}C_r = (k^2 - 3)^n C_{r+1}$ and $k \in \mathbb{R}^+$, then least value of $5[k]$ is (where $[\]$ represents greatest integer function)	(P) 10
(B)	$\sum_{i=0}^m {}^{20}C_i \cdot {}^{40}C_{m-i}$, where ${}^nC_r = 0$ if $r > n$, is maximum when $\frac{m}{5}$ is	(Q) 5
(C)	Number of non-negative integral solutions of inequation $x + y + z \leq 4$ is	(R) 35
(D)	Let $A = \{1, 2, 3, 4, 5\}$, $f: A \rightarrow A$. The number of onto functions such that $f(x) = x$ for atleast 3 distinct $x \in A$, is not a multiple of	(S) 6 (T) 12

1.



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	Column-I	Column-II
(A)	If the sum of first 84 terms of the series $\frac{4 + \sqrt{3}}{1 + \sqrt{3}} + \frac{8 + \sqrt{15}}{\sqrt{3} + \sqrt{5}} + \frac{12 + \sqrt{35}}{\sqrt{5} + \sqrt{7}} + \dots$ is $549k$, then k is equal to	(P) 3

2.

(B)	If $x, y \in \mathbb{R}$, $x^2 + y^2 - 6x + 8y + 24 = 0$, the greatest value of $\frac{16}{5} \cos^2(\sqrt{x^2 + y^2}) - \frac{24}{5} \sin(\sqrt{x^2 + y^2})$ is	(Q) 2
(C)	If $(\sqrt{3} + 1)^6 + (\sqrt{3} - 1)^6 = 416$, if $xyz = [(\sqrt{3} + 1)^6]$, $x, y, z \in \mathbb{N}$, (where $[\]$ denotes the greatest integer function), then the number of ordered triplets (x, y, z) is	(R) 5
(D)	If $(1+x)(1+x^2)(1+x^4) \dots (1+x^{128}) = \sum_{r=0}^n x^r$, then $\frac{n}{85}$ is equal to	(S) 9



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Exercise 4 Subjective Type Problems

1. The sum of series $3 \cdot {}^{2007}C_0 - 8 \cdot {}^{2007}C_1 + 13 \cdot {}^{2007}C_2 - 18 \cdot {}^{2007}C_3 + \dots$ upto 2008 terms is K, then K is :

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2. In the polynomial $(x - 1)(x^2 - 2)(x^3 - 3)\dots(x^{11} - 11)$, the coefficient of x^{60} is :

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3. If $\sum_{r=0}^{2n} a_r(x - 2)^r = \sum_{r=0}^{2n} b_r(x - 3)^r$ and $a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$.

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4. If $3^{101} - 2^{100}$ is divided by 11, the remainder is

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5. Find the hundred's digit in the co-efficient of x^{17} in the expansion of $(1 + x^5 + x^7)^{20}$.

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6. Let $x = (3\sqrt{6} + 7)^{89}$. If $\{x\}$ denotes the fractional part of 'x' then find the remainder when $x\{x\} + (x\{x\})^2 + (x\{x\})^3$ is divided by 31.

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7. Let $n \in N$, $S_n = \sum_{r=0}^{3n} ({}^{3n}C_r)$ and $T_n = \sum_{r=0}^n ({}^{3n}C_{3r})$. Find $|S_n - 3T_n|$.

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8. Find the sum of possible real values of x for which the sixth term of

$$\left(3^{\log_3 \sqrt{9^{|x-2|}}} + 7^{\frac{1}{5} \log_7 (3^{|x-2|-9})} \right)^7 \text{ equal } 567:$$

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9. Let q be a positive with $q \leq 50$.

If ${}^{98}C_{30} + 2 \cdot {}^{97}C_{30} + 3 \cdot {}^{96}C_{30} + \dots + 68 \cdot {}^{31}C_{30} + 69 \cdot {}^{30}C_{30} = 100q$ the sum

Find the sum of the digits of q .

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10. The remainder when $\left(\sum_{k=1}^5 {}^{20}C_{2k-1} \right)^6$ is divided by 11, is :

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11. Let $a = 3^{\frac{1}{223}} + 1$ and for all $n \geq 3$, let $f(n) = {}^n C_0 a^{n-1} - {}^n C_1 a^{n-2} + {}^n C_2 a^{n-3} - \dots + (-1)^{n-1} {}^n C_{n-1}$.
 $f(2007) + f(2008) = 3^k$ where $k \in \mathbb{N}$, then the value of k is.

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12. In the polynomial $(x-1)(x^2-2)(x^3-3)\dots(x^{11}-11)$, the coefficient of x^{60} is :

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13. Let the sum of all divisors of the form $2^p \cdot 3^q$ (with p, q positive integers) of the number $19^{88} - 1$ be λ . Find the unit digit of λ .

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14. about to only mathematics

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15. Let $1 + \sum_{r=1}^{10} (3^r \cdot {}^{10}C_r + r \cdot {}^{10}C_r) = 2^{10}(\alpha \cdot 4^5 + \beta)$ where $\alpha, \beta \in N$ and $f(x) = x^2 - 2x - k^2 + 1$. If α, β lies between the roots of $f(x) = 0$, then find the smallest positive integral value of k .

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16. Let $S_n = {}^nC_0 {}^nC_1 + {}^nC_1 {}^nC_2 + \dots + {}^nC_{n-1} {}^nC_n$. If $\frac{S_{n+1}}{S_n} = \frac{15}{4}$, find the sum of all possible values of n ($n \in N$)

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