MATHS

BOOKS - VK JAISWAL ENGLISH

CONTINUITY, DIFFERENTIABILITY AND DIFFERENTIATION

Exercise Single Choice Problems

1. Let 'f' be a fifferentiable real valued function satisfying $f(x+2y)=f(x)+f(2y)+6xy(x+2y)\,\forall x, y \in R.$ Then $f'(0), f(1), f'(2)\ldots..$ are in

A. AP

B. GP

C. HP

D. None of these

Answer: A

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2. Find the number of points of non-diferentiability for $f(x) = \max\{||x| - 1|, 1/2\}$.

A. 4

B. 3

C. 2

D. 5

Answer: D

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3. Number of points of discontinuity of $f(x) - \left\{\frac{x}{5}\right\} + \left[\frac{x}{2}\right]$ in $x \in [0, 100]$ is/are (where [.] denotes greatest integer function and {.}

denotes fractionalpart function)

A. 50

B. 51

C. 52

D. 61

Answer: A

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4. If $f(x)$ has isolated point of discontinuity at $x = a$ such that $|f(x)|$ is continuous at $x = a$ then :

A. $\lim_{x \to a} f(x)$ doesn not exist

B. $\lim_{x \to a} f(x) + f(a) = 0$

C. $f(a) = 0$

D. None of these

Answer: B

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5. If $f(x)$ is a thrice differentiable function such that $\lim_{x \to 0} \frac{f(4x) - 3f(3x) + 3f(2x) - f(x)}{x^3} = 12$ then the vlaue of $f'(0)$ equais to :

A. 0

B. 1

C. 12

D. None of these

Answer: C

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6.

$$y = \frac{1}{(1+\tan\theta)^{\sin\theta-\cos\theta} + (\cot\theta)^{\cos\theta-\cot\theta}} + \frac{1}{(1+\tan\theta)^{\cos\theta-\sin\theta} + (\cot\theta$$

then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$ is

A. 0

B. 1

C. $\sqrt{3}$

D. None of these

Answer: A

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7. Let $f'(x) = \sin(x^2)$ and $y = f(x^2+1)$ then $\frac{dy}{dx}$ at $x = 1$ is

A. $2\sin 2$

B. $2\cos 2$

C. $2\sin 4$

D. $\cos 2$

Answer: C

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8. If $f(x) = |\sin x - |\cos x|\,|$, then $f'\left(\frac{7\pi}{6}\right) =$

A. $\frac{\sqrt{3}+1}{2}$

B. $\frac{1-\sqrt{3}}{2}$

C. $\frac{\sqrt{3}-1}{2}$

D. $\frac{-1-\sqrt{3}}{2}$

Answer: C

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9. If $2\sin x.\cos y = 1$, then $\frac{d^2y}{dx^2} at \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is .....

A. -4

B. -2

C. -6

D. 0

Answer: A

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10. f is a differentiable function such that $x = f(t^2), y = f(t^3)$ and $f'(1) \neq 0$ if $\left(\frac{d^2y}{dx^2}\right)_{t=1}$ =

A. $\frac{3}{4}\left(\frac{f''(1) + f(1)}{(f'(1))^2}\right)$

B. $\frac{3}{4}\left(\frac{f(1).f'(1) - f'(1)}{f(f'(1))^2}\right)$

C. $\frac{4}{3}\frac{f'(1)}{(f'(1))^2}$

D. $\frac{4}{3}\left(\frac{f'(1)f''(1) - f'(1)}{(f'(1))^2}\right)$

Answer: A

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11. Let $f(x) = \begin{cases} ax+1 & \text{if} \quad x<1 \\ 3 & \text{if} \quad x=1 \\ bx^2+1 & \text{if} \quad x>1 \end{cases} Iff(x)$ is continuous at $x=1$ then (a-b) is equal to :

A. 0

B. 1

C. 2

D. 4

Answer: A

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12. If

$$y = 1 + \frac{\alpha}{\left(\frac{1}{x}-\alpha\right)} + \frac{\beta/x}{\left(\frac{1}{x}-\alpha\right)\left(\frac{1}{x}-\beta\right)} + \frac{\gamma/x^2}{\left(\frac{1}{x}-\alpha\right)\left(\frac{1}{x}-\beta\right)\left(\frac{1}{x}-\gamma\right)},$$

find $\frac{dy}{dx}$

A. $y\left(\frac{\alpha}{1/x-\alpha}+\frac{\beta}{1/x-\beta}+\frac{\gamma}{1/x-\gamma}\right)$

B. $\frac{y}{x}\left(\frac{\alpha}{1/x-\alpha}+\frac{\beta}{1/x-\beta}+\frac{\gamma}{1/x-\gamma}\right)$

C. $y\left(\frac{\alpha}{1/x-\alpha}+\frac{\beta}{1/x-\beta}+\frac{\gamma}{1/x-\gamma}\right)$

D. $y\left(\frac{\alpha/x}{1/x-\alpha}+\frac{\beta/x}{1/x-\beta}+\frac{\gamma/x}{1/x-\gamma}\right)$

Answer: B

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13. If $f(x)=\sqrt{\frac{1+\sin^{-1}x}{1-\tan^{-1}x}}$, then f (0) is equal to :

A. 4

B. 3

C. 2

D. 1

Answer: D

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14. Let $f(x) = \begin{cases} \sin^2 x, & \text{x is rational} \\ -\sin^2 x, & \text{x is irrational} \end{cases}$, then set of points, where f (x) is continuous, is:

A. $\left\{(2n+1)\frac{\pi}{2} \in I\right\}$

B. a null set

C. $\{n\pi, n \in I\}$

D. set of all rational numbers

Answer: C

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15. The number of values of x in $(0, 2\pi)$ where the function $f(x) = \frac{\tan x + \cot x}{2} - \left|\frac{\tan x - \cot x}{2}\right|$ continuous but non-derivable :

A. 3

B. 4

C. 0

D. 1

Answer: B

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16. If $f(x) = |x - 1|$ and $g(x) = f(f(f(x)))$, then for $x > 2, g'(x)$ is equal to

A. 1 for $x > 2$

B. 1 for $2 < x < 3$

C. -1 for $2 < x < 3$

D. -1 for $x > 3$

Answer: C

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17. If $f(x)$ is continuous function $\forall x \in R$ and the range of $f(x) is (2, \sqrt{26})$ and $g(x) = \left[\frac{f(x)}{c}\right]$ is continuous $\forall x \in R$, then find the least positive integral value of c, where $[.]$ denotes the greatest integer function.

A. 3

B. 5

C. 6

D. 7

Answer: C

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18. If $y = x + e^x$, then $\left(\frac{d^2y}{dy^2}\right)_{x=\ln 2}$ is :

A. $-\frac{1}{9}$

B. $-\frac{2}{27}$

C. $\frac{2}{27}$

D. $\frac{1}{9}$

Answer: B

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19. Let $f(x) = x^3 + 4x^2 + 6x$ and $g(x)$ be inverse then the vlaue of $g'(-4)$:

A. -2

B. 2

C. $\frac{1}{2}$

D. None of these

Answer: C

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20. If $f(x) = 2 + |x| - |x-1| - |x+1|$, then $f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) + f'\left(\frac{5}{2}\right)$ is equal to:

A. 1

B. -1

C. 2

D. -2

Answer: D

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21. If $f(x) = \cos\left(x^2 - 4[x]\right), 0 < x < 1,$ (whre [.] denotes greatest integer function) then $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is equal to:

A. $-\sqrt{\frac{\pi}{2}}$

B. $\sqrt{\frac{\pi}{2}}$

C. 0

D. $\sqrt{\frac{\pi}{4}}$

Answer: A

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22. Let g (x) be then inverse of f (x) such that $f'(x) = \frac{1}{1+x^5}$, then $\frac{d^2(g(x))}{dx^2}$ is equal to:

A. $\frac{1}{1+(g(x))^5}$

B. $\frac{g'(x)}{1+(g(x))^5}$

C. $f(g(x))^4(1+g(x))^5)$

D. $1+(g(x))^5$

Answer: C

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23. Let $f(x) = \begin{cases} \min(x, x^2) & x \geq 0 \\ \max(2x, x-1) & x < 0 \end{cases}$, then which of the following is not true ?

A. f (x) is not differentiable at $x = 0$

B. f (x) is not differentiable are exactly two points

C. f (x) is continous everywhere

D. f (x) is strictly increasing $\forall x \in R$

Answer: B

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24. if $f(x) = \lim_{x \to \infty} \left(\prod_{i=l}^{n} \cos\left(\frac{x}{2^l}\right) \right)$ then f '(x) is equal to:

A. $\frac{\sin x}{x}$

B. $\frac{x}{\sin x}$

C. $\frac{x\cos - \sin x}{x^2}$

D. $\frac{\sin x - x\cos x}{\sin^2 x}$

Answer: C

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25. Let $f(x) = \begin{cases} \frac{1-\tan x}{4x-\pi} & x \neq \frac{\pi}{4} \\ \lambda & x = \frac{\pi}{4} \end{cases}, x \in \left[0, \frac{\pi}{2}\right)$, If f (x) is continuous in $\left[0, \frac{\pi}{2}\right)$ then λ is equal to:

A. 1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -1

Answer: C

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26. Let $f(x) = \begin{cases} e^{\frac{1}{x^2}} \sin\frac{1}{x} & x \neq 0 \\ \lambda & x = \frac{\pi}{4} \end{cases}, then f'(0)$

A. 1

B. -1

C. 0

D. Does not exist

Answer: C

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27. Let f be a differentiable function satisfying $f'(x) = 2f(x) + 10 \forall x \in R$ and $f(0) = 0,$ then the number of real roots of the equation $f(x) + 5\sec^2 x = 0$ ln $(0, 2\pi)$ is:

A. 0

B. 1

C. 2

D. 3

Answer: A

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28. If $f(x) = \begin{cases} \frac{\sin\{\cos x\}}{x - \frac{\pi}{2}} & x \neq \frac{\pi}{2} \\ 1 & x = \frac{\pi}{2} \end{cases}$, where {k} represents the fractional park of k, then:

A. f (x) is continous at $x = \frac{\pi}{2}$

B. $\lim_{x \to \frac{\pi}{2}}$ f (x) does not exist

C. $\lim_{x \to \frac{\pi}{2}} f(x)$ exists, but f is not continuous at $x = \frac{\pi}{2}$

D. $\lim_{x \to \frac{\pi}{2}} f(x) = 1$

Answer: B

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29. Let f (x) be a polynomial in x. The second derivative of $f(e^x)$ w.r.t. x is :

A. $f''(e^x)e^x + f(e^x)$

B. $f''(e^x)e^{2x} + f'(e^x)e^{2n}$

C. $f''(e^x)e^x + f'(e^x)e^{2x}$

D. $f''(e^x)e^{2x} + e^x f'(e^x)$

Answer: D

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30. If $e^{f(x)} = \log x$ and g(x) is the inverse function of f(x), then $g'(x)$ is

A. $e^x + x$

B. $e^{e^{e^x}} e^{e^x}$

C. $e^{e^x + z}$

D. e^{e^x}

Answer: C

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31. If $y = f(x)$ is differentiable $\forall x \in R,$ then

A. $y = |f(x)|$ is differentiable $\forall x \in R$

B. $y = f^2(x)$ is not-differentiable for atleast one x

C. $y = f(x)|f(x)|$ is non-differentiable for atleast one x

D. $y = |f(x)|^3$ is differentiable $Ax \in R$

Answer: D

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32. If $f(x) = (x-1)^4(x-2)^3(x-3)^2$ then the value of $f'(1) + f''(2) + f''(3)$ is:

A. 0

B. 1

C. 2

D. 6

Answer: A

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33. Indicate all correct alternatives: if $f(x) = \frac{x}{2} - 1$, then on the interval $[0, \pi]$:

A. $\tan(f(x))$ and $\frac{1}{f(x)}$ are both continous

B. $\tan(f(x))$ and $\frac{1}{f(x)}$ are both discontinous

C. $tna(f(x))$ and $f^{-1}(x)$ are both continous

D. $\tan f(x)$ is continous but $f^{-1}(x)$ is not

Answer: C

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34. Let f (x)= $\begin{cases} \frac{\left(e^{\frac{1}{x-2}}-3\right)}{\frac{1}{3^{x-2}}+1} & x>2 \\ \frac{b\sin\{-x\}}{\{-x\}} & x<2 \\ c & x=2 \end{cases}$, where {.} denotes fraction part function, is continuous at $x=2,$ then $b+c=$

A. 0

B. 1

C. 2

D. 4

Answer: A

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35. Let $f(x)=\frac{e^{\tan x}-e^{x}+\ln(\sec x+\tan x)-x}{\tan x-x}$ be a continous function at $x=0.$ The value of $f(0)$ equals:

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 2

Answer: C

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36. Let $f(x) = \begin{cases} (1+ax)^{1/x} & x < 0 \\ \frac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1} & x > 0 \end{cases}$, is continous at $x = 0$, then $3(e^a + b + c)$ is equal to:

A. 3

B. 6

C. 7

D. 8

Answer: C

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37. If $\sqrt{x+y}+\sqrt{y-x}=5$, then $\frac{d^2y}{dx^2}=$

A. $\frac{2}{5}$

B. $\frac{4}{25}$

C. $\frac{2}{25}$

D. $\frac{1}{25}$

Answer: C

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38. If $f(x)=x^2+x^4+\log x$ and g is the inverse of f, then $g'(2)$ is:

A. 8

B. $\frac{1}{8}$

C. 2

D. $\frac{1}{4}$

Answer: B

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39. The number of points at which the function,

$$f(x) = \begin{cases} \min\{|x|, x^2\} \\ \min(2x-1, x^2\} \end{cases} \quad \text{if} \quad x \in (-\infty, 1) \quad \text{otherwise} \quad \text{is} \quad \text{not}$$

differentiable is:

A. 0

B. 1

C. 2

D. 3

Answer: B

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40. If $f(x)$ is a function such that $f(x) + f''(x) = 0$ and $g(x) = (f(x))^2 + (f'(x))^2$ and $g(3) = 8$, then $g(8) =$

A. 0

B. 3

C. 5

D. 8

Answer: D

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41. Let f is twice differerntiable on R such that $f(0) = 1, f''(0) = 0$ and $f'(0) = -1$, then for

$a \in R, \lim_{x \to \infty} \left(f\left(\frac{a}{\sqrt{x}} \right) \right)^x =$

A. e^{-e^2}

B. $e^{\frac{a^2}{4}}$

C. $e^{\frac{a^2}{2}}$

D. e^{-2a^2}

Answer: C

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42. Let $f_1(x) = e^x$ and $f_{n+1}(x) = e^{f_n(x)}$ for any $n \geq 1, n \in N$. Then for any fixed n, the vlaue of $\frac{d}{dx} f)(n)(x)$ equals:

A. $f_n(x)$

B. $f_n(x) f_{n-1}(x) \ldots . f_2(x) f_2(x)$

C. $f_n(x) f_{n-1}(x)$

D. $f_n(x) f_{n-1}(x) \ldots . f_2(x) f_1(x) e^x$

Answer: B

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43. If $y = \tan^{-1}\left(\frac{x^{1/3} - a^{1/3}}{1 + x^{1/3}a^{1/3}}\right), x > 0, a > 0, then \frac{dy}{dx}$ is:

A. $\frac{1}{x^{2/3}\left(1 + x^{2/3}\right)}$

B. $\frac{3}{x^{2/3}\left(1 + x^{2/3}\right)}$

C. $\frac{1}{3x^{2/3}\left(1 + x^{2/3}\right)}$

D. $\frac{1}{3x^{1/3}\left(1 + x^{2/3}\right)}$

Answer: C

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44. The value of $k + f(0)$ so that $f(x) = \begin{cases} \frac{\sin(4k-1)x}{3x}, & x < 0 \\ \frac{\tan(4k+1)x}{5x}, & 0 < x < \frac{\pi}{2} \\ 1, & x = 0 \end{cases}$

can bemade continous at $x = 0$ is:

A. 1

B. 2

C. `-1

D. `-2

Answer: B

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45. If $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right), |x| \leq 1$, then $\frac{dy}{dx} at \left(\frac{1}{2}\right)$ is:

A. $\frac{1}{\sqrt{3}}$

B. 3

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2}{\sqrt{3}}$

Answer: A

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46. Let $f(x) = \frac{e^x x \cos x - x \log_e(1+x)x}{x^2}, x \neq 0 If f(x)$ is continous at $x = 0,$ then f (0) is equal to:

A. 0

B. 1

C. -1

D. 2

Answer: A

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47. A function $f(x) = \max(\sin x, \cos x, 1 - \cos x)$ is non-derivable for n values of $x \in [0, 2\pi]$. Then the vaue of n is:

A. 2

B. 1

C. 3

D. 4

Answer: C

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48. Let g be the inverse function of a differentiable function f and $G(x) = \frac{1}{g(x)}$. If $f(4) = 2$ and $f'(4) = \frac{1}{16}$, then the value of $(G'(2))^2$ equals to:

A. 1

B. 4

C. 16

D. 64

Answer: A

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49. If $f(x) = \max . \left(x^4, x^2, \frac{1}{81}\right) \forall x \in [0, \infty)$, then the sum of square of reciprocal of all the values of x where f (x) is non-differentiable, is equal to:

A. 1

B. 81

C. 82

D. $\frac{82}{81}$

Answer: C

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50. If $f(x)$ is derivable at $x = 2$ such that $f(2) = 2$ and $f'(2) = 4$, then the value of $\lim_{h \to 0} \frac{1}{h^2}\left(\ln f\left(2 + h^2\right) - \ln f\left(2 - h^2\right)\right)$ is equal to

A. 1

B. 2

C. 3

D. 4

Answer: D

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51. Let $f(x) = \left(x^2 - 3x + 2\right)\left|\left[x^3 - 6x^2 + 11x - 6\right)\right| + \left|\sin\left(x + \frac{\pi}{4}\right)\right|$. Number of points at which the function f (x) is non-differentiable in $[0, 2\pi]$, is:

A. 5

B. 4

C. 3

D. 2

Answer: C

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52. Let f and g be differrntiable functions on R (the set of all real numbers) such that $g(1) = 2 = g'(1)$ and $f'(0) = 4$. $If h(x) = f(2xg(x) + \cos \pi x - 3)$ then $h'(1)$ is equal to:

A. 28

B. 24

C. 32

D. 18

Answer: C

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53. If $f(x) = \frac{(x+1)^7\sqrt{1+x^2}}{(x^2-x+1)^6}$, then the value of $f'(0)$ is equal to:

A. 10

B. 11

C. 13

D. 15

Answer: C

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54. Statement.1 : The function $f(x) = \lim_{n\to\infty} \frac{\log_e(1+x) - x^{2n}\sin(2x)}{1+x^{2n}}$

is discontinuous at $x = 1$

Statement.2: $L.H.L. = R.H.L. \neq f(1).$

A. Statent:1 is ture , Statement:2 is true and Statement:2 is correct explanation for Statement-1

B. Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation for Statemetn-1

C. Statement-1 is true, Statement-2 is false

D. Statement-1 is false, Statement-2 is true

Answer: C

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55. If $f(x) = \begin{bmatrix} x & \text{if x is rational} \\ 1-x & \text{if x is irrational} \end{bmatrix}$, then number of points for $x \in R$, where $y = f(f(x))$ discontinous is:

A. 0

B. 1

C. 2

D. Infinitely many

Answer: A

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56. Number of points where $f(x) = \begin{cases} \max\left(|x^2 - x - 2, x^2 - 3x\right) & x \geq 0 \\ \max\left(\ln(-x), e^x\right) & x < 0 \end{cases}$ is non-differentiable will be:

A. 1

B. 2

C. 3

D. None of these

Answer: C

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57. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ and $g(x) = f^{-1}(x)$, then the value of $g'\left(\frac{-7}{6}\right)$ equals :

A. $\frac{1}{5}$

B. $-\frac{1}{5}$

C. $\frac{6}{7}$

D. $-\frac{6}{7}$

Answer: A

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58. Find k, if possible, so that

$$f(x) = \begin{cases} \frac{\ln(2-\cos 2x)}{\ln^2(1+\sin 3x)} & x < 0 \\ k & x = 0 \\ \frac{e^{\sin 2x}-1}{\ln(1+\tan 9x)} & x > 0 \end{cases}$$ is continious at $x = 0$.

A. $\frac{2}{3}$

B. $\frac{1}{9}$

C. $\frac{2}{9}$

D. not possible

Answer: C

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59. A function is reprersented parametrically by the equations $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t}\dot{T}henthevalueof\left|\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3\right|$ is_____

A. 2

B. 0

C. -1

D. -2

Answer: C

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60. If $y^{-2} = 1 + 2\sqrt{2}\cos 2x,$ then :

$\frac{d^2y}{dx^2} = y(py^2 + 1)(qy^2 - 1)$ then the vlaue of $(p + q)$ equals to:

A. 7

B. 8

C. 9

D. 10

Answer: D

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61. If $y^{-2} = 1 + 2\sqrt{2}\cos 2x,$ then :

$\frac{d^2y}{dx^2} = y(py^2 + 1)(qy^2 - 1)$ then the vlaue of $(p + q)$ equals to:

A. 7

B. 8

C. 9

D. 15

Answer: B

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62. Let $f: R \to R$ is not identically zero, differentiable function and satisfy the equals $d(xy) = f(x)f(y)$ and $f(x+z) + f(x) + f(a)$, then $f(5) =$

A. 3

B. 5

C. 10

D. 15

Answer: B

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63. If $y\left(x+\sqrt{1+x^2}\right)^n$ then $\left(1+x^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is:

A. n^2y

B. y^{-n^2}

C. $-y$

D. $2x^2y$

Answer: A

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64. Let $g(x)=f\left(x-\sqrt{1-x^2}\right)$ and $f'(x)=1-x^2$ then $g'(x)$ equal to:

A. $1-x^2$

B. $\sqrt{1-x^2}$

C. $2x\left(x+\sqrt{x-x^2}\right)$

D. $2x\left(x-\sqrt{1-x^2}\right)$

Answer: C

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65. Let $f(x)=\lim_{n\to\infty}\frac{\log(2+x)-x^{2n}\sin x}{1+x^{2n}}$. then

A. f (x) is continuous at $x=1$

B. $\lim_{x\to 1} f(x)=\log,3$

C. $\lim_{x\to 1^+} f(x)=-\sin 1$

D. $\lim_{x\to 1^+} f(x)$ does not exist

Answer: C

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66. Let $f(x+y)=f(x)f(y)$ for all x and y, and $f(5)=2, f'(0)=3,$ then $f'(5)$ is equal to:

A. 3

B. 1

C. -6

D. 6

Answer: C

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67. Let $f(x)\ \lim_{x\to\infty}\frac{\log,(2+x)-x^{2x}\sin x}{1+x^{2n}}$ then:

A. f (x) continuous at $x=1$

B. $\lim_{x\to 1^-} f(x)=\log_e 3$

C. $\lim_{x\to 1^+} f(x)=-\sin 1$

D. $\lim_{x \to 1^-} f(x)$ does not exist

Answer: C

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68. If $f(x) = \begin{cases} \frac{x - e^x + 1 - (1 - \cos 2x)}{x^2} & x \neq 0 \\ k & x = 0 \end{cases}$ is continous at $x = 0$ then which of the following statement is false ?

A. $k = \frac{-5}{2}$

B. $\{k\} = \frac{1}{2}$

C. $[k] = -2$

D. $[k]\{k\} = \frac{-3}{2}$

Answer: C

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69. Let $f(x) = \left||x^2 - 10x + 21| - p\right|$, then the exhausive set of values of for which f (x) has exactly 6 points of non-derivability, is:

A. $(4, \infty)$

B. $(0, 4)$

C. $[0, 4]$

D. $(-4, 4)$

Answer: B

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70. If $f(x) = \sqrt{\frac{1 + \sin^{-1} x}{1 - \tan^{-1} x}}$, then $f'(0)$ is equal to:

A. 4

B. 3

C. 2

D. 1

Answer: D

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71. For $t \in (0, 1), Let x = \sqrt{2^{\sin^{-1}(t)}}$ and $y = \sqrt{2^{\cos^{-1}(t)}}$ then $1 + \left(\frac{dy}{dx}\right)^2$ equals :

A. $\frac{x^2}{y^2}$

B. $\frac{y^2}{x^2}$

C. $\frac{x^2 + y^2}{y^2}$

D. $\frac{x^2 + y^2}{x^2}$

Answer: D

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72. Let $f(x) = -1 + |x-2|$ and $g(x) = 1 - |x|$ then set of all possible value (s) of for which (fog) (x) is discontinuous is:

A. $\{0, 1, 2\}$

B. $\{0, 2\}$

C. $\{0\}$

D. an empty set

Answer: D

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73. If $f(x) = [x]\tan(\pi x)$ then $f'(k')$ is equal to $(k \in$ and $[.]$ denotes greatest integer function):

A. $(k-1)\pi(-1)^k$

B. $k\pi$

C. $k\pi(-1)^{k+1}$

D. $(k-1)\pi(-1)^{k-1}$

Answer: B

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74. If $f(x) = \begin{cases} \frac{ae^{\sin x} + be^{-\sin x} - c}{x^2} & x \neq 0 \\ 2 & x = 0 \end{cases}$ is continous at $x = 0$, then:

A. $a = b = c$

B. $a = 2b = 3c$

C. $a = b = 2c$

D. $2a = 2b = c$

Answer: D

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75. If tan x cot $y = \sec\alpha$ where α is constant and $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) then \frac{d^2y}{dx^2} at \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ equal to:

A. 0

B. 1

C. 2

D. 3

Answer: A

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76. If $y = (x-3)(x-2)(x-1) \times (x+1)(x+2)(x+3),$ then $\frac{d^2y}{dx^2} at x = 1$ is:

A. -101

B. 48

C. 56

D. 190

Answer: C

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77. Leg $f(x+y)=f(x)+f(y)f$ or $allx, y \in R,$ If $f(x)iscont \in ueatx=0, showthatf(x)$ is continuous at all $x\cdot$

A. all natural numbers only

B. all integers only

C. all rational numbers only

D. all real numbers

Answer: D

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78. If

$f(x) = 3x^9 - 2x^4 + 2x^3 - 3x^2 + x + \cos x + 5$ and $g(x) = f^{-1}(x)$,

then the value of $g'(6)$ equals:

A. 1

B. $\frac{1}{2}$

C. 2

D. 3

Answer: A

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79. about to only mathematics

A. $\frac{d'f'' - f'g''}{(g')^2}$

B. $\frac{g'f'' - f'g''}{(g')^3}$

C. $\frac{fg'' - g'f''}{(g')^3}$

D. None of these

Answer: B

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80. Let $f(x) = \begin{bmatrix} x+1 & x<0 \\ (x-1) & x \geq 0 \end{bmatrix}$ and $g(x) = \begin{bmatrix} x+1 & x<0 \\ (x-1)^2 & x \geq 0 \end{bmatrix}$ then the number of points where $g(f(x))$ is not differentiable.

A. 0

B. 1

C. 2

D. None of these

Answer: C

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81. f(x)=[sinx]+[cosx] ,$x\varepsilon[0, 2\pi]$, where[.] denotes the greatest integer function. Total number of point where f (x) is non-differentiable is equal to

A. 2

B. 3

C. 4

D. 5

Answer: D

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82. Let $f(x) = \cos x, g(x) = \begin{cases} \min\{f(t): 0 \le t \le x\}, & x \in [0,\pi] \\ (\sin x) - 1, & x > \pi \end{cases}$

Then

A. g (x) is discontinuous at $x = \pi$

B. $g(x)$ is continous for $x \in [0, \infty)$

C. $g(x)$ is differentiable at $x = \pi$

D. $g(x)$ is differentiable for $x \in [0, \infty)$

Answer: B

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83. If $f(x) = (4 + x)^n, n \in N$ and $f'(0)$ represents then r^{th} derivative of $f(x) at x = 0,$ then the value of $\sum_{r=0}^{\infty} \frac{f'(0)}{r!}$ is equal to :

A. 2^n

B. 3^n

C. 5^n

D. 4^n

Answer: C

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84. Let $f(x) = \begin{cases} \frac{x}{1+|x|}, & |x| \geq 1 \\ \frac{x}{1-|x|}, & |x| < 1 \end{cases}$, then domain of $f'(x)$ is:

A. $(-\infty, \infty)$

B. $(-\infty, \infty) - \{-1, 0, 1\}$

C. $(-\infty, \infty) - \{-1, 1\}$

D. $(-\infty, \infty) - \{0\}$

Answer: C

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85. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ and $g(x) = f^{-1}(x)$, then the value of $g'\left(\frac{-7}{6}\right)$ equals :

A. $\frac{1}{5}$

B. $-\frac{1}{5}$

C. $\frac{6}{7}$

D. $-\frac{6}{7}$

Answer: A

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86. The number of points at which the function $f(x) = (x - |x|)^2(1 - x + |x|)^2$ is not differentiable in the interval $(-3, 4)$ is ___

A. Zero

B. One

C. Two

D. Three

Answer: A

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87. If $f(x) = \sqrt{\frac{1+\sin^{-1}x}{1-\tan^{-1}x}}$, then $f'(0)$ is equal to:

A. 4

B. 3

C. 2

D. 1

Answer: D

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88. Use the following table and the fact that $f(x)$ is invertible and differentiable everywhere to find $\left(f^{-1}(3)\right)'$:

x	$f(x)$	$f'(x)$
3	1	7
6	2	10
9	3	5

A. 0

B. $\frac{1}{5}$

C. $\frac{1}{10}$

D. $\frac{1}{7}$

Answer: B

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89. Let $f(x) = \begin{cases} x^n\left(\sin\frac{1}{x},\right) & x \neq 0 \\ 0, & x = 0 \end{cases}$ Such that f (x) is continuous at $x = 0$, $f'(0)$ is real and finite, and $\lim_{x \to 0^+} f'(x)$ does not exist. The holds true for which of the following values of n ?

A. 0

B. 1

C. 2

D. 3

Answer: C

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Exercise One Or More Than One Answer Is Are Correct

1. If $f(x) = \tan^{-1}\left(sgn\left(x^2 - \lambda x + 1\right)\right)$ has exactly one point of discontinuity, then the value of λ can be:

A. 1

B. -1

C. 2

D. -2

Answer: C::D

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2. $f(x) = \begin{cases} 2(x+1) & x \leq -1 \\ \sqrt{1-x^2} & -1 < x < 1 \\ |||x|-1|-1| & x \geq 1 \end{cases}$, then:

A. f (x) is non-differentialbe at exactly three points

B. f (x) is continous in $(-\infty, 1]$

C. $f(x)$ is differentiable in $(-\infty, -1]$

D. f (x) is finite type of discontinuity at $x = 1$, but continous at $x = -1$

Answer: A::C::D

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3. Let $f(x) = \begin{cases} \frac{x\left(3e^{1/x}+4\right)}{2-e^{1/x}} & (x \neq 0) \; x \neq \frac{1}{\ln 2} \\ 0 & x = 0 \end{cases}$ which of the following statement (s) is/are correct ?

A. f (x) is continous $atx = 0$

B. $f(x)$ is non-dervable at $x = 0$

C. $f'\left(0^+\right) = -3$

D. $f'\left(0^-\right)$ does not exist

Answer: A::B::C

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4. Let $|f(x)| \leq \sin^2 x,\ \forall x \in R,$ then

A. f (x) is continous at $x = 0$

B. f (x) is differentiable at $x = 0$

C. $f(x)$ is continous but not differentiable at $x = 0$

D. $f(0) = 0$

Answer: A::B::D

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5. Let $f(x) = \begin{cases} \frac{a(1 - x\sin x) + b\cos x + 5}{x^2} & x < 0 \\ \left(1 + \left(\frac{dx + dx^3}{dx^2}\right)\right)^{\frac{1}{x}} & x > 0 \end{cases}$

If f is continous at $x = 0$ then correct statement (s) is/are:

A. $a + c = -1$

B. $b + x = -4$

C. $a + b = -5$

D. $c + d = an$ irrational number

Answer: A::B::C::D

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6. If $f(x) = |||x| - 2| + p|$ have more than 3 points non-derivability then the value of p can be :

A. 0

B. -1

C. -2

D. 2

Answer: B::C

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7. Identify the options having correct statement:

A. $f(x) = \sqrt[3]{x^2|x|} - 1|x|$ is nowhere non-differentiable

B. $\lim_{x \to \infty} (x+5)\tan^{-1}(x+1)\Big) - \big((x+1)\tan^{-1}(x+1)\big) = 2\pi$

C. $f(x) = \sin\left(\ln\left(x + \sqrt{x^2+1}\right)\right)$ is an odd function

D. $f(x) = \dfrac{4 - x^2}{4x - x^3}$ is discontinous at exactly one point

Answer: A::B::C

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8. A twice differentiable function f(x) is denined for all real numbers and satisfies the following conditions : $f(0) = 2, f'(0) = -5$ and $f''(0) = 3$. The function g(x) is defined by $g(x) = e^{ax} + f(x)\, \forall x \in R$, where 'a' is any constant. If $g'(0) + g''(0) = 0$. Then the value/values of a is/are

A. 1

B. -1

C. 2

D. -2

Answer: A::D

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9. If $f(x) = |\sin x|$, then

A. differentiable everwhere

B. not differentiable at $x = n\pi n, \in I$

C. not diffentiable at $x = 0$

D. continous at $x = 0$

Answer: A::D

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10. Let [] donots the greatest integer function and $f(x) = \left[\tan^2 x\right]$, then

A. $\lim_{x \to 0} f(x)$ does not exist

B. f (x) is continuous at $x = 0$

C. f (x) is not differentiable at $x = 0$

D. $f'(0) = 0$

Answer: B::D

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11. Let $f: R \to R$ be a function, such that $|f(x)| \leq x^{4n}, n \in N \forall n \in R$ then $f(x)$ is:

A. discontinous at $x = 0$

B. continous at $x = 0$

C. non-differentiable at $x = 0$

D. differentiable at $x = 0$

Answer: B::D

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12. Let $f(x) = [x]$ and $g(x) = 0$ when x is an integer and $g(x) = x^2$ when x is not an integer ([] is ghe greatest integer function) then:

A. $\lim_{x \to 1} g(x)$ exists, but g (x) is not continous at $x = 1$

B. $\lim_{x \to 1} f(x)$ does not exist

C. g(g(x)) is continous for all x

D. d(g(x)) is continous for all x

Answer: A::B

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13. let the function f be defined by $f(x) = \begin{cases} p + qx + x^2, & x < 2 \\ 2px + 3qx^2, & x \geq 2 \end{cases}$,

Then:

A. f (x) is continous in R if $3p + 10q = 4$

B. f (x) is differentiable in R if $p = q = \frac{4}{13}$

C. If $p = -2, q = 1,$ then f(x) is continuos in R

D. f (x) is differentiable in R if $1p + 11q = 4$

Answer: A::B::C

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14. Let $f(x) = |2x - 9| + |2x + 9|$. Which of the following are true ?

A. f (x) is not differerntiable at $x = \frac{9}{2}$

B. f (x) is not differentiable at $x = \frac{-9}{2}$

C. f (x) is not diffentiable at $x = 0$

D. f (x) is differentiable at $x = \frac{-9}{2}, 0, \frac{9}{2}$

Answer: A::B::C

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15. Let $f(x) = \max\left(x, x^2 x^3\right) \in -2 \leq x \leq 2$. Then:

A. f (x) is continous in $-2 \leq x \leq 2$

B. $f(x)$ is not differentiable at $x = 1$

C. $f(-1) + f\left(\frac{3}{2}\right) = \frac{35}{8}$

D. $f(-1) f'\left(\frac{3}{2}\right) = \frac{-35}{4}$

Answer: A::B::C

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16. If $f(x)$ be a differentiable function satisfying $f(y) f\left(\frac{x}{y}\right) = f(x) \forall, xy \in R, y \neq 0$ and $f(1) \neq 0, f(1) = 3$, then

A. sgn $(f(x))$ is non-differentiable at exactly one point

B. $\lim_{x \to 0} \frac{x^2(\cos x - 1)}{f(x)} = 0$

C. $f(x) = x$ has 3 solutions

D. $f(f(x)) - f^3(x) = 0$ has infinitely many solutions

Answer: A::B::C::D

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17. Let $f(x) = \left(x^2 - 3x + 2\right)\left(x^2 + 3x + 2\right)$ and α, β, γ satisfy $\alpha < \beta\gamma$ are the roots of $f'(x) = 0$ then which of the following is/are correct ([.] denots greatest integer function) ?

A. $[\alpha] = -2$

B. $[\beta] = -1$

C. $[\beta]0$

D. $[\alpha] = 1$

Answer: A::C

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18. let the function f be defined by $f(x) = \begin{cases} p + qx + x^2, & x < 2 \\ 2px + 3qx^2, & x \geq 2 \end{cases}$,

Then:

A. f (x) is continous in R if $3p + 10q = 4$

B. $f(x)$ is differentiable in R is $p = q = \frac{4}{13}$

C. If $p = -2, q = 1,$ then f(x) is continous in R

D. f (x) is differentiable in R is $2p + 11q = 4$

Answer: A::B::C

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19. Let $y = e^x s \in x^3 + (tanx)^x \dot{F} \in d\frac{dy}{dx}$.

A.

$$e^{x\sin\left(x^3\right)}\left[3x^3\cos\left(x^3\right)+\sin\left(x^3\right)\right]+(\tan x)^x[\ln\tan x+2x\cos ec2x]$$

B.

$$e^{x\sin\left(x^3\right)}\left[x^3\cos\left(x^3\right)+\sin\left(x^3\right)\right]+(\tan x)^x[\ln\tan x+2x\cos ec2x]$$

C. $e^{x\sin\left(x^3\right)}\left[x^3\sin\left(x^3\right)+\sin\left(x^3\right)\right]+(\tan x)^x[\ln\tan x+2\cos ec2x]$

D. $e^{x\sin\left(x^3\right)}\left[3x^3\cos\left(x^3\right)+\sin\left(x^3\right)\right]+(\tan x)^x\left[\ln\tan x+\frac{x\sec^2}{\tan x}\right]$

Answer: A::D

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20. Let

$f(x)=x+(1-x)x^3+(1-x)\left(1-x^2\right)x^3+\ldots\ldots+(1-x)\left(1-x^2\right)..$

then :

A. $f(x)=-\prod_{r=1}^{n}(1-n^r)$

B. $f(x)=1-\prod_{r=1}^{n}(1-x^r)$

C. $f(x) = f(x)\left(\prod_{r=1}^{n} \frac{rx^{r-1}}{(1-x^r)}\right)$

D. $f'(x) = f(x)\left(\left(\prod_{r=1}^{n} \frac{rx^{r-4}}{(1-x^r)}\right)\right.$

Answer: B::C

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21. Let $f(x) = \begin{bmatrix} x^2 + a & 0 \le x < 1 \\ 2x + b & 1 \le x \le 2 \end{bmatrix}$ and $g(x) = \begin{bmatrix} 3x + b & 0 \le x < 1 \\ x^3 & 1 \le x \le 2 \end{bmatrix}$

If derivative of $f(x)$ w.r.t. $g(x) at x = 1$ exists and is equal to λ, then which of the followig is/are correct?

A. $a + b = -3$

B. $a - b = 1$

C. $\frac{ab}{\lambda} = 3$

D. $\frac{-b}{\lambda} = 3$

Answer: A::B::C::D

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22. If $f(x) = \begin{cases} \frac{\sin[x^2]\pi}{x^2-3x+8} + ax^3 + b & 0 \le x \le 1 \\ 2\cos \pi x + \tan^{-1} x & 1 < x \le 2 \end{cases}$ is differentiable in $[0, 2]$ then: ([.] denotes greatest integer function)

A. $a = \frac{1}{3}$

B. $a = \frac{1}{6}$

C. $b = \frac{\pi}{4} - \frac{13}{6}$

D. $b = \frac{\pi}{4} - \frac{7}{3}$

Answer: B::C

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23. If $f(x) = \begin{cases} 1 + x & 0 \le x \le 2 \\ 3x - 2 & 2 < x \le 3 \end{cases}$, then $f(f(x))$ is not differentiable at:

A. $x = 1$

B. $x = 2$

C. $x = \frac{5}{2}$

D. $x = 3$

Answer: A::B

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24. Let

$f(x) = (x+1)(x+2)(x+3)\ldots..(x+100)$ and $g(x) = f(x)f''(x) - f$

Let n be the numbers of real roots of $g(x) = 0$, then:

A. $n < 2$

B. $n > 2$

C. $n < 100$

D. $n > 100$

Answer: A::C

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25. If

$f(x) = \{|x| - 3x < 1|x - 2| + ax \geq 1 \& g(x) = \{2 - |x| x < 2 sgn(x) - bx$

is discontinuous at exactly one point, then -

A. $a = -3, b = 0$

B. $a = -3, b = -1$

C. $a = 2, b = 1$

D. $a = 0, b = 1$

Answer: A::B::C::D

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26. Let f (x) be a continous function in $[-1, 1]$ such that

$$f(x) = \begin{cases} \frac{\ln(ax^2+bx+c)}{x^2} & -1 \leq x < 0 \\ 1 & x = 0 \\ \frac{\sin\left(e^{x^2}-1\right)}{x^2} & 0 < x \leq 1 \end{cases}$$

Then which of the following is/are corrent

A. $a+b+c=0$

B. $b=a+c$

C. $c=1+b$

D. $b^2+c^2=1$

Answer: C::D

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27. f (x) is differentiable function satisfying the relationship $f^2(x)+f^2(y)+2(xy-1)=f^2(x+y)\,\forall x,y\in R$

Also $f(x)>0\forall x\in R$ and $f\left(\sqrt{2}\right)=2$. Then which of the following statement (s) is/are correct about $f(x)$?

A. $[f(3)]=3$([.] denotes greatest integer function)

B. $f\left(\sqrt{7}\right)=3$

C. $f(x)$ is even

D. $f'(0)=0$

Answer: A::B::C::D

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28. The function $f(x) = \left[\sqrt{1-\sqrt{1-x^2}}\right]$, (where [.] denotes greatest integer function):

A. has domain $[-1, 1]$

B. is discontinous at two points in its domain

C. is discontinous at $x = 0$

D. is discontinous at $x = 1$

Answer: A::B::D

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29. A function f(x) satisfies the relation $f(x+y) = f(x) + f(y) + xy(x+y)$, $\forall x, y \in R$. If f'(0) = - 1, then

A. f (x) is a polynomial funciton

B. f (x) is an exponetial function

C. f (x) is twice differentiable for all $x \in R$

D. $f'(3) = 8$

Answer: A::C::D

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30. The points of discontinuities of $f(x) = \left[\frac{6x}{\pi}\right] \cos\left[\frac{3x}{\pi}\right] \text{in}\left[\frac{\pi}{6}, \pi\right]$ is/are:(where [.] denotes greattest integer function)

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. π

Answer: B::C

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31. Check the continuity of f(x) = $\begin{cases} \frac{x^2}{2} & \text{if } 0 \leq x \leq 1 \\ 2x^2 - 3x + \frac{3}{2} & \text{if } 1 < x \leq 2 \end{cases}$ at $x = 1$

A. $f(x), f'(x)$ are continous

B. $f'(x)$ is continous, $f''(x)$ is not continous

C. $f''(x)$ is continous

D. $f''(x)$ is non differentiable

Answer: A::B::D

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32. If $x = \phi(t), y = \psi(t),$ then $\frac{d^2y}{dx^2} =$

A. $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^2}$

B. $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^3}$

C. $\frac{\psi''}{\phi'} - \frac{\psi'\phi''}{(\phi')^2}$

D. $\frac{\psi''}{(\phi')^2} - \frac{\psi'\phi''}{(\phi')^3}$

Answer: B::D

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33. $f(x) = [x]$ and $g(x) = \begin{cases} 0, & x \in I \\ x^2, & x \notin I \end{cases}$ where [.] dentoes the greatest integer function. Then

A. gof is continous for all x

B. gof is not continous for all x

C. fog is continous everywhere

D. fog is not continous everywhere

Answer: A

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34. Let $f: R^+ \to R$ defined as $f(x) = e^x + \ln x$ and $g = f^{-1}$ then correct statement (s) is/are:

A. $g''(e) = \dfrac{1-e}{(1+e)^3}$

B. $g''(e)\dfrac{e-1}{(1+e)^3}$

C. $g'(e) = e+1$

D. $g'(e) = \dfrac{1}{e+1}$

Answer: A::D

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35. Let $f(x) = \begin{cases} \frac{3x-x^2}{2} & x < 2 \\ [x-1] & 2 \le x < 3 \\ x^2 - 8x + 17 & x \ge 3 \end{cases}$: then which of the following hold(s) good ?

A. $\lim_{x \to 2} f(x) = 1$

B. $f(x)$ is differentiable at $x = 2$

C. $f(x)$ is continous at $x = 2$

D. $f(x)$ is discontinous at $x = 3$

Answer: A::C::D

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Exercise Comprehension Type Problems

1. Let $f(x) = \lim_{n \to \infty} n^2 \tan\left(\ln\left(\sec\frac{x}{n}\right)\right)$ and $g(x) = \min(f(x), \{x\})$

(where {.} denotes fractional part function)

Left derivative of $\phi(x) = e^{\sqrt{2f(x)}}$ at $x = 0$ is:

A. 0

B. 1

C. -1

D. Does not exist

Answer: C

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2. Let $f(x) = \lim_{n \to \infty} n^2 \tan\left(\ln\left(\sec\frac{x}{n}\right)\right)$ and $g(x) = \min(f(x), \{x\})$

(where {.} denotes fractional part function)

Number of points in $x \in [-1, 2]$ at which g (x) is discontinous :

A. 2

B. 1

C. 0

D. 3

Answer: A

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3. Let $f(x)$ and $g(x)$ be two differentiable functions, defined as:

$f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$.

The value of $f(1) + g(-1)$ is:

A. 0

B. 1

C. 2

D. 3

Answer: D

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4. Let $f(x)$ and $g(x)$ be two differentiable functions, defined as:

$f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$.

The number of integers in the domain of the function $F(x) = \sqrt{-\frac{f(x)}{g(x)}} + \sqrt{3-x}$ is:

A. 0

B. 1

C. 2

D. Infinite

Answer: C

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5. Define: $f(x) = \left|x^2 - 4x + 3\right| \ln x + 2(x-2)^{1/3}, x > 0$

$h(x) = \begin{cases} x - 1, & x \in Q \\ x^2 - x - 2, & x \notin Q \end{cases}$

$f(x)$ is non-differentiable at..... points and the sum of corresponding x value (s) is .....

A. $3, 6$

B. $2, 3$

C. $2, 4$

D. $2, 5$

Answer: D

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6. Define: $f(x) = |x^2 - 4x + 3| \ln x + 2(x-2)^{1/3}, x > 0$

$$h(x) = \begin{cases} x - 1, & x \in Q \\ x^2 - x - 2, & x \notin Q \end{cases}$$

h (x) is discontinous at $x = \ldots\ldots$

A. $1 + \sqrt{2}$

B. $\tan \frac{3\pi}{8}$

C. $\tan \frac{7\pi}{8}$

D. $\sqrt{2} - 1$

Answer: D

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7. Consider a function defined in $[-2, 2]$

$$f(x) = \begin{cases} \{x\} & -2 \leq x < -1 \\ |sgnx| & -1 \leq x \leq 1 \\ \{-x\} & 1 < x \leq 2 \end{cases}$$, where {.} denotes the fractional part function.

The total number of points of discontinuity of $f(x)$ for $x \in [-2, 2]$ is:

A. 0

B. 1

C. 2

D. 4

Answer: B

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8. Consider a function defined in $[-2, 2]$

$$f(x) = \begin{cases} \{x\} & -2 \leq x < -1 \\ |sgnx| & -1 \leq x \leq 1 \\ \{-x\} & 1 < x \leq 2 \end{cases}$$, where {.} denotes the fractional

part function.

The total number of points of discontinuity of $f(x)$ for $x \in [-2, 2]$ is:

A. 0

B. 1

C. 2

D. 3

Answer: D

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9. Consider a function $f(x)$ in $[0, 2\pi]$ defined as :

$$f(x) = \begin{cases} [\sin x] + [\cos x] & 0 \leq x \leq \pi \\ [\sin x] - [\cos x] & \pi < x \leq 2\pi \end{cases}$$

where {.} denotes greatest integer function then.

Number of points where $f(x)$ is non-derivable :

A. 2

B. 3

C. 4

D. 5

Answer: B

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10. Consider a function $f(x)$ in $[0, 2\pi]$ defined as :

$$f(x) = \begin{cases} [\sin x] + [\cos x] & 0 \leq x \leq \pi \\ [\sin x] - [\cos x] & \pi < x \leq 2\pi \end{cases}$$

where [.] denotes greatest integer function then.

$\lim_{x \to \left(\frac{3\pi}{2}\right)^+}$, $f(x)$ equals

A. 0

B. 1

C. -1

D. 2

Answer: C

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11. Let $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1) & 2 \leq x \leq 3 \end{cases}$ where [x]= greatest integer less than or equal to x, then:

The number of values of x for $x \in [0, 3]$ where $f(x)$ is dicontnous is:

A. 0

B. 1

C. 2

D. 3

Answer: C

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12. Let $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1) & 2 \leq x \leq 3 \end{cases}$ where [x]= greatest integer less than or equal to x, then:

The number of values of x for $x \in [0, 3]$ where $f(x)$ is non-differentiable is :

A. 0

B. 1

C. 2

D. 3

Answer: C

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13. Let $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1) & 2 \leq x \leq 3 \end{cases}$ where [x]= greatest integer less than or equal to x, then:

The number of integers in the range of $y = f(x)$ is:

A. 3

B. 4

C. 5

D. 6

Answer: A

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14. Let $f: R \to R$ be a continous and differentiable function such that $f(x+y) = f(x).F(y) \forall x, y, f(x) \neq 0$ and $f(0) = 1$ and $f'(0) = 2.$

Let $f(xy) = g(x).G(y) \forall x, y$ and $g'(1) = 2.g(1) \neq = 0$

Identify the correct option:

A. $f(2) = e^4$

B. $f(2) = 2e^2$

C. $f(1) < 4$

D. $f(3) > 729$

Answer: A

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15. Let $f: R \to R$ be a continous and differentiable function such that $f(x+y) = f(x).\, F(y)\, \forall x, y, f(x) \neq 0$ and $f(0) = 1$ and $f'(0) = 2.$

Let $g(xy) = g(x).\, g(y)\, \forall x, y$ and $g'(1) = 2.\, g(1) \neq = 0$

Identify the correct option:

A. $g(2) = 2$

B. $g(3) = 3$

C. $g(3) = 9$

D. $g(3) = 6$

Answer: C

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16. Let $f: R \to R$ be a continous and differentiable function such that $f(x+y) = f(x).\, F(y)\, \forall x, y, f(x) \neq 0$ and $f(0) = 1$ and $f'(0) = 2.$

Let $g(xy) = g(x).\, g(y)\, \forall x, y$ and $g'(1) = 2.\, g(1) \neq = 0$

The number of values of x, where $f(x)g(x)$:

A. 0

B. 1

C. 2

D. 3

Answer: B

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17. $f(x) = \dfrac{\cos^2 x}{1 + \cos x + \cos^2 x}$ and $g(x) = k\tan x + (1-k)\sin x - x$,

where $k \in R, g'(x) =$

A. $\dfrac{(1-\cos x)(f(x)-k)}{\cos x}$

B. $\dfrac{(1-\cos x)(k-f(x))}{\cos x}$

C. $\dfrac{(1-\cos x)(k-f(x))}{f(x)}$

D. $\frac{(1-\cos x)(k-f(x))}{(f(x))^2}$

Answer: C

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18. Let $f(x)\frac{\cos^2 x}{1+\cos+\cos^2 x}$ and $g(x)\lambda\tan x+1(1-\lambda)\sin x-x$, where $\lambda \in R$ and $x \in [0, \pi/2]$.

$g'(x)$ equals

A. $[1, \infty)$

B. $[0, \infty)$

C. $\left[\frac{1}{2}, \infty\right)$

D. $\left[\frac{1}{3}, \infty\right)$

Answer: D

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19. Let f and g be two differentiable functins such that:

$f(x) = g'(1)\sin x + (g''(2) - 1)x$

$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f'\left(-\frac{\pi}{2}\right)$

The number of solution (s) of the equation $f(x) = g(x)$ is/are :

A. 1

B. 2

C. 3

D. infinite

Answer: B

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20. Let f and g be two differentiable functins such that:

$f(x) = g'(1)\sin x + (g''(2) - 1)x$

$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f'\left(-\frac{\pi}{2}\right)$

If $\int \frac{g(\cos x)}{f(x)-x}dx = \cos x + \ln(h(x)) + C$ where C is constant and $h\left(\frac{\pi}{2}\right) = 1$ then $\left|h\left(\frac{2\pi}{3}\right)\right|$ is:

A. $3\sqrt{2}$

B. $2\sqrt{3}$

C. $\sqrt{3}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

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21. Let f and g be two differentiable functins such that:

$f(x) = g'(1)\sin x + (g''(2) - 1)x$

$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f'\left(-\frac{\pi}{2}\right)$

If $\phi(x) = f^{-1}(x)$ then $\phi'\left(\frac{\pi}{2} + 1\right)$ equals to :

A. $\frac{\pi}{2} + 1$

B. $\frac{\pi}{2}$

C. 1

D. 0

Answer: C

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22. Suppose a function f(x) satisfies the following conditions

$$f(x+y)=\frac{f(x)+f(y)}{f+f(x)f(y)}\text{ , }\forall x,y\in R\text{ and }f'(0)=1$$

Also $-1<f(x)<1,\ \forall x\in R$

f(x) increases in the complete interval :

A. $(-\infty,\ -1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$

B. $(-\infty,\infty)$

C. $(-\infty,1)\cup(-1,0)$

D. $(0,1)\cup(1,\infty)$

Answer: B

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23. Suppose a function f(x) satisfies the following conditions

$f(x+y)=\frac{f(x)+f(y)}{f+f(x)f(y)}$ ' $\forall x, y \in R$ and $f'(0)=1$

Also $-1<f(x)<1, \forall x \in R$

The value of the limit $lt_{x \to \infty}(f(x))^x$ is:

A. 0

B. 1

C. e

D. e^2

Answer: B

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24. Let $f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^4 f(x)}{x^8+1}=3$

$f(2)=5, f(3)=10, f(-1)=2, f(-6)=37$

The value of $\lim_{x \to -6} \frac{f(x) - x^2 - 1}{3(x+6)}$ equals to:

A. $-\left|\frac{6}{-}\right.$

B. $\left|\frac{6}{-}\right.$

C. $\frac{\left|\frac{6}{-}\right.}{2}$

D. $\frac{\left|\frac{-6}{-}\right.}{2}$

Answer: D

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25. Let $f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^4 f(x)}{x^8 + 1} = 3$

$f(2) = 5, f(3) = 10, f(-1) = 2, f(-6) = 37$

The number of points of discontinuity of discontinuity of $f(x) = \frac{1}{x^2 + 1 - f(x)}$ in $\left[\frac{-15}{2}, \frac{5}{2}\right]$ equals:

A. 4

B. 3

C. 1

D. 0

Answer: B

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26. Consider $f(x) = x^{\ln x}$, and $g(x) = e^2 x$. Let α and β be two values of x satisfying $f(x) = g(x)(\alpha < \beta)$

$\lim_{x \to \beta} \frac{f(x) - \beta^2}{g(x) - \beta^2} = l$ then the value of x -l equals to:

A. $4 - e^2$

B. $e \% (2) - 4$

C. $4 - e$

D. $e - 4$

Answer: B

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27. Consider $f(x) = x^{\ln x}$, and $g(x) = e^2 x$. Let α and β be two values of x satisfying $f(x) = g(x) (\alpha < \beta)$

If $h(x) = \dfrac{f(x)}{g(x)}$ then $h'(\alpha)$ equals to:

A. e

B. $-e$

C. $3e$

D. $-3e$

Answer: D

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28. Let $f_n x + f_n(y) = \dfrac{x^n + y^n}{x^n y^n} \forall x, y \in R - \{0\}$. where $n \in N$ and $g(x) = \max \left\{ f_2(x), f_3(x), \dfrac{1}{2} \right\} \forall x \in R - \{0\}$

The number of values of x for which g(x) is non-differentiable $(x \in R - \{0\})$:

A. 1

B. 2

C. $\sqrt{2}$

D. 4

Answer: B

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29. Let $f_n x + f_n(y) = \frac{x^n + y^n}{x^n y^n} \forall x, y \in R - \{0\}.$ where $n \in N$ and $g(x) = \max\left\{f_2(x), f_3(x), \frac{1}{2}\right\} \forall x \in R - \{0\}$

The number of values of x for which g(x) is non-differentiable $(x \in R - \{0\})$:

A. 3

B. 4

C. 5

D. 1

Answer: A

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Exercise Matching Type Problems

1. Let a function $f(x) = [x]\{x\} - |x|$ where $[.]$, $\{.\}$ are greatest integer and fractional part respectively then match the following List-I with List-II.

	Column-I		Column-II
(A)	$f(x)$ is continuous at x equal to	(P)	3
(B)	$\frac{4}{3}\left\lvert\int_2^3 f(x)\,dx\right\rvert$ is equal to	(Q)	1
(C)	If $g(x) = x - 1$ and if $f(x) = g(x)$ where $x \in (-3, \infty)$, then number of solutions	(R)	4
(D)	If $l = \lim_{x \to 4^+} f(x)$, then $-l$ is equal to	(S)	2

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2.

	Column-I		Column-II
(A)	$\lim_{x\to\infty}\left(\frac{x^2+2x-1}{2x^2-3x-2}\right)^{\frac{2x+1}{2x-1}} =$	(P)	$\frac{1}{2}$
(B)	$\lim_{x\to 0}\frac{\log_{\sec x/2}\cos x}{\log_{\sec x}\cos\frac{x}{2}} =$	(Q)	2
(C)	Let $f(x) = \max.(\cos x, x, 2x-1)$ where $x \geq 0$ then number of points of non-differentiability of $f(x)$ is	(R)	5
(D)	If $f(x) = [2+3\sin x]$, $0 < x < \pi$ then number of points at which the function is discontinuous, is	(S)	16

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3. The function $\begin{aligned} f(x) &= ax(x-1)+b & x &< 1 \\ &= x-1 & 1 &\leq x \leq 3 \\ &= px^2+qx+2 & x &> 3 \end{aligned}$

if (i) f (x) is continous for all x

(ii) $f'(1)$ does not exist

(iii) $f'(x)$ is continous at $x = 3,$ then

	Column-I		Column-II
(A)	a cannot has value	(P)	1/3
(B)	b has value	(Q)	0
(C)	p has value	(R)	–1
(D)	q has value	(S)	1

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Exercise Subjective Type Problems

1. Let $f(x) = \begin{cases} ax(x-1)+b & x<1 \\ x+2 & 1 \le x \le 3 \\ px^2+qx+2 & x>3 \end{cases}$ is continous $\forall x \in R$ except $x=1$ but $|f(x)|$ is differentiable everywhere and $f'(x)$ is continous at $x=3$ and $|a+p+q|=k$, then k=

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2. If $y = \sin\left(8\sin^{-1}x\right)$ then $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -ky$, where k =

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3. If $y^2 = 4ax$, then $\frac{d^2y}{dx^2} = \frac{ka^2}{y^2}, where k^2 =$

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4. The number of values of $x, x \in [-2, 3]$ where $f(x) = \left[x^2\right]\sin(\pi x)$ is discontinous is (where [.] denotes greatest integer function)

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5. If $f(x)$ is continous and differentiable in $[-3, 9]$ and $f'(x) \in [-2, 8] \forall x \in (-3, 9)$. Let N be the number of divisors of the greatest possible value of $f(9) - f(-3)$, then find the sum of digits of N.

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6. In $f(x) = \left[\left(\cos x^2, , x < 0\right), \left(\sin x^3 - \left|x^3 - 1\right|, , x \geq 0\right)\right.$ then find the number of points where $g(x) = f(|x|)$ is non-differentiable.

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7. Consider

$f(x) = x^2 + ax + 3$ and $g(x) = x + b$ and $F(x) = \lim_{n \to \infty} \frac{f(x) + x^{2n} g(x}{1 + x^{2n}}$

If F(x) is continuous at x=-1, then

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8. Let $f(x) = \begin{cases} 2 - x, & -3 \leq x \leq 0 \\ x - 2, & 0 < x < 4 \end{cases}$ Then f^{-1} (x) is discontinous at x=

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9. If $f(x) + 2f(1 - x) = x^2 + 2 \forall x \in R$ and $f(x)$ is a differentiable function, then the value of $f'(8)$ is

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10. Let $f(x) =$ signum (x) and $g(x) = x\left(x^2 - 10x + 21\right)$, then the number of points of discontinuity of $f[g(x)]$ is

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11. If $\frac{d^2}{dx^2}\left(\frac{\sin^4 x+\sin^2 x+1}{\sin^2 x+sinx+1}\right)=a\sin^2 x+b\sin x+c$ then the value of $b+c-a$ is

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12. $f(x)=a\cos(\pi x)+b, f'\left(\frac{1}{2}\right)=\pi$ and $\int_{1/2}^{3/2} f(x)dx=\frac{2}{\pi}+1,$ then find the value of $-\frac{12}{\pi}\left(\frac{\sin^{-1}a}{3}+\cos^{-1}b\right).$

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13. Let $\alpha(x)=f(x)-f(2x)$ and $\beta(x)=f(x)-f(4x)$ and $\alpha'(1)=5\alpha'(2)=$

then find the vlaue of $\beta'(1)-10$

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14. Let $f(x) = -4.e^{\frac{1-x}{2}} + \frac{x^3}{3} + \frac{x^2}{2} + x + 1$ and g be inverse function of f and $h(x) = \frac{a + bx^{3/2}}{x^{5/4}}, h'(5) = 0,$ then $\frac{a^2}{5b^2 g'\left(\frac{-7}{6}\right)} =$

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15. If $y = 3^{2\sin^{-1}}$ then $\left|\frac{(x^2-1)y'' + xy'}{y}\right|$ is equal to

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16. Let $f(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$ and let $g(x) = f^{-1}(x)$. Find $g''(o)$.

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17. In $f(x) = [(\cos x^2, , x < 0), (\sin x^3 - |x^3 - 1|, , x \geq 0)$ then find the number of points where $g(x) = f(|x|)$ is non-differentiable.

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18. For the curve $\sin x + \sin y = 1$ lying in first quadrant. If $\lim_{x \to 0} x^{\alpha} \frac{d^2y}{dx^2}$ exists and non-zero than $2\alpha =$

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19. Let $f(x) = x \tan^{-1}(x^2) + x^4$ Let $f^k(x)$ denotes k^{th} derivative of $f(x)$ w.r.t. x, $k \in N$. If

f^{2m} (0) != 0, m belongs to N, then m =

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20. If $x = \cos\theta$ and $y = \sin^3\theta$, then $\left| \frac{yd^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right| at\theta = \frac{\pi}{2}$ is:

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21. The value of $x, x \in (2, \infty)$ where $f(x) = \sqrt{x\sqrt{8x-16}} + \sqrt{x-\sqrt{8x-16}}$ is not differentiable is:

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22. The number of non differentiability of runction $f(x) = \min\left(|x|, \{x\}, \left|x - \frac{3}{2}\right|\right)$ for $x \in (0, 2)$, where [.] and {.} denote greatest integer function and fractional part function respectively.

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