# MATHS

## BOOKS - VK JAISWAL ENGLISH

## COMPLEX NUMBERS

### Exercise 1 Single Choice Problems

**1.** Let $z_1, z_2$ and $z_3$ be three points on $|z| = 1$. If $\theta_1, \theta_2$ and $\theta_3$ be the arguments of $z_1, z_2, z_3$ respectively, then $\cos(\theta_1 - \theta_2) + \cos(\theta_2 - \theta_3) + \cos(\theta_3 - \theta_1)$

A. $\geq -\frac{3}{2}$

B. $\leq -\frac{3}{2}$

C. $\geq \frac{3}{2}$

D. $\leq 2$

**Answer: A**

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**2.** The number of points of intersection of the curves represented by $arg(\pi - 2 - 7i) = \cot^{-1}(2)$ and arg $\left(\frac{z - 5i}{z + 2 - i}\right) = \pm\frac{\pi}{2}$

A. 0

B. 1

C. 2

D. None of these

**Answer: A**

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**3.** Let $z_1$ and $z_2$ be theroots of the equation $z^2 + az + b = 0$ z being compex. Further, assume that the origin $z_1$ and $z_2$ form an equilatrasl triangle then

A. $a^2 = b$

B. $a^2 = 2b$

C. $a^2 = 3b$

D. $a^2 = 4b$

**Answer: C**

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**4.** If $z and w$ are two complex number such that $|zw| = 1 and arg(z) - arg(w) = \frac{\pi}{2}$, then show that $zw = -i$.

A. 1

B. $-1$

C. i

D. $-i$

**Answer: D**

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**5.** If $\omega$ be an imaginary $n^{th}$ root of unity , then $\sum_{r=1}^{n}(ar+b)\omega^{r-1}$ is equal to :

A. $\dfrac{n(n+1)a}{2\omega}$

B. $\dfrac{nb}{1-n}$

C. $\dfrac{na}{\omega-1}$

D. None of these

**Answer: C**

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**6.** If $\alpha, \beta$ are complex numbers then the maximum value of $\dfrac{\alpha\bar{\beta}+\bar{\alpha}\beta}{|\alpha\beta|}$ is equal to :

A. 1

B. 2

C. greater than 2

D. less than 1

**Answer: B**

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**7.** let $\pi_1, \pi_2, \pi_3$ and $z_4$ be the roots of the equation $z^4 + z^3 + 2 = 0$ , then the value of $\prod_{r=1}^{4} (2\pi_r + 1)$ is equal to :

A. 28

B. 29

C. 30

D. 31

**Answer: D**

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**8.** If $\arg\left(\frac{z-6-3i}{z-3-6i}\right)=\frac{\pi}{4}$, then maximum value of |z| :

A. 28

B. 29

C. 30

D. 31

**Answer: B**

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**9.** If $z_1 \neq -z_2$ and $|z_1+z_2|=\left|\frac{1}{z_1}+\frac{1}{z_2}\right|$ then :

A. at least one of $z_1, z_2$ is unimodular

B. both $z_1, z_2$ are unimodular

C. $z_1 . z_2$ is unimodular

D. $z_1 - z_2$ is unimodular

**Answer: C**

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**10.** For a complex number Z, if $|Z - i| \leq 2$ and $Z_1 = 5 + 3i$, then the maximum value of $|iZ + Z_1|$ is (where, $i^2 = -1$)

A. $5 + \sqrt{13}$

B. $5 + \sqrt{2}$

C. 7

D. 8

**Answer: C**

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**11.** If $z_1, z_2, z_3$ are three collinear points in the argand plane such that $|(z_3 - z_1)(z_3 - z_2)| = A^2 and z_2 - z_3 = \frac{\lambda \dot{A}^2}{(z)_1 - (z)_3}$, then value of $|\lambda|$ is

A. $\pm \frac{\pi}{3}$

B. 0

C. $\pm \frac{\pi}{2}$

D. $\pm \frac{\pi}{6}$

**Answer: C**

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**12.** It is given that complex numbers $z_1$ and $z_2$ satisfy $|z_1| = 2$and $|z_2| = 3$. If the included angled of their corresponding vectors is $60^0$ , then find the value of $\left|\frac{z_1 + z_2}{z_1 - z_2}\right|$.

A. 126

B. 119

C. 133

D. 19

**Answer: D**

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**13.** If all the roots of $z^3 + az^2 + bz + c = 0$ are of unit modulus, then (A) $|a| \leq 3$ (B) $|b| \leq 3$ (C) $|c| = 1$ (D) none of these

A. $|a| \leq 3$

B. $|b| \leq 3$

C. $|c| = 1$

D. All of the above

**Answer: D**

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**14.** Let z be a complex number satisfying $\frac{1}{2} \leq |z| \leq 4$ , then sum of greatest and least values of $\left|z + \frac{1}{z}\right|$ is :

A. $\frac{65}{4}$

B. $\frac{65}{16}$

C. $\frac{17}{4}$

D. 17

**Answer: C**

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**15.** if $|z - 2i| \leq \sqrt{2}$ , then the maximum value of |3+i(z-1)| is :

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $2 + \sqrt{2}$

D. $3+2\sqrt{2}$

**Answer: B**

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**16.** Let $x-\frac{1}{x}=(\sqrt{2})i$ where $i=\sqrt{-1}$. Then the value of $x^{2187}-\frac{1}{x^{2187}}$ is :

A. $i\sqrt{2}$

B. $-i\sqrt{2}$

C. $-2$

D. $\frac{i}{\sqrt{2}}$

**Answer: A**

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**17.** If $z = re^{i\theta}$ ( $r > 0$ & $0 \leq \theta < 2\pi$) is a root of the equation $z^8 - z^7 + z^6 - z^5 + z^4 - z^3 + z^2 - z + 1 = 0$ then number of values of $'\theta'$ is : (a) 6 (b) 7 (c) 8 (d) 9

A. 6

B. 7

C. 8

D. 9

**Answer: C**

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**18.** Let P and Q be two points on the circle |w|=r represented by $w_1$ and $w_2$ respectively, then the complex number representing the point of intersection of the tangents of P and Q is :

A. $\frac{w_1 w_2}{2(w_1 + w_2)}$

B. $\dfrac{2w_1\overline{w}_2}{w_1 + w_2}$

C. $\dfrac{2w_1w_2}{w_1 + w_2}$

D. $\dfrac{2\overline{w}_1w_2}{w_1 + w_2}$

**Answer: C**

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**19.** If $z_1, z_2, z_3$ are complex number , such that $|z_1| = 2, |z_2| = 3, |z_3| = 4$ , the maximum value $|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$ is : (a) 58 (b) 29 (c) 87 (d) none of these

A. 58

B. 29

C. 87

D. None of these

**Answer: C**

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**20.** If $Z = \dfrac{7+i}{3+4i}$ , then find $Z^{14}$.

A. $2^7$

B. $(-2)^7$

C. $(2^7)i$

D. $(-2^7)i$

**Answer: C**

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**21.** If |Z-4| + |Z+4|=10 , then the difference between the maximum and the minimum values of |Z| is :

A. 2

B. 3

C. $\sqrt{41}-5$

D. 0

**Answer: A**

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## Exercise 2 One Or More Than One Answer Is Are Correct

**1.** Let $Z_1$ and $Z_2$ are two non-zero complex number such that $|Z_1+Z_2|=|Z_1|=|Z_2|$, then $\frac{Z_1}{Z_2}$ may be :

A. $1+\omega$

B. $1+\omega^2$

C. $\omega$

D. $\omega^2$

**Answer: C::D**

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**2.** If $z_1, z_2, z_3$ are three complex numbers such that $|z_1| = |z_2| = 1$, find the maximum value of $|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 + z_1|^2$

A. If arg $\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ then arg $\left(\frac{z - z_1}{z - z_2}\right) > \frac{\pi}{4}$ where |z| gt 1

B. $|z_1 z_2 + z_2 z_3 + z_3 z_1| = |z_1 + z_2 + z_3|$

C. $lm\left(\frac{(z_1 + z_2)(z_2 + z_3)(z_3 + z_1)}{z_1 .\ z_2 .\ z_3}\right) = 0$

D. If $|z_1 - z_2| = \sqrt{2}|z_1 - z_3| = \sqrt{2}|z_2 - z_3|$, then Re$\left(\frac{z_3 - z_1}{z_3 - z_2}\right) = 0$

**Answer: B::C::D**

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**3.** The triangle formed by the complex numbers z, iz , $i^2 z$ is :

A. equilateral

B. isosceles

C. right angled

D. isosceles but not right angled

**Answer: B::C**

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**4.** if $A(z_1), B(z_2), C(z_3), D(z_4)$ lies on |z|=4 (taken in order) , where $z_1 + z_2 + z_3 + z_4 = 0$ then :

A. Max. area of quadrilateral ABCD=32

B. Max. area of quadrilateral ABCD=16

C. The triangle $\Delta ABC$ is right angled

D. The quadrilateral ABCD is rectangle

**Answer: A::C::D**

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**5.** Let $z_1, z_2$ and $z_3$ be three distinct complex numbers , satisfying $|z_1| = |z_2| = |z_3| = 1$. Which of the following is/are true :

A. If $\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ then arg $\left(\frac{z - z_1}{z - z_2}\right) > \frac{\pi}{4}$ where | z| gt 1

B. $|z_1 z_2 + z_2 z_3 + z_3 z_1| = |z_1 + z_2 + z_3|$

C. $\lim \left(\frac{(z_1 + z_2)(z_2 + z_3)(z_3 + z_1)}{z_1 .\ z_2 .\ z_3}\right) = 0$

D. If $|z_1 - z_2| = \sqrt{2}|z_1 - z_3| = \sqrt{2}|z_2 - z_3|$, then Re $\left(\frac{z_3 - z_1}{z_3 - z_2}\right)$ = 0

**Answer: B::C::D**

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**6.** Let $z_1 = a + ib$ and $z_2 = c + id$ are two complex number such that $|z_1| = r$ and $Re(z_1 z_2) = 0$ . If $w_1 = a + ic$ and $w_2 = b + id,$ then $|w_2| = r$ (b) $|w_2| = r\ Re(w_1 w_2) = 0$ (d) $Im(w_1 w_2) = 0$

A. $Im(w_1\overline{w}_2) = 0$

B. $Im(\overline{w}_1 w_2) = 0$

C. $Im\left(\frac{(z_1+z_2)(z_2+z_3)(z_3+z_1)}{z_1.\,z_2.\,z_3}\right) = 0$

D. Re $\left(\frac{w_1}{w_2}\right) = 0$

**Answer: A::B::C**

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**7.** The solutions of the equation $z^4 + 4iz^3 - 6x^2 - 4iz - I = 0$ represent vertices of a convex polygon in the complex plane. The area of the polygon is :

A. $2^{1/2}$

B. $2^{3/2}$

C. $2^{5/2}$

D. $2^{5/4}$

**Answer: D**

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**8.** Least positive argument of the $4^{th}$ root of the complex number $2 - i\sqrt{12}$is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{12}$

C. $\frac{5\pi}{12}$

D. $\frac{7\pi}{12}$

**Answer: C**

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**9.** Let $\omega$ be the imaginary cube root of unity and $(a + b\omega + c\omega^2)^{2015} = (a + b\omega^2 + c\omega)$ where a,b,c are unequal real numbers . Then the value of $a^2 + b^2 + c^2 - ab - bc - ca$ equals.

A. 0

B. 1

C. 2

D. 3

**Answer: B**

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**10.** Let n be a positive integer and a complex number with unit modulus is a solution of the equation $z^n + z + 1 = 0$ then then the value of n can be :

A. 62

B. 155

C. 221

D. 196

**Answer: A::B::C**

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## Exercise 3 Comprehension Type Problems

**1.** Let f(z) is of the form $\alpha z + \beta$ , where $\alpha, \beta, z$ are complex numbers such that $|\alpha| \ne |\beta|$.f(z) satisfies following properties :

(i)If imaginary part of z is non zero, then $f(z) + \overline{f(z)} = f(\bar{z}) + \overline{f(z)}$

(ii)If real part of of z is zero , then $f(z) + \overline{f(z)} = 0$

(iii)If z is real , then $\overline{f(z)}f(z) > (z+1)^2 \forall z \in R$

$\frac{4x^2}{(f(1) - f(-1))^2} + \frac{y^2}{(f(0))^2} = 1, x, y \in R$, in (x,y) plane will represent :

A. hyperbola

B. circle

C. ellipse

D. pair of line

**Answer: A**

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**2.** Let $z_1$ and $z_2$ be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots $\alpha$ and $\beta$ of $x^2 + z_1 x + z_2 + m = 0$ for some complex number $m$ satisfies $|\alpha - \beta| = 2\sqrt{7}$.

The locus of the complex number $m$ is a curve

A. a square with side 7 and centre (4,5)

B. a circle with radius 7 and centre (4,5)

C. a circle with radius 7 and centre (-4,5)

D. a square with side 7 and centre (-4,5)

**Answer: B**

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**3.** Let $z_1$ and $z_2$ be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots $\alpha$ and $\beta$ of $x^2 + z_1x + z_2 + m = 0$ for some complex number $m$ satisfies $|\alpha - \beta| = 2\sqrt{7}$. The value of $|m|$,

A. $5\sqrt{21}$

B. $5 + \sqrt{23}$

C. $7 + \sqrt{43}$

D. $7 + \sqrt{41}$

**Answer: D**

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**4.** Let $z_1$ and $z_2$ be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots $\alpha$ and $\beta$ of $x^2 + z_1x + z_2 + m = 0$ for some complex number

$m$ satisfies $|\alpha - \beta| = 2\sqrt{7}$.

The locus of the complex number $m$ is a curve

A. $7 - \sqrt{41}$

B. $7 - \sqrt{43}$

C. $5 - \sqrt{23}$

D. $5 + \sqrt{21}$

**Answer: A**

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**5.** Let $z_1 = 3$ and $z_2 = 7$ represent two points A and B respectively on complex plane . Let the curve $C_1$ be the locus of pint P(z) satisfying $|z - z_1|^2 + |z - z_2|^2 = 10$ and the curve $C_2$ be the locus of point P(z) satisfying $|z - z_1|^2 + |z - z_2|^2 = 16$

Least distance between curves $C_1$ and $C_2$ is :

A. 4

B. 3

C. 2

D. 1

**Answer: D**

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**6.** Let $z_1 = 3$ and $z_2 = 7$ represent two points A and B respectively on complex plane . Let the curve $C_1$ be the locus of pint P(z) satisfying $|z - z_1|^2 + |z - z_2|^2 = 10$ and the curve $C_2$ be the locus of point P(z) satisfying $|z - z_1|^2 + |z - z_2|^2 = 16$

The locus of point from which tangents drawn to $C_1$ and $C_2$ are perpendicular , is :

A. |z-5|=4

B. |z-3|=2

C. |z-5|=3

D. |z-5|=$\sqrt{5}$

**Answer: D**

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**7.** In the Argand plane $Z_1, Z_2$ and $Z_3$ are respectively the verticles of an isosceles triangle ABC with AC=BC and $\angle CAB = \theta$. If $I(Z_4)$ is the incentre of triangle , then :

The value of $(Z_4 - Z_1)^2(1 + \cos\theta)\sec\theta$ is :

A. $\left|\frac{(Z_2 - Z_1)(Z_1 - Z_3)}{(Z_4 - Z_1)}\right|$

B. $\left|\frac{(Z_2 - Z_1)(Z_3 - Z_1)}{(Z_4 - Z_1)}\right|$

C. $\left|\frac{(Z_2 - Z_1)(Z_3 - Z_1)}{(Z_4 - Z_1)^2}\right|$

D. $\left|\frac{(Z_2 - Z_1)(Z_3 - Z_1)}{(Z_4 + Z_1)}\right|$

**Answer: C**

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**8.** In the Argand plane $Z_1, Z_2$ and $Z_3$ are respectively the verticles of an isosceles triangle ABC with AC=BC and $\angle CAB = \theta$. If $I(Z_4)$ is the incentre of triangle , then :

The value of $(Z_4 - Z_1)^2 (1 + \cos\theta) \sec\theta$ is :

A. $(Z_2 - Z_1)(Z_3 - Z_1)$

B. $\dfrac{(Z_2 - Z_1)(Z_3 - Z_1)}{Z_4 - Z_1}$

C. $\dfrac{(Z_2 - Z_1)(Z_3 - Z_1)}{(Z_4 - Z_1)^2}$

D. $(Z_2 - Z_1)(Z_3 - Z_1)^2$

**Answer: A**

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## Exercise 4 Matching Type Problems

**1.** Let ABCDEF is a regular hexagon $A(z_1), B(z_2), C(z_3), D(z_4), E(z_5), F(z_6)$ in argand plane where

A,B,C,D,E and F are taken in anticlockwise manner. If $z_1 = -2, z_3 = 1 - \sqrt{3}i$.

| | Column-I | | Column-II |
|---|---|---|---|
| (A) | If $z_2 = a + ib$, then $2a^2 + b^2$ is equal to | (P) | 8 |
| (B) | The square of the inradius of hexagon is | (Q) | 7 |
| (C) | The area of region formed by point $P(z)$ lying inside the incircle of hexagon and satisfying $\frac{\pi}{3} \le \arg(z) \le \frac{5\pi}{6}$ is $\frac{m}{n}\pi$, where $m, n$ are relatively prime natural numbers, then $m + n$ is equal to | (R) | 5 |
| (D) | The value of $z_4^2 - z_1^2 - z_2^2 - z_3^2 - z_5^2 - z_6^2$ is equal to | (S) | 3 |
| | | (T) | 2 |

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**2.** Match the following Column I to Column II

| | Column-I | | Column-II |
|---|---|---|---|
| (A) | Let $\omega$ be a non real cube root of unity then the number of distinct elements in the set $\{(1 + \omega + \omega^2 + \ldots + \omega^n)^m;$ $n, m \in N\}$ is : | (P) | 3 |
| (B) | Let $\omega$ and $\omega^2$ be non real cube root of unity. The least possible degree of a polynomial with real co-efficients having roots $2\omega, (2 + 3\omega), (2 + 3\omega)^2, (2 - \omega - \omega^2)$ is | (Q) | 4 |
| (C) | Let $\alpha = 6 + 4i$ and $\beta = 2 + 4i$ are two complex numbers on Argand plane. A complex number $z$ satisfying amp $\left(\frac{z - \alpha}{z - \beta}\right) = \frac{\pi}{6}$ moves on a major segment of a circle whose radius is | (R) | 5 |
| (D) | Let $z_1, z_2, z_3$ are complex numbers denoting the vertices of an equilateral triangle $ABC$ having circumradius equals to unity. If $P$ denotes any arbitrary point on its circumcircle then the value of $\frac{1}{2}((PA)^2 + (PB)^2 + (PC)^2)$ equals to | (S) | 7 |

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## Exercise 5 Subjective Type Problems

**1.** Let complex number 'z' satisfy the inequality $2 \leq |x| \leq 4$. A point P is selected in this region at random. The probability that argument of P lies in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is $\frac{1}{K}$, then K =

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**2.** Let Z be a complex number satisfying $|Z-1| \leq |Z-3|, |Z-3| \leq |Z-5|, |Z+i| \leq |Z-i|, |Z-i| \leq |Z-5i$. Then area of region in which Z lies is A square units, Where A is equal to :

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**3.** If $z_1 and z_2$ both satisfy $z+\bar{z}r=2|z-1|$ and $arg(z_1-z_2)=\frac{\pi}{4}$ , then find I m(z_1+z_2) .

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**4.** If $|z_1|=1, |z_2|=2, |z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=36,$ then $|z_1+z_2+z_3|$ is equal to ______.

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**5.** If $|z_1|$ and $|z_2|$ are the distance of points on the curve $5z\bar{z}-2i(z^2-\bar{z}^2)-9=0$ which are at maximum and minimum distance from the origin, then the value of $|z_1|+|z_2|$ is equal to :

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**6.** Let $\dfrac{1}{a_1+\omega}+\dfrac{1}{a_2+\omega}+\dfrac{1}{a_3+\omega}+\ldots.+\dfrac{1}{a_n+\omega}=i$

where $a_1, a_2, a_3$ .... $a_n \in R$ and $\omega$ is imaginary cube root of unity , then evaluate $\sum_{r=1}^{n} \dfrac{2a_r-1}{a_r^2-a_r+1}$.

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**7.** If $|z_1|=2, |z_2|=3, |z_3|=4$ and $|2z_1+3z_2+4z_3|=9$ , then value of $|8z_2z_3+27z_3z_1+64z_1z_2|^{1/3}$ is :

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**8.** The sum of maximum and minimum modulus of a complex number z satisfying $|z-25i| \leq 15, i=\sqrt{-1}$ is S , then $\dfrac{S}{10}$ is :

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