# MATHS

## BOOKS - VK JAISWAL ENGLISH

## TRIGONOMETRIC EQUATIONS

### Exercise 1 Single Choice Problems

**1.** Let x and y be 2 real numbers which satisfy the equations $\left(\tan^2 x - \sec^2 y\right) = \frac{5a}{6} - 3$ and $\left(-\sec^2 x + \tan^2 y\right) = a^2$ , then the product of all possible value's of a can be equal to :

A. 0

B. $-\frac{2}{3}$

C. -1

D. $-\frac{3}{2}$

**Answer: C**

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**2.** The general solution of the equation $\tan^2(x+y)+\cot^2(x+y)=1-2x-x^2$ lie on the line is :

A. $x=-1$

B. $x=-2$

C. $y=-1$

D. $y=-2$

**Answer: A**

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**3.** General solution of the equation

$\sin x+\cos x=\min_{a\in R}\left\{1,a^2-4a+6\right\}$ is :

A. $\frac{n\pi}{2} + (-1)^n \frac{\pi}{4}$

B. $2n\pi + (-1)^n \frac{\pi}{4}$

C. $n\pi + (-1)^{n+1} \frac{\pi}{4}$

D. $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

**Answer: D**

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**4.** The number of solutions of the equation $\left(2\sin\left(\frac{\sin x}{2}\right)\right)\left(\cos\left(\frac{\sin x}{2}\right)\right)\left(\sin\left(2\tan\frac{x}{2}\ \cos^2\frac{x}{2}\right) - 3\right) + 2 = 0$ in $[0, 2\pi]$ is :

A. 0

B. 1

C. 2

D. 4

**Answer: A**

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**5.** Number of solution of $\tan(2x) = \tan(6x)$ in $(0, 3\pi)$ is :

A. 4

B. 5

C. 3

D. None of these

**Answer: B**

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**6.** The number of values of $x$ in the in interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is 0 (b) 5 (c) 6 (d) 10

A. 0

B. 2

C. 6

D. 8

**Answer: C**

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**7.** The number of different values of $\theta$ satisfying the equation $\cos\theta + \cos 2\theta = -1$, and at the same time satisfying the condition $0 < \theta < 360^\circ$ is :

A. 1

B. 2

C. 3

D. 4

**Answer: D**

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**8.** The total number of solutions of the equation $\max(\sin x, \cos x) = \frac{1}{2}$ for $x \in (-2\pi, 5\pi)$ is equal to :

A. 3

B. 6

C. 7

D. 8

**Answer: C**

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**9.** The general value of x satisfying the equation

$2\cot^2 x + 2\sqrt{3}\cot x + 4\text{cosec}x + 8 = 0$ is : (where $n \in I$ )

A. $n\pi - \frac{\pi}{6}$

B. $n\pi + \frac{\pi}{6}$

C. $2n\pi - \frac{\pi}{6}$

D. $2n\pi + \frac{\pi}{6}$

**Answer: C**

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**10.** The general solution of the equation $\sin^2 x + \cos^2 3x = 1$ is equal to :

(where $n \in I$)

A. $x = \frac{n\pi}{2}$

B. $x = n\pi + \frac{\pi}{4}$

C. $x = \frac{n\pi}{4}$

D. $x = n\pi + \frac{\pi}{2}$

**Answer: C**

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**11.** Values of x between 0 and $2\pi$ which satisfy the equation $\sin x\sqrt{8\cos^2 x} = 1$ are in A.P. whose common difference is :

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

**Answer: A**

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**12.** Number of solution of $\sum_{r=1}^{5} \cos rx = 5$ in the interval $[0, 4\pi]$ is :

A. 0

B. 2

C. 3

D. 7

**Answer: C**

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**13.** The number of solutions of the equation $4\sin^2 x + \tan^2 x + \cot^2 x + \text{cosec}^2 x = 6$ in $[0, 2\pi]$

A. $n\pi \pm \frac{\pi}{4}$

B. $2n\pi \pm \frac{\pi}{4}$

C. $n\pi + \frac{\pi}{3}$

D. $n\pi - \frac{\pi}{6}$

**Answer: A**

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**14.** Smallest positive x satisfying the equation $\cos^3 3x + \cos^3 5x = 8\cos^3 4x \cdot \cos^3 x$ is :

A. $15^\circ$

B. $18^\circ$

C. $22.5^\circ$

D. $30^\circ$

**Answer: B**

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**15.** The general solution of the equation $\sin x - \cos x = 1$ is (where $n \in I$) :

A. $2n\pi + \frac{\pi}{2}$

B. $n\pi + \frac{\pi}{2}$

C. $2n\pi - \frac{\pi}{2}$

D. $n\pi$

**Answer:**

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**16.** Number of solution (s) of equation $\sin\theta = \sec^2 4\theta$ in $[0, \pi]$ is/are :

A. 0

B. 1

C. 2

D. 3

**Answer: B**

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**17.** The number of solutions of the equation $4\sin^2 x + \tan^2 x + \cot^2 x + \text{cosec}^2 x = 6$ in $[0, 2\pi]$ :

A. 1

B. 2

C. 3

D. 4

**Answer: D**

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**18.** The number of solutions of the equation $\sin^4 \theta - 2\sin^2 \theta + 1 = 0$ which lie between 0 and $2\pi$ is :

A. 0

B. 2

C. 4

D. 8

**Answer: A**

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**19.** Find the smallest positive number p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has a solution $x \in [0, 2\pi]$.

A. $\frac{\pi}{\sqrt{2}}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{2\sqrt{2}}$

D. $\frac{3\pi}{2\sqrt{2}}$

**Answer: C**

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**20.** The total number of ordered pairs (x, y) satisfying $|x| + |y| = 2, \sin(\pi x^2/3) = 1$, is equal to

A. 2

B. 4

C. 6

D. 8

**Answer: B**

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**21.** Set of values of x lying in $[0, 2\pi]$ satisfying the inequality $|\sin x| > 2\sin^2 x$ contains

A. $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$

B. $\left(-\frac{\pi}{2}, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

C. $\left(-\frac{\pi}{2}, -\frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$

D. $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$

**Answer: D**

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**22.** The general solution of the equation $\sin^4 x + \cos^4 x = \sin x \cos x$ is

A. 2

B. 4

C. 6

D. 8

**Answer: A**

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**23.** The number of solutions of ewuation $\sin.\frac{5x}{2} - \sin.\frac{x}{2} = 2$ in $[0, 2\pi]$ is

A. 1

B. 2

C. 0

D. Infinite

**Answer: C**

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**24.** In interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the equation $\log_{\sin\theta}\left(\cos^2\theta\right) = 2$ has

A. No solution

B. One solution

C. Two solution

D. Infinite solution

**Answer: B**

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**25.** If $\alpha$ and $\beta$ are 2 distinct roots of equation $a\cos\theta + b\sin\theta = C$ then $\cos(\alpha+\beta) =$

A. $\frac{2ab}{a^2+b^2}$

B. $\frac{2ab}{a^2-b^2}$

C. $\frac{a^2+b^2}{a^2-b^2}$

D. $\frac{a^2-b^2}{a^2+b^2}$

**Answer: D**

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Exercise 2 One Or More Than One Answer Is Are Correct

**1.** If $2\cos\theta + 2\sqrt{2} = 3\sec\theta$, where $\theta \in (0, 2\pi)$, then which of the following can be correct ?

A. $\cos\theta = \frac{1}{\sqrt{2}}$

B. $\tan\theta = 1$

C. $\sin\theta = -\frac{1}{\sqrt{2}}$

D. $\cot\theta = -1$

**Answer: A::B::C::D**

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**2.** In a triangle ABC if $\tan C < 0$ then :

A. $\tan A \tan B < 1$

B. $\tan A \tan B > 1$

C. $\tan A + \tan B + \tan C < 0$

D. $\tan A + \tan B + \tan C > 0$

**Answer: A::C**

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**3.** The inequality $4\sin 3x + 5 \geq 4\cos 2x + 5\sin x$ true for $x \in$

A. $\left[-\pi, \frac{3\pi}{2}\right]$

B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C. $\left[\frac{5\pi}{8}, \frac{13\pi}{8}\right]$

D. $\left[\frac{23\pi}{14}, \frac{41\pi}{14}\right]$

**Answer: A::B::C::D**

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**4.** The least difference between the roots of the equation $4\cos x\left(2 - 3\sin^2 x\right) + \cos 2x + 1 = 0 \forall x \in R$ is :

A. equal to $\frac{\pi}{2}$

B. $> \frac{\pi}{10}$

C. $< \frac{\pi}{2}$

D. $< \frac{\pi}{3}$

**Answer: B::C::D**

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**5.** General solution of the equation,

$\cos x \cdot \cos 6x = -1$ is =

A. has 50 solutions in $[0, 100\pi]$

B. has 3 solutions in $[0, 3\pi]$

C. has even number of solutions in $(3\pi, 13\pi)$

D. has one solution in $\left[\frac{\pi}{2}, \pi\right]$

**Answer: A::C::D**

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**6.** Identify the correct option

A. $\frac{\sin 3\alpha}{\cos 2\alpha} > 0$ for $\alpha \in \left(\frac{3\pi}{8}, \frac{23\pi}{48}\right)$

B. $\frac{\sin 3\alpha}{\cos 2\alpha} < 0$ for $\alpha \in \left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$

C. $\frac{\sin 2\alpha}{\cos \alpha} < 0$ for $\alpha \in \left(-\frac{\pi}{2}, 0\right)$

D. $\frac{\sin 2\alpha}{\cos \alpha} > 0$ for $\alpha \in \left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$

**Answer: A::B::C::D**

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**7.** The equation $\sin^4 x + \cos^4 x + \sin 2x + k = 0$ must have real solutions if :

A. $k = 0$

B. $|k| \leq \frac{1}{2}$

C. $-\frac{3}{2} \leq k \leq \frac{1}{2}$

D. $-\frac{1}{2} \leq k \leq \frac{3}{2}$

**Answer: A::B::C**

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**8.** Let $f(\theta) = \left(\cos\theta - \cos\frac{\pi}{8}\right)\left(\cos\theta - \cos\frac{3\pi}{8}\right)\left(\cos\theta - \cos\frac{5\pi}{8}\right)\left(\cos\theta - \cos\frac{7\pi}{8}\right)$ then :

A. maximum value of $f(\theta)\,\forall\theta \in R$ is $\frac{1}{4}$

B. maximum value of $f(\theta)\,\forall\theta \in R$ is $\frac{1}{8}$

C. $f(0) = \frac{1}{8}$

D. Number of principle solutions of $f(\theta) = 0$ is 8

**Answer: B::C::D**

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**9.** Solve $\frac{\sin^2 2x + 4\sin^4 x - 4\sin^2 x \cos^2 x}{4 - \sin^2 2x - 4\sin^2 x} = \frac{1}{9}$.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

**Answer: B::D**

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**10.** The possible value(s) of $'\theta'$ satisfying the equation

$\sin^2\theta \tan\theta + \cos^2\theta \cot\theta - \sin 2\theta = 1 + \tan\theta + \cot\theta$

where $\theta \in [0, \pi]$ is/are :

A. $\frac{\pi}{4}$

B. $\pi$

C. $\frac{7\pi}{12}$

D. None of these

**Answer: C**

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**11.** $\sin\theta + \sqrt{3}\cos\theta = 6x - x^2 - 11, 0 \leq \theta \leq 4\pi, x \in R,$ (a) hold for no values of $x$ and $\theta$ (b) one value of $x$ and two values of $\theta$ (c) two values of $x$ and two values of $\theta$ (d) two point of values of $(x, \theta)$

A. no values of x and $\theta$

B. one value of x and two values of $\theta$

C. two values of x and two values of $\theta$

D. two pairs of values of $(x, \theta)$

**Answer: B::D**

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## Exercise 3 Comprehension Type Problems

**1.** Consider f, g and h be three real valued function defined on R. Let $f(x) = \sin 3x + \cos x, g(x) = \cos 3x + \sin x$ and $h(x) = f^2(x) + g^2(x)$. Then,

The length of a longest interval in which the function g=h(x) is increasing, is

A. $\frac{\pi}{8}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

**Answer: B**

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**2.** Consider f, g and h be three real valued function defined on R.

Let $f(x) = \sin 3x + \cos x, g(x) = \cos 3x + \sin x$ and

$h(x) = f^2(x) + g^2(x)$

Q. General solution of the equation $h(x) = 4$, is :

[where $n \in I$ ]

A. $(4n + 1)\frac{\pi}{8}$

B. $(8n + 1)\frac{\pi}{8}$

C. $(2n + 1)\frac{\pi}{4}$

D. $(7n + 1)\frac{\pi}{4}$

**Answer: A**

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**3.** Consider f, g and h be three real valued function defined on R. Let

$f(x) = \sin 3x + \cos x, g(x) = \cos 3x + \sin x$ and

$h(x) = f^2(x) + g^2(x)$. Then,

Number of point (s) where the graphs of the two function, y=f(x) and y=g(x) intersects in $[0, \pi]$, is

A. 2

B. 3

C. 4

D. 5

**Answer: C**

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Exercise 4 Matching Type Problems

**1.** Match the following Column I to Column II

| | Column-I | | Column-II |
|---|---|---|---|
| (A) | If $\sin x + \cos x = \frac{1}{5}$; then $\lvert 12\tan x \rvert$ is equal to | (P) | 2 |
| (B) | Number of values of $\theta$ lying in $(-2\pi, \pi)$ and satisfying $\cot\frac{\theta}{2} = (1+\cot\theta)$ is | (Q) | 6 |
| (C) | If $2-\sin^4 x + 8\sin^2 x = \alpha$ has solution, then $\alpha$ can be | (R) | 9 |
| (D) | Number of integral values of $x$ satisfying $\log_4(2x^2+5x+27) - \log_2(2x-1) \geq 0$ | (S) | 14 |
| | | (T) | 16 |

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**2.** Match the Column - I with the Column-II to form the correct pair

| | Column-I | | Column-II |
|---|---|---|---|
| (A) | Minimum value of $y = 4\sec^2 x + \cos^2 x$ for permissible real values of $x$ is equal to | (P) | 2 |
| (B) | If $m$, $n$ are positive integers and $m + n\sqrt{2} = \sqrt{41+24\sqrt{2}}$ then $(m+n)$ is equal to : | (Q) | 7 |
| (C) | Number of solutions of the equation : $\log_{\left(\frac{9x-x^2-14}{7}\right)}(\sin 3x - \sin x) = \log_{\left(\frac{9x-x^2-14}{7}\right)}\cos 2x$ is equal to : | (R) | 4 |
| (D) | Consider an arithmetic sequence of positive integers. If the sum of the first ten terms is equal to the 58th term, then the least possible value of the first term is equal to : | (S) | 5 |
| | | (T) | 3 |

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## Exercise 5 Subjective Type Problems

**1.** Find the number of solutions of the equations

$(\sin x - 1)^3 + (\cos x - 1)^3 + (\sin x)^3 = (2\sin x + \cos x - 2)^3$ in $[0, 2\pi]$.

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**2.** If $x + \sin y = 2014$ and $x + 2014\cos y = 2013, 0 \leq y \leq \frac{\pi}{2}$, then find the value of $[x + y] - 2005$

(where [.] denotes greatest integer function)

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**3.** The complete set of values of x satisfying $\frac{2\sin 6x}{\sin x - 1} < 0$ and $\sec^2 x - 2\sqrt{2}\tan x \leq 0$ in $\left(0, \frac{\pi}{2}\right)$ is $[a, b) \cup (c, d]$, then find the value of $\left(\frac{cd}{ab}\right)$.

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**4.** The range of value's of k for which the equation $2\cos^4 x - \sin^4 x + k = 0$ has atleast one solution is $[\lambda, \mu]$. Find the value of $(9\mu + \lambda)$.

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**5.** The number of points in interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, where the graphs of the curves $y = \cos x$ and $y = \sin 3x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ intersects is

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**6.** Find the number of solutions of the equation $2\sin^2 x + \sin^2 2x = 2, \sin 2x + \cos 2x = \tan x$ in $[0, 4\pi]$ satisfying the condition $2\cos^2 x + \sin x \leq 2$.

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**7.** If the sum of all the solutions of the equation $3\cot^2\theta + 10\cot\theta + 3 = 0$ in $[0, 2\pi]$ is $k\pi$ where $k \in I$, then find the value of k.

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**8.** If the sum of all values of $\theta, 0 \leq \theta \leq 2\pi$ satisfying the equation $(8\cos 4\theta - 3)(\cot\theta + \tan\theta - 2)(\cot\theta + \tan\theta + 2) = 12$ is $k\pi$, then k is equal to :

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**9.** Find the number of solutions of the equation $2\sin^2 x + \sin^2 2x = 2, \sin 2x + \cos 2x = \tan x$ in $[0, 4\pi]$ satisfying the condition $2\cos^2 x + \sin x \leq 2$.

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