doubtnut
India's Number 1 Education App

MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

AREA OF BOUNDED REGIONS

Examples

1. Mark the region represtented by $3x + 4y \leq 12.$

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2. Sketch the curve $y = x^3.$

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3. Sketch the curve $y = x^3 - 4x$.

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4. Sketch the curve $y = (x - 1)(x - 2)(x - 3)$.

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5. Sketch the graph for $y = x^2 - x$.

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6. Sketch the curve $y = \sin 2x$.

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7. Sketch the curve $y = \sin^2 x$.

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8. Construct the graph for $f(x) = \frac{x^2 - 1}{x^2 + 1}$.

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9. Construct the graph for $f(x) = x + \frac{1}{x}$.

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10. Construct the graph for $f(x) = \frac{1}{1 + e^{1/x}}$.

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11. Sketch the graph $y = |x + 1|$. Evaluate $\int_{-4}^{2} |x + 1| dx$. What does the value of this integral represent on the graph?

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12. The area inside the parabola $5x^2 - y = 0$ but outside the parabola $2x^2 - y + 9 = 0$ is (a) $12\sqrt{3}$ sq units (b) $6\sqrt{3}$ sq units (c) $8\sqrt{3}$ sq units (d) $4\sqrt{3}$ sq units

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13. The area enclosed by $y = x(x-1)(x-2)$ and x-axis, is given by

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14. The area between the curve $y = 2x^4 - x^2$, the axis, and the ordinates of the two minima of the curve is 11/60 sq. units (b) 7/120 sq. units 1/30 sq. units (d) 7/90 sq. units

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15. Sketch the curves and identify the region bounded by the curves $x = \frac{1}{2}, x = 2, y = \log x any = 2^x$. Find the area of this region.

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16. Find the area of region : $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$.

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17. The area common to the region determined by $y \geq \sqrt{x}$ and $x^2 + y^2 < 2$ has the value

A. π sq units

B. $(2\pi - 1)$sq units

C. $\left(\frac{\pi}{4} - \frac{1}{6}\right)$sq units

D. None of these

Answer: C

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18. Find the area of the figure enclosed by the curve $5x^2 + 6xy + 2y^2 + 7x + 6y + 6 = 0.$

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19. If $f(x) = \begin{cases} \sqrt{\{x\}} & x \notin Z \\ 1 & x \in Z \end{cases}$ and $g(x) = \{x\}^2$ then area bounded by f(x) and g(x) for $x \in [0, 10]$ is

A. $\frac{5}{3}$ sq units

B. 5 sq units

C. $\frac{10}{3}$ sq units

D. None of these

Answer: C

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20. Find the area of the region bounded by the curves $y=x^2, y=|2-x^2|,$ and $y=2,$ which lies to the right of the line x=1.

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21. The area enclosed by the curve $|y|=\sin 2x,$ where $x \in [0, 2\pi].$ is

A. 1 sq unit

B. 2 sq unit

C. 3 sq unit

D. 4 sq unit

Answer: D

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22. Let $f(x) = x^2, g(x) = \cos x$ and $\alpha, \beta(\alpha < \beta)$ be the roots of the equation $18x^2 - 19\pi x + \pi^2 = 0$. Then the area bounded by the curves $u = \text{fog}(x)$, the ordinates $x = \alpha, x = \beta$ and the X-asis is

A. $\frac{1}{2}(\pi - 3)$ sq units

B. $\frac{\pi}{3}$ sq units

C. $\frac{\pi}{4}$ sq units

D. None of these

Answer: D

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23. Find the area bounded by the curves $x^2 + y^2 = 25, 4y = |4 - x^2|$, and $x = 0$ above the x-axis.

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24. Find area enclosed by $|x|+|y|=1$.

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25. Let $f(x)=\max\left\{\sin x, \cos x, \frac{1}{2}\right\}$, then determine the area of region bounded by the curves $y=f(x)$, X-axis, Y-axis and $x=2\pi$.

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26. If A denotes the area bounded by $f(x)=\left|\frac{\sin x+\cos x}{x}\right|$, X-axis, $x=\pi$ and $x=3\pi$,then

A. $1 < A < 2$

B. $0 < A < 2$

C. $2 < A < 3$

D. None of these

Answer: B

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27. If $y = f(x)$ makes positive intercepts of 2 and 1 unit on x and y-coordinates axes and encloses an area of $\frac{3}{4}$ sq unit with the axes, then $\int_0^2 x f'(x)\,dx$, is

A. $\frac{3}{4}$

B. 1

C. $\frac{5}{4}$

D. $-\frac{3}{4}$

Answer: D

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28. The area of the region included between the regions satisfying $\min(|x|, |y|) \geq 1$ and $x^2 + y^2 \leq 5$ is

A. $\frac{5}{2}\left(\frac{\sin^{-1}(2)}{\sqrt{5}} - \frac{\sin^{-1}1}{\sqrt{5}}\right) - 4$

B. $10\left(\frac{\sin^{-1}(2)}{\sqrt{5}} - \frac{\sin^{-1}(1)}{\sqrt{5}}\right) - 4$

C. $\frac{2}{5}\left(\frac{\sin^{-1}(2)}{\sqrt{5}} - \frac{\sin^{-1}(1)}{\sqrt{5}}\right) - 4$

D. $15\left(\frac{\sin^{-1}(2)}{\sqrt{5}} - \frac{\sin^{-1}(1)}{\sqrt{5}}\right) - 4$

Answer: B

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29. The area of the region bounded by the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines x=0 and $x = \frac{\pi}{4}$ is

A. $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

B. $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

C. $\int_0^{\sqrt{2}=1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

D. $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

Answer: B

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30. Let T be the triangle with vertices $(0, 0)$, $(0, c^2)$ and (c, c^2) and let R be the region between $y = cx$ and $y = x^2$ where $c > 0$ then

A. Area $(R) = \frac{c^3}{6}$

B. Area of $R = \frac{c^3}{3}$

C. $\lim_{c \to 0^+} \frac{Area(T)}{Area(R)} = 3$

D. $\lim_{c \to 0^+} \frac{Area(T)}{Area(R)} = \frac{3}{2}$

Answer: A::C

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31. Suppose fis defined from $R \to [-1, 1]$ as $f(x) = \frac{x^2 - 1}{x^2 + 1}$ where R is the set of real number .then the statement which does not hold is

A. f is many-one onto

B. f increases for $x > 0$ and decreases for $x < 0$

C. minimum value is not attained even though f is bounded

D. the area included by the curve $y - f(x)$ and the line $y = 1$ is π sq units

Answer: A::C::D

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32. Consider $f(x) = \begin{cases} \cos x & 0 \le x < \frac{\pi}{2} \\ \left(\frac{\pi}{2} - x\right)^2 & \frac{\pi}{2} \le x < \pi \end{cases}$ such that f is periodic with period π. Then which of the following is not true?

A. the range of f is $\left[0, \frac{\pi^2}{4}\right)$

B. f is continuous for all real x, but not defferentiable for some real x

C. f is continuous fo all real x

D. the area bounded by $y = f(x)$ and the X-axis for $x = n\pi$ to $x = n\pi$ is $2n\left(1 + \frac{\pi^2}{24}\right)$ for a given $n \in N$

Answer: A::D

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33. Consider the functions f(x) and g(x), both defined from $R \to R$ and are defined as $f(x) = 2x - x^2$ and $g(x) = x^n$ where $n \in N$. If the area between f(x) and g(x) is 1/2, then the value of n is

A. 12

B. 15

C. 20

D. 30

Answer: B::C::D

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34. The area of the region bounded by the curve $y = e^x$ and lines $x = 0 and y = e$ is $e - 1$ (b) $\int_1^e 1n(e+1-y)dy$ $e - \int_0^1 e^x dx$ (d) $\int_1^e 1nydy$

A. $e - 1$

B. $\int_1^e In(e+1-y)dy$

C. $e - \int_0^1 e^x dx$

D. $\int_0^e Inydy$

Answer: B::C::D

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35. Consider the function $f(x) = x^3 - 8x^2 + 20x - 13$

The function $f(x)$ defined for $R \rightarrow R$

A. (a)is one-one onto

B. (b)is many-one onto

C. (c)has 3 real roots

D. (d)is such that $f(x_1) \cdot f(x_2) < 0$ where x_1 and x_2 are the roots of $f'(x) = 0$

Answer: B

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36. Consider the function $f(x) = x^3 - 8x^2 + 20x - 13$

Area enclosed by $y = f(x)$ and the coordinate axes is

A. $65/12$

B. $13/12$

C. $71/12$

D. None of these

Answer: A

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37. Let $h(x) - f(x) - g(x)$ where $f(x) = \sin^4 \pi x$ and $g(x) = In x$. Let $x_0, x_1, x_2, \ldots\ldots, x_{n-1}$ be the roots of $f(x) = g(x)$ in increasing oder.Then the absolute area enclosed by $y = f(x)$ and $y = g(x)$ is given by

A. $\sum_{r=0}^{n} \int_{x_r}^{x_{r+1}} (-1)^r h(x) dx$

B. $\sum_{r=0}^{n} \int_{x_r}^{x_{r+1}} (-1)^{r+1} h(x) dx$

C. $2\sum_{r=0}^{n} \int_{x_r}^{x_{r+1}} (-1)^r h(x) dx$

D. $\frac{1}{2}\sum_{r=0}^{n} \int_{x_r}^{x_{r+1}} (-1)^{r+1} h(x) dx$

Answer: A

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38. Let $h(x) = f(x) = f_x - g_x$, where $f_x = \sin^4 \pi x$ and $g(x) = Inx$. Let $x_0, x_1, x_2, \ldots, x_{n+1}$ be the roots of $f_x = g_x$ in increasing order.

In the above question, the value of n is

A. 1

B. 2

C. 3

D. 4

Answer: B

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39. Let $h(x) = f(x) = f_x - g_x$, where $f_x = \sin^4 \pi x$ and $g(x) = Inx$. Let $x_0, x_1, x_2, \ldots, x_{n+1}$ be the roots of $f_x = g_x$ in increasing order.

The absolute area enclosed by $y = f_x$ and $y = g(x)$ is given by

A. $\frac{11}{8}$

B. $\frac{8}{3}$

C. 2

D. $\frac{13}{3}$

Answer: A

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40. Consider the function defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real-valued defferentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real-valud diferentiable function $y - g(x)$ satisfying $g_0 = 0$.

If $f\left(-10\sqrt{2}\right) = 2\sqrt{2}$, then $f\left(-10\sqrt{2}\right)$ is equal to

A. $\frac{4\sqrt{2}}{7^3 3^2}$

B. $-\frac{4\sqrt{2}}{7^3 3^2}$

C. $\frac{4\sqrt{2}}{7^3 3}$

D. $-\frac{4\sqrt{2}}{7^3 3}$

Answer: B

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41. Consider the function defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real-valued defferentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real-valud diferentiable function $y - g(x)$ satisfying $g_0 = 0$.

The area of the region bounded by the curve $y = f(x)$, the X-axis and the line $x = a$ and $x = b$, where $-\infty < a < b < -2$ is

A. $\int_a^b \frac{x}{3\left[\{f(x)\}^2 - 1\right]} dx + by(b) - af(a)$

B. $-\int_a^b \frac{x}{3\left[\{f(x)\}^2-1\right]}dx - by(b) + af(a)$

C. $\int_a^b \frac{x}{3\left[\{f(x)\}^2-1\right]}dx - by(b) + af(a)$

D. $-\int_a^b \frac{x}{3\left[\{f(x)\}^2-1\right]}dx + by(b) = af(a)$

Answer: A

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42. Consider the function defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real-valued defferentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real-valud diferentiable function $y - g(x)$ satisfying $g_0 = 0$.

If $f\left(-10\sqrt{2}\right) = 2\sqrt{2}$, then $f\left(-10\sqrt{2}\right)$ is equal to

A. $2g(-1)$

B. 0

C. $-2g(1)$

D. $2g(1)$

Answer: D

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43. A curve $y=f(x)$ passes through point $P(1,1)$. The normal to the curve at P is a $(y-1)+(x-1)=0$. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point, then the equation of the curve is (a) $(b)(c)y=(d)e^{(e)(f)K((g)(h)x-1(i))(j)}(k)(l)$ (m) (b) $(n)(o)y=(p)e^{(q)(r)Ke(s)}(t)(u)$ (v) (c) $(d)(e)y=(f)e^{(g)(h)K((i)(j)x-2(k))(l)}(m)(n)$ (o) (d) None of these

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44. Sketch the region bounded by the curves $y = x^2 and y = \frac{2}{1+x^2}$.

Find the area.

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45. The area enclosed between the curves $y = \log_e(x+e), x = \log_e\left(\frac{1}{y}\right)$, and the x-axis is

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46. Find the area of the region bounded by the curve C : y=tan x ,tangent drawn to C at x=pi/4, and the x-axis.

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47. Find all the possible values of $b > 0$, so that the area of the bounded region enclosed between the parabolas $y = x - bx^2 and y = \frac{x^2}{b}$ is maximum.

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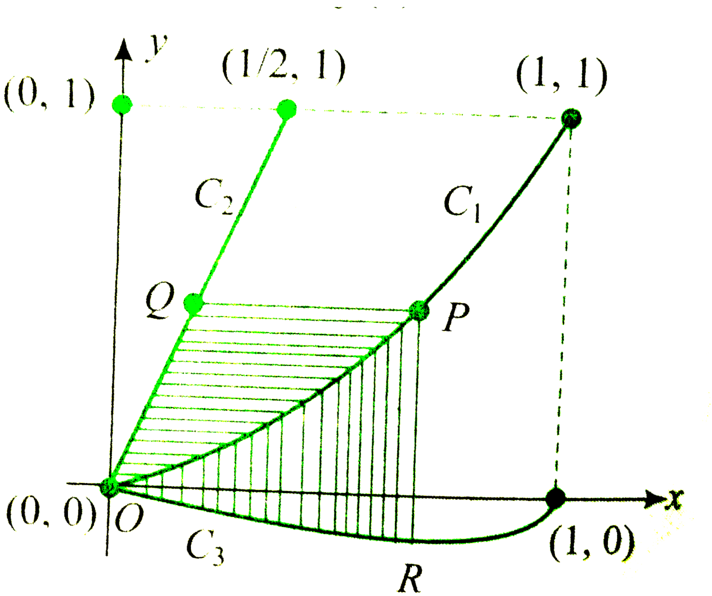
48. Let C_1 and C_2 be the graphs of the functions $y = x^2$ and $y = 2x$, respectively, where $0 \leq x \leq 1$. Let C_3 be the graph of a function y=f(x), where $0 \leq x \leq 1, f(0) = 0$. For a point P on C_1, let the lines through P, parallel to the axes, meet C_2 and C_3 at Q and R, respectively (see figure). If for every position of $P(on C_1)$, the areas of the shaded regions OPQ and ORP are equal, determine the function f(x).



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49. Compute the area of the region bounded by the curves $y - ex(\log)_e x and y = \frac{\log x}{ex}$

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50. If A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0,\ y = 0,\ x = \pi / 4$, then for $n > 2$.

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51. Consider a square with vertices at $(1, 1), (- 1, 1), (- 1, - 1)$ and $(1, - 1)$. Let S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area.

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52. The area of the region included between the curves $x^2+y^2=a^2$ and $\sqrt{|x|}+\sqrt{|y|}=\sqrt{a}(a>0)$ is

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53. Show that the area included between the parabolas $y^2=4a(x+a)$ and $y^2=4b(b-x)$ is $\frac{8}{3}\sqrt{ab}(a+b)$.

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54. Determine the area of the figure bounded by two branches of the curve $(y-x)^2=x^3$ and the straight line $x=1$.

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55. Prove that the areas S_0, S_1, S_2...bounded by the x-axis and half-waves of the curve $y=e^{-ax}\sin\beta x, x\geq 0$ form a geometric progression with the common ratio $r=e^{-\pi\alpha/\beta}$.

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56. Let $b \neq 0$ and for $j = 0, 1, 2, \ldots, n$. Let S_j be the area of the region bounded by Y_axis and the curve $x \cdot e^{ay} = \sin by, \frac{j\pi}{b} \leq y \leq \frac{(j+1)\pi}{b}$. Show that $S_0, S_1, S_2, \ldots S_n$ are in geometric progression. Also, find their sum for a=-1 and $b = \pi$.

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57. For any real $t, x = \frac{1}{2}\left(e^t + e^{-t}\right), y = \frac{1}{2}\left(e^t - e^{-t}\right)$ is a point on the hyperbola $x^2 - y^2 = 1$ Show that the area bounded by the hyperbola and the lines joining its centre to the points corresponding to $t_1 and - t_1$ ist_1.

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58. Find the area enclosed by circle $x^2 + y^2 = 4$, parabola $y = x^2 + x + 1$, the curve $y = \left[\frac{\sin^2 x}{4} + \frac{\cos x}{4}\right]$ and X-axis (where,[.] is the greatest integer function.

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59. Let $f(x) = Ma\xi\mu m\left\{x^2, (1-x)^2, 2x(1-x)\right\}$, where $0 \leq x \leq 1$. Determine the area of the region bounded by the curves $y = f(x), x - a\xi s, x = 0,$ and $x = 1$.

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60. Let $f(x) = \begin{cases} -2 & -3 \leq x \leq 0 \\ x - 2 & 0 < x \leq 3 \end{cases}$, where $g(x) = \min\{f(|x|) + |f(x)|, f(|x|) - |f(x)|\}$. Find the area bounded by the curve $g(x)$ and the X-axis between the ordinates at $x = 3$ and $x = -3$.

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61. Let $O(0, 0), A(2, 0), andB\left(1\frac{1}{\sqrt{3}}\right)$ be the vertices of a triangle. Let R be the region consisting of all those points P inside OAB which satisfy $d(P, OA) \leq \min[d(p, OB), d(P, AB)]$, where d denotes the distance from the point to the corresponding line. Sketch the region R and find its area.

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62. A curve $y = f(x)$ passes through the origin. Through any point (x, y) on the curve, lines are drawn parallel to the co-ordinate axes. If the curve divides the area formed by these lines and co-ordinates axes in the ratio $m : n,$ find the curve.

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63. Find the ratio of the areas in which the curve $y = \left[\frac{x^3}{100} + \frac{x}{35}\right]$ divides the circle $x^2 + Y^2 - 4x + 2y + 1 = 0$. (where, [.] denotes the greated integer function).

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64. Area bounded by the line y=x, curve $y = f(x), (f(x) > x, \forall x > 1)$ and the lines x=1,x=t is $\left(t - \sqrt{1 + t^2} - (1 + \sqrt{2})\right)$ for all $t > 1$. Find $f(x)$.

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65. If the area bounded by the curve y=f(x), x-axis and the ordinates x=1 and x=b is (b-1) sin(3b+4), then find f(x).

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66. Let f (x) be a function which satisfy the equatio $f(xy) = f9x) + f(y)$ for all $x > 0, y > 0$ such that $f'(1) = 2$. Let A be the area of the region bounded by the curves

$y = f(x), y = \left|x^3 - 6x^2 + 11x - 6\right|$ and $x = 0$, then find value of $\frac{28}{17}A$.

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67. Find the area of the region which is inside the parabola satisfying the condition $|x - 2y| + |x + 2y| \leq 8$ and $xy \geq 2$.

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68. Consider the function $f(x) = \begin{cases} x - [x] - \frac{1}{2} & x \notin \\ 0 & x \in I \end{cases}$ where [.] denotes the fractional integral function and I is the set of integers. Then find $g(x) \max . \left[x^2, f(x), |x|\right\}, -2 \leq x \leq 2.$

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69. Find the area of the region bounded by the curves $y = x^2$ and $y = \sec^{-1}\left[-\sin^2 x\right]$, where [.] denotes G.I.F.

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70. Draw a graph of the function $f(x) = \cos^{-1}(4x^3 - 3x), x \in [-1, 1]$ and find the ara enclosed between the graph of the function and the x-axis varies from 0 to 1.

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71. Let $f(x)$ be continuous function given by $f(x) = \{2x, |x| \leq 1$and $x^2 + ax + b, |x| > 1\}$.

Find the area of the region in the third quadrant bounded by the curves $x = -2y^2 andy = f(x)$ lying on the left of the line $8x + 1 = 0$.

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72. Let $[x]$ denotes the greatest integer function. Draw a rough sketch of the portions of the curves $x^2 = 4[\sqrt{x}]y$ and $y^2 = 4[\sqrt{y}]x$ that lie within the square $\{(x, y) \mid 1 \leq x \leq 4, 1 \leq y \leq 4\}$. Find the area of the part of the square that is enclosed by the two curves and the line $x + y = 3$.

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73. Find all the values of the parameter $a(a \leq 1)$ for which the area of the figure bounded by pair of straight lines $y^2 - 3y + 2 = 0$ and curves $y = [a]x^2, y = \frac{1}{2}[a]x^2$ is greatest , where [.] denotes the greatest integer function.

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74. Find the area in the 1* quadrant bounded by $[x] + [y] = n$, where $n \in N$ and $y = k$(where $k \in n \forall k \leq n + 1$), where [.] denotes the greatest integer less than or equal to x.

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Exercise For Session 1

1. Draw a rough sketch of $y = \sin 2x$ and determine the area enclosed by the curve. X-axis and the lines $x = \pi/4$ and $x = 3\pi/4$.

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2. Find the area under the curve $y = (x^2 + 2)^2 + 2x$ between the ordinates x =0 and x=2`

A. $\frac{236}{15}$ sq units

B. $\frac{136}{14}$ sq units

C. $\frac{430}{14}$ sq units

D. $\frac{436}{14}$ sq units

Answer: $\frac{436}{14}$ **sq units**

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3. Find by integration the area of the region bounded by the curve $y = 2x - x^2$ and the x-axis.

A. $\frac{1}{3}$ sq units

B. $\frac{2}{3}$ sq units

C. $\frac{4}{3}$ sq units

D. $\frac{5}{3}$ sq units

Answer: $\frac{4}{3}$ **sq units**

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4. Find the area of the region bounded by the curve $y^2 = 2y - x$ and the y-axis.

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5. Find the area bounded by the curve $y = 4 - x^2$ and the lines $y = 0, \ y = 3$.

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6. Find the area of the region bounded by the curve $x = at^2, y = 2at$ between the ordinates corresponding $t = 1 and t = 2$.

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7. Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

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8. Find the area bounded by $y = 1 + 2\sin^2 x$, X-axis, $X = 0$ and $x = \pi$.

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9. Sketch the graph of $y = \sqrt{x} + 1 \text{in}[0, 4]$ and determine the area of the region enclosed by the curve, the axis of X and the lines $x = 0, x = 4.$

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10. Find the area of the region bounded by the curve $xy - 3x - 2y - 10 = 0$, x-axis and the lines $x = 3, \ x = 4.$

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Exercise For Session 2

1. Find the area of the region bounded by parabola $y^2 = 2x + 1$ and the line $x - y - 1 = 0.$

A. $2/3$

B. $4/3$

C. $8/3$

D. $16/3$

Answer: D

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2. Find the area bounded by the curve $y=2x-x^2$, and the line $y=x$

A. $9/2$

B. $43/6$

C. $35/6$

D. None of these

Answer: A

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3. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ & $x = 1$ is:

A. 0

B. $1/3$

C. $2/3$

D. None of these

Answer: C

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4. Area of the region bounded by the curves $y = 2^x, y = 2x - x^2, x = 0$ and $x = 2$ is given by :

A. $\frac{3}{\log 2} - \frac{4}{3}$

B. $\frac{3}{\log 2} + \frac{4}{3}$

C. $3 \log 2 - \frac{4}{3}$

D. None of these

Answer: A

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5. Find the area (in sq. unit) bounded by the curves : $y = e^x, y = e^{-x}$ and the straight line x =1.

A. $e + \frac{1}{e}$

B. $e - \frac{1}{e}$

C. $e + \left(\frac{1}{e}\right) - 2$

D. None of these

Answer: A

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6. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is 2 b. $\frac{9}{4}$ c. $\frac{9}{3}$ d. $\frac{9}{2}$

A. 2

B. $\frac{9}{4}$

C. $6\sqrt{3}$

D. None of these

Answer: B

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7. The area of the region bounded by $y = \sin x$, $y = \cos x$ in the first quadrant is

A. $2\left(\sqrt{2 - 1}\right)$

B. $\sqrt{3} + 1$

C. $2\left(\sqrt{3} - 1\right)$

D. None of these

Answer: A

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8. The area bounded by the curves $y = xe^x, y = xe^{-x}$ and the line x=1 is

A. $\frac{2}{e}$

B. $1 - \frac{2}{e}$

C. $\frac{1}{e}$

D. $1 - \frac{1}{e}$

Answer: A

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9. The figure into which the curve $y^2 = 6x$ divides the circle $x^2 + y^2 = 16$ are in the ratio

A. $\frac{2}{3}$

B. $\frac{4\pi - \sqrt{3}}{8\pi + \sqrt{3}}$

C. $\frac{4\pi + \sqrt{3}}{8\pi - \sqrt{3}}$

D. None of these

Answer: C

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10. Find the area bounded by the y-axis, $y = \cos x$,and $y = \sin x$ when $0 \le x \le \frac{\pi}{2}$.

A. $2(\sqrt{2-1})$

B. $\sqrt{2} - 1)$

C. $(\sqrt{2} + 1)$

D. $\sqrt{2}$

Answer: B

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11. The area bounded by the curves $y = -x^2 + 2$ and $y = 2|x| - x$ is

A. $2/3$

B. $8/3$

C. $4/3$

D. None of these

Answer: D

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12. The are bounded by the curve $y^2 = 4x$ and the circle $x^2 + y^2 - 2x - 3 = 0$ is

A. $2\pi + \frac{8}{3}$

B. $4\pi + \frac{8}{3}$

C. $\pi + \frac{8}{3}$

D. $\pi - \frac{8}{3}$

Answer: A

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13. A point P moves inside a triangle formed by $A(0, 0), B(1, \sqrt{3}), C(2, 0)$ such that min $\{PA, PB, PC) = 1$, then the area bounded by the curve traced by P, is

A. (a)$3\sqrt{3} - \frac{3\pi}{2}$

B. (b)$\sqrt{3} + \frac{\pi}{2}$

C. (c)$\sqrt{3}-\frac{\pi}{2}$

D. (d)$3\sqrt{3}+\frac{3\pi}{2}$

Answer: C

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14. The graph of $y^2+2xy+40|x|=400$ divides the plane into regions. Then the area of the bounded region is $200 sq units$ (b) $400 sq units$ $800 sq units$ (d) $500 sq units$

A. 400

B. 800

C. 600

D. None of these

Answer: B

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15. The area of the region defined by $||x| - |y|| \leq 1$ and $x^2 + y^2 \leq 1$ in the xy plane is

A. $\pi - 2$

B. $2\pi - 1$

C. 3π

D. 1

Answer: A

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16. The area of the region defined by $1 \leq |x - 2| + |y + 1| \leq 2$ is (a) 2 (b) 4 (c) 6 (d) non of these

A. 2

B. 4

C. 6

D. None of these

Answer: C

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17. The area of the region enclosed by the curve $|y| = -(1-|x|)^2 + 5$, is

A. $\frac{8}{3}(7+5\sqrt{5})$ sq units

B. $\frac{2}{3}(7+5\sqrt{5})$ sq units

C. $\frac{2}{3}(5\sqrt{5}-7)$ sq units

D. None of these

Answer: A

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18. The area bounded by the curve $f(x) = ||\tan x + \cot x| - |\tan x - \cot x||$ between the lines $x = 0, x = \frac{\pi}{2}$ and the X-axis is

A. log 4

B. $\log\sqrt{2}$

C. $2\log 2$

D. $\sqrt{2}\log 2$

Answer: A

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19. If $f(x) = \max\left\{\sin x, \cos x, \frac{1}{2}\right\}$, then the area of the region bounded by the curves $y = f(x)$, x-axis, Y-axis and $x = \frac{5\pi}{3}$ is

A. $\left(\sqrt{2} - \frac{\sqrt{3}}{2} + \frac{5\pi}{12}\right)$sq units

B. $\left(\sqrt{2} + \sqrt{3} + \frac{5\pi}{2}\right)$sq units

C. $\left(\sqrt{2}+\sqrt{3}+\frac{5\pi}{2}\right)$sq units

D. None of these

Answer: B

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Exercise Single Option Correct Type Questions

1. A point $P(x, y)$ moves such that $[x+y+1]=[x]$. Where [.] denotes greatest intetger function and $x \in (0, 2)$, then the area represented by all the possible position of P , is

A. (a)$\sqrt{2}$

B. (b)$2\sqrt{2}$

C. (c)$4\sqrt{2}$

D. (d)2

Answer: D

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2. If $f:[-1,1] \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]: f(x)=\frac{x}{1+x^2}$, then find the area bounded by $y=f^{-1}(x)$, the x-axis and the lines $x=\frac{1}{2}, x=-\frac{1}{2}$.

A. $\frac{1}{2}\log e$

B. $\log\left(\frac{e}{2}\right)$

C. $\frac{1}{2}\frac{\log e}{3}$

D. $\frac{1}{2}\log\left(\frac{e}{2}\right)$

Answer: B

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3. If the length of latusrectum of ellipse $E_1: 4(x+y+1)^2+2(x-y+3)^2=8$ and

$E_2 = \frac{x^2}{p} + \frac{y^2}{p^2} = 1, (0 < p < 1)$ are equal , then area of ellipse E_2, is

A. $\frac{\pi}{2}$

B. $\frac{\pi}{\sqrt{2}}$

C. $\frac{\pi}{2\sqrt{2}}$

D. None of these

Answer: B

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4. The area of bounded by the curve $4\left|x - 2017^{2017}\right| + 5\left|y - 2017^{2017}\right| \leq 20$, is

A. (a)60

B. (b)50

C. (c)40

D. (d)30

Answer: C

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5. If the area bounded by the corve $y = x^2 + 1, y = x$ and the pair of lines $x^2 + y^2 + 2xy - 4x - 4y + 3 = 0$ is K units, then the area of the region bounded by the curve $y = x^2 + 1, y = \sqrt{x - 1}$ and the pair of lines $(x + y - 1)(x + y - 3) = 0$ is

A. (a)K

B. (b)2K

C. (c)$\frac{K}{2}$

D. (d)None of these

Answer: B

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6. Suppose $y = f(x)$ and $y = g(x)$ are two functions whose graphs intersect at the three point (0, 4), (2,2) and (4, 0) with f(x) gt g(x) for 0 lt x lt 2 and f(x) lt g(x) for 2 lt x lt 4.

If $\int_0^4 [f(x) - g(x)]dx = 10$ and $\int_2^4 [g(x) - f(x)]dx = 5$, the area between two curves for 0 lt x lt 2, is

A. 5

B. 10

C. 15

D. 20

Answer: C

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7. Let 'a' be a positive constant number. Consider two curves $C_1 : y = e^x, C_2 : y = e^{a-x}$. Let S be the area of the part surrounding by C_1, C_2 and the y axis, then $\lim_{a \to 0} \frac{s}{a^2}$ equals (A) 4 (B) $\frac{1}{2}$ (C) 0 (D) $\frac{1}{4}$

A. 4

B. $\frac{1}{2}$

C. 0

D. 1.4

Answer: D

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8. 3 point O$(0, 0)$, $P(a, a^2)$, $Q(-b, b^2)(a > 0, b > 0)$ are on the parabola $y = x^2$. Let S_1 be the area bounded by the line PQ and parabola let S_2 be the area of the ΔOPQ, the minimum value of S_1 / S_2 is

A. (a)$2/3$

B. (b)$5/3$

C. (c)2

D. (d)73

Answer: A

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9. Area enclosed by the graph of the function $y = In^2 x - 1$ lying in the 4^{th} `quadrant is

A. $\frac{2}{e}$

B. $\frac{4}{e}$

C. $2\left(e + \frac{1}{e}\right)$

D. $4\left(e - \frac{1}{e}\right)$

Answer: B

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10. The area bounded by $y = 2 - |2 - x| \text{ and } y = \frac{3}{|x|}$ is:

A. (a) $\dfrac{4+3\ln 3}{2}$

B. (b) $\dfrac{19}{8}-3\ln 2$

C. (c) $\dfrac{3}{2}+\ln 3$

D. (c) $\dfrac{1}{2}+\ln 3$

Answer: B

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11. Suppose $g(x)=2x+1$ and $h(x)=4x^2+4x+5$ and $h(x)=(fog)(x)$. The area enclosed by the graph of the function $y=f(x)$ and the pair of tangents drawn to it from the origin is:

A. (a) $\dfrac{8}{3}$

B. (b) $\dfrac{16}{3}$

C. (c) $\dfrac{32}{3}$

D. (d) None of these

Answer: B

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12. The area bounded by the curves $y = -\sqrt{-x}$ and $x = -\sqrt{-y}$ where $x, y \leq 0$

A. cannot be determined

B. is $\frac{1}{3}$

C. is $\frac{2}{3}$

D. is same as that of the figure bounded by the curves $y = \sqrt{-x}, x \leq 0 \text{ and } x = \sqrt{-y}, y \leq 0$

Answer: B

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13. $y = f(x)$ is a function which satisfies $f(0) = 0$, $f''(x) = f'(x)$ and $f'(0) = 1$ then the area bounded by the graph of $y = f(x)$, the lines $x = 0$, $x - 1 = 0$ and $y + 1 = 0$ is

A. e

B. e-2

C. e-1

D. e+1

Answer: C

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14. The area of the region enclosed between the curves $x = y^2 - 1$ and $x = |x|\sqrt{1 - y^2}$ is

A. 1

B. $4/3$

C. $2/3$

D. 2

Answer: D

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15. The area bounded by the curve $y = xe^{-x}, y = 0 \text{ and } x = c,$ where c is the x-coordinate to the curve's inflection point, is

A. $1 - 3e^{-2}$

B. $1 - 2e^{-2}$

C. $1 - e^{-2}$

D. 1

Answer: A

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16. If $(a, 0)$, agt 0, is the point where the curve $y = \sin 2x - \sqrt{3} \sin x$ cuts the x-axis first, A is the area bounded by this part of the curve, the origin and the positive x-axis. Then

A. $4A + 8 \cos a = 7$

B. $4A + 8 \sin a = 7$

C. $4A - 8 \sin a = 7$

D. $4A - 8 \cos a = 7$

Answer: A

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17. The curve $y = ax^2 + bx + c$ passes through the point $(1, 2)$ and its tangent at origin is the line $y = x$. The area bounded by the curve, the ordinate of the curve at minima and the tangent line is

A. $\frac{1}{24}$

B. $\frac{1}{12}$

C. $\frac{1}{8}$

D. $\frac{1}{6}$

Answer: A

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18. A function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} - y = \cos x - \sin x$ with initial condition that y is bounded when $x_{>}\infty$. The area enclosed by $y = f(x), y = \cos x$ and the y-axis is

A. (a)$\sqrt{2} - 1$

B. (b)$\sqrt{2}$

C. (c)1

D. (d)$1 / \sqrt{2}$

Answer: A

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19. The ratio between masses of two planets is 3 : 5 and the ratio between their radii is 5 : 3. The ratio between their acceleration due to gravity will be

A. (a)4 or -2

B. (b)two values are in (2,3) and one in (-1,0)

C. (c)two values are in (3,4) and one in (-2,-1)

D. (d)None of the above

Answer: C

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20. Area bounded by $y = f^{-1}(x)$ and tangent and normal drawn to it at points with abscissae π and 2π, where $f(x) = \sin x - x$ is

A. a) $\frac{\pi^2}{2} - 1$

B. b) $\frac{\pi^2}{2} - 2$

C. c) $\frac{\pi^2}{2} - 4$

D. d) $\frac{\pi^2}{2}$

Answer: B

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21. If $f(x) = x - 1$ and $g(x) = |f|(x)| - 2|$, then the area bounded by $y = g(x)$ and the curve $x^2 - 4y + 8 = 0$ is equal to

A. $\frac{4}{3}(4\sqrt{2} - 5)$

B. $\frac{4}{3}(4\sqrt{2} - 3)$

C. $\frac{8}{3}(4\sqrt{2} - 3)$

D. $\frac{8}{3}(4\sqrt{2} - 5)$

Answer: A

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22. Let

$S = \left\{(x, y) : \frac{y(3x-1)}{x(3x-2)} < 0\right\}, S' = \{(x, y) \in A \times B : -1 \leq A \leq 1, -$

then the area of the region enclosed by all points in $S \cap S'$ is

A. 1

B. 2

C. 3

D. 4

Answer: B

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23. The area of the region bounded between the curves

$y = e||x|In|x| \;\; |, x^2 + y^2 - 2(|x| + |y|) + 1 \geq 0$ and X-axis where

$|x| \leq 1$, if α is the x-coordinate of the point of intersection of curves in 1st quadrant, is

A. $4\left[\int_0^\alpha exInxdx + \int_\alpha^1 \left(1 - \sqrt{1 - (x-1)^2}\right) dx\right]$

B. $4\left[\int_0^\alpha exInxdx + \int_1^\alpha \left(1 - \sqrt{1 - (x-1)^2}\right) dx\right]$

C. $4\left[-\int_0^\alpha exInxdx + \int_\alpha^1 \left(1 - \sqrt{1 - (x-1)^2}\right) dx\right]$

D. $2\left[\int_0^\alpha exInxdx + \int_\alpha^1 \left(1 - \sqrt{1 - (x-1)^2}\right) dx\right]$

Answer: D

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24. A point P lying inside the curve $y = \sqrt{2ax - x^2}$ is moving such that its shortest distance from the curve at any position is greater than its distance from X-axis. The point P enclose a region whose area is equal to

A. (a) $\frac{\pi a^2}{2}$

B. (b) $\frac{a^2}{3}$

C. (c) $\frac{2a^2}{3}$

D. (d) $\left(\frac{3\pi - 4}{6}\right)a^2$

Answer: C

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Exercise More Than One Correct Option Type Questions

1. The triangle formed by the normal to the curve $f(x) = x^2 - ax + 2a$ at the point (2,4) and the coordinate axes lies in second quadrant, if its area is 2 sq units, then a can be

A. 2

B. $17/4$

C. 5

D. None of these

Answer: B::C

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2. Let f and g be continuous function on $a \le x \le b$ and set $p(x) = \max\{f(x), g(x)\}$ and $q(x) = \min\{f(x), g(x)\}$. Then the area bounded by the curves $y = p(x), y = q(x)$ and the ordinates $x = a$ and $x = b$ is given by

A. (a) $\int_a^b |f(x) - g(x)| dx$

B. (b) $\int_a^b |p(x) - q(x)| dx$

C. (c) $\int_a^b \{f(x) - g(x)\} dx$

D. (d) $\int_a^b \{p(x) - a(x)\} dx$

Answer: A::B::D

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3. The area bounded by the parabola $y = x^2 - 7x + 10$ and X-axis

A. $9/2$ sq units

B. $1/6$ sq units

C. $5/6$ sq units

D. None of these

Answer: A

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4. Area bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is equal to

A. 6π sq units

B. 3π sq units

C. 12πsq units

D. area bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Answer: A::D

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5. There is curve in which the length of the perpendicular from the orgin to tangent at any point is equal to abscissa of that point. Then,

A. $x^2 + y^2 = 2$ is one such curve

B. $y^2 = 4x$ is one such curve

C. $x^2 + y^2 = 2cx$ (c parameters) are such curve

D. there are no such curves

Answer: A::C

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Exercise Statement I And Ii Type Questions

1. Statement I- The area of the curve $y = \sin^2 x \text{from} 0 \text{to} \pi$ will be more than that of the curve $y = \sin x \text{from} 0 \text{to} \pi$.

Statement II -$x^2 > x, \quad \text{if} \quad x > 1$.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true , Statement II is false

D. Statement I is false , Statement II is true

Answer: D

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2. Statement I- The area of bounded by the curves $y = x^2 - 3$ and $y = kx + 2$ is least if $k = 0$.

Statement II- The area bounded by the curves $y = x^2 - 3$ and $y = kx + 2is\sqrt{k^2 + 20}$.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true , Statement II is false

D. Statement I is false , Statement II is true

Answer: C

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3. Statement I- The area of region bounded parabola $y^2 = 4x$ and $x^2 = 4y$ is $\frac{32}{3}$ sq units.

Statement II- The area of region bounded by parabola $y^2 = 4ax$ and $x^2 = 4by$ is $\frac{16}{3}ab$.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true , Statement II is false

D. Statement I is false , Statement II is true

Answer: D

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4. Statement I- The area by region $|x+y|+|x-y| \leq 2 \text{is} 4$ sq units. Statement II- Area enclosed by region $|x+y|+|x-y| \leq 2$ is symmetric about X-axis.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true , Statement II is false

D. Statement I is false , Statement II is true

Answer: B

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5. Statement I- Area bounded by $y = x(x-1)$ and $y = x(1-x)$ is $\frac{1}{3}$.

Statement II- Area bounded by $y = f(x)$ and $y = g(x)$ "is" $\left|\int_a^b (f(x) - g(x))dx\right|$ is true when $f(x)$ and $g(x)$ lies above X-axis. (Where a and b are intersection of $y = f(x)$ and $y = g(x)$).

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true , Statement II is false

D. Statement I is false , Statement II is true

Answer: C

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Exercise Passage Based Questions

1. Let $f(x)=\dfrac{ax^2+bx+c}{x^2+1}$ such that y=-2 is an asymptote of the curve $y=f(x)$. The curve $y=f(x)$ is symmetric about Y-axis and its maximum values is 4. Let $h(x)=f(x)-g(x)$,where $f(x)=\sin^4\pi x$ and $g(x)=\log_e x$. Let $x_0, x_1, x_2 ... x_{n+1}$ be the roots of $f(x)=g(x)$ in increasing order

Then, the absolute area enclosed by $y=f(x)$ and $y=g(x)$ is given by

A. $\sum_{r=0}^{n}\int_{x_r}^{x_{r+1}}(-1)^r\cdot h(x)dx$

B. $\sum_{r=0}^{n}\int_{x_1}^{x_{r+1}}(-1)^{r+1}\cdot h(x)dx$

C. $2\sum_{r=0}^{n}\int_{x_r}^{x_{r\_r+1}}(-1)^r\cdot h(x)dx$

D. $\frac{1}{2}\cdot\sum_{r=0}^{n}\int_{x_1}^{x_{r+1}}(-1)^{r+1}\cdot h(x)dx$

Answer: A

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2. Let $h(x)=f(x)=f_x-g_x$, where $f_x=\sin^4\pi x$ and $g(x)=Inx$. Let $x_0, x_1, x_2, \ldots., x_{n+1}$ be the roots of $f_x=g_x$ in increasing order.

In the above question, the value of n is

A. 1

B. 2

C. 3

D. 4

Answer: B

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3. Let $f(x) = \frac{ax^2 + bx + c}{x^2 + 1}$ such that y=-2 is an asymptote of the curve $y = f(x)$. The curve $y = f(x)$ is symmetric about Y-axis and its maximum values is 4. Let $h(x) = f(x) - g(x)$,where $f(x) = \sin^4 \pi x$ and $g(x) = \log_e x$. Let $x_0, x_1, x_2 \ldots x_{n+1}$ be the roots of $f(x) = g(x)$ in increasing order

Then, the absolute area enclosed by $y = f(x)$ and $y = g(x)$ is given by

A. $\frac{11}{8}$

B. $\frac{8}{3}$

C. 2

D. $\frac{13}{3}$

Answer: A

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4. Consider the function $f:(-\infty,\infty)\to(-\infty,\infty)$ defined by $f(x)=\dfrac{x^2-ax+1}{x^2+ax+1}; 0<a<2$. Which of the following is true ?

A. $(2-a)^2 f(1)+(2-a)^2 f(-1)=0$

B. $(2-a)^2 f(1)$-(2-a)^(2)f$(-1)=0$

C. $f'(1)f'(-1)=(2-a)^2$

D. $f'(1)f'(-1)=-(2+a)^2$

Answer: A

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5. Consider the function $f:(-\infty,\infty)\to(-\infty,\infty)$ defined by $f(x)=\dfrac{x^2-ax+1}{x^2+ax+1}; 0<a<2$. Which of the following is true ?

A. f(x) is decreasing on $(-1,1)$ and has a local minimum at x=1

B. f(x) is increasing on (-1,1) and has maximum at x=1

C. f(x) is increasing on (-1,1) but has neither a local maximum nor a local minimum at x=1`

D. f(x) is decreasing on (-1,1) but has neither a local maximum nor a local minimum at x=1.

Answer: A

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6. Let $g(x) = \int_0^{e^x} \frac{f'(t)dt}{1+t^2}$. Which of the following is true?

A. $g'(x)$ is positive on$(-\infty, 0)$ and negative on $(0, \infty)$

B. $g'(x)$ is negative on$(-\infty, 0)$ and positive on $(0, \infty)$

C. $g'(x)$ change sign on both $(-\infty, 0)$ and $(0, \infty)$

D. $g'(x)$ does not change sign on $(-\infty, \infty)$.

Answer: B

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7. Computing area with parametrically represented boundaries

If the boundary of a figure is represented by parametric equations $x = x(t), y = y(t)$, then the area of the figure is evaluated by one of the three formulae

$$S = -\int_{\alpha}^{\beta} y(t)x'(t)dt,\ S = \int_{\alpha}^{\beta} x(t)y'(t)dt$$

$$S = \frac{1}{2}\int_{\alpha}^{\beta} (xy' - yx')dt$$

where α and β are the values of the parameter t corresponding respectively to the beginning and the end of traversal of the contour .

The area enclosed by the astroid $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$ is

A. (a) $\frac{3}{4}a^2\pi$

B. (b) $\frac{3}{18}\pi a^2$

C. (c) $\frac{3}{8}\pi a^2$

D. (d) $\frac{3}{4}a\pi$

Answer: C

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8. Computing area with parametrically represented boundaries : If the boundary of a figure is represented by parametric equation, i.e., $x = x(t), y = (t),$ then the area of the figure is evaluated by one of the three formulas :

$$S = -\int_{\alpha}^{\beta} y(t)x'(t)dt,$$

$$S = \int_{\alpha}^{\beta} x(t)y'(t)dt,$$

$$S = \frac{1}{2}\int_{\alpha}^{\beta} (xy' - yx')dt,$$

Where α and β are the values of the parameter t corresponding respectively to the beginning and the end of the traversal of the curve corresponding to increasing t.

The area of the region bounded by an are of the cycloid $x = a(t - \sin t), y = a(1 - \cos t)$ and the x-axis is

A. $6\pi a^2$

B. $3\pi a^2$

C. $4\pi a^2$

D. None of these

Answer: B

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9. Computing area with parametrically represented boundaries : If the boundary of a figure is represented by parametric equation, i.e., $x = x(t), y = (t),$ then the area of the figure is evaluated by one of the three formulas :

$$S = -\int_{\alpha}^{\beta} y(t)x'(t)dt,$$

$$S = \int_{\alpha}^{\beta} x(t)y'(t)dt,$$

$$S = \frac{1}{2}\int_{\alpha}^{\beta} (xy' - yx')dt,$$

Where α and β are the values of the parameter t corresponding respectively to the beginning and the end of the traversal of the curve corresponding to increasing t.

The area of the loop described as

$x = \frac{t}{3}(6-t), y = \frac{t^2}{8}(6-t)$ is

A. $\frac{27}{5}$

B. $\frac{24}{5}$

C. $\frac{27}{6}$

D. $\frac{21}{5}$

Answer: A

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Area Of Bounded Regions Exercise 5 Matching Type Questions

1. Match the statement of Column I with value of Column II.

	Column I		Column II
(A)	The area bounded by the curve $y = x + \sin x$ and its inverse function between the ordinates $x = 0$ to $x = 2\pi$ is $4s$. Then, the value of s is	(p)	2
(B)	The area bounded by $y = xe^{\lvert x\rvert}$ and lies $x = 1,\ y = 0$ is	(q)	1
(C)	The area bounded by the curves $y^2 = x^3$ and $y = 2x$ is	(r)	$\frac{16}{5}$
(D)	The smaller are included between the curves $\sqrt{\lvert x\rvert} + \sqrt{\lvert y\rvert} = 1$ and $\lvert x\rvert + \lvert y\rvert = 1$ is	(s)	$\frac{1}{3}$

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2. Match the following

	Column I		Column II
(A)	Area enclosed by $y = \lvert x\rvert$, $\lvert x\rvert = 1$ and $y = 0$ is	(p)	2
(B)	Area enclosed by the curve $y = \sin x$, $x = 0, x = \pi$ and $y = 0$ is	(q)	4
(C)	If the area of the region bounded by $x^2 \leq y$ and $y \leq x + 2$ is $\frac{k}{4}$, then k is equal to	(r)	27
(D)	Area of the quadrilateral formed by tangents at the ends of latusrectum of ellipse of ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is	(s)	18

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Exercise Single Integer Answer Type Questions

1. Consider $f(x) = x^2 - 3x + 2$ The area bounded by $|y| = |f(|x|)|, x \geq 1$ is A, then find the value of $3A + 2$.

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2. If S is the sum of cubes of possible value of c for which the area of the figure bounded by the curve $y = 8x^2 - x^5$, then straight lines $x = 1 and x = c$ and the abscissa axis is equal to $\frac{16}{3}$, then the value of $[S], where[.]$ denotest the greatest integer function, is___

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3. If the area bounded by $y = 2 - |2 - x|$ and $y = \frac{3}{|x|}$ is $\frac{k - 3\ln 3}{2}$, then k is equal to ____.

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4. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A(2, 0), B (4, 5) and C (6, 3).

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5. A point 'P' moves in xy plane in such a way that $[|x|] + [|y|] = 1$ where [.] denotes the greatest integer function. Area of the region representing all possible positions of the point 'P' is equal to:

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6. Let $f : [0, 1] \to \left[0, \frac{1}{2}\right]$ be a function such that $f(x)$ is a polynomial of 2nd degree, satisfty the following condition :

(a) $f(0) = 0$

(b) has a maximum value of $\frac{1}{2} at x = 1.$

If A is the area bounded by $y = f(x) = f^{-1}(x)$ and the line $2x + 2y - 3 = 0$ in 1st quadrant, then the value of 24A is equal to ....

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7. Let $f(x) = \min\left\{\sin^{-1}x, \cos^{-1}x, \frac{\pi}{6}\right\}, x \in [0, 1]$. If area bounded by $y = f(x)$ and X-axis, between the lines $x = 0$ and $x = 1 is \frac{a}{b\left(\sqrt{3}+1\right)}$. Then , (a-b) is ....

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8. Let $f(x)$ be a real valued function satisfying the relation $f\left(\frac{x}{y}\right) = f(x) - f(y)$ and $\lim_{x \to 0} \frac{f(1+x)}{x} = 3$. The area bounded by the curve $y = f(x)$, y-axis and the line $y = 3$ is equal to

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Exercise Subjective Type Questions

1. Find the continuous function f where $(x^4 - 4x^2) \leq f(x) \leq (2x^2 - x^3)$ such that the area bounded by $y = f(x), y = x^4 - 4x^2.$ then y-axis, and the line $x = t,$ where $(0 \leq t \leq 2)$ is k times the area bounded by $y = f(x), y = 2x^2 - x^3$,y-axis ,and line $x = t(where 0 \leq t \leq 2).$

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2. Let $f(t) = |t - 1| - |t| + |t + 1|, \forall t \in R.$ Find $g(x) = \max\{f(t) : x + 1 \leq t \leq x + 2\}, \forall x \in R.$ Find $g(x)$ and the area bounded by the curve $y = g(x),$ the X-axis and the lines $x = -3/2$ and $x = 5.$

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3. Let f(x)= minimum $\{e^x, 3/2, 1 + e^{-x}\}, 0 \leq x \leq 1.$ Find the area bounded by $y = f(x),$ X-axis and the line x=1.

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4. Find the area bounded by $y = f(x)$ and the curve $y = \frac{2}{1 + x^2}$ satisfying the condition $f(x), f(y) = f(xy) \forall x, y \in R$ and $f'(1) = 2, f(1) = 1,$

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5. The value of

$\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}dt$, is

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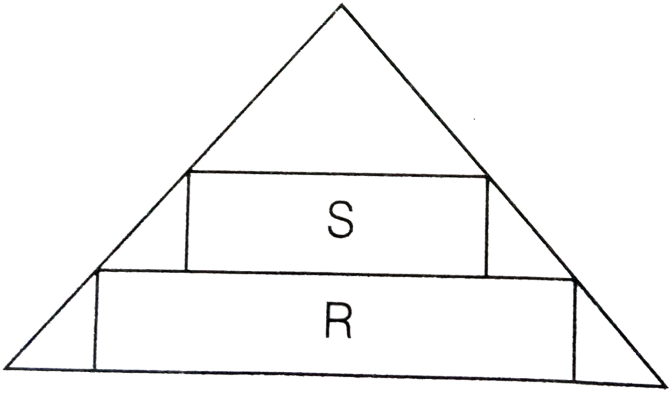
6. Let T be an acute triangle Inscribe a pair R,S of rectangle in T as shown: Let $A(x)$ denote the area of polygon X find the maximum value (or show that no maximum exists), of $\frac{A(R) + A(S)}{A(T)}$, where T ranges over all triangles and R,S over all rectangle as above.



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7. Find the maximum area of the ellipse that can be inscribed in an isoceles triangles of area A and having one axis lying along the perpendicular from the vertex of the triangles to its base.

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8. Find the area of the region bounded by curve $y = 25^x + 16$ and the curve $y = b.5^x + 4$, whose tangent at the point x=1 make an angle $\tan^{-1}$ (40 In 5) with the X-axis.

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9. If the circles of the maximum area inscriabed in the region bounded by the curves $y = x^2 - 2x - 3$ and $y = 3 + 2x - x^2$, then the area of region $y - x^2 + 2x + 3 \leq 0, y + x^2 - 2x - 3 \leq 0$ and $s \leq 0.$

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10. Find limit of the ratio of the area of the triangle formed by the origin and intersection points of the parabola $y = 4x^2$ and the line $y = a^2$ to the area between the parabola and the line as a approaches to zero.

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11. Find the area of curve enclosed by $|x + y| + |x - y| \leq 4, |x| \leq 1, y \geq \sqrt{x^2 - 2x + 1}.$

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12. Calculate the area enclosed by the curve $4 \leq x^2 + y^2 \leq 2(|x| + |y|)$.

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13. Find the area enclosed by the curve $[x] + [y] - 4$ in 1st quadrant (where [.] denotes greatest integer function).

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14. Sketch the region and find the area bounded by the curves $|y + x| \leq 1, |y - x| \leq 1$ and $2x^2 + 2y^2 \geq 1.$

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15. Find the area of the region bounded by the curve $2^{|x|}|y| + 2^{|x|-1} \leq 1,$ with in the square formed by the lines $|x| \leq 1/2, |y| \leq 1/2.$

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16. The value of the parameter $a(a \geq 1)$ for which the area of the figure bounded by the pair of staight lines $y^2 - 3y + 2 = 0$ and the curves $y = [a]x^2, y = \frac{1}{2}[a]x^2$ is greatest is (Here [.] denotes the greatest integer function). (A) $[0, 1)$ (B) $[1, 2)$ (C) $[2, 3)$ (D) $[3, 4)$

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Area Of Bounded Regions Exercise 7 Subjective Type Questions

1. If f(x) is positive for all positive values of X and $f'(x) < 0, f(x) > 0, \forall x \in R^+,$ prove that $\frac{1}{2}f(1) + \int_1^n f(x)dx < \sum_{r=1}^{n} f(r) < \int_1^n f(x)dx + f(1).$

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Exercise Questions Asked In Previous 13 Years Exam

1. Area of the region

$\left\{(x, y) \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq x+9 \leq 15\right\}$ is equal to

A. $\frac{1}{6}$

B. $\frac{4}{3}$

C. $\frac{3}{2}$

D. $\frac{5}{3}$

Answer: C

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2. about to only mathematics

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3. about to only mathematics

A. 3

B. 6

C. 9

D. 15

Answer: D

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4. The area enclosed by the curve $y = \sin x + \cos x and y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is $(a) 4\left(\sqrt{2} - 2\right)$ (b) $2\sqrt{2}$ ($\sqrt{2}$ -1) $(c) 2\left(\sqrt{2}\right.$ +1) (d) $2\sqrt{2}\left(\sqrt{2} + 1\right)$

A. $4\left(\sqrt{2} - 1\right)$

B. $2\sqrt{2}\left(\sqrt{2} - 1\right)$

C. $2\left(\sqrt{2} + 1\right)$

D. $2\sqrt{2}\left(\sqrt{2} + 1\right)$

Answer: B

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5. Let S be the area of the region enclosed by $y - e^{-x^2}, y = 0, x = 0$ and $x = 1$. Then

A. $S \geq \frac{1}{e}$

B. $S \geq 1 - \frac{1}{e}$

C. $S \leq \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$

D. $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

Answer: B::D

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6. Let $f: [-1, 2] \rightarrow [0, \infty)$ be a continuous function such that $f(x) = f(1-x)f \text{ or } all x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x f(x) dx$, and R_2

be the area of the region bounded by $y = f(x), x = -1, x = 2,$ and the x- axis . Then $R_1 = 2R_2$ (b) $R_1 = 3R_2$ (c) $2R_1 = R_2$ (d) $3R_1 = R_2$

A. $R_1 = 2R_2$

B. $R_1 = 3R_2$

C. $2R_1 = R_2$

D. $3R_1 = R_2$

Answer: C

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7. Let the straight line x= b divide the area enclosed by $y = (1-x)^2, y = 0,$ and $x = 0$ into two parts $R_1(0 \leq x \leq b)$ and $R_2(b \leq x \leq 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Answer: B

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8. The area of the region bounded by the curve $y = e^x$ and lines $x = 0 and y = e$ is $e - 1$ (b) $\int_1^e 1n(e + 1 - y)dy$ $e - \int_0^1 e^x dx$ (d) $\int_1^e 1nydy$

A. $e - 1$

B. $\int_1^e In(e + 1 - y)dy$

C. $e - \int_0^1 e^x dx$

D. $\int_0^e Inydy$

Answer: B::C

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9. The area of the region bounded by the curves $y = \sqrt{\frac{1 + \sin x}{\cos x}}$ and $y = \sqrt{\frac{1 - \sin x}{\cos x}}$ bounded by the lines x=0 and $x = \frac{\pi}{4}$ is

A. A. $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

B. B. $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

C. C. $\int_0^{\sqrt{2}=1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

D. D. $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

Answer: B

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10. Consider the function defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real-valued defferentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real-valud diferentiable function $y - g(x)$

satisfying $g_0 = 0$.

If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f(-10\sqrt{2})$ is equal to

A. (a) $\frac{4\sqrt{2}}{7^3 3^2}$

B. (b) $-\frac{4\sqrt{2}}{7^3 3^2}$

C. (c) $\frac{4\sqrt{2}}{7^3 3}$

D. (d) $-\frac{4\sqrt{2}}{7^3 3}$

Answer: B

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11. Consider the function defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real-valued defferentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real-valud diferentiable function $y - g(x)$ satisfying $g_0 = 0$.

The area of the region bounded by the curve $y = f(x)$, the X-axis and the line $x = a$ and $x = b$, where $-\infty < a < b < -2$ is

A. $\int_a^b \frac{x}{3\left[\{f(x)\}^2 - 1\right]} dx + by(b) - af(a)$

B. $-\int_a^b \frac{x}{3\left[\{f(x)\}^2 - 1\right]} dx - by(b) + af(a)$

C. $\int_a^b \frac{x}{3\left[\{f(x)\}^2 - 1\right]} dx - by(b) + af(a)$

D. $-\int_a^b \frac{x}{3\left[\{f(x)\}^2 - 1\right]} dx + by(b) = af(a)$

Answer: A

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12. Consider the function defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real-valued defferentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real-valued diferentiable function $y - g(x)$

satisfying $g_0 = 0$.

$\int_{-1}^{1} g'(x)dx$ is equal to

A. $2g(-1)$

B. 0

C. $-2g(1)$

D. $2g(1)$

Answer: D

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13. The area (in sqaure units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is

A. $\frac{5}{2}$

B. $\frac{59}{12}$

C. $\frac{3}{2}$

D. $\frac{7}{3}$

Answer: A

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14. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \leq 0, y \geq 0\}$ is

A. $\pi - \frac{4}{3}$

B. $\pi - \frac{8}{3}$

C. $\pi - \frac{4\sqrt{2}}{3}$

D. $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

Answer: B

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15. The area (in sq units) of the region described by $\{(x, y): y^2 \leq 2x \text{ and } y \geq 4x - 1\}$ is

A. $\frac{7}{32}$

B. $\frac{5}{64}$

C. $\frac{15}{64}$

D. $\frac{9}{32}$

Answer: D

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16. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latus rectum to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is (a) $\frac{27}{4}$ (b) 18 (c) $\frac{27}{2}$ (d) 27

A. $\frac{27}{4}$

B. 18

C. $\frac{27}{2}$

D. 27

Answer: D

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17. The area of the region described by $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is

A. $\frac{\pi}{2} + \frac{4}{3}$

B. $\frac{\pi}{2} - \frac{4}{3}$

C. $\frac{\pi}{2} - \frac{2}{3}$

D. $\frac{\pi}{2} + \frac{2}{3}$

Answer: A

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18. The area bounded by the curves $y = \sqrt{x}, 2y + 3 = x,$ and x-axis in the 1st quadrant is 18 sq. units (b) $\frac{27}{4}\mathrm{s}\,\mathrm{qu}\,\mathrm{n}\,\dot{\mathrm{i}}\,\mathrm{t}\,\mathrm{s}$ $\frac{4}{3}\mathrm{s}\,\mathrm{qu}\,\mathrm{n}\,\dot{\mathrm{i}}\,\mathrm{t}\,\mathrm{s}$ (d) 9 sq. units

A. 9

B. 36

C. 1

D. $\frac{27}{4}$

Answer: A

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19. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line y=2 is

A. $20\sqrt{2}$

B. $\frac{10\sqrt{2}}{3}$

C. $\frac{20\sqrt{2}}{3}$

D. $10\sqrt{2}$

Answer: C

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20. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x-axis is

A. 1 sq unit

B. $\frac{3}{2}$ sq units

C. $\frac{5}{2}$sq units

D. $\frac{1}{2}$ sq unit

Answer: B

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21. The area bounded by the curves y=cos x and y= sin x between the ordinates x=0 and $x = 3\pi / 2$ is

A. $(4\sqrt{2} - 2)$sq units

B. $(4\sqrt{2} + 2)$sq units

C. $(4\sqrt{2} - 1)$sq units

D. $(4\sqrt{2} + 1)$sq units

Answer: A

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22. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point $(2, 3)$ and the x-axis is

A. 6 sq units

B. 9 sq units

C. 12 sq units

D. 3 sq units

Answer: B

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23. The area of the plane region bounded by the curves $x+2y^2=0$ and $x+3y^2=1$ is equal to (1) $\frac{5}{3}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

A. $\frac{5}{3}$ sq units

B. $\frac{1}{3}$ sq unit

C. $\frac{2}{3}$ sq unit

D. $\frac{4}{3}$ sq units

Answer: D

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