MATHS

BOOKS - VK JAISWAL ENGLISH

MATRICES

Exercise 1 Single Choice Problems

1. Let $A = BB^T + CC^T$, where $B = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}, C = \begin{bmatrix} \sin\theta \\ -\cos\theta \end{bmatrix}, \theta \in R.$

Then prove that a is unit matrix.

A. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Answer: C

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2. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ Then only correct statement about the matrix A is (A) A is a zero matrix (B) $A^2 = 1$ (C) A^{-1} does not exist (D) $A = (-1)$ I where I is a unit matrix

A. A is a zero matrix

B. $A^2 = I$, where I is a unit matrix

C. A^{-1} does not exist

D. $A = (-1)I$, where I is a unit matrix

Answer: B

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3. Let $A = [a_{ij}]_{3 \times 3}$ be such that $a_{ij} = \begin{bmatrix} 3 & \text{when} & \hat{i} = \hat{j} \\ 0 & & \hat{i} \neq \hat{j} \end{bmatrix}$ then $\left\{ \dfrac{\det(\text{adj}(\text{adj A}))}{5} \right\}$ equals :

(where {.} denotes fractional part function)

A. $\frac{2}{5}$

B. $\frac{1}{5}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer: B

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4. If

$$A^{-1} = \begin{bmatrix} \sin^2\alpha & 0 & 0 \\ 0 & \sin^2\beta & 0 \\ 0 & 0 & \sin^2\gamma \end{bmatrix} \text{ and } B^{-1} = \begin{bmatrix} \cos^2\alpha & 0 & 0 \\ 0 & \cos^2\beta & 0 \\ 0 & 0 & \cos^2\gamma \end{bmatrix}$$

where α, β, γ are any real numbers and

$C = (A^{-5} + B^{-5}) + 5A^{-1}B^{-1}(A^{-3} + B^{-3}) + 10A^{-2}B^{-2}(A^{-1} + B^{-1}$

then find |C|.

A. 0

B. 1

C. 2

D. 3

Answer: B

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5. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $A^{-1} =$

A. A

B. A^2

C. A^3

D. A^4

Answer: C

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6. Let $M = [a_{ij}]_{3 \times 3}$ where $a_{ij} \in \{-1, 1\}$. Find the maximum possible value of det(M).

A. 3

B. 4

C. 5

D. 6

Answer: B

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7. Let matrix $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if $xyz = 2\lambda$ and $8x + 4y + 3x = \lambda + 28$, then (adj

A) A equals :

A. $\begin{bmatrix} \lambda+1 & 0 & 0 \\ 0 & \lambda+1 & 0 \\ 0 & 0 & \lambda+1 \end{bmatrix}$

B. $\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

C. $\begin{bmatrix} \lambda^2 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & \lambda^2 \end{bmatrix}$

D. $\begin{bmatrix} \lambda+2 & 0 & 0 \\ 0 & \lambda+2 & 0 \\ 0 & 0 & \lambda+2 \end{bmatrix}$

Answer: B

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8. If the trace of matrix

$A = [[x-2, e^x, -\sin x], [\cos x^2, x^2 - x + 3, \ln|x|], [0, \tan^{-1} x, x - 7]]$

is zero, then x is equal to :

A. -2 or 3

B. -3 or -2

C. -3 or 2

D. 2 or 3

Answer: C

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9. if $A=[a_{ij}]_{2\cdot 2}$ where $a_{ij}=\{i+j, i\neq j$ and $i^2-2j, i=j\}$ then A^{-1} is equal to

A. $\frac{1}{9}\begin{bmatrix}0 & 3\\ 3 & 1\end{bmatrix}$

B. $\frac{1}{9}\begin{bmatrix}0 & -3\\ 3 & -1\end{bmatrix}$

C. $\frac{1}{9}\begin{bmatrix}0 & -3\\ -3 & -1\end{bmatrix}$

D. $\frac{1}{3}\begin{bmatrix}0 & 3\\ 3 & 1\end{bmatrix}$

Answer: A

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10. If $\begin{pmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{pmatrix}\begin{pmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{pmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

A. $a = b = 1$

B. $a = \cos 2\theta, b = \sin 2\theta$

C. $a = \sin 2\theta, b = \cos 2\theta$

D. $a = 1, b = \sin 2\theta$

Answer: B

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11. A square matrix P satisfies $P^2 = I - P$, where I is identity matrix. If $P^n = 5I - 8P$, then n is :

A. 4

B. 5

C. 6

D. 7

Answer: C

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12. Let matrix $A = \begin{bmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ where $x, y, z \in N$. If det. (adj. (adj. A)) $= 2^8 \cdot 3^4$ then the number of such matrices A is :

[Note : adj. A denotes adjoint of square matrix A.]

A. 220

B. 45

C. 55

D. 110

Answer: C

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13. If A is a 2×2 non singular matrix, then adj(adj A) is equal to :

A. A^2

B. A

C. A^{-1}

D. $(A^{-1})^2$

Answer: B

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14. $A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ and $MA = A^{2m}$, $m \in N$ for some matrix M, then which one of the following is correct ?

A. $M = \begin{bmatrix} a^{2m} & b^{2m} \\ b^{2m} & -a^{2m} \end{bmatrix}$

B. $M = (a^2 + b^2)^m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C. $M = (a^m + b^m) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $M = (a^2 + b^2)^{m-1} \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

Answer: D

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15. Let A be a square matrix satisfying $A^2 + 5A + 5I = 0$. The inverse of $A + 2I$ is equal to :

A. $A - 2I$

B. $A + 3I$

C. $A - 3I$

D. non-existent

Answer: B

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16. Let $A=\begin{bmatrix}3 & -5 \\ 7 & -12\end{bmatrix}$ and $B=\begin{bmatrix}12 & -5 \\ 7 & -3\end{bmatrix}$ bc two given matrices, then $(AB)^{-1}$ is :

A. $\begin{bmatrix}1 & 0 \\ 0 & -1\end{bmatrix}$

B. $\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$

C. $\begin{bmatrix}-1 & 0 \\ 0 & 1\end{bmatrix}$

D. $\begin{bmatrix}0 & 1 \\ 1 & 0\end{bmatrix}$

Answer: B

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17. If matrix $A=\begin{bmatrix}2 & 2 \\ 2 & 3\end{bmatrix}$ then the value of [adj. A] equals to :

A. 2

B. 3

C. 4

D. 6

Answer: A

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18. If for the matrix $A = \begin{bmatrix} \cos\theta & 2\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, A^{-1} = A^T$ then number of possible value(s) of θ in $[0, 2\pi]$ is :

A. 2

B. 3

C. 1

D. 4

Answer: B

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19. Let M be a column vector (not null vector) and $A = \frac{MM^T}{M^TM}$ the matrix A is :

(where M^T is transpose matrix of M)

A. idempotant

B. nilpotent

C. involutary

D. none of these

Answer: A

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20. If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, P = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, Q = P^TAP,$ find $PQ^{2014}P^T$: (a) $\begin{pmatrix} 1 & 2^{2014} \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 4028 \\ 0 & 1 \end{pmatrix}$ (c) $(P^T)^{2013}A^{2014}P^{2013}$ (d) $P^TA^{2014}P$

A. $\begin{pmatrix} 1 & 2^{2014} \\ 0 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 4028 \\ 0 & 1 \end{pmatrix}$

C. $\left(P^T\right)^{2013} A^{2014} P^{2013}$

D. $P^T A^{2014} P$

Answer: B

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21. If M be a square matrix of order 3 such that $|M| = 2,$ then $\left|adj\left(\frac{M}{2}\right)\right|$ equals to :

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{16}$

Answer: D

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22. If A is matrix of order 3 such that $|A| = 5$ and $B =$ adj A, then the value of $\left||A^{-1}|(AB)^T\right|$ is equal to (where |A| denotes determinant of matrix A. A^T denotes transpose of matrix A, A^{-1} denotes inverse of matrix A. adj A denotes adjoint of matrix A)

A. 5

B. 1

C. 25

D. $\dfrac{1}{25}$

Answer: B

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Exercise 2 One Or More Than One Answer Is Are Correct

1. If A and B are two orthogonal matrices of order n and $\det(A) + \det(B) = 0$, then which of the following must be correct ?

A. $\det(A + B) = \det(A) + \det(B)$

B. $\det(A + B) = 0$

C. A and B both are singular matrices

D. $A + B = 0$

Answer: A::B

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2. Let M be a 3×3 matrix satisfying $M^3 = 0$. Then which of the following statement(s) are true: (a) $|M^2 + M + I| \neq 0$ (b) $|M^2 - M + I| = 0$ (c) $|M^2 + M + I| = 0$ (d) $|M^2 - M + I| \neq 0$

A. $|M^2 + M + I| \neq 0$

B. $|M^2 - M + I| = 0$

C. $\left|M^2 + M + I\right| = 0$

D. $\left|M^2 - M + I\right| \neq 0$

Answer: A::D

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3. Let $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then :

A. $A_{\alpha+\beta} = A_\alpha A_\beta$

B. $A_\alpha^{-1} = A_{-\alpha}$

C. $A_\alpha^{-1} = -A_\alpha$

D. $A_\alpha^2 = -I$

Answer: A::B

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4. $A^3 - 2A^2 - A + 2I = 0$ if $A =$

A. I

B. 2I

C. $\begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Answer: A::B::C::D

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5. Let A be 3×3 symmetric invertible matrix with real positive elements. Then the number of zero elements in A^{-1} are less than or equal to :

A. 0

B. 1

C. 2

D. 3

Answer: D

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Exercise 3 Matching Type Problems

1. Consida square matrix A of order 2 which has its elements as 0, 1, 2 and 4. Let N denotes the number of such matrices.

	Column-I		Column-II
(A)	Possible non-negative value of $\det(A)$ is	(P)	2
(B)	Sum of values of determinants corresponding to N matrices is	(Q)	4
(C)	If absolute value of $(\det(A))$ is least, then possible value of $\lvert \operatorname{adj}(\operatorname{adj}(\operatorname{adj} A))\rvert$	(R)	–2
(D)	If $\det(A)$ is least, then possible value of $\det(4A^{-1})$ is	(S)	0
		(T)	8

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2.

	Column-I		Column-II
(A)	If A is an idempotent matrix and I is an identify matrix of the same order, then the value of n, such that $(A+I)^n = I + 127A$ is	(P)	9
(B)	If $(I-A)^{-1} = I + A + A^2 + \ldots\ldots A^7$, then $A^n = O$ where n is	(Q)	10
(C)	If A is matrix such that $a_{ij} = (i+j)(i-j)$, then A is singular if order of matrix is	(R)	7
(D)	If a non-singular matrix A is symmetic, such that A^{-1} is also symmetric, then order of A can be	(S)	8

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3.

(C)	If $P = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10Q = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & t \\ 1 & -2 & 3 \end{bmatrix}$ and $Q = P^{-1}$, then the value of t is equal to	(R)	4
(D)	If $y = \tan u$ where $u = v - \frac{1}{v}$ and $v = \ln x$, then the value of $\frac{dy}{dx}$ at $x = e$ is equal to λ then $[\lambda]$ is equal to (where [·] denotes greatest integer function)	(S)	5

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4.

	Column-I		Column-II
(A)	If P and Q are variable points on $C_1 : x^2 + y^2 = 4$ and $C_2 : x^2 + y^2 - 8x - 6y + 24 = 0$ respectively then the maximum value of PQ, is equal to	(P)	1
(B)	Let P, Q, R be invertible matrices of second order such that $A = PQ^{-1}, B = QR^{-1}, C = RP^{-1}$, then the value of det. $(ABC + BCA + CAB)$ is equal to	(Q)	2
(C)	The perpendicular distance of the point whose position vector is (1, 3, 5) from the line $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ is equal to	(R)	9
(D)	Let $f(x)$ be a continuous function in $[-1, 1]$ such that $f(x) = \begin{cases} \frac{\ln(px^2 + qx + r)}{x^2} & ; -1 \le x < 0 \\ 1 & ; x = 0 \\ \frac{\sin(e^{x^2} - 1)}{x^2} & ; 0 < x \le 1 \end{cases}$ then the value of $(p + q + r)$, is equal to	(S)	8

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Exercise 4 Subjective Type Problems

1. A and B are two matrices. Such that $A^2B = BA$ and if $(AB)^{10} = A^k \cdot B^{10}$. Find the value of k 1020.

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2. Let A_n and B_n be square matrices of order 3, which are defined as :

$A_n = [a_{ij}]$ and $B_n = [b_{ij}]$ where $a_{ij} = \dfrac{2i + j}{3^{2n}}$ and $b_{ij} = \dfrac{3i - j}{2^{2n}}$ for all i and j, $1 \leq i, j \leq 3$.

If

$l = \lim_{n \to \infty} Tr. \left(3A_1 + 3^2 A_2 + 3^3 A_3 + \ldots\ldots. + 3^n A_n\right)$ and $m = \lim_{n \to \infty} T$

, then find the value of $\dfrac{(l + m)}{3}$

[Note : Tr (P) denotes the trace of matrix P.]

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3. Let A be a 2×3 matrix, whereas B be a 3×2 amtrix. If $\det.(AB) = 4$, then the value of $\det.(BA)$ is

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4. Find the maximum value of the determinant of an arbitrary 3×3 matrix A, each of whose entries $a_{ij} \in \{-1, 1\}$.

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5. The set of natural numbers is divided into array of rows and columns in the form of matrices as $A_1 = [1], A_2 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, A_3 = \begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$ and so on. Let the trace of A_{10} be λ . Find unit digit of λ ?

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