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## MATHS

## BOOKS - VK JAISWAL ENGLISH

## SOLUTION OF TRIANGLES

## Exercise 1 Single Choice Problems

1. In a $\triangle A B C$ if $9\left(a^{2}+b^{2}\right)=17 c^{2}$ then the value of the $\frac{\cot A+\cot B}{\cot C}$ is
A. $\frac{13}{4}$
B. $\frac{7}{4}$
C. $\frac{5}{4}$
D. $\frac{9}{4}$

## Answer: D

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2. Let $H$ be the orthocentre of triangle $A B C$. Then angle subtended by side BC at the centre of incircle of $\triangle C H B$ is
A. $\frac{A}{2}+\frac{\pi}{2}$
B. $\frac{B+C}{2}+\frac{\pi}{2}$
C. $\frac{B-C}{2}+\frac{\pi}{2}$
D. $\frac{B+C}{2}+\frac{\pi}{4}$

Answer: B
3. Circum radius of a $\triangle A B C$ is 3 units, let O be the circum and $H$ be the orthocentre then the value of $\frac{1}{64}\left(A H^{2}+B C^{2}\right)\left(B H^{2}+A C^{2}\right)\left(C H^{2}+A B^{2}\right)$ equals :
A. $3^{4}$
B. $9^{3}$
C. $27^{6}$
D. $81^{4}$

## Answer: B

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4. The angles $A, B$ and $C$ of a triangle $A B C$ are in arithmetic progression. If $2 b^{2}+3 c^{2}$ then the angle A is :
A. $15^{\circ}$
B. $60^{\circ}$
C. $75^{\circ}$
D. $90^{\circ}$

## Answer: C

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5. In a triangle ABC if $\tan \cdot \frac{A}{2} \tan \cdot \frac{B}{2}=\frac{1}{3}$ and $\mathrm{ab}=4$, then the value of c can be
A. 1
B. 2
C. 4
D. 6

## Answer: B

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6. In a triangle $A B C$ the expression $a \cos B \cos C+b \cos C \cos A+c \cos A \cos B$ equals to :
A. $\frac{r s}{R}$
B. $\frac{r}{s R}$
C. $\frac{R}{r s}$
D. $\frac{R s}{r}$

Answer: A
(D) Watch Video Solution
7. The set of all real numbers $a$ such that $a^{2}+2 a, 2 a+3, a^{2} d a^{2}+3 a+8$ are the sides of a triangle is
A. $(0, \infty)$
B. $(5,8)$
C. $\left(-\frac{11}{3}, \infty\right)$
D. $(5, \infty)$

## Answer: D

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8. In a $\triangle A B C, \angle B=\frac{\pi}{3}$ and $\angle C=\frac{\pi}{4}$ let D divide BC internally in the ratio $1: 3$, then $\frac{\sin (\angle B A D)}{\sin (\angle C A D)}$ is equal to :
A. $\frac{1}{\sqrt{6}}$
B. $\frac{1}{3}$
C. $\frac{1}{\sqrt{3}}$
D. $\frac{\sqrt{2}}{3}$

## Answer: A

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9. Let $A D, B E, C F$ be the lengths of internal bisectors of angles $A$, $B, C$ respectively of triangle $A B C$. Then the harmonic mean of $A D \sec \frac{A}{2}, B E \sec \frac{B}{2}, C F \sec \frac{C}{2}$ is equal to :
A. Harmonic mean of sides of $\triangle A B C$
B. Geometric mean of sides of $\triangle A B C$
C. Arithmetic mean of sides of $\triangle A B C$
D. Sum of reciprocals of the sides of $\triangle A B C$

## Answer: A

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10. In $\triangle A B C$, if $2 b=a+c$ and $A-C=90^{\circ}$, then $\sin \mathrm{B}$ equal

All symbols used have usual meaning in $\triangle A B C$.]
A. $\frac{\sqrt{7}}{5}$
B. $\frac{\sqrt{5}}{8}$
C. $\frac{\sqrt{7}}{4}$
D. $\frac{\sqrt{5}}{3}$

## Answer: C

11. In a triangle ABC , if $2 a \cos \left(\frac{B-C}{2}\right)=b+c$, then $\sec A$ is equal to :
A. $\frac{2}{\sqrt{3}}$
B. $\sqrt{2}$
C. 2
D. 3

## Answer: C

## - Watch Video Solution

12. In a triangle ABC if $B C=1$ and $A C=2$, then what is maximum possible value of angle $A$ ?
A. $\frac{\pi}{6}$
B. $\frac{\pi}{4}$
C. $\frac{\pi}{3}$
D. $\frac{\pi}{2}$

## Answer: A

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13. $\Delta I_{1} I_{2} I_{3}$ is an excentral triangle of an equilateral triangle $\triangle A B C$ such that $I_{1} I_{2}=4$ unit, if $\triangle D E F$ is pedal triangle of $\Delta A B C$, then $\frac{\operatorname{Ar}\left(\Delta I_{1} I_{2} I_{3}\right)}{\operatorname{Ar}(\Delta D E F)}=$
A. 16
B. 4
C. 2

## Answer: A

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14. Let ABC be a triangle with $\angle B A C=2 \pi / 3$ and $A B=x$
such that $(A B)(A C)=1$. If $x$ varies, then find the longest possible length of the angle bisector $A D$
A. $\frac{1}{3}$
B. $\frac{1}{2}$
C. $\frac{2}{3}$
D. $\frac{\sqrt{2}}{3}$

## Answer: B

15. In an equilateral $\triangle A B C$, (where symbols used have usual meanings), then $r, R$ and $r_{1}$ form :
A. an A.P.
B. a G.P.
C. an H.P.
D. none of these

## Answer: A

## - Watch Video Solution

16. If $\frac{\sin A}{\sin C}=\frac{\sin (A-B)}{\sin (B-C)}$, prove that $a^{2}, b^{2}, c^{2}$ are in A.P.
A. A.P.
B. G.P.
C. H.P.
D. none of these

## Answer: A

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17. In $\triangle A B C, \tan A=2, \tan B=\frac{3}{2}$ and $c=\sqrt{65}$, then circumradius of the triangle is: (a) 65 (b) $\frac{65}{7}$ (c) $\frac{65}{14}$ (d) none of these
A. 65
B. $\frac{65}{7}$
C. $\frac{65}{14}$
D. none of these

## Answer: C

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18. If the sides $a, b, c$ of a triangle $A B C$ are the roots of the equation $x^{3}-13 x^{2}+54 x-72=0$, then the value of $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}$ is equal to :
A. $\frac{61}{144}$
B. $\frac{61}{72}$
C. $\frac{169}{144}$
D. $\frac{59}{144}$
19. In $\triangle A B C$, if $\angle C=90^{\circ}$, then $\frac{a+c}{b}+\frac{b+c}{a}$ is equal to :
A. $\frac{c}{r}$
B. $\frac{1}{2 R r}$
C. 2
D. $\frac{R}{r}$

Answer: A

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20. In a $\Delta A B C$, if $a^{2} \sin B=b^{2}+c^{2}$, then :
A. $\angle A$ is obtuse
B. $\angle A$ is acute
C. $\angle B$ is abtuse
D. $\angle A$ is right angle

## Answer: A

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21. If $R$ and $R^{\prime}$ are the circumradii of triangles $A B C$ and $O B C$, where $O$ is the orthocenter of triangle $A B C$, then :
A. $R^{\prime}=\frac{R}{2}$
B. $R^{\prime}=2 R$
C. $R^{\prime}=R$
D. $R^{\prime}=3 R$

## Answer: C

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22. The acute angle of a rhombus whose side is the geometric mean of its diagonals is
A. $15^{\circ}$
B. $20^{\circ}$
C. $30^{\circ}$
D. $60^{\circ}$

## Answer: C

23. In a $\triangle A B C$ right angled at A , a line is drawn through A to meet BC at D dividing BC in $2: 1$. If $\tan (\angle A D C)=3$ then $\angle B A D$ is: (a) $30^{\circ}$ (b) $45^{\circ}$ (c) $60^{\circ}$ (d) $75^{\circ}$
A. $30^{\circ}$
B. $45^{\circ}$
C. $60^{\circ}$
D. $75^{\circ}$

## Answer: B

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24. A circle is cirumscribed in an equilateral triangle of side ' $l$ '.

The area of any square inscribed in the circle is :
A. $\frac{4}{3} l^{2}$
B. $\frac{2}{3} l^{2}$
C. $\frac{1}{3} l^{2}$
D. $l^{2}$

## Answer: B

## - Watch Video Solution

25. if the sides of a triangle are in the ratio $2: \sqrt{6}: \sqrt{3}+1$, then the largest ange of the trangle will be (1) 60 (2) 72 (3) 75 (4) 90
A. $60^{\circ}$
B. $72^{\circ}$
C. $75^{\circ}$
D. $90^{\circ}$

## Answer: C

## - Watch Video Solution

26. In a triangle ABC if $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are in A.P. and $C-A=120^{\circ}$, then
$\frac{s}{r}=$
(where notations have their usual meaning)
A. $\sqrt{15}$
B. $2 \sqrt{15}$
C. $3 \sqrt{15}$
D. $6 \sqrt{15}$

## Answer: C

27. In $\Delta A B C$, let $a=5, b=4$ and $\cos \left(A-B=\frac{31}{32}\right)$, then which of the following statement (s) is (are) correct ?
[Note All symbols used have usuall meaning in a triangle]
A. $\sqrt{6}$
B. $6 \sqrt{6}$
C. 6
D. $(216)^{1 / 4}$

## Answer: C

- Watch Video Solution

28. If semiperimeter of a triangle is 15 , then the value of
$(b+c) \cos (B+C)+(c+a) \cos (C+A)+(a+b) \cos (A+B)$
is equal to :
(where symbols used have usual meanings)
A. -60
B. -15
C. -30
D. can not be determined

## Answer: C

## - Watch Video Solution

29. Let triangle $A B C$ be an isosceles with $A B=A C$. Suppose that the angle bisector of its angle $B$ meets the side $A C$ at a point $D$
and that $B C=B D+A D$. Measure of the angle A in degrees, is :
A. 80
B. 100
C. 110
D. 130

## Answer: B

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30. 

$A: B: C=1: 2: 4, \quad$ then $\left(a^{2}-b^{2}\right)\left(b^{2}-c^{2}\right)\left(c^{2}-a^{2}\right)=\lambda a^{2} b^{2} c^{2}$ , where $\lambda=$
(where notations have their usual meaning)
A. 1
B. 2
C. 4
D. 9

## Answer: A

## D Watch Video Solution

31. In a triangle ABC with altitude $A D$, $\angle B A C=45^{\circ}, D B=3$ and $C D=2$. The area of the triangle $A B C$ is : (a) 6 (b) 15 (c) $\frac{15}{4}$ (d) 12
A. 6
B. 15
C. $15 / 4$
D. 12

## Answer: B

## - Watch Video Solution

32. A triangle has base 10 cm long and the base angles of $50^{\circ}$ and $70^{\circ}$. If the perimeter of the triangle is $x+y \cos z^{\circ}$ where $z \in(0,90)$ then the value of $x+y+z$ equals :
A. 60
B. 55
C. 50
D. 40

## Answer: D

33. Let $H$ be the orthocentre of triangle $A B C$. Then angle subtended by side BC at the centre of incircle of $\triangle C H B$ is
A. $\frac{A}{2}+\frac{\pi}{2}$
B. $\frac{B+C}{2}+\frac{\pi}{2}$
C. $\frac{B-C}{2}+\frac{\pi}{2}$
D. $\frac{B+C}{2}+\frac{\pi}{4}$

## Answer: B

## - Watch Video Solution

34. Triangel $A B C$ is right angles at $A$. The points $P$ and $Q$ are on hypotenuse $B C$ such that $B P=P Q=Q C$. If $A P=3$ and $A Q=4$, then
length $B C$ is equal to
A. $\sqrt{27}$
B. $\sqrt{36}$
C. $\sqrt{45}$
D. $\sqrt{54}$

## Answer: C

## - Watch Video Solution

35. In a $\triangle A B C$ if $b=a(\sqrt{3}-1)$ and $\angle C=30^{\circ}$ then the measure of the angle $A$ is
A. $15^{\circ}$
B. $45^{\circ}$
C. $75^{\circ}$
D. $105^{\circ}$

## Answer: D

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36. Through the centroid of an equilateral triangle, a line parallel to the base is drawn. On this line, an arbitrary point $P$ is taken inside the triangle. Let h denote the perpendicular distance of P from the base of the triangle. Let $h_{1}$ and $h_{2}$ be the perpendicular distance of $P$ from the other two sides of the triangle. Then :
A. $h=\frac{h_{1}+h_{2}}{2}$
B. $h=\sqrt{h_{1} h_{2}}$
C. $h=\frac{2 h_{1} h_{2}}{h_{1}+h_{2}}$
D. $h=\frac{\left(h_{1}+h_{2}\right) \sqrt{3}}{4}$

Answer: A

## - Watch Video Solution

37. The angles $A, B$ and $C$ of a triangle $A B C$ are in arithmetic progression. $A B=6$ and $B C=7$. Then $A C$ is :
A. $\sqrt{41}$
B. $\sqrt{39}$
C. $\sqrt{42}$
D. $\sqrt{43}$

## Answer: D

38. In $\triangle A B C$, if $A-B=120^{\circ}$ and $R=8 r$, then the value of $\frac{1+\cos C}{1-\cos C}$ equals :
(All symbols used hav their usual meaning in a triangle)
A. 12
B. 15
C. 21
D. 31

## Answer: B

39. The lengths of the sides $C B$ and $C A$ of a triangle $A B C$ are given by $a$ and $b$ and the angle $C$ is $\frac{2 \pi}{3}$. The line $C D$ bisects the angle $C$ and meets $A B$ at $D$. Then the length of $C D$ is: (a)
$\frac{1}{a+b}$ (b) $\frac{a^{2}+b^{2}}{a+b}$ (c) $\frac{a b}{2(a+b)}$ (d) $\frac{a b}{a+b}$
A. $\frac{1}{a+b}$
B. $\frac{a^{2}+b^{2}}{a+b}$
C. $\frac{a b}{2(a+b)}$
D. $\frac{a b}{a+b}$

## Answer: D

## - Watch Video Solution

40. In $\triangle A B C$, angle A
$120^{\circ}, B C+C A=20$, and $A B+B C=21$ Find the length of the side $B C$
A. 13
B. 15
C. 17
D. 19

## Answer: A

## D Watch Video Solution

41. A triangle has sides 6,7 and 8 . The line through its incenter parallel to the shortest side is drawn to meet the other two sides at $P$ and $Q$. Then find the length of the segment $P Q$
A. $\frac{12}{5}$
B. $\frac{15}{4}$
C. $\frac{30}{7}$
D. $\frac{33}{9}$

## Answer: C

## D Watch Video Solution

42. The perimeter of a $\triangle A B C$ is 48 cm and one side is 20 cm . Then remaining sides of $\triangle A B C$ must be greater than : (a) 8 cm
(b) 9 cm (c) 12 cm (d) 4 cm
A. 8 cm
B. 9 cm
C. 12 cm
D. 4 cm

## Answer: D

## - Watch Video Solution

43. In an equilateral $\triangle A B C$, (where symbols used have usual meanings), then $r, R$ and $r_{1}$ form :
A. an A.P.
B. a G.P.
C. an H.P.
D. neither an A.P., G.P. nor H.P.

## Answer: A

$$
\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4 b^{2} c^{2}}=\sin ^{2}
$$

A. $\cos ^{2} A$
B. $\sin ^{2} A$
C. $\cos A \cos B \cos C$
D. $\sin A \sin B \sin C$

## Answer: B

## - Watch Video Solution

45. Circumradius of an isosceles $\triangle A B C$ with $\angle A=\angle B$ is 4 times its in radius, then $\cos A$ is root of the equation :
A. $x^{2}-x-8=0$
B. $8 x^{2}-8 x+1=0$
C. $x^{2}-x-4=0$
D. $4 x^{2}-4 x+1=0$

## Answer: B

## - Watch Video Solution

46. A is the orthocentre of $\Delta A B C$ and D is reflection point of A w.r.t. perpendicualr bisector of BC , then orthocenter of $\triangle D B C$ is :
A. D
B. C
C. B
D. A

## Answer: A

## - Watch Video Solution

47. If $a \neq b \neq c$ are all positive, then the value of the determinant $\left|\begin{array}{lll}a & b & c \\ b & c & a \\ c & a & b\end{array}\right|$ is
A. $\geq 0$
B. $>0$
C. $\leq-1$
D. $<0$

Answer: D
48.

In
triangle
ABC
$A: B: C=1: 2: 4, \quad$ then $\left(a^{2}-b^{2}\right)\left(b^{2}-c^{2}\right)\left(c^{2}-a^{2}\right)=\lambda a^{2} b^{2} c^{2}$
, where $\lambda=$
(where notations have their usual meaning)
A. 1
B. 2
C. 3
D. $\frac{1}{3}$

## Answer: A

## - Watch Video Solution

49. In any triangle, the minimum value of $r_{1} r_{2} r_{3} / r^{3}$ is equal to
A. 1
B. 3
C. 8
D. 27

## Answer: D

## - Watch Video Solution

50. In a triangle $A B C$, side $B C=3, A C=4$ and $A B=5$. The value of $\sin A+\sin 2 B+\sin 3 C$ is equal to :
A. $\frac{24}{25}$
B. $\frac{14}{25}$
C. $\frac{64}{25}$
D. None

## Answer: B

## D Watch Video Solution

51. In any triangle $A B C$, the value of $\frac{r_{1}+r_{2}}{1+\cos C}$ is equal to (where notation have their usual meaning) :
A. $2 R$
B. $2 r$
C. R
D. $\frac{2 R^{2}}{r}$
52. In a triangle $A B C$, medians $A D$ and $B E$ are drawn. If $A D=4, \angle D A B=\frac{\pi}{6}$ and $\angle A B E=\frac{\pi}{3}$ then the area of the triangle $A B C$ is :
A. $\frac{8}{3 \sqrt{3}}$
B. $\frac{16}{3 \sqrt{3}}$
C. $\frac{32}{3 \sqrt{3}}$
D. $\frac{64}{3 \sqrt{3}}$

## Answer: C

53. The sides of the triangle are $\sin \alpha, \cos \alpha$ and $\sqrt{1+\sin \alpha \cos \alpha}$ for some $0<\alpha<\frac{\pi}{2}$. Then the greatest angle of the triangle is
А. $\frac{\pi}{3}$
B. $\frac{\pi}{2}$
C. $\frac{2 \pi}{3}$
D. $\frac{5 \pi}{6}$

## Answer: C

## - Watch Video Solution

54. Let ABC be a right with $\angle B A C=\frac{\pi}{2}$, then $\left(\frac{r^{2}}{2 R^{2}}+\frac{r}{R}\right)$ is equal to :
(where symbols used have usual meaning in a striangle)
A. $\sin B \sin C$
B. $\tan B \tan C$
C. $\sec B \sec C$
D. $\cot B \cot C$

## Answer: A

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55. Find the radius of the circle escribe to the triangle $A B C$ (Shown in the figure below) on the side BC if
$\angle N A B=30^{\circ}, \angle B A C=30^{\circ}, \mathrm{AB}=\mathrm{AC}=5^{\circ}$.

A. $\frac{(10 \sqrt{2}+5 \sqrt{3}-5)(2-\sqrt{3})}{2 \sqrt{2}}$
B. $\frac{(10 \sqrt{2}+5 \sqrt{3}+5)}{2 \sqrt{2}}(2-\sqrt{3})$
C. $\frac{(10 \sqrt{2}+5 \sqrt{3}-5)}{2 \sqrt{2}}(2+\sqrt{3})$
D. $\frac{(10 \sqrt{2}+5 \sqrt{2}+1)}{2 \sqrt{3}}(\sqrt{3}-1)$

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56. In a $\triangle A B C$, with usual notations, if $b>c$ then distance between foot of median and foot of altitude both drawn from vertex $A$ on $B C$ is :
A. $\frac{a^{2}-b^{2}}{2 c}$
B. $\frac{b^{2}-c^{2}}{2 a}$
C. $\frac{b^{2}+c^{2}-a^{2}}{2 a}$
D. $\frac{b^{2}+c^{2}-a^{2}}{2 c}$

## Answer: B

57. In a triangle $A B C$ the expression $a \cos B \cos C+b \cos A \cos B$ equals to :
A. $\frac{r s}{R}$
B. $\frac{r}{s R}$
C. $\frac{R}{r s}$
D. $\frac{R s}{r}$

## Answer: A

## (D) Watch Video Solution

58. A circle of area 20 sq. units is centered at the point $O$.

Suppose $\triangle A B C$ is inscribed in that circle and has area 8 sq.
units. The central angles $\alpha, \beta$ and $\gamma$ are as shown in the figure.

The value of $(\sin \alpha+\sin \beta+\sin \gamma)$ is equal to
A. $\frac{4 \pi}{5}$
B. $\frac{3 \pi}{4}$
C. $\frac{2 \pi}{5}$
D. $\frac{\pi}{4}$

## Answer: A

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## Exercise 2 One Or More Than One Answer Is Are Correct

1. In $\triangle A B C, \angle A=60^{\circ}, \angle B=90^{\circ}, \angle C=30^{\circ}$. Let H be its orthocentre, then :
(where symbols used have usual meanings)
A. $A H=c$
B. $C H=a$
C. $A H=a$
D. $B H=0$

## Answer: A::B::D

## - Watch Video Solution

2. In an equilateral triangle, if inradius is a rational number then which of the following is/are correct ?
A. circumradius is always rational
B. exradii are always rational
C. area is always ir-rational
D. perimeter is always rational

## Answer: A::B::C

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3. Let $A, B, C$ be angles of a triangle $A B C$ and let
$D=\frac{5 \pi+A}{32}, E=\frac{5 \pi+B}{32}, F=\frac{5 \pi+C}{32}$, then :
(where $D, E, F \neq \frac{n \pi}{2}, n \in I, I$ denote set of integers)
A. $\cot D \cot E+\cot E \cot F+\cot D \cot F=1$
B. $\cot D+\cot E+\cot F=\cot D \cot E \cot F$
C. $\tan D \tan E+\tan E \tan F+\tan F \tan D=1$
D. $\tan D+\tan E+\tan F=\tan D \tan E \tan F$

## - Watch Video Solution

4. In a triangle ABC , if $\mathrm{a}=4, \mathrm{~b}=8$ and $\angle C=60^{\circ}$, then :
(where symbols used have usual meanings)
A. $c=6$
B. $c=4 \sqrt{3}$
C. $\angle A=30^{\circ}$
D. $\angle B=90^{\circ}$

## Answer: B::C::D

(D) Watch Video Solution
5. Find the perimeter of an equilateral triangle with side 9 cm .

## D Watch Video Solution

6. $A B C$ is a triangle whose circumcentre, incentre and orthocentre are $\mathrm{O}, \mathrm{I}$ and H respectively which lie inside the triangle, then :
A. $\angle B O C=A$
B. $\angle B I C=\frac{\pi}{2}+\frac{A}{2}$
C. $\angle B H C=\pi-A$
D. $\angle B H C=\pi-\frac{A}{2}$

## Answer: B::C

7. In a triangle $A B C$, $\tan A$ and $\tan B$ satisfy the inequality $\sqrt{3} x^{2}-4 x+\sqrt{3}<0$, then which of the following must be correct ?
(where symbols used have usual meanings)
A. $a^{2}+b^{2}-a b<c^{2}$
B. $a^{2}+b^{2}>c^{2}$
C. $a^{2}+b^{2}+a b>c^{2}$
D. $a^{2}+b^{2}<c^{2}$

## Answer: A::C

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8. If in $\triangle A B C, \angle C=\frac{\pi}{8}, a=\sqrt{2}$ and $b=\sqrt{2+\sqrt{2}}$ then find the measure of angle $A$ (in degree).
A. $45^{\circ}$
B. $135^{\circ}$
C. $30^{\circ}$
D. $150^{\circ}$

## Answer: A

## - Watch Video Solution

9. In triangle ABC, $a=3, b=4, c=2$. Point D and E trisect the side $B C$. If $\angle D A E=\theta$, then $\cot ^{2} \theta$ is divisible by :
A. 2
B. 3
C. 5
D. 7

## Answer: B::C

## - Watch Video Solution

10. In a $\triangle A B C, 3 \sin A+4 \cos B=6 \quad$ and
$3 \cos A+4 \sin B=1$, then $\angle C$ can be
A. $\frac{\pi}{4}$
B. $\frac{\pi}{6}$
C. $\frac{\pi}{3}$
D. $\frac{5 \pi}{6}$

Answer: B

- Watch Video Solution

11. If the line joining the incentre to the centroid of a triangle
$A B C$ is parallel to the side $B C$. Which of the following are correct
?
A. $2 b=a+c$
B. $2 a=b+c$
C. $\cot \frac{A}{2} \cot \frac{C}{2}=3$
D. $\cot \frac{B}{2} \cot \frac{C}{2}=3$

## Answer: B::D

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12. In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P, then the length of the third
side can be (a) $5-\sqrt{6}$ (b) $3 \sqrt{3}$ (c) 5 (d) $5+\sqrt{6}$
A. $5-\sqrt{6}$
B. $5+\sqrt{6}$
C. $6-\sqrt{5}$
D. $6+\sqrt{5}$

## Answer: A::B

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13. If area of $\Delta A B C(\Delta)$ and angle $C$ are given and if c opposite to given angle is minimum, then
A. $a=\sqrt{\frac{2 \Delta}{\sin C}}$
B. $b=\left(\frac{2 \Delta}{\sin C}\right)$
C. $a=\frac{4 \Delta}{\sin C}$
D. $b=\frac{4 \Delta}{\sin ^{2} C}$

## Answer: A::B

## - Watch Video Solution

14. In a triangle ABC,
$\tan A=2 \sin 2 C$ and $3 \cos A=2 \sin B \sin C$ then possible values of $C$ is/are
A. $\frac{\pi}{8}$
B. $\frac{\pi}{6}$
C. $\frac{\pi}{4}$
D. $\frac{\pi}{3}$

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## Exercise 3 Comprehension Type Problems

1. Internal bisectors of $\triangle A B C$ meet the circumcircle at point D ,

## E, and F

Area of $\triangle D E F$ is
A. $2 R^{2} \cos ^{2}\left(\frac{A}{2}\right) \cos ^{2}\left(\frac{B}{2}\right) \cos ^{2}\left(\frac{C}{2}\right)$
B. $2 R^{2} \sin \left(\frac{A}{2}\right) \sin \left(\frac{B}{2}\right) \sin \left(\frac{C}{2}\right)$
C. $2 R^{2} \sin ^{2}\left(\frac{A}{2}\right) \sin ^{2}\left(\frac{B}{2}\right) \sin ^{2}\left(\frac{C}{2}\right)$
D. $2 R^{2} \cos \left(\frac{A}{2}\right) \cos \left(\frac{B}{2}\right) \cos \left(\frac{C}{2}\right)$

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2. Internal angle bisecotors of $\triangle A B C$ meets its circum circle at
$D, E$ and $F$ where symbols have usual meaning.
Q. The ratio of area of triangle $A B C$ and triangle DEF is :
A. $\geq 1$
B. $\leq 1$
C. $\geq 1 / 2$
D. $\leq 1 / 2$

## Answer: B

3. Let triangle $A B C$ is right triangle right angled at $C$ such that $A<B$ and $r=8, R=41$.
Q. Area of $\triangle A B C$ is:
A. 720
B. 1440
C. 360
D. 480

Answer: A

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4. Let triangle $A B C$ is right triangle right angled at $C$ such that
$A<B$ and $r=8, R=41$.
Q. $\tan \frac{A}{2}=$
A. $\frac{1}{18}$
B. $\frac{1}{3}$
C. $\frac{1}{6}$
D. $\frac{1}{9}$

## Answer: D

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5. Let the incircle of $\triangle A B C$ touches the sides $\mathrm{BC}, \mathrm{CA}, \mathrm{AB}$ at $A_{1}, B_{1}, C_{1}$ respectively. The incircle of $\Delta A_{1} B_{1} C_{1}$ touches its sides of $B_{1} C_{1}, C_{1} A_{1}$ and $A_{1} B_{1}$ at $A_{2}, B_{2}, C_{2}$ respectively and so on.
Q. $\lim _{n \rightarrow \infty} \angle A_{n}=$
A. 0
B. $\frac{\pi}{6}$
C. $\frac{\pi}{4}$
D. $\frac{\pi}{3}$

## Answer: D

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6. Let the incircle of $\triangle A B C$ touches the sides $\mathrm{BC}, \mathrm{CA}, \mathrm{AB}$ at $A_{1}, B_{1}, C_{1}$ respectively. The incircle of $\Delta A_{1} B_{1} C_{1}$ touches its sides of $B_{1} C_{1}, C_{1} A_{1}$ and $A_{1} B_{1}$ at $A_{2}, B_{2}, C_{2}$ respectively and so on.
Q. In $\Delta A_{4} B_{4} C_{4}$, the value of $\angle A_{4}$ is:
A. $\frac{3 \pi+A}{6}$
B. $\frac{3 \pi-A}{8}$
C. $\frac{5 \pi-A}{16}$
D. $\frac{5 \pi+A}{16}$

## Answer: D

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7. Let $A B C$ be a given triangle. Points $D$ and $E$ are on sides $A B$ and AC respectively and point $F$ is on line segment $D E$. Let $\frac{A D}{A B}=x, \frac{A E}{A C}=y, \frac{D F}{D E}=z$. Let area of $\Delta B D F=\Delta_{1}$, Area of $\triangle C E F=\Delta_{2}$ and area of $\Delta A B C=\Delta$.
Q. $\frac{\Delta_{2}}{\Delta}$ is equal to :
A. $(1-x) y(1-z)$
B. $(1-x)(1-y) z$
C. $x(1-y)(1-z)$
D. $(1-x) y z$

## Answer: C

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8. $\mathrm{a}, \mathrm{b}, \mathrm{c}$ ar the length of sides $\mathrm{BC}, \mathrm{CA}, \mathrm{AB}$ respectively of $\triangle A B C$
satisfying $\log \left(1+\frac{c}{a}\right)+\log a-\log b=\log 2$. a, b, c are in :
A. A.P.
B. G.P.
C. H.P.
D. None

Answer: A
9. $a, b, c$ are the length of sides $B C, C A, A B$ respectively of $\triangle A B C$ satisfying $\log \left(1+\frac{c}{a}\right)+\log a-\log b=\log 2$.

Also the quadratic equation $a\left(1-x^{2}\right)+2 b x+c\left(1+x^{2}\right)=0$ has two equal roots.
. Measure of angle C is :
A. $30^{\circ}$
B. $45^{\circ}$
C. $60^{\circ}$
D. $90^{\circ}$

## Answer: D

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10. If $a, b, c$ are the sides of triangle $A B C$ satisfying $\log \left(1+\frac{c}{a}\right)+\log a-\log b=\log 2$.

Also
$a\left(1-x^{2}\right)+2 b x+c\left(1+x^{2}\right)=0$ has two equal roots. Find the value of $\sin A+\sin B+\sin C$.
A. $\frac{5}{2}$
B. $\frac{12}{5}$
C. $\frac{8}{3}$
D. 2

## Answer: B

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11. Let $A B C$ be a triangle inscribed in a circle and let $l_{a}=\frac{m_{a}}{M_{a}}, l_{b}=\frac{m_{b}}{M_{b}}, l_{c}=\frac{m_{c}}{M_{c}} \quad$ where $\quad m_{a}, m_{b}, m_{c} \quad$ are the
lengths of the angle bisectors of angles $A, B$ and $C$ respectively, internal to the triangle and $M_{a}, M_{b}$ and $M_{c}$ are the lengths of these internal angle bisectors extended until they meet the circumcircle.
Q. $l_{a}$ equals:
A. $\frac{\sin A}{\sin \left(B+\frac{A}{2}\right)}$
B. $\frac{\sin B \sin C}{\sin ^{2}\left(\frac{B+C}{2}\right)}$
C. $\frac{\sin B \sin C}{\sin ^{2}\left(B+\frac{A}{2}\right)}$
D. $\frac{\sin B+\sin C}{\sin ^{2}\left(B+\frac{A}{2}\right)}$

## Answer: C

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12. Let $A B C$ be a triangle inscribed in a circle and let $l_{a}=\frac{m_{a}}{M_{a}}, l_{b}=\frac{m_{b}}{M_{b}}, l_{c}=\frac{m_{c}}{M_{c}} \quad$ where $\quad m_{a}, m_{b}, m_{c} \quad$ are the lengths of the angle bisectors of angles $\mathrm{A}, \mathrm{B}$ and C respectively, internal to the triangle and $M_{a}, M_{b}$ and $M_{c}$ are the lengths of these internal angle bisectors extended until they meet the circumcircle.
Q. The maximum value of the product
$\left.\left(l_{a} l_{b} l_{c}\right) \times \cos ^{2}\left(\frac{B-C}{2}\right) \times \frac{\cos ^{2}(C-A)}{2}\right) \times \cos ^{2}\left(\frac{A-B}{2}\right)$
is equal to :
A. $\frac{1}{8}$
B. $\frac{1}{64}$
C. $\frac{27}{64}$
D. $\frac{27}{32}$

## Exercise 4 Matching Type Problems



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2. In $\triangle A B C$, If $r_{1}=21, r_{2}=24, r_{3}=28$, then


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## Exercise 5 Subjective Type Problems

1. If the median $A D$ of triangle $A B C$ makes an angle $\frac{\pi}{4}$ with the side BC , then find the value of $|\cot B-\cot C|$.

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2. In a $\triangle A B C$; inscribed circle with centre $I$ touches sides $A B, A C$ and $B C a t D, E, F \quad$ respectively.Let area of quadrilateral ADIE is 5 units and area of quadrilteral BFID is 10 units. Find the value of $\frac{\cos \left(\frac{C}{2}\right)}{\sin \left(\frac{A-B}{2}\right)}$.

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3. If $\Delta$ be are of incricle of a triangle ABC and $\Delta_{1}, \Delta_{2}, \Delta_{3}$ be the area of excircles then find the least value of $\frac{\Delta_{1} \Delta_{2} \Delta_{3}}{729 \Delta^{3}}$.

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4. In an acute angled triangle $\mathrm{ABC}, \angle A=20^{\circ}$, let DEF be the feet of altitudes through $A, B, C$ respectively and $H$ is the
orthocentre of $\triangle A B C$. Find $\frac{A H}{A D}+\frac{B H}{B E}+\frac{C H}{C F}$.

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5. If the quadratic equation $a x^{2}+b x+c=0$ has equal roots where $a, b, c$ denotes the lengths of the sides opposite to vertex $\mathrm{A}, \mathrm{B}$ and C of the $\triangle A B C$ respectively. Find the number of integers in the range of $\frac{\sin A}{\sin C}+\frac{\sin C}{\sin A}$.

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6. if ABC is a triangle and $\tan \left(\frac{A}{2}\right), \tan \left(\frac{B}{2}\right), \tan \left(\frac{C}{2}\right)$ are in H.P. Then find the minimum value of $\cot \left(\frac{B}{2}\right)$
7. Sides $A B$ and $A C$ in an equilateral triangle $A B C$ with side length 3 is extended to form two rays from point $A$ as shown in the figure. Point $P$ is chosen outside the triangle $A B C$ and between the two rays such that $\angle A B P+\angle B C P=180^{\circ}$. If the maximum length of $C P$ is $M$, then $M^{2} / 2$ is equal to :

8. Let $\mathrm{a}, \mathrm{b}, \mathrm{c}$ be sides of a triangle ABC and $\Delta$ denotes its area .

If $a=2, \Delta=\sqrt{3}$ and $a \cos C+\sqrt{3} a \sin C-b-c=0$, then find the value of $(b+c)$.
(symbols used have usual meaning in $\triangle A B C$ ).

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9. Circumradius of $\triangle A B C$ is 3 cm and its area is $6 \mathrm{~cm}^{2}$. If DEF is the triangle formed by feet of the perpendicular drawn from $A, B$ and $C$ on the sides $B C, C A$ and $A B$, respectively, then the perimeter of $\triangle D E F($ in cm$)$ is $\qquad$

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