MATHS

BOOKS - VK JAISWAL ENGLISH

DETERMINANTS

Exercise 1 Single Choice Problems

1. If $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ then the value of $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ is : (a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{9}{4}$

A. 1

B. $\frac{1}{2}$

C. $\frac{3}{8}$

D. $\frac{9}{4}$

Answer: A

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2. Let the following system of equations

$$kx + y + z = 1$$
$$x + ky + z = k$$
$$x + y + kz = k^2$$

has no solution . Find |k|.

A. 0

B. 1

C. 2

D. 3

Answer: C

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3. If a,b, and c are all different and if

$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$ =0 Prove that abc =-1.

A. 2

B. -1

C. 1

D. 0

Answer: B

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4. if the system of linear equations

$x + 2ay + az = 0$
$x + 3by + bz = 0$
$x + 4cy + cz = 0$

has a non zero solution then a,b,c are in

A. are in A.P.

B. are in G.P.

C. are in H.P.

D. satisfy a+2b+3c=0

Answer: C

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5. if the number of quadratic polynomials $ax^2 + 2bx + c$ which satisfy the following conditions :

(i)a,b,c are distinct

(ii)a,b,c $\in$ {1,2,3,..... , 2001,2002}

(iii)x+1 divides $ax^2 + 2bx + c$ is equal to 1000λ , then find the value of λ .

A. 2002

B. 2001

C. 2003

D. 2004

Answer: A

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6. If the system of equations $2x+ay+6z=8$, $x+2y+z=5$, $2x+ay+3x=4$ has a unique solution then 'a' cannot be equal to :

A. 2

B. 3

C. 4

D. 5

Answer: C

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7. If one root of the equation $\begin{vmatrix} 7 & 6 & x^2-13 \\ 2 & x^2-13 & 2 \\ x^2-13 & 3 & 7 \end{vmatrix}$ =0 is x=2 the sum of all other five roots is

A. -2

B. 0

C. $2\sqrt{3}$

D. $\sqrt{15}$

Answer: A

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8. The system of equations

$$kx+(k+1)y+(k-1)z=0$$
$$(k+1)x+ky+(k+2)z=0$$
$$(k-1)x+(k+2)y+kz=0$$

has a nontrivial solution for :

A. Exactly three real value of k

B. Exactly two real values of k

C. Exactly one real value of k

D. Infinite number of values of k

Answer: C

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9. if $a_1, a_2, \dots\dots a_n, \dots\dots$ form a G.P. and $a_1 > 0,$ for all $I \geq 1$

$$\begin{vmatrix} \log a_n, & \log a_n + \log a_{n+2}, & \log a_{n+2} \\ \log a_{n+3}, & \log a_{n+3} + \log a_{n+5}, & \log a_{n+5} \\ \log a_{n+6}, & \log_{n+6} + \log a_{n+8}, & \log a_{n+8} \end{vmatrix}$$

A. 0

B. $\left(\sum_{i=1}^{n^2+n} a_i\right)$

C. 1

D. 2

Answer: A

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10. if $D_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D_2 = \begin{vmatrix} a_1 + 2a_2 + 3a_3 & 2a_3 & 5a_2 \\ b_1 + 2b_2 + 3b_3 & 2b_3 & 5b_2 \\ c_1 + 2c_2 + 3c_3 & 2c_3 & 5c_2 \end{vmatrix}$ then $\frac{D_2}{D_1}$ is equal to :

A. 10

B. -10

C. 20

D. -20

Answer: B

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11. If $A = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}, B = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$, then

A. $\Delta_1 = \Delta_2$

B. $\Delta_1 = 2\Delta_2$

C. $\Delta_1 + \Delta_2 = 0$

D. $\Delta_1 + 2\Delta_2 = 0$

Answer: C

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12. The value of the determinant $\begin{vmatrix} 1 & 0 & -1 \\ a & 1 & 1-a \\ b & a & 1+a-b \end{vmatrix}$ depends on :

A. only a

B. only b

C. neither a nor b

D. both a and b

Answer: C

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13. Sum of solution of the equation $\begin{vmatrix} 1 & 2 & x_2 \\ 2 & 3 & x^2 \\ 3 & 5 & 2 \end{vmatrix} = 10$ is :

A. 1

B. -1

C. 2

D. 4

Answer: B

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14. if $D = \begin{vmatrix} x+d & x+e & x+f \\ x+d+1 & x+e+1 & x+f+1 \\ x+a & x+b & x+c \end{vmatrix}$ then D does not depend on :

A. a

B. e

C. d

D. x

Answer: D

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15. Value of $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix}$

A. $xyz(x+y+z)^2$

B. $(x+y+z)(x+y+z)^2$

C. $(x+y+z)^3$

D. $(x+y+z)^2$

Answer: C

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16. A rectangle ABCD is inscribed in a circle . Let PQ be the diameter of the circle parallel to the side AB. If $\angle BPC = 30^\circ$, then the ratio of the area of rectangle to the area of circle is :

A. $\frac{\sqrt{3}}{x}$

B. $\frac{\sqrt{3}}{2x}$

C. $\frac{3}{\pi}$

D. $\frac{\sqrt{3}}{9\pi}$

Answer: A

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17. Show that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

A. 3

B. 9

C. 27

D. 81

Answer: C

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18. Let det $A = \begin{vmatrix} l & m & n \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ and if $(l-m)^2 + (p-q)^2 = 9, (m-n)^2 + (q-r)^2 = 16, (n-l)^2 + (r-p)^2 =$

, then the value $(\det.A)^2$ equals :

A. 36

B. 100

C. 144

D. 160

Answer: C

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19. A solution set of the equations $x+2y+z=1$, $x+3y+4z=k$, $x+5y+10z=k^2$ is

A. 0

B. 4

C. 2

D. 3

Answer: C

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20. If $\begin{vmatrix} (x+1) & (x+1)^2 & (x+1)^3 \\ (x+2) & (x+2)^2 & (x+2)^3 \\ (x+3) & (x+3)^2 & (x+3)^3 \end{vmatrix}$ is expressed as a polynomial in x, then the term independent of x is :

A. 0

B. 2

C. 12

D. 16

Answer: C

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21. If A,B,C are the angles of triangle ABC, then the minimum value of $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ is equal to :

A. 0

B. -1

C. 1

D. -2

Answer: B

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22. if the system of linear equations

$$x + 2ay + az = 0$$
$$x + 3by + bz = 0$$
$$x + 4cy + cz = 0$$

has a non zero solution then a,b,c are in

A. A.P

B. G.P

C. H.P

D. None of these

Answer: C

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23. If a,b and c are the roots of the equation $x^3 + 2x^2 + 1 = 0$, find

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}:$$

A. 8

B. -8

C. 0

D. 2

Answer: A

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24. The system of homogeneous equation

$\lambda x + (\lambda + 1)y + (\lambda - 1)z = 0, (\lambda + 1)x + \lambda y + (\lambda + 2)z = 0, (\lambda - 1)x -$

has non-trivial solution for :

A. exactly three real value of λ

B. exactly two real values of λ

C. exactly three real value of λ

D. infinitely many real value of λ

Answer: C

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25. If one root of the equation $\begin{vmatrix} 7 & 6 & x^2-13 \\ 2 & x^2-13 & 2 \\ x^2-13 & 3 & 7 \end{vmatrix}$ =0 is x=2 the sum of all other five roots is

A. -2

B. 0

C. $2\sqrt{5}$

D. $\sqrt{15}$

Answer: A

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Exercise 2 One Or More Than One Answer Is Are Correct

1. $\Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a+b & (a+b) \\ 0 & 1 & 2a+3b \end{vmatrix}$ is divisible by a.$a+b$ b. $a+2b$ c. $2a+3b$ d. a^2

A. (2a+b) is a factor of f(a,b)

B. (a+2b) is a factor of f(a,b)

C. (a+b) is a factor of f(a,b)

D. a is factor of f (a,b)

Answer: B::C::D

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2. If $\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 2\sqrt{3}\tan\theta \\ \cos^2\theta & 1+\sin^2\theta & 2\sqrt{3}\tan\theta \\ \cos^2\theta & \sin^2\theta & 1+2\sqrt{3}\tan\theta \end{vmatrix}$=0 then θ may be :

A. $\frac{\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{7\pi}{6}$

D. $\frac{11\pi}{6}$

Answer: B::D

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3. Let $\Delta = \begin{vmatrix} a & a+d & a+3d \\ a+d & a+2d & a \\ a+2d & a & a+d \end{vmatrix}$ then :

A. Δ depends on a

B. Δ depends on d

C. Δ is independent of a,d

D. $\Delta = 0$

Answer: A::B

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4. The value(s) of λ for which the system of equations

$$(1-\lambda)x + 3y - 4z = 0$$

$$x - (3+\lambda)y + 5z = 0$$

$$3x + y - \lambda z = 0$$

possesses non-trivial solutions .

A. -1

B. 0

C. 1

D. 2

Answer: A::B

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5. Let $D(x) = \begin{vmatrix} x^2+4x-3 & 2x+4 & 13 \\ 2x^2+5x-9 & 4x+5 & 26 \\ 8x^2-16x+1 & 16x-6 & 104 \end{vmatrix} = \alpha x^3 + \beta x^2 + \gamma x + \delta$

then :

A. $\alpha+\beta=0$

B. $\beta+\gamma=0$

C. $\alpha+\beta+\gamma+\delta=0$

D. $\alpha+\beta+\gamma=0$

Answer: A::B::D

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6. if the system of equations

$$ax+y+2z=0$$
$$x+2y+z=b$$
$$2x+y+az=0$$

has no solution then (a+b) can be equals to :

A. -1

B. 2

C. 3

D. 4

Answer: B::C::D

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Exercise 3 Comprehension Type Problems

1. Consider the system of equations

$2x + \lambda y + 6z = 8$
$x + 2y + \mu z = 5$
$x + y + 3z = 4$

The system of equations has :

No solution if :

A. $\lambda = 2, \mu = 3$

B. $\lambda \neq 2, \mu = 3$

C. $\lambda \neq 2, \mu \neq 3$

D. $\lambda = 2, \mu \in R$

Answer: B

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2. Consider the system of equations

$2x + \lambda y + 6z = 8$

$x + 2y + \mu z = 5$

$x + y + 3z = 4$

The system of equations has : Exactly one solution if :

A. $\lambda = 2, \mu = 3$

B. $\lambda \neq 2, \mu = 3$

C. $\lambda \neq 2, \mu \neq 3$

D. $\lambda = 2, \mu \in R$

Answer: A

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3. Consider the system of equations

$2x + \lambda y + 6z = 8$

$x + 2y + \mu z = 5$

$x + y + 3z = 4$

The system of equations has :

Infinitely many solutions if :

A. $\lambda = 2, \mu = 3$

B. $\lambda \neq 2, \mu = 3$

C. $\lambda \neq 2, \mu \neq 3$

D. $\lambda = 2, \mu \in R$

Answer: D

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Exercise 4 Subjective Type Problems

1. If 3^n is a factor of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ .^n C_1 & .^{n+3} C_1 & .^{n+6} C_1 \\ .^n C_2 & .^{n+3} C_2 & .^{n+6} C_2 \end{vmatrix}$ then the maximum value of n is ....

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2. Find the value of λ for which

$$\begin{vmatrix} 2a_1+b_1 & 2a_2+b_2 & 2a_3+b_3 \\ 2b_1+c_1 & 2b_2+c_2 & 2b_3+c_3 \\ 2c_1+a_1 & 2c_2+a_2 & 2c_3+a_3 \end{vmatrix} = \lambda \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

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3. Find the co-efficient of x in the expansion of the determinant

$$\begin{vmatrix} (1+x)^2 & (1+x)^4 & (1+x)^6 \\ (1+x)^3 & (1+x)^6 & (1+x)^9 \\ (1+x)^4 & (1+x)^8 & (1+x)^{12} \end{vmatrix}$$

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4. Let

$$\begin{vmatrix} y^5z^6(z^3-y^3) & x^4z^6(x^3-z^3) & x^4y^5(y^3-x^3) \\ y^2z^3(y^6-z^6) & xz^3(z^6-x^6) & xy^2(x^6-y^6) \\ y^2 \hat{} (3)(z^3-y^3) & xz^3(x^3-z^3) & xy^2(y^3-x^3) \end{vmatrix} \text{ and } \Delta_2 = \begin{vmatrix} x \\ x^4 \\ x^7 \end{vmatrix}$$

.Then $\Delta_1 \Delta_2$ is equal to

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5. if the system of equations :

$$2x + 3y - z = 0$$
$$3x + 2y + kz = 0$$
$$4x + y + z = 0$$

have a set of non-zero integral solutions then , find the smallest positive value of z.

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6. Find $a \in R$ for which the system of equations 2ax-2y+3z=0 , x+ay + 2z=0 and 2x+az=0 also have a non-trivial solution.

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7. If three non-zero distinct real numbers form an arithmatic progression and the squares of these numbers taken in the same order constitute a geometric progression. Find the sum of all possible common ratios of the geometric progression.

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8. Let $\Delta_1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} 6a_1 & 2a_2 & 2a_3 \\ 3b_1 & b_2 & b_3 \\ 12c_1 & 4c_2 & 4c_3 \end{vmatrix}$ and

$$\Delta_3 = \begin{vmatrix} 3a_1 + b_1 & 3a_2 + b_2 & 3a_3 + b_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix}$$

then $\Delta_3 - \Delta_2 = k\Delta_1$, find k.

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9. The minimum value of determinant

$\Delta = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 2 \end{vmatrix} \forall \theta \in R$ is :

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10. For a unique value of μ & λ , the system of equations given by

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$2x + 5y + \lambda z = \mu$$

has infinitely many solutions , then $\dfrac{\mu - \lambda}{4}$ is equal to

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11. Let $\lim_{n \to \infty}$ n sin (2π e $\lfloor n$)= k π, where $n\pi N$. Find k :

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12. If the system of linear equations

$$(\cos\theta)x + (\sin\theta)y + \cos\theta = 0$$
$$(\sin\theta)x + (\cos\theta)y + \sin\theta = 0$$
$$(\cos\theta)x + (\sin\theta)y - \cos\theta = 0$$

is consistent , then the number of possible values of θ, $\theta \in [0, 2\pi]$ is :

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