MATHS

BOOKS - VK JAISWAL ENGLISH

FUNCTION

Single Choice Problems

1. Range of the function $f(x) = \log_{\sqrt{2}}\left(2 - \log_2 16\sin^2 x + 1\right)$ is:

A. $[0, 1]$

B. $(-\infty, 1]$

C. $[-1, 1]$

D. $(-\infty, \infty)$

Answer: B

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2. The value of a and b for which $\left|e^{|x-b|}-a\right|=2,$ has four distinet solutions, are :

A. $a \in (-2, \infty), b=0$

B. $a \in (2, \infty), b=0$

C. $a \in (3, \infty), b \in R$

D. $a \in (2, \infty) b=0$

Answer: C

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3. The range of the function : $f(x)=\tan^{-1}x+\frac{1}{2}\sin^{-1}x$

A. $(-\pi/2, \pi/2)$

B. $[-\pi/2, \pi/2]-\{0\}$

C. $[-\pi/2, \pi/2]$

D. $(-3\pi/4, 3\pi/4)$

Answer: C

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4. Find the number of real ordered pair (s) (x,y) for which :

$$16^{x^2+y}+16^{x+y^2}=1$$

A. 0

B. 1

C. 2

D. 3

Answer: B

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5. The range of values of 'a' such that $\left(\frac{1}{2}\right)^{|x|} = x^2 - a$ is satisfied for maximum number of values of 'x'

A. $(-\infty, -1)$

B. $(-\infty, \infty)$

C. $(-1, 1)$

D. $(-1, \infty)$

Answer: D

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6. For a real number x let [x] denoutes the greatest interger less than or equal to x, let $f: R \to R$ be defined by $f(x) = 2x + [x] + \sin\cos x$, then f is :

A. One-one but not onto

B. onto but not one-one

C. Both one-one and onto

D. Neither one-one nor onto

Answer: A

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7. The maximum value of $\sec^{-1}\left(\frac{7-5(x^2+3)}{2(x^2+2)}\right)$ is:

A. $\frac{5\pi}{6}$

B. $\frac{5\pi}{12}$

C. $\frac{7\pi}{12}$

D. $\frac{2\pi}{3}$

Answer: D

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8. Number of ordered pair (a,b) the set $A = \{1, 2, 3, 4, 5\}$ so that the functon $f(x) = \frac{x^3}{3} + \frac{a}{2}x^2 + bx + 10$ is an injective mapping $\forall x \in R$:

A. `13

B. 14

C. 15

D. 16

Answer: C

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9. let A be the greatest value of the function $f(x) = \log_x [x]$, (where $[.]$ denotes gratest integer function) and B be the least value of the function $g(x) = |\sin x| + |\cos x|$, then :

A. $A > B$

B. $A < B$

C. $A = B$

D. $2A + B = 4$

Answer: C

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10. Let $A = [a, \infty)$ denotes domain, then $f : [a, \infty) \to B, f(x) = 2x^3 + 6$ will have an inverse for then smallest real values of a, if:

A. $a = 1, B = [5, \infty)$

B. $a = 2, B = [10, \infty)$

C. $a = 0, B = [6, \infty)$

D. $a = -1, B = [1, \infty)$

Answer: A

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11. Solution of the inequation $\{x\}(\{x\}-1)(\{x\}+2)\geq 0$

where $\{.\}$ denots fractin part function) is :

A. $x \in (-2, 1)$

B. $x \in I$ (I denote set of integers)

C. $x \in [0, 1)$

D. $x \in [-2, 0)$

Answer: B

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12. Let $f(x), g(x)$ be two real valued functions then the function $h(x) = 2 \max \{f(x) - g((x), 0\}$ is equal to :

A. $f(x) - g(x) - |g(x) - f(x)|$

B. $f(x) + g(x) - |g(x) - f(x)|$

C. $f(x) - g(x) + |g(x) - f(x)|$

D. $f(x) + g(x) + |g(x) - f(x)|$

Answer: C

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13. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation the set $A = \{1, 2, 3, 4\}$. The relation R is (a). a function (b). reflexive (c). not symmetric (d). transitive

A. a function

B. reflexive

C. not symmetric

D. transitive

Answer: C

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14. The true set of valued of 'K' for which $\sin^{-1}\left(\frac{1}{1+\sin^2 x}\right) = \frac{k\pi}{6}$ may have a solution is : $(a).\ \left[\frac{1}{4}, \frac{1}{2}\right]$ $(b).\ [1, 3]$ $(c).\ \left[\frac{1}{6}, \frac{1}{2}\right]$ $(d).\ [2, 4]$

A. $\left[\frac{1}{4}, \frac{1}{2}\right]$

B. $[1, 3]$

C. $\left[\frac{1}{6}, \frac{1}{2}\right]$

D. $[2, 4]$

Answer: B

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15. A real valued function f(x) satisfies the functional equation $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$, where a is a given constant and f(0)=1 , f(2a-x) =?

A. $-f(x)$

B. $f(x)$

C. $f(a) + f(a - x)$

D. $f(- x)$

Answer: A

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16. Let $g: R \to R$ be given by $g(x) = 3 + 4x$ if $g^n(x) =$ gogogo......og (x) n times. Then inverse of $g^n(x)$ is equal to :

A. $(x + 1 - 4^n). 4^{-n}$

B. $(x - 1 + 4^n)4^{-n}$

C. $(x + 1 + 4^n)4^{-n}$

D. None of these

Answer: A

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17. Let $f: D \rightarrow R$ bge defined as : $f(x) = \frac{x^2 + 2x + a}{x^2 + 4x + 3a}$ where D and R denote the domain of f and the set of all the real numbers respectively. If f is surjective mapping. Then the complete range of a is :

A. $0 < a \leq 1$

B. $0 < a \leq 1$

C. $0 \leq a < 1$

D. $0 < a < 1$

Answer: D

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18. Let $f: [2, \infty) \rightarrow X$ be defined by $f(x) = 4x - x^2$. Then, f is invertible, if $X =$ $[2, \infty)$ (b) $(-\infty, 2]$ (c) $(-\infty, 4]$ (d) $[4, \infty)$

A. $2 - \sqrt{4 - x}$

B. $2 + \sqrt{4 - x}$

C. $-2 + \sqrt{4 - x}$

D. $-2 - \sqrt{4 - x}$

Answer: A

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19. IF $[5 \sin x] + [\cos x] + 6 = 0,$ then range of $f(x) = \sqrt{3} \cos x + \sin$ x corresponding to solution set of the given equation is: (where $[.]$ denotes greatest integer function)

A. $[-2, -1]$

B. $\left(-\frac{3\sqrt{3}+2}{5}, -1\right)$

C. $[-2, -\sqrt{3})$

D. $\left(-\frac{3\sqrt{3}+4}{5}, -1\right)$

Answer: D

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20. If $f: R \to R$ where $f(x) = ax + \cos x$ is an invertible function, then

$(a). (-2, -1] \cup [1, 2)$; $(b). [-1, 1]$; $(c). (-\infty, -1] \cup [1, \infty)$;

$(d). (-\infty, -2] \cup [2, \infty)$.

A. $(-2, -1] \cup [1, 2)$

B. $[-1, 1]$

C. $(-\infty, -1] \cup [1, \infty)$

D. $(-\infty, -2] \cup [2, \infty)$

Answer: C

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21. The range of

$f(x) = [1 + \sin x] + \left[2 + sin\frac{x}{2}\right] + \left[3 + sin\frac{x}{3}\right] + + \left[n + sin\frac{x}{n}\right] \forall x \in [$

, where [.] denotes the greatest integer function, is,

$\left\{\frac{n+n-2^2}{2}, \frac{n(n+1)}{2}\right\}$ $\left\{\frac{n(n+1)}{2}\right\}$ $\left\{\frac{n(n+1)}{2}, \frac{n^2+n+2}{2}\right\}$
$\left[\frac{n(n+1)}{2}, \frac{n^2+n+2}{2}\right]$

A. $\left\{\frac{n^2+n-2}{2}, \frac{n(n+1)}{2}\right\}$

B. $\left\{\frac{n(n+1)}{2}\right\}$

C. $\left\{\frac{n(n+1)}{2}, \frac{n^2+n+2}{2}, \frac{n^2+n+4}{2}\right\}$

D. $\left\{\frac{n(n+1)}{2}, \frac{n^2+n-2}{2}\right\}$

Answer: D

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22. Let $f: R \to R,$ where $f(x) = \frac{x^2+ax+1}{x^2+x+1}$. Then the complete set of values of 'a' such that f(x) is onto is :

A. $(-\infty, \infty)$

B. $(-\infty, 0)$

C. $(0, \infty)$

D. not possible

Answer: D

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23. Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$

what is the number of non empty relations from A to B

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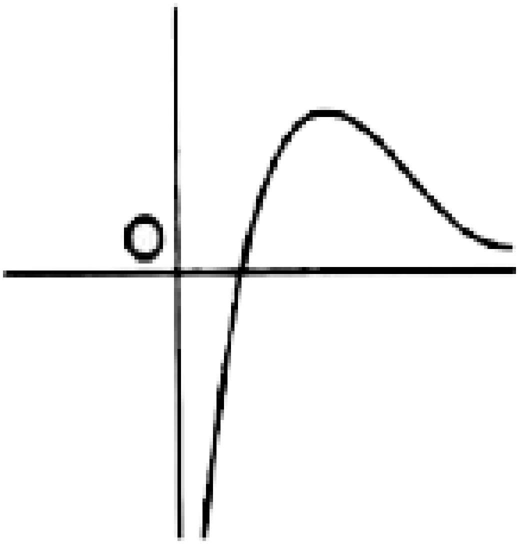
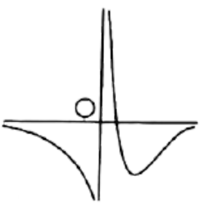
24. The graph of function $f(x)$ is shown below :

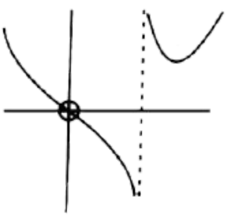


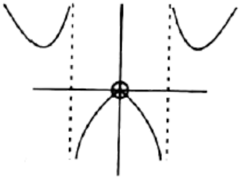
Then the graph of $g(x) = \dfrac{1}{f(|x|)}$ is:

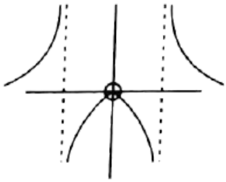
A.



B.



C. 

D. 

Answer: C

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25. Which of the following function is homogeneous ?

A. $f(x) = x\sin y + y\sin x$

B. $g(x) = xe^{\frac{y}{x}} + ye^{\frac{x}{y}}$

C. $h(x) = \dfrac{xy}{x+y^2}$

D. $\phi(x) = \dfrac{x - y\cos x}{y\sin x + y}$

Answer: B

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26. Let $f(x) = \begin{bmatrix} 2x+3 & x>1 \\ \alpha^2 x+1 & x \leq 1 \end{bmatrix}$ If range of $f(x) = R$ (set of real numbers) then number orf integral value(s), which α any take :

A. 2

B. 3

C. 4

D. 5

Answer: C

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27. The maximum integral values of x in the domain of $f(x) = \log_{10}\left(\log_{1/3}(\log_4(x-5)\right))$ is : (a). 5 (b). 7 (c). 8 (d). 9

A. 5

B. 7

C. 8

D. 9

Answer: C

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28. Range of the function f(x)=$\log_2\left(\frac{4}{\sqrt{x+2}+\sqrt{2-x}}\right)$ is $(a).\ (0, \infty)$ $(b).\ \left[\frac{1}{2}, 1\right]$ $(c).\ [1, 2]$ $(d).\ \left[\frac{1}{4}, 1\right]$

A. $(0, \infty)$

B. $\left[\frac{1}{2}, 1\right]$

C. $[1, 2]$

D. $\left[\frac{1}{4}, 1\right]$

Answer: B

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29. Number of integers stastifying the equation $\left|x^2+5x\right|+\left|x-x^2\right|=|6x|$ is:

A. 3

B. 5

C. 7

D. 9

Answer: C

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30. If $A=\{2,1\}$, find $A\times A\times A$

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31. Which of the following function is periodic with fundamental period π ?

A. $f(x) = \cos x\left|\frac{\sin x}{2}\right|$, where $[.]$ denotes greatest integer function

B. $g(x) = \frac{\sin x + \sin 7x}{\cos x + \cos 7x} + |\sin x|$

C. $h(x) = \{x\} + |\cos x|$, where $\{.\}$ denotes functional part function

D. $\phi(x) = |\cos x| + \ln(\sin x)$

Answer: B

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32. Let $f: N \to Z$ and $f(x) = \begin{bmatrix} \frac{x-1}{2} & \text{when x is odd} \\ -\frac{x}{2} & \text{when x is even} \end{bmatrix}$, then:

(a). f (x) is bijective (b).f (x) is injective but not surjective (c).f (x) is not injective but surjective (d).f (x) is neither injective nor subjective

A. f (x) is bijective

B. f (x) is injective but not surjective

C. f (x) is not injective but surjective

D. f (x) is neither injective nor subjective

Answer: A

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33. Let $g(x)$ be the inverse of $f(x) = \dfrac{2^{x+1} - 2^{1-x}}{2^x + 2^{-x}}$ then g (x) be :

A. $\frac{1}{2}\log_2\left(\frac{2+x}{2-x}\right)$

B. $-\frac{1}{2}\log_2\left(\frac{2+x}{2-x}\right)$

C. $\log_2\left(\frac{2+x}{2-x}\right)$

D. $\log_2\left(\frac{2-x}{2+x}\right)$

Answer: C

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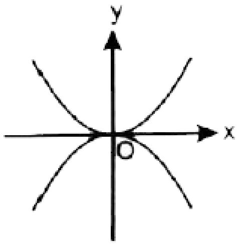
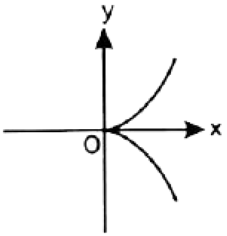
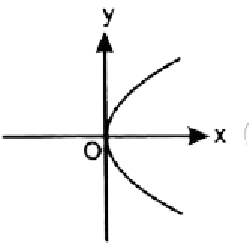
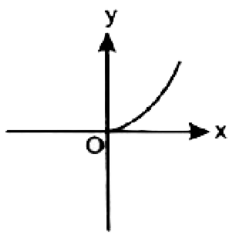
34. Which of the following is the graph of the curve $\sqrt{|y|} = x$ is ?

A. 

B. 

C. 

D. 

Answer: B

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35. Domain of $f(x) = \log_{(x)}\left(9 - x^2\right)$ is :

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36. If $e^x + e^{f(x)} = e,$ then for $f(x)$ domain is:

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37. If high voltage current is applied on the field given by the graph $y + |y| - x - |x| = 0.$ on which of the following curve a person can move so that the remains safe ?

A. $y = x^2$

B. $y = sgn\left(-e^2\right)$

C. $y = \log_{1/3} x$

D. $y = m + |x|, m > 3$

Answer: D

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38. If $f(x) + 6 - x^2] = |f(x)| + |4 - x^2| + 2,$ then f(x) is necessarily non-negative for:

A. $x \in [-2, 2]$

B. $xn(-\infty, -2) \cup (2, \infty)$

C. $x \in [-\sqrt{6}, \sqrt{6}]$

D. $x \in [-5, -2] \cup [2, 5]$

Answer: A

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39. Let $f(x) = \cos x + \sin px$ be periodic, then p must be :

(a).Positive real number (b). Negative real number (c).Rational (d).Prime

A. Positive real number

B. Negative real number

C. Rational

D. Prime

Answer: C

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40. The domain of $f(x) is (0, 1)$. Then the domain of $(f(e^x) + f(1n|x|)$ is

$(a)(-1, e)$ (b). $(1, e)$ $(c). (-e, -1)$ (d) $(-e, 1)$

A. $\left(\frac{1}{e}, 1\right)$

B. $(-e, 1)$

C. $\left(-1, -\frac{1}{e}\right)$

D. $(-e, -1) \cup (1, e)$

Answer: B

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41. Let $A=\{1, 2, 3, 4\}$ and $f: A \rightarrow$ A satisfy $f(1)=2, f(2)=3, f(3)=4, f(4)=1$. Suppose $g: A \rightarrow A$ satisfies $g(1)=3$ and $fog=gof$, then g =

(a). $\{(1, 3), (2, 1), (3, 2), (4, 4)\}$ (b). $\{(1, 3), (2, 4), (3, 1), (4, 2)\}$

(c). $\{(1, 3), (2, 2), (3, 4), (4, 3)\}$ (d). $\{(1, 3), (2, 4), (3, 2), (4, 1)\}$

A. $\{(1, 3), (2, 1), (3, 2), (4, 4)\}$

B. $\{(1, 3), (2, 4), (3, 1), (4, 2)\}$

C. $\{(1, 3), (2, 2), (3, 4), (4, 3)\}$

D. $\{(1, 3), (2, 4), (3, 2), (4, 1)\}$

Answer: B

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42. Number of solutions of the equation, $[y+[y]]=2\cos x$ is: (where $y=1/3)[\sin x+[\sin x+[\sin x]]]$ and $[]=$ greatest integer function)

(a). 0 (b) 1 (c) 2 (d) ∞

A. 0

B. 1

C. 2

D. Infinite

Answer: A

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43. The function $f(x)=\left\{\frac{(x^{2n})}{(x^{2n} sgn x)^{2n+1}}\left(\frac{e^{\frac{1}{x}}-e^{\frac{1}{x}}}{e^{\frac{1}{x}}+e^{-\frac{1}{x}}}\right)x\neq 0 n\in N\right.$ is:

A. Odd function

B. Even function

C. Neither odd nor even function

D. Constant function

Answer: B

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44. Let $A = \{1, 2\}, B = \{1, 2, 3\}, c = \{5, 6\}$ and $d = \{5, 6, 7\}$ verify that

$A \times C$ is a subset of $B \times D$

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45. Let $f(x) = \frac{x}{\sqrt{1 + x^2}}$ then ubrace(fo fo fo ......of)(x) ` is :

A. $\frac{x}{\sqrt{1 + \left(\sum_{r=1}^{n} r\right) x^2}}$

B. $\dfrac{x}{\sqrt{1+\left(\sum_{r=1}^{n} 1\right)x^2}}$

C. $\left(\dfrac{x}{\sqrt{1+x^2}}\right)^n$

D. $\dfrac{n\pi}{\sqrt{1+\pi x^2}}$

Answer: B

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46. Let $f: R \rightarrow R,$ then $f(x) = 2x + |\cos x|$ is:

(a).One-one into (b).One-one and onto

(c).May-one and into (d).Many-one and onto

A. One-one into

B. One-one and onto

C. May-one and into

D. Many-one and onto

Answer: B

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47. Let $f: R \rightarrow R$ be a function defined by $f(x) = x^3 + x^2 + 3x + \sin x$.

Then f is

A. One-one end into

B. One-one and onto

C. Many-one and into

D. many-one and onto

Answer: B

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48. If $f(x) = \{x\} + \{x+1\} + \{x+2\} \ldots \ldots \ldots \{x+99),$ then the value of $\left[f\left(\sqrt{2}\right)\right]$ is, where (.) denotes fractional part function & [.] denotes the greatest integer function (a).5050 (b).4950 (c).41 (d).14

A. 5050

B. 4950

C. 41

D. 14

Answer: C

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49. If $|\cot x + \cos ecx| = |\cot x| + |\cos ecx|, x \in [0, 2\pi]$, then complete set of values of x is : $(a). [0, \pi]$ $(b). \left(0, \frac{\pi}{2}\right]$ $(c). \left(0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right)$ $(d). \left(\pi, \frac{3\pi}{2}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$

A. $[0, \pi]$

B. $\left(0, \frac{\pi}{2}\right]$

C. $\left(0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right)$

D. $\left(\pi, \frac{3\pi}{2}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$

Answer: C

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50. The function $f(x)=0$ has eight dinstict real solution and f also satisfy $f(4+x)=f(4-x)$. The sum of all the eight solution of $f(x)=0$ is :

(a). 12 (b). 32 (c). 16 (d). 15

A. 12

B. 32

C. 16

D. 15

Answer: B

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51. Let f(x) polynomial of degree 5 with leading coefficient unity such that f(1)=5, f(2)=4,f(3)=3,f(4)=2,f(5)=1, then f(6) is equal to (a).0 (b). 24 (c). 120 (d). 720

A. 0

B. 24

C. 120

D. 720

Answer: C

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52. Let $f: A \rightarrow B$ be a function such that $f(x) = \frac{(c)}{\sqrt{x-2}} + \sqrt{4-x,}$ is invertible, then which of the following is not possible ?

A. $A = [3, 4]$

B. $A = [2, 3]$

C. $A = [2, 2\sqrt{3}]$

D. $\{2, 2\sqrt{2}]$

Answer: C

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53. Find the number of positive integral values of x satisfying $\left[\frac{x}{9}\right] = \left[\frac{x}{11}\right]$ is where [.] -=GI.F) (a). 21 (b). 22 (c). 23 (d). 24

A. 21

B. 22

C. 23

D. 24

Answer: D

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54. The domain of function $f(x) = \log_{\left[x+\frac{1}{2}\right]}\left(2x^2 + x - 1\right)$, where $[.]$ denotes the greatest integer function is :

A. $\left[\frac{3}{2}, \infty\right)$

B. $(2, \infty)$

C. $\left(-\frac{1}{2}, \infty\right) - \left\{\frac{1}{2}\right\}$

D. $\left(\frac{1}{2}, 1\right) \cup (1, \infty)$

Answer: A

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55. The solution set of the equation $[x]^2 + [x+1] - 2 = 0$, where $[.]$ represents greatest integeral function is :

A. $[-1, o) \cup [1, 2)$

B. $[-2, -1) \cup [1, 2]$

C. $[1, 2]$

D. $[-3, -2) \cup [2, 3)$

Answer: B

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56. Which among the following relations is a function ?

A. $x^2 + y^2 = r^2$

B. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = r^2$

C. $y^2 = 4ax$

D. $x^2 = dxy$

Answer: D

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57. A function $f: R \to R$ is defined as $f(x) = 3x^2 + 1$. then $f^{-1}(x)$ is :

A. $\frac{\sqrt{x-1}}{3}$

B. $\left(\frac{1}{2}\sqrt{x}-1\right.$

C. f^{-1} does not exist

D. $\sqrt{\frac{x-1}{3}}$

Answer: C

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58. If $\begin{cases} 2+x, & x \geq 0 \\ 4-x, & x < 0 \end{cases}$, a then $f(f(x))$ is given by :

A. 1) $f(f(x)) = \begin{cases} 4+x, & x \geq 0 \\ 6-x, & x < 0 \end{cases}$

B. 2) $f(f(x)) = \begin{cases} 4+x, & x \geq 0 \\ x, & x < 0 \end{cases}$

C. 3) $f(f(x)) = \begin{cases} 4-x, & x \geq 0 \\ x, & x < 0 \end{cases}$

D. 4) $f(f(x)) = \begin{cases} 4-x, & x \geq 0 \\ x+2x, & x < 0 \end{cases}$

Answer: A

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59. The function $f: R \rightarrow R$ defined as $f(x) = \frac{3x^2 + 3x - 4}{3 + 3x - 4x^2}$ is :

(a) One to one but not onto

(b) Onto but not one to one

(c) Both one to one and onto

(d)Neither one to one nor onto

A. One ot one buty not onto

B. Onto but not one to one

C. Both one to one and onto

D. Neither one to one nor onto

Answer: B

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60. The number of solutions of the equation $e^x - \log|x| = 0$ is :

A. 0

B. 1

C. 2

D. 5

Answer: B

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61. If complete solution set of $e^{-x} \leq 4 - xis[\alpha, \beta]$ is equal to : (where [.] denotes greatest integer function)

A. 0

B. 2

C. 1

D. 4

Answer: C

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62. Range of $f(x) = \sqrt{\sin(\log_7(\cos(\sin x)))}$ is:

A. $[0, 1]$

B. $\{0, 1\}$

C. $\{0\}$

D. $[1, 7]$

Answer: C

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63. If the domain of $y = f(x) is[-3, 2]$, then find the domain of $g(x) = f(|[x]|), where$ $[.]$ denotes the greatest integer function.

A. $[-3, 2]$

B. $[-2, 3)$

C. $[-3, 3]$

D. $[-2, 3]$

Answer: B

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64. Range of the function $f(x) = \cot^{-1}\{-x\} + \sin^{-1}\{x\} + \cos^{-1}\{x\}$, where {.} denotes fractional part function:

A. $\left(\frac{3\pi}{4}, \pi\right)$

B. $\left[\frac{3\pi}{4}, \pi\right)$

C. $\left[\frac{3\pi}{4}, \pi\right]$

D. $\left(\frac{3\pi}{4}, \pi\right]$

Answer: D

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65. Let

$f: R - \left\{\frac{3}{2}\right\} \to R,\ f(x) = \frac{3x+5}{2x-3}.\ Let f_1(x) = f(x),\ f_n(x) = f(f_{n-1}(x)$

for $\pi \geq 2,\ n \in N,$ then $f_{2008}(x) + f_{2009}(x) =$

A. $\frac{2x^2+5}{2x-3}$

B. $\frac{x^2+5}{2x-3}$

C. $\frac{2x^2-5}{2x-3}$

D. $\frac{x^2-5}{2x-3}$

Answer: A

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66. Range of the function, $f(x) = \frac{(1+x+x^2)(1+c^4)}{x^3}$, for $x > \ = 0$

is :

A. $[0, \infty]$

B. $[2, \infty]$

C. $[4, \infty]$

D. $[6, \infty]$

Answer: D

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67. The function $f: (-\infty, 3] \rightarrow (o, e^7]$ defined by $f(x) = e^{x^3 - 3x^2 - 9x + 2}$ is

A. Many one and onto

B. Many one and into

C. One to one and onto

D. One to one and into

Answer: A

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68. Find the domain and range of function

$$f(x) = \sin\left(\log_e\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)$$

A. $[-1, 1]$

B. $[0, 1]$

C. $[-1, 1)$

D. None of these

Answer: A

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69. Set of values of 'a' for which the function $f: R \to R$, given by $f(x) = x^3 + (a+2)x^2 + 3ax + 10$ is one-one is given by :

A. $(-\infty, 1] \cup [4, \infty)$

B. $[1, 4]$

C. $[1, \infty]$

D. $[-\infty, 4]$

Answer: B

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70. If the range of the function $f(x) = \tan^{-1}(3x^2 + bx + c)$ is $\left[0, \frac{\pi}{2}\right)$, (domain is R), then:

A. $b^2 = 3c$

B. $b^2 = 4c$

C. $b^2 = 12c$

D. $b^2 = 8c$

Answer: C

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71. Let $f(x) = \sin^{-1} x - \cos^{-1}$ x, then the set of values of k for which of $|f(x)| = k$ has exactly two distinct solutions is :

A. $(0, \pi]$

B. $\left(0, \frac{\pi}{2}\right]$

C. $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right)$

D. $\left[\pi, \frac{3\pi}{2}\right]$

Answer: A

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72. Let $f: R \rightarrow R$ is defined by $f(x) = \begin{cases} (x+1)^3 & x \leq 1 \\ \ln x + (b^2 - 3b + 10) & x > 1 \end{cases}$ If f (x) is invertible, then the set of all values of 'b' is :

A. $\{1, 2\}$

B. ϕ

C. $\{2, 5\}$

D. None of these

Answer: A

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73. If f(x) is continuous such that $|f(x)| \leq 1, \forall x \in R$ and $g(x) = \frac{e^{f(x)} - e^{-|f(x)|}}{e^{f(x)} + e^{-|f(x)|}}$, then range of g(x) is

A. $[0, 1]$

B. $\left[0, \frac{e^2+1}{e^2-1}\right]$

C. $\left[0, \frac{e^2-1}{e^2+1}\right]$

D. $\left[\frac{e^2+1}{e^2+1}, 0\right]$

Answer: D

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74. Consider all function $f: \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ which are one-one, onto and satisfy the following property :

If $f(k)$ is odd then $f(k+1)$ is even, $K = 1, 2, 3$. The number of such function is :

A. 4

B. 8

C. 12

D. 16

Answer: C

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75. Consider the function $f: R - \{1\}$ given by $f(x) = \dfrac{2x}{x-1}$ Then $f^{-1}(x)$=

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76. If rang of fraction $f(x)$ whose domain is set of all real numbers is $[-2, 4]$, then range of function $g(x) = \frac{1}{2} f(2x+1)$ is equal to :

A. $[-2, 4]$

B. $[-1, 2]$

C. $[-3, 9]$

D. $[-2, 2]$

Answer: B

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77. Let $f: R \rightarrow$ and $f(x) = \frac{x(x^4+1)(x+1)+x^4+1}{x^2+x+1}$, then $f(x)$ is :

A. one-one, into

B. Many -one onto

C. One-one, onto

D. Many one, into

Answer: D

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78. Let f (x) be defined as

$$f(x) = \begin{cases} |x| & 0 \le x < 1 \\ |x-1| + |x-2| & 1 \le x < 2 \\ |x-3| & 2 \le x < 3 \end{cases}$$

The range of function $g(x) = \sin(7(f(x))$ is :

A. $[0, 1]$

B. $[-1, 0]$

C. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

D. $[-1, 1]$

Answer: D

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79. If $[x]^2 - 7[x] + 10 < 0$ and $4[y]^2 - 16[y] + 7 < 0$, then $[x + y]$ cannot be ([.]` denotes greatest integer function):

A. 7

B. 8

C. 9

D. both (b) and (c)

Answer: C

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80. Let $f: R \rightarrow R$ be a functino defined by $f(x) = e^x - e^{-x}, then f^{-1}(x) =$

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81. The function $f(x)$ satisfy the equation $f(1-x)+2f(x)=3x\,\forall x \in R,$ then $f(0)=$

A. -2

B. -1

C. 0

D. 1

Answer: B

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82. Let $f:[0,5]\to[0,5)$ be an invertible function defined by $f(x)=ax^2+bx+C,$ where $a,b,c\in R, abc\neq 0,$ then one of the root of the equation $cx^2+bx+a=0$ is:

A. a

B. b

C. c

D. $a + b + c$

Answer: A

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83. Let $f(x) = x^2 + \lambda x + \mu \cos x$, λ being an integer and μ is a real number. The number of ordered pairs (λ, μ) for which the equation $f(x) = 0$ and $f(f(x)) = 0$ have the same (non empty) set of real roots is:

A. 2

B. 1

C. 4

D. 6

Answer: C

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84. Consider all function $f:\{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$ which are one-one, onto and satisfy the following property :

If $f(k)$ is odd then $f(k+1)$ is even, $K = 1, 2, 3$. The number of such function is :

A. 4

B. 8

C. 12

D. 16

Answer: C

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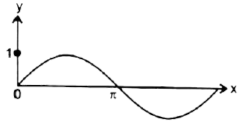
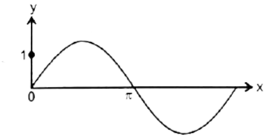
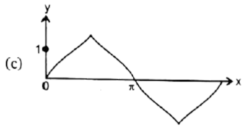
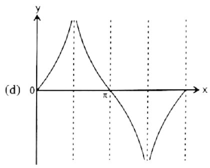
85. Which of the following is closest to the graph of $y = \tan(\sin x), x > 0$?

A. 

B. 

C. 

D. 

Answer: B

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86. Consider the function $f: R-\{1\} \rightarrow R-\{2\}$ given by $f(x)=\dfrac{2x}{x-1}$ Then :

A. f is one-one but not onto

B. f is onto but not one-one

C. f is one-one nor onto

D. f is both one-one and onto

Answer: D

Watch Video Solution

87. If rangr of funtion $f(x)$ whose domain is set of all real numbers is $[-2, 4]$, then range of function $g(x)=\frac{1}{2}f(2x+1)$ is equal to

A. $[-2, 4]$

B. $[-1, .2]$

C. $[-3, 9]$

D. $[-2, 2]$

Answer: B

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88. Let $f: R \to$ and $f(x) = \dfrac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$, then $f(x)$ is

:

A. One-one, into

B. Many one, onto

C. One-one, onto

D. Many one, into

Answer: D

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89. Let f (x) be defined as

$$f(x) = \begin{cases} |x| & 0 \le x < 1 \\ |x-1| + |x-2| & 1 \le x < 2 \\ |x-3| & 2 \le x < 3 \end{cases}$$

The range of function $g(x) = \sin(7(f(x))$ is :

A. $[0, 1]$

B. $[-1, 0]$

C. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

D. $[-1, 1]$

Answer: D

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90. Number of integral values of x in the domain of function $f(x) = \sqrt{\ln(|\ln|x||)} + \sqrt{7|x| - (|x|)^2 - 10}$ is equal to

A. 5

B. 6

C. 7

D. 8

Answer: B

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91. The number of integral ordered pair (x,y) that satisfy the system of equatin $|x+y-4|=5$ and $|x-3|+|y-1|=5$ is/are:

A. 2

B. 4

C. 6

D. 12

Answer: D

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92. Let $f: R \to R$, where $f(x) = \frac{x^2+ax+1}{x^2+x+1}$. Then the complete set of values of 'a' such that f(x) is onto is :

A. $(-\infty, \infty)$

B. $(-\infty, 0)$

C. $(0, \infty)$

D. Empty set

Answer: D

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93. If $A = \{1, 2, 3, 4\}$ and $f: A \to A,$ then total number of invertible function 'f' such that $f(2) \neq 2, f(4) \neq 4, f(1) = 1$ is equal to :

A. 1

B. 2

C. 3

D. 4

Answer: C

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94. The domian of definition of $f(x) = \log_{(x^2 - x - 1)}(2x^2 - 7x + 9)$ is :

A. R

B. $R - \{0\}$

C. $R - \{0, 1\}$

D. $R - \{1\}$

Answer: C

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95. If $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ are two sets and function $f: A \rightarrow B$ is defined by $f(x) = x + 2, \forall x \in A$, then the function f is

A. bijective

B. one one into

C. many oneonto

D. None of these

Answer: B

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96. Let $f(x) = x^2 - 2x - 3, x \geq 1$ and $g(x) = 1 + \sqrt{x + 4}, x \geq -4$ then the number of real solution os equation $f(x) = g(x)$ is/are

A. 0

B. 1

C. 2

D. 4

Answer: B

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One Or More Than One Answer Is Are Correct

1. f(x) is an even periodic function with period 10. In $[0, 5] f(x) = \begin{cases} 2x & 0 \leq x < 2 \\ 3x^2 - 8 & 2 \leq x < 4 \\ 10x & 4 \leq x \leq 5 \end{cases}$ Then :

A. $f(-4) = 40$

B. $\dfrac{f(-13) - f(11)}{f(13) + f(-11)} = \dfrac{17}{21}$

C. $f(5)$ is not defined

D. Range of $f(x)$ is $[0, 50]$

Answer: A::B::D

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2. Let $f(x) = \left| |x^2 - 4x + 3| - 2 \right|$. Which of the following is/are correct ?

A. $f(x) =$ m has exactly two real solutios of different sign $\forall m > 2$

B. $f(x) = m$ has exactly two real solution $\forall m \in (2, \infty) \cup \{0\}$

C. $f(x) = m$ has no solutions $\forall m < 0$

D. $f(x) = m$ has four distinct real solution $\forall m \in (0, 1)$

Answer: A::B::C

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3. Let $f(x) = \cos^{-1}\left(\frac{1 - \tan^2(x/2)}{1 + \tan^2(x/2)}\right)$

Which of the following statement (s) is/are correct about $f(x)$?

A. Domain is R

B. Range is $[0, \pi]$

C. $f(x)$ is even

D. $f(x)$ is dervable in $(\pi, 2\pi)$

Answer: C::D

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4. $|\log_e|x|| = |k-1| - 3$ has four distict roots then k satisfies : (where $|x| < d^2, x \neq 0)$

A. $(-4, -2)$

B. $(4, 6)$

C. (e^{-1}, e)

D. (d^{-2}, e^{-1})

Answer: A::B

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5. Which of the following funjctions are difined for all $x \in R$?

(Where$[.]$ = denotes greatest integer function)

A. $f(x) = \sin[x] + \cos[x]$

B. $f(x) = \sec^{-1}(1 + \sin^2 x)$

C. $f(x) = \sqrt{\frac{9}{8} + \cos x + \cos 2x}$

D. $f(x) = \tan(ln(1 + |x|))$

Answer: A::B::C

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6. Let $f(x) = \begin{cases} x^2 & 0 < x < 2 \\ 2x - 3 & 2 \leq x < 3 \\ x + 2 & x \geq 3 \end{cases}$ then the tuue equations:

A. $f\left(f\left(f\left(\frac{3}{2}\right)\right)\right) = f\left(\frac{3}{2}\right)$

B. $1 + f\left(f\left(f\left(\frac{5}{2}\right)\right)\right) = f\left(\frac{5}{2}\right)$

C. $f(f(f(2)) = f(1)$

D. $\underbrace{f(f(f(......f(4))...))} = 2012$

Answer: A::B::C::D

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7. Let $f:\left[\frac{2\pi}{3},\frac{5\pi}{3}\right]\to[0,4]$ be a function difined as $f(x)=\sqrt{3}\sin x-\cos x+2$, then :

A. $f^{-1}(1)=\frac{4\pi}{3}$

B. $f^{-1}(1)=\pi$

C. $f^{-1}(2)=\frac{5\pi}{6}$

D. $f^{-1}(2)=\frac{7\pi}{6}$

Answer: A::D

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8. Let $f(x)$ be invertible function and let $f^{-1}(x)$ be is inverse. Let equation $f\left(f^{-1}(x)\right)=f^{-1}(x)$ has two real roots α and β (with in domain of $f(x)$), then :

A. $f(x)=x$ also have same two rreal roots

B. $f^{-1}(x)=x$ also have same two real roots

C. $f(x) = f^{-1}(x)$ also have same two real roots

D. Area of triangle formed by $(0, 0), (\alpha, f(\alpha))$, and $(\beta, f(\beta))$ is 1 unit

Answer: A::B::C

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9. Find the value of $\cos^{-1}(x) + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3 - 3x^2}}{2}\right)$

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10. Let $f : R \to R$ defined by $f(x) = \cos^{-1}(-\{-x\})$, where {x} denotes fractional part of x. Then, which of the following is/are correct?

A. f is many coe but not even function

B. Eange of f contains two prime numbers

C. f is a periodic

D. Graph of f does not lie below x-axis

Answer: A::B::D

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11. Which option (s) is/are ture ?

A. $f: R \rightarrow R,\ f(x) = e^{|x|} - e^{-x}$ is many-one into function

B. $f: R \rightarrow R,\ f(x) = 2x + |\sin x|$ is one-one onto

C. $f: R \rightarrow R,\ f(x) = \dfrac{x^2 + 4x + 30}{x^2 - 8x + 18}$ is many-one onto

D. $f: R \rightarrow R,\ f(x) = \dfrac{2x^2 - x + 5}{7x^2 + 2x + 10}$ is many-one into

Answer: A::B::D

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12. If $f(x) = \left[\frac{\ln(x)}{e}\right] + \left[\frac{\ln(e)}{x}\right]$, where [.] denotes greatest interger function, the which of the following are ture ?

A. range of $f(x) is \{-1, 0\}$

B. If $f(x) = -1,$ then x can be rational as well as irrational

C. If $f(x) = -1,$ then x can be rational as well as irrational

D. $f(x)$ is periodic function

Answer: A::C

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13. If $f(x) = \begin{cases} x^3 & x = Q \\ -x^3 & x \neq Q \end{cases}$, then :

A. f (x) is periodic

B. f (x) is many-one

C. f (x) is one-one

D. range of the function is R

Answer: C::D

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14. Let $f(x)$ be a real vaued continuous function such that

$f(0)=\frac{1}{2}$ and $f(x+y)=f(x)f(4-y)+f(y)f(4-x)\,\forall x, y \in R,$

then for some real a:

A. f (x) is perodic function

B. f (x) is a constant function

C. $f(x)=\frac{1}{2}$

D. $f(x)=\frac{\cos x}{2}$

Answer: A::B::C

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15. f(x) is an even periodic function with period 10. In $[0, 5] f(x) = \begin{cases} 2x & 0 \le x < 2 \\ 3x^2 - 8 & 2 \le x < 4 \\ 10x & 4 \le x \le 5 \end{cases}$ Then :

A. $f(-4) = 40$

B. $\dfrac{f(-13) - x(11)}{f(13) + f(-11)} = \dfrac{17}{21}$

C. $f(5)$ is not defined

D. Range of $f(x)$ is $[0, 50]$

Answer: A::B::D

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16. For the equation $\dfrac{e^{-x}}{1+x} = \lambda$ which of the following statement (s) is/are correct ?

A. when $\lambda \in (0, \infty)$ equation has 2 real and distinct roots

B. when $\lambda \in \left(-\infty, -e^2\right)$ equation has 2 real anddistinct roots

C. when $\lambda \in (0, \infty)$ equatio has 1 real root

D. when $\lambda \in (-e, 0)$ equation has no real root

Answer: B::C::D

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17. For $x \in R^+$, if $x, [x], \{x\}$ are in harmonic progesssion then the vaue of x can not be equal to :

A. $\frac{1}{\sqrt{2}}\tan\frac{\pi}{8}$

B. $\frac{1}{\sqrt{2}}\cot\frac{\pi}{8}$

C. $\frac{1}{\sqrt{2}}\tan\frac{\pi}{12}$

D. $\frac{1}{\sqrt{2}}\cot\frac{\pi}{12}$

Answer: A::C::D

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18. The equation $||x-1|+a|=4, a \in R,$ has :

A. 3 distinet real roots for unique value of a.

B. 4 distinct real roots for $a \in (-\infty, -4)$

C. 2 distinct real roots for $|a| < 4$

D. no real roots for $a > 4$

Answer: A::B::C::D

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19. The numer of real values of x satisfying the equation , $\left[\frac{2x+1}{3}\right]+\left[\frac{4x+5}{6}\right]=\frac{3x-1}{2}$ are greater than or equal to {[.]` denotes greatest integer functio}:

A. 7

B. 8

C. 9

D. 10

Answer: A::B::C

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20. Let $f\left(x = \sin^6\left(\frac{x}{4}\right) + \cos^6\left(\frac{x}{4}\right). Iff f^n(x)\right.$ denotes n^{th} derivative of f evaluated at x. Then which of the following hold ?

A. $f^{2014}(0) = -\frac{3}{8}$

B. $f^{2015}(0) = \frac{3}{8}$

C. $f^{2010}\left(\frac{\pi}{2}\right) = 0$

D. $f^{2011}\left(\frac{\pi}{2}\right) = \frac{3}{8}$

Answer: A::C::D

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21. Which of the following is (are) incorrect ?

A. If $f(x) = \sin x$ and $g(x) =$ in x then range of $g(f(x))$ is $[-1, 1]$

B.

C. If $f(x) = \left(2011 - x^{2012}\right)^{\frac{1}{2012}}$ then $f(f(2)) = \frac{1}{2}$

D. The function $f: R \to R$ defined as $f(x) = \frac{x^2 + 4x + 30}{x^2 - 8x + 18}$ is not surjective.

Answer: A::B

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22. If [x] denotes the integral part of x for real x, and

$$S = \left[\frac{1}{4}\right] + \left[\frac{1}{4} + \frac{1}{100}\right] + \left[\frac{1}{4} + \frac{1}{100}\right] + \left[\frac{1}{4} + \frac{3}{200}\right] \ldots . + \left[\frac{1}{4} + \frac{199}{200}\right.$$

then

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23. Let $f(x) = \log_{\{x\}}[x]$

$g(x) = \log_{\{x\}}\{x\}$

$h(x) = \log_{\{x\}}\{x\}$

where $[], \{\}$ denotes the greatest integer function and fractional part function respectively.

Domine of $h(x)$ is :

A. 5

B. 4

C. 3

D. 2

Answer: C

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Comprehension Type Problems

1. Let $f(x) = \log_{\{x\}}[x]$

$g(x) = \log_{\{x\}} - \{x\}$

$h(x)\log_{\{x\}}\{x\}$

where $[], \{\}$ denotes the greatest integer function and fractional part function respectively.

If $A = \{x : x \in \text{ domine of } f(x)))$ and $B\{x : x$ domine of $g(x)\}$ then $\forall x \in (1, 5)$, $A - B$ will be :

A. $(2, 3)$

B. $(1, 3)$

C. $(1, 2)$

D. None of these

Answer: D

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2. Let $f(x) = \log_{\{x\}}[x]$

$g(x) = \log_{\{x\}}\{x\}$

$h(x) = \log_{\{x\}}\{x\}$

where $[], \{\}$ denotes the greatest integer function and fractional part function respectively.

Domine of $h(x)$ is :

A. $[2, \infty)$

B. $[1, \infty)$

C. $[2, \infty) - \{I\}$

D. $R^{+} - \{I\}$

Answer: C

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3. θ is said to be well behaved if it lies in interval $\left[0, \frac{\pi}{2}\right]$. They are intelligent if they make domain of $f + g$ and g equal. The vlaue of θ for which $h(\theta)$ is defined are handosome. Let

$$f(x) = \sqrt{\theta x^2 - 2(\theta^2 - 3)x - 12\theta}, g(x) = \ln(x^2 - 49),$$

$$h(\theta)\ln\left[\int_0^\theta 4\cos^2 t dt - \theta^2\right],$$ where θ is in radians.

Complete set of vlaues of θ which are well behaved as well as intelligent is:

A. $\left[\frac{3}{4}, \frac{\pi}{2}\right]$

B. $\left[\frac{3}{5}, \frac{7}{8}\right]$

C. $\left[\frac{5}{6}, \frac{\pi}{2}\right]$

D. $\left[\frac{6}{7}, \frac{\pi}{2}\right]$

Answer: D

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4. θ is said to be well behaved if it lies in interval $\left[0, \frac{\pi}{2}\right]$. They are intelligent if they make domain of $f + g$ and g equal. The vlaue of θ for which $h(\theta)$ is defined are handosome. Let

$f(x) = \sqrt{\theta x^2 - 2(\theta^2 - 3)x - 12\theta}, g(x) = \ln(x^2 - 49),$

$h(\theta)\ln\left[\int_0^\theta 4\cos^2 t dt - \theta^2\right]$, where θ is in radians.

Complete set of alues of θ which are intelligent is :

A. $\left[\frac{6}{7}, \frac{7}{2}\right]$

B. $\left(0, \frac{\pi}{3}\right]$

C. $\left[\frac{1}{4}, \frac{6}{7}\right]$

D. $\left[\frac{1}{2}, \frac{\pi}{2}\right]$

Answer: A

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5. θ is said to be well behaved if it lies in interval $\left[0, \frac{\pi}{2}\right]$. They are intelligent if they make domain of $f + g$ and g equal. The vlaue of θ for which $h(\theta)$ is defined are handosome. Let

$f(x) = \sqrt{\theta x^2 - 2(\theta^2 - 3)x - 12\theta}, g(x) = \ln(x^2 - 49),$

$h(\theta)\ln\left[\int_0^\theta 4\cos^2 t dt - \theta^2\right]$, where θ is in radians.

Complete set of vlaues of θ which are well behaved as well as intelligent is:

A. $\left(0, \frac{\pi}{2}\right]$

B. $\left[\frac{6}{7}, \frac{\pi}{2}\right]$

C. $\left[\frac{3}{4}, \frac{\pi}{2}\right]$

D. $\left[\frac{3}{5}, \frac{\pi}{2}\right]$

Answer: B

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6. Let $f(x) = 2 - |x - 3|, 1 \leq x \leq 5$ and for rest of the values f(x) can be obtained by using the relation $f(5x) = \alpha f(x) \forall x \in R$ The maximum value of $f(x)$ in $\left[5^4, 5^5\right]$ for $\alpha = 2$ is

A. 16

B. 32

C. 64

D. 8

Answer: B

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7. Let $f(x) = 2 - |x - 3|, 1 \leq x \leq 5$ and for rest of the values f(x) can be obtained by using the relation $f(5x) = \alpha f(x) \forall x \in R$ The maximum value of $f(x)$ in $[5^4, 5^5]$ for $\alpha = 5$ is

A. 1118

B. 2007

C. 1250

D. 132

Answer: A

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8. An even periodic functin $f: R \to R$ with period 4 is such that

$$f(x) = \begin{bmatrix} \max.(|x|, x^2) & 0 \le x < 1 \\ x & 1 \le x \le 2 \end{bmatrix}$$

The value of $\{f(5.12)\}$ (where {.} denotes fractional part function), is :

A. $\{f(3.26)\}$

B. $\{f(7.88)\}$

C. $\{f(2.12)\}$

D. $\{f(5.88)\}$

Answer: B

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9. An even periodic functin $f: R \to R$ with period 4 is such that

$$f(x) = \begin{bmatrix} \max.(|x|, x^2) & 0 \le x < 1 \\ x & 1 \le x \le 2 \end{bmatrix}$$

The number of solution of $f(x) = |3\sin x|$ for $x \in (-6, 6)$ are :

A. 5

B. 3

C. 7

D. 9

Answer: C

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10. Let $f(x) = \frac{2|x| - 1}{x - 3}$

Range of $f(x)$:

A. $R - \{3\}$

B. $\left(-\infty, \frac{1}{3}\right] \cup (2, \infty)$

C. $\left(-2, \frac{1}{3}\right] \cup (2, \infty)$

D. R

Answer: B

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11. Let $f(x) = \frac{2|x| - 1}{x - 3}$

Range of the values of 'k' for which $f(x) = k$ has exactly two distinct solutions:

A. $\left(-2, \frac{1}{3}\right)$

B. $(-2, 1]$

C. $\left(0, \frac{2}{3}\right]$

D. $(-\infty, -2)$

Answer: A

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12. Let $f(x)$ be a continuous function (define for all x) which satisfies $f^3(x) - 5f^2(x) + 10f(x) - 12 \geq 0$, $f^2(x) + 3 \geq 0$ and $f^2(x) - 5f(x) +$

If distinct positive number b_1, b_2 and b_3 ar in G.P. then $f(1) + \ln b_1), f(2) + \ln b_2, f(3) + \ln b_3$ are in :

A. A.P.

B. G.P.

C. H. P.

D. A. G. P.

Answer: A

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13. Let $f(x)$ be a continuous function (define for all x) which satisfies $f^3(x)-5f^2(x)+10f(x)-12\geq 0, f^2(x)+3\geq 0$ and $f^2(x)-5f(x)+$

The equation of tangent that can be drawn from $(2, 0)$ on the curve $y=x^2 f(\sin x)$ is :

A. $y=24(x+2)$

B. $y=12(x+2)$

C. $y=24(x-2)$

D. $y=12(x-2)$

Answer: C

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14. Let $f:[2,\infty)\rightarrow\{1,\infty)$ defined by $f(x)=2^{x^4-4x^2}$ and $g:\left[\frac{\pi}{2},\pi\right]\rightarrow A$ defined by $g(x)=\frac{\sin x+4}{\sin x-2}$ be two invertible functions, then

$f^{-1}(x)$ is equal to

A. $\sqrt{2+\sqrt{4-\log_2 x}}$

B. $\sqrt{2+\sqrt{4+\log_2 x}}$

C. $\sqrt{4+\sqrt{4+\log_2 x}}$

D. $\sqrt{4-\sqrt{2+\log_2 x}}$

Answer: B

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15. Let $f:[2,\infty)\to\{1,\infty)$ defined by $f(x)=2^{x^4-4x^3}$ and $g:\left[\frac{\pi}{2},\pi\right]\to A$ defined by $g(x)=\frac{\sin x+4}{\sin x-2}$ be two invertible functions, then

The set "A" equals to

A. $[5, 2]$

B. $[-2, 5]$

C. $[-5, 2]$

D. $[-5, -2]$

Answer: D

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Matching Type Problems

1. If $x, y, z\in R$ satisfies the system of equations $x+(y)+(s)=12.7, [x]+\{y\}+z=4.1$ and $\{x\}+y+[z]=2$

where {.} and [.] denotes the fractional and integral parts respectively)

then match the following

	Column-I		Column-II
(A)	$\{x\}+\{y\}=$	(P)	7.7
(B)	$[z]+[x]=$	(Q)	1.1
(C)	$x+\{z\}=$	(R)	1
(D)	$z+[y]-\{x\}=$	(S)	3
		(T)	4

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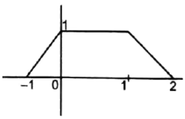
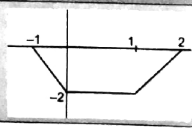
2. Given the graph of $y = f(x)$



	Column-I		Column-II
(A)	$y=f(1-x)$	(P)	

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3.

	Column-I		Column-II
(A)	$f(x)=\sin^2 2x-2\sin^2 x$	(P)	Range contains no natural number
(B)	$f(x)=\frac{4}{\pi}(\sin^{-1}(\sin \pi x))$	(Q)	Range contains atleast one integer
(C)	$f(x)=\sqrt{\ln(\cos(\sin x))}$	(R)	Many one but not even function
(D)	$f(x)=\tan^{-1}\left(\frac{x^2+1}{x^2+\sqrt{3}}\right)$	(S)	Both many one and even function
		(T)	Periodic but not odd function

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4.

	Column-I		Column-II
(A)	If $\lvert x^2-x\rvert \ge x^2+x$, then complete set of values of x is	(P)	$(0,\infty)$
(B)	If $\lvert x+y\rvert > x-y$, where $x>0$, then complete set of values of y is	(Q)	$(-\infty,0]$
(C)	If $\log_2 x \ge \log_2(x^2)$, then complete set of values of x is	(R)	$[-1,\infty)$
(D)	$[x]+2\ge\lvert x\rvert$, (where $[\cdot]$ denotes the greatest integer function) then complete set of values of x is	(S)	$(0,1]$
		(T)	$[1,\infty)$

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5.

	Column-I		Column-II
(A)	Domain of $f(x) = \ln \tan^{-1}\{(x^3 - 6x^2 + 11x - 6)x(e^x - 1)\}$ is	(P)	$\left[-1, \frac{5}{4}\right]$
(B)	Range of $f(x) = \sin^2\frac{x}{4} + \cos\frac{x}{4}$ is	(Q)	$[2, \infty)$
(C)	The domain of function $f(x) = \sqrt{\log_{(\lvert x\rvert-1)}(x^2 + 4x + 4)}$ is	(R)	$(1, 2) \cup (3, \infty)$
(D)	Let $f(x) = \begin{cases} x^2 & x < 1 \\ x+1 & x \geq 1 \end{cases}; g(x) = \begin{cases} x+2 & x < 1 \\ x^2 & x \geq 1 \end{cases}$	(S)	$[0, \infty)$
	Then range of function $f(g(x))$ is	(T)	$(-\infty, -3) \cup (-2, -1) \cup (2, \infty)$

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6. Let $f(x)\begin{bmatrix} 1 + x, & 0 \leq x \leq 2 \\ 3 - x, & 2 < x \leq 3 \end{bmatrix}$:

$g(x) = f(f(x))$:

	Column-I		Column-II
(A)	If domain of $g(x)$ is $[a, b]$ then $b - a$ is	(P)	1
(B)	If range of $g(x)$ is $[c, d]$ then $c + d$ is	(Q)	2
(C)	$f(f(f(2))) + f(f(f(3)))$, is	(R)	3
(D)	m = maximum value of $g(x)$ then $2m - 2$ is :	(S)	4

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Subjective Type Problems

1. Let $f(x)$ be a polynomial of degree 6 with leading coefficient 2009. Suppose further that f(1) =1, f(2)=3, f(3)=5, f(4)=7, f(5) =9, f'(2)=2. Then the sum of all the digits of f(6) is

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2. Let $f(x) = x^3 - 3x$ Find $f(f(x))$

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3. If $f(x+y+1) = \left\{\sqrt{f(x)} + \sqrt{f(y)}\right\}^2$ and $f(0) = 1 \forall x, y \in R, determ \in ef(n), n \in N.$

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4. If the domain of $f(x) = \sqrt{12 - 3^x - 3^{3-x}} + \sin^{-1}\left(\frac{2x}{3}\right) is[a, b]$, then $a = \ldots\ldots$

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5. The number of elements in the range of functions: $y = \sin^{-1}\left[x^2 + \frac{5}{9}\right] + \cos^{-1}\left[x^2 - \frac{4}{9}\right]$ where where [.] denotes the greatest integer function is:

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6. The number of integers in the range of function $f(x) = [\sin x] + [\sin x + \cos x]$ is (where [.]= denotes greatet integer funcion)

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7. If $P(x)$ is polynomial of degree 4 such than $P(-1) = P(1) = 5 \text{ and } P(-2) = P(0) = P(2) = 2$ find the maximum vaue of P (x).

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8. The number of integral vlaue (s) of k for which the curve $y=\sqrt{-x^2-2x}$ and $x+y-k=0$ intersect at 2 distinct points is/are

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9. Let the solution set of the equation :

$\sqrt{\left[x+\left[\frac{x}{2}\right]\right]}+\sqrt{(x)+\left[\frac{x}{3}\right]}=3$ is (a,b) Find the product ab.

(where $[.]$ and $\{.\}$ denote greatest integer and fractional part function repectively),

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10. For the real number x, let $f(x)=\frac{1}{2011\sqrt{1-x^{2011}}}$. Find the number of real roots of the equation

$f(f(\ldots..(f(x))\ldots..)=(\{-x\}$

where f is applies 2013 times and {.} denotes fractional part function.

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11. Find the number of elements contained in the range of the function $f(x) = \left[\frac{x}{6}\right]\left[\frac{-6}{x}\right] \forall x \in$ (0,30)]$where[.]`$ denotes greatest integer function)

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12. Let $f(x, y) = x^2 - y^2$ and $g(x, y) = 2xy.$ such that $(f(x, y))^2 - (g(x, y))^2 = \frac{1}{2}$ and $f(x, y). G(x, y) = \frac{\sqrt{3}}{4}$ Find the number of ordered pairs (x, y) ?

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13. Let $f(x) = \frac{x+5}{\sqrt{x^2+1}} \forall x \in R,$ then the smallest integral value of k for which $f(x) \leq k \forall x \in R$is

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14. The number of integral values of a for which $f(x) = x^3 + (a+2)x^2 + 3ax + 5$ is monotonic in $\forall x \in R$.

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15. The number of roots of equation $\left(\frac{(x-1)(x-3)}{(x-2)(x-4)} - e^x\right)\left(\frac{(x+1)(x+3)e^x}{(x+2)(x+4)} - 1\right)(x^3 - \cos x) = 0$:

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16. The number of solutions of the equation $\cos^{-1}\left(\frac{1-x^2-2x}{(x+1)^2}\right) = \pi(1 - \{x\},$ for $x \in [0, 76]$ is equal to (where $\{.\}$ denote fraction part function)

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17. Let $f(x) = x^2 - bx + c$, b is an odd positive integer. Given that f(x)=0 has two prime numbers as roots and b+c=35. If the least value of $f(x) \forall x \in R$ is λ, then $\left[\left|\frac{\lambda}{3}\right|\right]$ is equal to (where [.] denotes greatest integer function)

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18. Let $f(x)$ be a continuous function such that $f(0) = 1$ and $f(x) = f\left(\frac{x}{7}\right) = \frac{x}{7} \forall x \in R$, then $f(42)$ is

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19. If $f(x) = 4x^3 - x^2 - 2x + 1$ and $g(x) = \begin{cases} \min\{f(t) : 0 \le t \le x\} & 0 \le \\ 3 - x & 1 < \end{cases}$

and if $\lambda = g\left(\frac{1}{4}\right) + g\left(\frac{3}{4}\right) + g\left(\frac{5}{4}\right)$, then $2\lambda =$

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20. If $x = 10\sum_{r=3}^{100} \frac{1}{(r^2 - 4)}$, then $[x] =$

(where [.] denotes gratest integer function)

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21. Let $f(x) = \frac{ax+b}{xc+d}$, where a,b,c d are non zero If $f(7) = 7, f(11) = 11$ and $f(f(x)) = x$ for all x expect $-\frac{d}{c}$. The unique number which is not is the range of f is

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22. Let $A = \{x \mid x^2 - 4x + 3 < 0, x \in R\}$

If $A \subset B$, then the rang of real number $p \in [a, b]$ where, a,b are integers. Find the value of $(b - a)$.

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23. Let the maximum value of expression $y = \frac{x^4 - x^2}{x^6 + 2x^3 - 1}$ for $x > 1 is \frac{p}{1}$, where p and 1q are relatively prime natural numbers, then $p + q =$

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24. If $f(x)$ is an even function, then the number of distinect real numbers x such that $f(x) = f\left(\frac{x+1}{x+2}\right)$ is :

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25. The least integral value of m, m $\in R$ for which the range of function $f(x) = \frac{x+m}{x^2+1}$ contains the interval $[0, 1]$ is :

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26. Let x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in G.P. where $x(1), x_2, x_3$ are positive numbers. Then the maximum value of $[\beta] + [\gamma] + 4$ is where [.] denotes greatest integer function is :

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27. Let $A = \{1, 2, 3, 4\}$ and $B = \{0, 1, 2, 3, 4, 5\}$. If 'm' is the number of strictly increasing function f, $f: A \to B$ and n is the number of onto functions $g: B \to A$. Then the last digit of n-m is.

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28. If $\sum_{r=1}^{n} [\log_2 r] = 2010$ where [.] denotes greatest integer function, then the sum of the digits of n is:

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29. Let $f(x) = \frac{ax+b}{xc+d}$, where a,b,c d are non zero If $f(7) = 7, f(11) = 11$ and $f(f(x)) = x$ for all x expect $-\frac{d}{c}$. The unique number which is not is the range of f is

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30. It is pouring down rain and the amount of rain hitting point (x, y) is given by $f(x, y) = \left|x^3 + 2x^2y - 5xy^2 - 6y^3\right|$. If Mr. 'A' starts at $(0, 0)$, find number of possible value (s) for 'm' such that y= mx is a line along which Mr.' A could walk without any rain falling on him.

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31. Let P (x) be a cubic polynomical with leading co-efficient unity. Let the remainder when P (x) is divided by $x^2 - 5x + 6$ equals 2 times the remainder when P (x) is divided by $x^2 - 5x + 4$. If $P(0) = 100$, find the sum of the digits of $P(5)$,

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32. Let $f(x) = x^2 + 10x + 20$. Find the number of real solution of the equation $f(f(f(f(x)))) = 0$

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33. If range of $f(x) = \dfrac{(\ln x)\left(\ln x^2\right) + \ln x^3 + 3}{\ln^2 x + \ln x^2 + 2}$ can be expressed as $\left[\dfrac{a}{b}, \dfrac{c}{d}\right]$ where a,b,c and d are prime numbers (not nacessarily distinct) then find the value of $\dfrac{(a+b+c+d)}{2}$.

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34. Polynomial $P(x)$ is divided by $(x-3)$, the remainder if 6.If $P(x)$ is divided by $\left(x^2-9\right)$, then the remainder is $g(x)$. Then the value of $g(2)$ is __________.

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35. The equation $2x^3 - 3x^2 + p = 0$ has three real roots. Then find the minimum value of p.

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36. Find the number of integers in the domain of $f(x) = \frac{1}{\sqrt{\ln \cos^{-1} x}}$

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