MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

BIONOMIAL THEOREM

Examples

1. Expand $\left(2a-\frac{3}{b}\right)^{5}$ by binomial theorem

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2. Evaluate the following: $\left(x+\sqrt{x^{2}-1}\right)^{6}+\left(x-\sqrt{x^{2}-1}\right)^{6}$

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3. In the expansion of $(x+a)^n$ if the sum of odd terms is P and the sum of even terms is Q, tehn $P^2 - Q^2 = (x^2 - a^2)^n$

$4PQ = (x+a)^{2n} - (x-a)^{2n}$ $2(P^2 + Q^2) = (x+a)^{2n} + (x-a)^{2n}$

none of these

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4. Using binomial theorem, prove that $(101)^{50} > 100^{50} + 99^{50}$.

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5. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, find the value of $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$

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6. Find the number of dissimilar terms in

the expasion of $\left(1-3x+3x^2-x^3\right)^{33}$

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7. Find the sum of $\sum_{r=1}^{n} \frac{r^n C_r}{\hat{}\ nC_{r-1}}$.

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8. Prove that

$(C_0+C_1)(C_1+C_2)(C_2+C_3)(C_3+C_4)..........(C_{n-1}+C_n)$ = $\frac{C_0C_1C_2.....C_{n-1}(n+1)^n}{n!}$

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9.

$\sum_{r=0}^{n}(-1)^r\ \hat{}\ nC_r\left[\frac{1}{2^r}+\frac{3}{2^{2r}}+\frac{7}{2^{3r}}+\frac{15}{2^{4r}}+up\rightarrow mterms\right]=\frac{2^{mn}-}{2^{mn}(2^n-}$

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10. The seventh term in the expansion of $\left(4x - \frac{1}{2\sqrt{x}}\right)^{13}$ is

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11. Find the coefficient of x^8 in the expansion of $\left(x^2 - \frac{1}{x}\right)^{10}$

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12. Find the coefficient of x^7 in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$. (ii) the coefficient of x^{-7} in the expansion of $\left(ax + \frac{1}{bx^2}\right)^{11}$. Also , find the relation between a and b , so that these coefficients are equal .

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13. Find the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$.

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14. Write the 4th term from the end in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$.

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15. Find the (n+1)th term from the end in

the expansion of $\left(2x - \frac{1}{x}\right)^{3n}$

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16. Find the number of terms in the

expansion of $\left(\sqrt[2]{9} + \sqrt[2]{8}\right)^{500}$ which are integers .

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17. The sum of all rational terms in the expansion of $\left(3^{\frac{1}{5}}+2^{\frac{1}{3}}\right)^{15}$ is

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18. The number of irrational terms in the expansion of

$\left(\sqrt[8]{5}+\sqrt[6]{2}\right)^{100}$ is

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19. Let n be a positive integer . If the

cofficients of rth (r +1) th and (r +2)th terms in the expansion of $(1+x)^{n}$

are in AP, then find the relation between n and r .

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20. If a, b, c, d be four consecutive coefficients in the binomial expansion

of $(1+x)^{n}$, then value of the expression

$\left(\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right)$ (where $x > 0$ and $n \in N$) is

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21. If the 2nd, 3rd and 4th terms in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, find $x,\ a,\ n.$

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22. Find the middle term in the expansion of : $\left(\frac{a}{x} + \ bx\right)^{12}$

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23. Find the middle term (terms) in the expansion of

(i) $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$ (ii) $\left(3x - \frac{x^3}{6}\right)^9$

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24. Show that the middle term in the expansion of $(1+x)^{2n} is \frac{(1.\,3.\,5(2n-1))}{n!} 2^n x^n, where n$ is a positive integer.

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25. Find numerically greatest term in the expansion of $(2+3x)^9$, when x = 3/2.

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26. Find the numerically grates term in the expansion of $3-5x^{15} when x = 1/5.$

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27. Show that , if the greatest term in the expansion of $(1+x)^{2n}$ has also the greatest coefficient

then x lies between $\frac{n}{n+1}$ and $\frac{n+1}{n}$

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28. Find out the sum of the coefficients in the expansion of the binomial $(5p - 4q)^n$, where n is a +ive integer.

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29. In the expansion of $\left(3^{-x/4} + 3^{5x/4}\right)^{\pi}$ the sum of binomial coefficient is 64 and term with the greatest binomial coefficient exceeds the third by $(n-1)$, the value of x must be 0 b. 1 c. 2 d. 3

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30. Find the sum of $\dfrac{1}{1!(n-1)!} + \dfrac{1}{3!(n-3)!} + \dfrac{1}{5!(n-5)!} + \ldots,$

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31. Find the values of $\frac{1}{12!}+\frac{1}{10!2!}+\frac{1}{8!4!}+\ldots+\frac{1}{12!}$

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32. The sum of the coefficeints of the polynominal $\left(1+x-3x^2\right)^{2163}$ is

....

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33. If the sum of the coefficient in the expansion of

$\left(\alpha x^2-2x+1\right)^{35}$ is equal to the sum of the coefficient of the

expansion of $(x-\alpha y)^{35}$, then α =

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34. If $\left(1+x+2x^2\right)^{20}=a_0+a_1x+a_2x^2+\ldots+a_{40}x^{40}$.

The value of $a_0+a_2+a_4+\ldots+a_{38}$ is

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35. Show that the integral part of

$\left(5+2\sqrt{6}\right)^{n}$ is odd where n is natural number

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36. Show that the integral part of

$\left(5\sqrt{5}+11\right)^{2n+1}$ is even where $n \in N$.

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37. Let $\left(6\sqrt{6}+14\right)^{2n+1}=R$, if f be the fractional part of R, then prove that $Rf=20^{2n+1}$

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38. If $(7+4\sqrt{3})^n+5+t$, where n and s are positive integers and t is a proper fraction , show that (1 - t) (s + t) = 1

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39. If $x=(8+3\sqrt{7})^n$, where n is a natural number, power that the integral part of x is an odd integer and also show that $x-x^2+x[x]=1,$ where [.] denotes the greatest integer function .

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40. Show that

$1992^{1998}-1955^{1998}-1938^{1998}+1901^{1998}$ is divisible by 1998

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41. Prove that $2222^{5555} + 5555^{2222}$ is

divisible by 7 .

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42. If n is any positive integer , show that

$2^{3n+3} - 7n - 8$ is divisible by 49 .

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43. If 10^m divides the number $101^{100} - 1$ then, find the greatest value of

$m.$

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44. If 7^{103} is divided by 25 , find the remainder .

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45. Find the remainder when $x = 5^5 \hat{\ } 5 \hat{\dot{\ }} 5$ (24 times 5) is divided by 24.

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46. If 7 divides $32^{32^{32}}$, then find the remainder

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47. The last two digits of the number 3^{400} are:

(A) 81 (B) 43 (C) 29 (D) 01

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48. If the number is 17^{256}, find the last two digits

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49. If the number is 17^{256} , find the last digit

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50. Find (i) the last digit, (ii) the last two digits, and (iii) the last three digits of 17^{256}.

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51. Find the greater number is 100^{100} and (300)!.

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52. Find the greater number in 300 ! and

$$\sqrt{300^{\sqrt{300}}}$$

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53. If $(1+x)^n = C_0 + C_1x + C_2x^2$

$+C_3x^3 + C_4x^4 + \ldots,$ find the values of

(i) $C_0 - C_2 + C_4 - C_0 + \ldots$

(ii) $C_1 - C_3 + C_5 - C_7 + \ldots$

(iii) $C_0 + C_3 + C_6 + \ldots$

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54. Find the value of $\hat{} \ 4nC_0 +^{4n} C_4 +^{4n} C_8 + \ + {}^{4n}C_{4n}$.

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55. Find the coefficient of $a^2b^3c^4d$ in the expansion of $(a-b-c+d)^{10}$.

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56. Find the coefficient of $a^3b^4c^5$ in the

expasion of $(bc + ca + ab)^6$

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57. Find the totoal number of distnct or

dissimilar terms in the expansion of

$(x + y + z + w)^n, n \in N$

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58. Find the greatest coefficient in the expansion of $(a + b + c + d)^{15}$.

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59. Find the coefficient of x^7 in the expansion of $\left(1 + 3x - 2x^3\right)^{10}$.

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60. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, prove that

$C_1 + 2C_2 + 3C_3 + ... + nC_n = n \cdot 2^{n-1}$

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61. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that

$C_0 + 2C_1 + 3C_2 + \ldots + (n+1)C_n = (n+2)2^{n-1}$.

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62. If $(1+x)^n = c_0 + c_1x + c_2x^2 + ... + c_nx^n$ then the value of

$c_0 + 3c_1 + 5c_2 + ... + (2n+1)c_n$ is-

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63. If $(1+x)^n = C_0 + C_1x + C_2x^2$

$+ \ldots + C_nx^n, \quad \text{prove that} \quad 1^2 \cdot C_1 + 2^2 \cdot C_2 + 3^2 \cdot C_3 + \ldots + n^2 \cdot C_n$

$=n(n+1) \cdot 2^{n-2}$.

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64. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that

$(1 \cdot 2)C_2 + (2 \cdot 3)$

$C_3 + \ldots + \{(n-1) \cdot n\}C_n = n(n-1)2^{n-2}$.

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65. $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots + C_nx^n$, prove

that $C_0 - 2C_1 + 3C_2 - 4C_3 + \ldots + (-1)^n(n+1)C_n = 0$

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66. If

$(1+x)^n = c_0 + C_1 x + C_2 x^2 + \quad + C_n x^n, u \sin g derivtives prove that$

$C_1 - 2C_2 + 3C_3 + \quad + (-1)^{n-1} n C_n = 0$

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67. Prove that :$C_0 - 3C_1 + 5C_2 - \dots \dots \dots . (-1)^n (2n+1) C_n = 0$

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68. Prove that $.^n C_0 + \frac{.^n C_1}{2} + \frac{.^n C_2}{3} + \dots \dots + \frac{.^n C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}.$

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69. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n$, prove that

$C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \ldots + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}.$

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70. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n$ prove that

$$\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \ldots = \frac{2^n}{n+1}.$$

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71. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3$

$+ \ldots + C_n x^n$, prove that $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \ldots = \frac{2^{n+1}-1}{n+1}.$

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72. $3C_0 + 3^2 \frac{C_1}{2} + 3^3 \frac{C_2}{3} + \ldots\ldots\ldots\ldots 3^{n+1} \cdot \frac{C_n}{n+1}$ eqaul to

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73. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$,

Show that

$$\frac{2^2}{1 \cdot 2}C_0 + \frac{2^3}{2 \cdot 3}C_1 + \frac{2^4}{3 \cdot 4}C_2 + \dots + \frac{2^{n+2}C_n}{(n+1)(n+2)} = \frac{3^{n+2} - 2n - 5}{(n+1)(n+2)}$$

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74. Prove that

$$C_0C_r + C_1C_{r+1} + C_2C_{r+2} + \dots\dots\dots\dots\dots + c_{n-r}C_n = \frac{(2n)!}{(n-r)!(n+r)!}$$

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75. यदि $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots. + C_nx^n$. साबित कीजिए कि

$$C_0^2 + C_1^2 + C_2^2 + \dots\dots + C_n^2 = \frac{(2n)!}{n!n!} = \frac{1.3.5\dots(2n-1).2^n}{n!}$$

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76. Prove that

$$\left({}^{2n}C_0\right)^2 - \left({}^{2n}C_1\right)^2 + \left({}^{2n}C_2\right)^2 - \dots + \left({}^{2n}C_{2n}\right)^2 = (-1)^n \cdot {}^{2n}C_n.$$

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77. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that

$C_0^2 - C_1^2 + C_2^2 - \ldots + (-1)^n \cdot C_n^2 = 0$ or

$(-1)^{n/2} \cdot \dfrac{n!}{(n/2)!(n/2)!}$, according as n is odd or even

Also, evaluate $C_0^2 - C_1^2 + C_2^2 - \ldots + (-1)^n \cdot C_n^2$ for n

` = 10 and n= 11 .

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78. If m,n,r are positive integers such that r $<$ m,n, then

${}^mC_r + {}^mC_{r-1}{}^nC_1 + {}^mC_{r-2}{}^nC_2 + \ldots + {}^mC_1{}^nC_{r-1} + {}^nC_r$ equals

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79. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that

$C_0C_n - C_1C_{n-1} + C_2C_{n-2} - \ldots + (-1)^n C_nC_0 = 0$ or

$(-1)^{n/2} \dfrac{n!}{(n/2)!(n/2)!}$, according as n is odd or even .

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80. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that

$C_1 + 2C_2 + 3C_3 + \ldots + nC_n = n \cdot 2^{n-1}$

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81. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ then the value of $(C_0)^2 + \frac{(C_1)^2}{2} + \frac{(C_2)^2}{3} + \ldots + \frac{(C_n)^2}{n+1}$ is equal to

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82. Find the sum $\sum_{r=0}^{n} \hat{} \, (n+r)C_r$.

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83. Prove that $.^n C_0 .^{2n} C_n -^n C_1 .^{2n-2} C_n +^n C_2 .^{2n-4} C_n \equiv 2^n.$

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84. Prove that

$\hat{} \; nC_0^{2n}C_n -^n C_1^{2n-1}C_n +^n C_2 \times^{2n-2} C_n + \; + (-1)^n \; \hat{} \; nC_n^n C_n = 1.$

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85. If $C_0, C_1, C_2, \ldots, C_n$ denote the binomial coefficients in the expansion of $(1+x)^n$, then $\sum_{r=0}^{n} \sum_{s=0}^{n} (C_r + C_s)$

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86. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, find the values of the following . $\sum_{i=0}^{n} \sum_{j=0}^{n} (i+j) C_i C_j$

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87. If $(1+x)^n = C_0 = C_1x + C_2x^2 + \ldots + C_nx^n$,

find the values of the following

$$\sum_{0 \le i < j \le n}\sum jC_i$$

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88. If $(1+x)^n = C_0 = C_1x + C_2x^2 + \ldots + C_nx^n$, find the values of the following $\left(\sum\sum\right)_{0 \le i < j \le n} jC_i$

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89. If $(1+x)^n = C_0 = C_1x + C_2x^2 + \ldots + C_nx^n$,

find the values of the following

$$\sum_{0 \le i < j \le n}\sum jC_i$$

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90. If $(1+x)^n = C_0 = C_1x + C_2x^2 + \ldots + C_nx^n$,

find the values of the following

$$\sum_{0 \le i < j \le n}\sum (i.\,j)C_iC_j$$

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91. If $(1+x)^n = C_0 = C_1x + C_2x^2 + \ldots + C_nx^n$,

find the values of the following

$$\sum_{0 \le i \le j \le n}\sum (i+j)(C_i \pm C_j)^2$$

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92. If $(1+x)^n = C_0 = C_1x + C_2x^2 + \ldots + C_nx^n$,

find the values of the following

$$\left(\sum\sum\right)_{0 \le i \le j \le n}(i+j)(C_i \pm C_j)^2$$

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93. If $(1+x)^n = C_0 = C_1x + C_2x^2 + \ldots + C_nx^n$,

find the values of the following

$$\sum_{0 \le i \le j \le n} \sum C_iC_j$$

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94. If $\binom{2n+1}{0} + \binom{2n+1}{3} + \binom{2n+1}{6} + \ldots = 170$, then n equals

A. 2

B. 4

C. 6

D. 8

Answer: b

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95.

$\left({}^{m}C_0 + {}^{m}C_1 - {}^{m}C_2 - {}^{m}C_3\right) + \left({}^{m}C_4 + {}^{m}C_5 - {}^{m}C_6 - {}^{m}C_7\right) + \dots = 0$

if and only if for some positive integer $k, m =$ (a) 4k (b) 4k+1 (c) 4k-1 (d) 4k+2

A. 4k

B. $4k + 1$

C. $4k - 1$

D. $4k + 2$

Answer: c

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96. The coefficient of x^n in $(1 + x)^{101}(1 - x + x^2)^{100}$ is non zero, then n cannot be of the form a. $3r + 1$ b. $3r$ c. $3r + 2$ d. none of these

A. $3\lambda + 1$

B. 3λ

C. $3\lambda + 2$

D. $4\lambda + 1$

Answer: c

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97. The sum $\sum_{i=0}^{m}((10)c, (i))\binom{20}{m-1}$, where $\binom{p}{q} = 0$ if $p < q$, is maximum when m is equal to (A) 5 (B) 10 (C) 15 (D) 20

A. 5

B. 10

C. 15

D. 20

Answer: c

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98. If $\hat{} n - 1C_r = (k^2 - 3)^n C_{r+1}, then k \in (-\infty, -2]$ b. $[2, \infty)$ c. $[-\sqrt{3}, \sqrt{3}]$ d. $(\sqrt{3}, 2]$

A. $(-\infty, -2]$

B. $[2, \infty)$

C. $[-\sqrt{3}, \sqrt{3}]$

D. $(\sqrt{3}, 2]$

Answer: d

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99. If

$$\left(x + \frac{1}{x} + 1\right)^6 = a_0 + \left(a_1 x + \frac{b_i}{x}\right) + \left(a_2 x^2 + \frac{b_2}{x^2}\right) + \dots + \left(a_6 x^6 + \frac{b_6}{x^6}\right)$$

, then

A. 121

B. 131

C. 141

D. 151

Answer: c

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100. The coefficient of x^{50} in the series

$$\sum_{r=1}^{101} rx^{r-1}(1+x)^{101-r} \text{is}$$

A. ${}^{100}C_{50}$

B. ${}^{101}C_{50}$

C. ${}^{102}C_{50}$

D. ${}^{103}C_{50}$

Answer: b

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101. The largest integer λ such that 2^{λ} divides

$3^{2^n} - 1, n \in N$ is

A. $n - 1$

B. n

C. $n + 1$

D. $n + 2$

Answer:

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102. If the last tem in the binomial expansion of $\left(2^{\frac{1}{3}} - \frac{1}{\sqrt{2}}\right)^n is \left(\frac{1}{3^{\frac{5}{3}}}\right)^{\log_3 8}$, then 5th term from the beginning is 210 b. 420 c. 105 d. none of these

A. ${}^{10}C_6$

B. $2^{10}C_4$

C. $\frac{1}{2} \cdot {}^{10} C_4$

D. None of the above

Answer:

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103. If $f(x) = \sum_{r=1}^{n} \{ r^2 ({}^nC_r - {}^nC_{r-1}) + (2r+1) {}^nC_r \}$

and $f(30) = 30(2)^{\lambda}$, then the value of λ is

A. 3

B. 4

C. 5

D. 6

Answer:

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104. Let $a_n = \left(1 + \frac{1}{n}\right)^n$. Then for each $n \in N$

A. $a_n \geq 2$

B. $a_n < 3$

C. $a_n < 4$

D. $a_n < 2$

Answer: a, b, c

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105. Prove that $\sum_{r=0}^{n} \hat{} \, nC_r \sin rx \cos(n-r)x = 2^{n-1} \sin(nx)$.

A. $S_5\left(\frac{\pi}{2}\right) = 16$

B. $S_7\left(\frac{-\pi}{2}\right) = 64$

C. $S_{50}(\pi) = 0$

D. $S_{51}(-\pi) = -2^{50}$

Answer: a, b, c

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106. $If a + b = k,$ when $a, b > o$ and

$$S(k, n) = \sum_{r=0}^{n} r^2 ({}^nC_r) a^r \cdot b^{n-r}$$, then

A. $S(1, 3) = 3(3a^2 + ab)$

B. $S(2, 4) = 16(4a^2 + ab)$

C. $S(3, 5) = 25(5a^2 + ab)$

D. $S(4, 6) = 36(6a^2 + ab)$

Answer: a, b

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107. The value of x, for which the ninth term in the expansion of $\left\{\frac{\sqrt{10}}{\left(\sqrt{x}\right)^{5\log_{10}x}} + x \cdot x^{\frac{1}{2\log_{10}x}}\right\}^{10}$ is 450 is equal to

A. 10

B. 10^2

C. $\sqrt{10}$

D. $10^{-2/5}$

Answer: b, d

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108. For a positive integer n, if the expanison of $\left(\frac{5}{x^2} + x^4\right)$ has a term independent of x, then n can be

A. 18

B. 27

C. 36

D. 45

Answer: a, b, c, d

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109. Consider $\left(1+x+x^2\right)^n=\sum_{r=0}^{n}a_rx^r$, where $a_0, a_1, a_2, \ldots, a_{2n}$ are real number and n is positive integer.

If n is even, the value of $\sum_{r=0}^{n/2-1}a_{2r}$ is

A. $\dfrac{9^n-2a_{2n}-1}{4}$

B. $\dfrac{9^n-2a_{2n}+1}{4}$

C. $\dfrac{9^n+2a_{2n}-1}{4}$

D. $\dfrac{9^n+2a_{2n}+1}{4}$

Answer: b

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110. Consider $\left(1+x+x^2\right)^n = \sum_{r=0}^{n} a_r x^r$, where $a_0, a_1, a_2, \ldots, a_{2n}$ are real number and n is positive integer.

If n is odd , the value of $\sum_{r-1}^{2} a_{2r-1}$ is

A. $\dfrac{9^n - 1}{2}$

B. $\dfrac{9^n - 1}{4}$

C. $\dfrac{9^n + 1}{2}$

D. $\dfrac{9^n + 1}{4}$

Answer: b

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111. Consider $\left(1+x+x^2\right)^n = \sum_{r=0}^{2n} a_r x^r$, where $a_0, a_1,$ $a_2, \ldots a_{2n}$ are real numbers and n is a positive integer.

The value of a_2 is

A. ${}^{4n+1}C_2$

B. ${}^{3n+1}C_2$

C. ${}^{2n+1}C_2$

D. ${}^{n+1}C_2$

Answer: c

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112. Let $S=\sum_{r=1}^{30}\frac{{}^{30+r}C_r(2r-1)}{{}^{30}C_r(30+r)}, K=\sum_{r=0}^{30}\left({}^{30}C_r\right)^2$

and $G=\sum_{r=0}^{60}(-1)^r\left({}^{60}C_r\right)^2$

The value fo (G-S)is

A. 0

B. 1

C. 2^{30}

D. 2^{60}

Answer: b

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113. Let $S=\sum_{r=1}^{30}\frac{{}^{30+r}C_r(2r-1)}{{}^{30}C_r(30+r)}$, $K=\sum_{r=0}^{30}\left({}^{30}C_r\right)^2$

and $G=\sum_{r=0}^{60}(-1)^r\left({}^{60}C_r\right)^2$

The value (SK_SG) is

A. 0

B. 1

C. 2^{30}

D. 2^{60}

Answer: a

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114. Let $S = \sum_{r=1}^{30} \frac{{}^{30+r}C_r(2r-1)}{{}^{30}C_r(30+r)}, K = \sum_{r=0}^{30} \left({}^{30}C_r\right)^2$

and $G = \sum_{r=0}^{60} (-1)^r \left({}^{60}C_r\right)^2$

The value of K + G is

A. 2 S - 2

B. 2 S - 1

C. 2 S + 1

D. 2 S + 2

Answer: d

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115. The digit at units place in 2^9 ^ 100 is (A) 2 (B) 4 (C) 6 (D) 8

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116. If $(+x)^n = \sum_{r=0}^{n} a_r x^r \& b_r = 1 + \frac{a_r}{a_{r-1}} \& \prod_{r=1}^{n} b_r = \frac{(101)^{100}}{100!}$, then

equals to: 99 (b) 100 (c) 101 (d) None of these

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117. Statement-1 (Assertion) and Statement-2 (Reason)

Each of the these examples also has four laternative choices ,

only one of which is the correct answer. You have to select the correct

choice as given below .

$(7^9 + 9^7)$ is divisible by 16

Statement-2 $(x^y + y^x)$ is divisible by $(x + y)$, $\forall x, y$.

A. Statement-1 is ture ,Statement-2 is treu, Statement-2 is a correct explanation for Statement-1

B. Statement-1 is ture ,Statement-2 is treu, Statement-2 is not a correct explanation for Statement-1

C. Statement-1 is true ,Statement-2 is false

D. Statement-1 is true ,Statement-2 is ture

Answer: c

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118. Statement-1 (Assertion) and Statement-2 (Reason)

Each of the these examples also has four laternative choices , only one of which is the correct answer. You have to select the correct choice as given below .

Number of distincet terms in the sum of expansion $(1+ax)^{10}+(1-ax)^{10}$ is 22.

A. Statement-1 is ture ,Statement-2 is treu, Statement-2 is a correct explanation for Statement-1

B. Statement-1 is ture ,Statement-2 is treu, Statement-2 is not a correct explanation for Statement-1

C. Statement-1 is true ,Statement-2 is false

D. Statement-1 is true ,Statement-2 is ture

Answer: d

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119. Find the term independent of x in the expansion of $(1 + x + 2x^3)\left[(3x^2/2) - (1/3)\right]^9$

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120. $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$,

show that $\sum_{r=0}^{n} C_r^3$ is equal to the coefficient of $x^n y^n$ in the expansion of $\{(1+x)(1+y)(x+y)\}^n$.

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121. Let $\left(1+x^2\right)^2(1+x)^n=\sum_{k=0}^{n+4} a_k x^k$. If a_1, a_2 and a_3 are in arithmetic progression, then the possible value/values of n is/are a. 5 b. 4 c. 3 d. 2

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122. if $\left(1-x^3\right)^n=\sum_{r=0}^{n} a_r x^r(1-x)^{3n-2r}, where n \varepsilon N$ then find a_r.

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123. If $a_0, a_1, a_2, \ldots, a_{2n}$ are the coefficients in the expansion of $\left(1+xx^2\right)^n$ in ascending of x show that $a_0^2-a_1^2-a_2^2-\ldots+a_{2n}^2=a_n$.

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124. Show that no three consecutive binomial coefficients can be in (i) G.P., (ii) H.P.

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125. Show that no three consecutive binomial coefficients can be in GP .

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126. Evaluate $\sum_{i=0}^{n} \sum_{j=0}^{n} {}^{n}C_j \cdot {}^{j}C_i, i \leq j$.

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127. Find the remainder when 27^{40} is divided by 12.

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128. show that $\left[(\sqrt{3}+1)^{2n}\right]+1$ is divisible by 2^{n+1} $\forall n \in N$, where [.] denote the greatest integer function .

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129. Find number of rational terms in $\left(\sqrt{2}+3^{\frac{1}{3}}+5^{\frac{1}{6}}\right)^{10}$

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130. Find the remainder when $1690^{2608}+2608^{1690}$ is divided by 7.

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131. If $C_0, C_1, C_2, \ldots, C_n$ are the binomial coefficients

in the expansion of $(1+x)^n$, prove that

$(C_0+2C_1+C_2)(C_1+2C_2+C_3)\ldots(C_{n-1}+2C_n+C_{n+1})$

$\dfrac{(n-2)^n}{(n+1)!}\prod_{r=1}^{n}(C_{r-1}+C_r)$.

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132. If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r and a_k = 1$ for all $k \geq n,$ then show that $b_n = ^{2n+1} C_{n+1}$.

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133. If n is an odd natural number, then $\sum_{r=0}^{n} \frac{(-1)^r}{nC_r}$ is equal to

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134. If n is an even natural number , then $\sum_{r=0}^{n} \frac{(-1)^r}{^nC_r}$ equals

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135. If $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots + C_nx^n$, show that

$$C_1 - \frac{C_2}{2} + \frac{C_3}{3} - \ldots(-1)^{n-1}\frac{C_n}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}.$$

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136. If $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots + C_nx^n$, find the sum of the series

$$\frac{C_0}{2} - \frac{C_1}{6} + \frac{C_2}{10} + \frac{C_3}{14} - \ldots + (-1)^n \frac{C_n}{4n+2}.$$

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137. If $(1+x)^n = \sum_{r=0}^{n} C_r, x^r$, then prove that

$$\left(\sum\sum\right)_{0 \le i < j \le n} \left(\frac{i}{C_i} + \frac{j}{C_j}\right) = \frac{n^2}{2} \sum_{r=0}^{n} \frac{1}{C_r}.$$

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138. If $(1+x)^n = C_0 + C_1x + C_2x^2 + c_3x^3 + \ldots + C_nx^n$, show that

$$\sum_{r=0}^{n} \frac{C_r 3^{r+4}}{(r+1)(r+2)(r+3)(r+4)} =$$

$$\frac{1}{(n+1)(n+2)(n+3)(n+4)} \left(4^{n+4} - \sum_{t=0}^{3} {}^{n+4}C_t\right).$$

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139. Prove that $\sum_{k=0}^{9} x^k \text{divides} \sum_{k=0}^{9} x^{kkkk}$

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140. Prove that $\sum_{r=1}^{k} (-3)^{r-1} . {}^{3n} C_{2r-1} = 0$, where $k = 3n/2$ and n is an even integer.

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141. Prove that

$${}^nC_3 + {}^nC_7 + {}^nC_{11} + \ldots = \frac{1}{2}\left\{2^{n-1} - 2^{n/2} \sin \frac{n\pi}{4}\right\}$$

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142. Evaluate $\sum_{i=0}^{n} \sum_{j=0}^{n} {}^nC_j \cdot {}^jC_i, i \leq j$.

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143. If $\left(9+4\sqrt{5}\right)^{n}=I+f$, n and I being positive integers and f is a proper fraction, show that $(I-1)f+f^2$ is an even integer.

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144. If P_r is the coefficient of $x^{®}$ in the expansion of

$(1+x)^2\left(1+\frac{x}{2}\right)^2\left(1+\frac{x}{2^2}\right)^2\left(1+\frac{x}{2^3}\right)^2$... prove that

$$P_r=\frac{2^2}{(2^r-1)}(P_{r-1}+P_{r-2}) \text{ and } P_4=\frac{1072}{315}.$$

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Jee Type Solved Example Matching Type Questions

1.

	Column I		Column II
(A)	If m and n are the numbers of rational terms in the expansions of $(\sqrt{2}+3^{1/5})^{10}$ and $(\sqrt{3}+5^{1/8})^{256}$ respectively, then	(p)	$n-m=6$
(B)	If m and n are the numbers of irrational terms in the expansions of $(2^{1/2}+3^{1/5})^{40}$ and $(5^{1/10}+2^{1/6})^{100}$ respectively, then	(q)	$m+n=20$
(C)	If m and n are the numbers of rational terms in the expansions of $(1+\sqrt{2}+3^{1/3})^{6}$ and $(1+\sqrt[3]{2}+\sqrt[5]{3})^{15}$ respectively, then	(r)	$n-m=31$
		(s)	$m+n=35$
		(t)	$n-m=39$

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2. Match the following Column I to Column II

	Column I		Column II
(A)	If $S=\sum_{r=0}^{n}\lambda\, C_r$ and values of S are a, b, c for $\lambda=1, r, r^2$ respectively, then	(p)	$a=b+c$
(B)	If $S=\sum_{r=0}^{n}(-1)^r\ \lambda\, C_r$ and values of S are a, b, c for $\lambda=1, r, r^2$ respectively, then	(q)	$a+b=c+2$
(C)	If $S=\sum_{r=0}^{n}\frac{\lambda\, C_r}{(r+1)}$ and values of S are a, b, c for $\lambda=1, r, r^2$ respectively, then	(r)	$a^3+b^3+c^3=3abc$
		(s)	$b^{c-a}+(c-a)^b=1$
		(t)	$a+c=4b$

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Exercise For Session 1

1. The value of $\sum_{r=0}^{10} r^{10}C_r 3^r(-2)^{10-r}$ is 20 b. 10 c. 300 d. 30

A. 10

B. 20

C. 30

D. 300

Answer: c

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2. The expression $\left[x+\left(x^3-1\right)^{\frac{1}{2}}\right]^5+\left[x-\left(x^3-1\right)^{\frac{1}{2}}\right]^5$ is a polynomial of degree

A. 5

B. 6

C. 6

D. 8

Answer: c

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3. $\left(\sqrt{2}+1\right)^6-\left(\sqrt{2}-1\right)^6$ is equal to

A. 101

B. $70\sqrt{2}$

C. $140\sqrt{2}$

D. $120\sqrt{2}$

Answer: c

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4. about to only mathematics

A. 202

B. 51

C. 50

D. 101

Answer: b

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5. Find the number of nonzero terms in the expansion of $\left(1+3\sqrt{2}x\right)^9+\left(1-3\sqrt{2}x\right)^9$.

A. 0

B. 5

C. 9

D. 10

Answer: b

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6. If $(1+x)^n = \sum_{r=0}^{n} C_r x^r, \left(1+\frac{C_0}{C_1}\right)\left(1+\frac{C_2}{C_1}\right)...\left(1+\frac{C_n}{C_{n-1}}\right)$ is equal to

A. $\frac{n^{n-1}}{(n-1)!}$

B. $\frac{(n+1)^{n-1}}{(n-1)!}$

C. $\frac{(n+1)^n}{n!}$

D. $\frac{(n+1)^{n+1}}{n!}$

Answer: c

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7. If $.^{n+1}C_{r+1}:^{n}C_{r}:^{n-1}C_{r-1}=11:6:3$, find the values of n and r.

A. 20

B. 30

C. 0

D. 50

Answer: c

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Exercise For Session 2

1. about to only mathematics

A. 7

B. 9

C. 11

D. 13

Answer: b

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2. In $\left(33+\frac{1}{33}\right)^{n}$ if the ratio of 7th term from the beginning to the 7th term from the end is 1/6, then find the value of n.

A. 3

B. 5

C. 7

D. 9

Answer: d

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3. Find the number of integral terms in the expansion of $\left(5^{\frac{1}{2}}+7^{\frac{1}{8}}\right)^{1024}$.

A. 128

B. 129

C. 130

D. 131

Answer: b

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4. If the coefficients of three consecutive terms in the expansion of $(1+x)^{n}$ are 165,330 and 462 respectively , the value of n is is

A. 7

B. 9

C. 11

D. 13

Answer: c

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5. If the coefficients of 5th, 6th , and 7th terms in the expansion of $(1+x)^n$ are in A.P., then $n=$ a. 7 only b. 14 only c. 7 or 14 d. none of these

A. 7only

B. 14 only

C. 7 or 14

D. None of these

Answer: c

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6. If the middle term in the expansion $\left(x^2 + 1/x\right)^n$ is $924x^6$, then find the value of n.

A. 8

B. 12

C. 16

D. 20

Answer: b

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7. In the expansion of $(1 + x)\left(1 + x + x^2\right)\ldots\left(1 + x + x^2 + \ldots + x^{2n}\right)$, the sum of the coefficients is

A. 1

B. 2n!

C. 2n!+1

D. $(2n + 1)!$

Answer: d

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Exercise For Session 3

1. If $R = (7 + 4\sqrt{3})^{2n} = 1 + f$, where I $\in$ N and $0 < f < 1$, then R (1 - f) equals

A. (a)1

B. (b)0

C. (c)-1

D. (d)even integer

Answer: a

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2. Let $(5+2\sqrt{6})^n = I + f$, where n, $I \in N$ and $0 < f < 1$, then the value of $f^2 - f + I \cdot f - I$. Is

A. $\frac{1}{f} - f$

B. $\frac{1}{1+f} - f$

C. $\frac{1}{1-f} - f$

D. $\frac{1}{1+f} + f$

Answer: c

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3. If $n > 0$ is an odd integer and $x = (\sqrt{2}+1)^n,\ f = x - [x],\ \text{ then} \frac{1-f^2}{f}$ is

A. an irrational number

B. a non-integer rational number

C. an odd number

D. an even number

Answer: d

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4. Integral part of $(\sqrt{2}+1)^6$ is

A. (a)196

B. (b)197

C. (c)198

D. (d)199

Answer: b

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5. $(103)^{86}-(86)^{103}$ is divisible by

A. 7

B. 13

C. 17

D. 23

Answer: c

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6. fractional part of $\frac{2^{78}}{31}$ is:

A. $\frac{2}{31}$

B. $\frac{4}{31}$

C. (8)/(31)`

D. (16)/(31)`

Answer: c

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7. The unit digit of $17^{1983} + 11^{1983}$ is

A. 4

B. 2

C. 3

D. 0

Answer: a

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8. The last two digits of the number $(23)^{14}$ are 01 b. 03 c. 09 d. none of these

A. 1

B. 3

C. 9

D. 27

Answer: c

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9. The last four digits of the natural number 3^{100} are

A. 2001

B. 3211

C. 1231

D. 1

Answer: a

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10. The remainder when 23^{23} is divided by 53 is

A. 17

B. 21

C. 30

D. 47

Answer: c

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Exercise For Session 4

1. The coefficient of $a^8b^4c^9d^9$ in $(abc + abd + acdd + bcd)^{10}$ is $10!$ b. $\frac{10!}{8!4!9!9!}$ c. 2520 d. none of these

A. $10!$

B. $\frac{10!}{4!8!9!9!}$

C. 2520

D. Nono of these

Answer: c

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2. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + \ + a_{20}x^{20}, then a_1$ equals

10 b. 20 c. 210 d. none of these

A. 210

B. 20

C. 10

D. None of these

Answer: b

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3. If $(1 + x + x^2 + x^3)^5 = a_0 + a_1x + a_2x^2 + \ldots . + a_{15}x^{15}$, then a_{10} equals to

A. 99

B. 100

C. 101

D. 110

Answer: c

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4. Coefficient of x^{15} in $\left(1+x+x^3+x^4\right)^n$ is

A. A. $\sum_{r=0}^{5} {}^nC_{5-r} \cdot {}^nC_{3r}$

B. B. $\sum_{r=0}^{5} {}^nC_{5r}$

C. C. $\sum_{r=0}^{5} {}^nC_{2r}$

D. D. $\sum_{r=0}^{5} {}^nC_{3-r} \cdot {}^nC_{5r}$

Answer: a

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5. In the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n, n \in N,$

A. ${}^{n+2}C_2$

B. ${}^{n+3}C_2$

C. ${}^{2n+1}C_{2n}$

D. ${}^{3n+1}C_{3n}$

Answer: a

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6. If $(1+x)^{10} = a_0 + a_1x + a_2x^2 + \, + a_{10}x^{10}$, then value of $(a_0 - a_2 + a_4 - a_6 + a_8 - a_{10})^2 + (a_1 - a_3 + a_5 - a_7 + a_9)^2$ is

A. 2^9

B. 3^9

C. 2^{10}

D. 3^{10}

Answer: c

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7. If $(1+x)^n = C_0 + C_1 x C_2 x^2 + \ldots + C_n x^n$, then the sum $C_0 + (C_0 + C_1) + \ldots + (C_0 + C_1 + \ldots + C_{n-1})$ is equal to

A. $n.\ 2^n$

B. $n.\ 2^{n-1}$

C. $n.\ 2^{n-2}$

D. $n.\ 2^{n-3}$

Answer: b

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8. $\frac{C_0}{1.3}-\frac{C_1}{2.3}+\frac{C_2}{3.3}-\frac{C_3}{4.3}+\ldots\ldots+(-1)^n\frac{C_n}{(n+1)\cdot 3}$ is

A. $\frac{3}{n+1}$

B. $\frac{n+1}{3}$

C. $\frac{1}{3(n+1)}$

D. None of these

Answer: c

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9. The value of $\binom{50}{0}\binom{50}{1}+\binom{50}{1}\binom{50}{2}+\ldots\ldots\ldots\ldots+\binom{50}{49}\binom{50}{50}$ is

A. $\binom{100}{50}$

B. $\binom{100}{51}$

C. $\binom{50}{25}$

D. $\binom{50}{25}^2$

Answer: b

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10. If $c_r = nc_r$ then $\frac{C_1}{2} - \frac{C_2}{3} + \frac{C_3}{4} - \dots\dots\dots . - \frac{C_{100}}{101}$ is equal to

A. C_1

B. C_2

C. C_3

D. C_4

Answer: b

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11. The sum $\sum_{r=0}^{n} (r+1)(C_r)^2$ is equal to :

A. $\frac{(n+2)(2n-1)!}{n!(n-1)!}$

B. $\dfrac{(n+2)(2n+1)!}{n!(n-1)!}$

C. $\dfrac{(n+2)(2n+1)!}{n!(n+1)!}$

D. $\dfrac{(n+2)(2n-1)!}{n!(n+1)!}$

Answer: a

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12. $\sum_{r=1}^{n}\left\{\sum_{r1=0}^{r-1} {}^{n}C_{r}\,{}^{r}C_{r_1} 2^{r1}\right\}$ is equal to

A. $4^n - 3^n + 1$

B. $4^n - 3^n - 1$

C. $4^n - 3^n + 2$

D. $4^n - 3^n$

Answer: d

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13. The value of the expression $\left(\sum_{r=0}^{10} {}^{10}C_r\right)\left(\sum_{k=0}^{10}(-1)^k \frac{\hat{\ } 10C_k}{2^k}\right)$ is :

A. 1

B. 2^5

C. 2^{10}

D. 2^{20}

Answer: a

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14. The value of $\left(\sum\sum\sum\sum\right)_{0\le i<j<k<l\le n} 2$ is equal to

A. $2(n+1)^3$

B. $2\cdot {}^{n+1}C_4$

C. $2(n+1)^4$

D. $2\cdot {}^{n+2}C_3$

Answer: b

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Exercise Single Option Correct Type Questions

1.

$$\sum_{r=0}^{n}(-1)^{r}\ \hat{}\ nC_{r}\left[\frac{1}{2^{r}}+\frac{3}{2^{2r}}+\frac{7}{2^{3r}}+\frac{15}{2^{4r}}+up \rightarrow mterms\right]=\frac{2^{mn}-}{2^{mn}(2^{n}-}$$

A. -6

B. -3

C. 3

D. Cannot be determined

Answer: d

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2. The coefficient of $\left(x^3 \cdot b^6 \cdot C^8 \cdot d^9 \cdot e \cdot f\right)$ in the expansion of $(a+b+c-d-e-f)^{31}$ is

A. 12632

B. 23110

C. 3110

D. None of these

Answer: d

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3. Find the number of rational terms and also find the sum of rational terms in $\left(\sqrt{2}+\sqrt[3]{3}+\sqrt[6]{5}\right)^{10}$

A. 12632

B. 1260

C. 126

D. None of these

Answer: a

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4. If $(1 + x - 3x^2)^{2145} = a_0 + a_1x + a_2x^2 + \ldots$ then $a_0 - a_1 + a_2 - \ldots$ ends with

A. 1

B. 3

C. 7

D. 9

Answer: b

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5. In the expansion of $\left(\sqrt{\frac{q}{p}}+\sqrt[10]{\frac{p^7}{q^3}}\right)^n$, there is a term similar to pq , then that term is equl to

A. 45pq

B. 120 pq

C. 210 pq

D. 252 pq

Answer: d

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6. If $\left(5+2\sqrt{6}\right)^n=I+f$, where $I\in N, n\in$ N and $0\leq f\leq 1,$ then I equals

A. a natural number

B. a negateive integer

C. a prime number

D. an irrational number

Answer: b

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7. If $x+\frac{1}{x}=1$ and $p=x^{4000}+\frac{1}{x^{4000}}$ and q is the digit at unit place in the number $2^{2^n}+1, n \in N abd n>1$, then p + q is .

A. 8

B. 6

C. 7

D. None of these

Answer: b

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8. If the number of terms in $\left(x+1+\frac{1}{x}\right)^{n}(n \in I^{+}$ is $401,$

then n is greater then

A. 201

B. 200

C. 199

D. None of these

Answer: d

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9. The vaule of $\sum_{r=0}^{n-1}\left(\frac{C_r}{{}^{n}C_{r}+{}^{n}C_{r+1}}\right.$ is equal to

A. $\frac{n}{2}$

B. $\frac{n+1}{2}$

C. $\frac{n(n+1)}{2}$

D. $\frac{n(n-1)}{2(n+1)}$

Answer: a

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10. The largest term in the expansion of $\left(\frac{b}{2}+\frac{b}{2}\right)^{100}$ is

A. b^{100}

B. $\left(\frac{b}{2}\right)^{100}$

C. ${}^{100}C_{50}\left(\frac{b}{2}\right)^{100}$

D. ${}^{100}C_{50}b^{100}$

Answer: c

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11. If the fourth term in the expansion of $\left\{\sqrt{\frac{1}{\text{x^log(x+1)}}}' + \frac{1}{x^{12}}\right.^{6} isequal \rightarrow$ 200 and x >1,` then find x

A. $10\sqrt{2}$

B. 10

C. 10^4

D. $\frac{10}{\sqrt{2}}$

Answer: b

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12. The coefficient of x^m in $(1+x)^m + (1+m)^{m+1} + \dots + (1+x)^n, m \leq n$ is

A. ${}^{n+1}C_{m+1}$

B. ${}^{n-1}C_{m-1}$

C. ${}^{n}C_{m}$

D. ${}^{n}C_{m+1}$

Answer: a

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13. The number of values of 'r' satisfying the equation

${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is

A. 1

B. 2

C. 3

D. 4

Answer: b

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14. The sum S = ${}^{20}C_2 + 2 \cdot {}^{20}C_3 + 3 \cdot {}^{20}C_4 + \ldots + 19 \cdot {}^{20}C_{20}$ is equal to

A. $1 + 5 \cdot 2^{20}$

B. $1 + 2^{21}$

C. $1 + 9 \cdot 2^{20}$

D. 2^{20}

Answer: c

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15. The remainder, if $1 + 2 + 2^2 + \quad + 2^{1999}$ is divided by 5 is.

A. 0

B. 1

C. 2

D. 3

Answer: a

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16. The coefficient of $1/x$ in the expansion of $(1+x)^n(1+1/x)^n$ is (a). $\frac{n!}{(n-1)!(n+1)!}$ (b). $\frac{(2n)!}{(n-1)!(n+1)!}$ (c). $\frac{(2n)!}{(2n-1)!(2n+1)!}$ (d). none of these

A. $\frac{n!}{(n-1)!(n+1)!}$

B. $\frac{2n!}{(n-1)!(n+1)!}$

C. $\frac{n!}{(2n-1)!(1n+1)!}$

D. $\frac{2n!}{(2n-1)!(1n+1)!}$

Answer: b

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17. The last two digits of the number 19^{9^4} is

A. 19

B. 29

C. 39

D. 81

Answer: a

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18. If the second term of the expansion $\left[a^{\frac{1}{13}}+\frac{a}{\sqrt{a^{-1}}}\right]^{n}$ is $14a^{5/2}$, then the value of $\frac{\hat{\ } nC_3}{\hat{\ } nC_2}$ is.

A. 19

B. 29

C. 39

D. 81

Answer: a

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19. If $6^{83}+8^{83}$ is divided by 49 , the raminder is

A. 0

B. 14

C. 35

D. 42

Answer: c

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20. The sum of all rational terms in the expansion of

$\left(3^{1/4}+4^{1/3}\right)^{12}$ is

A. 91

B. 251

C. 273

D. 283

Answer: d

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21. Sum of last three digits of the number $N = 7^{100} - 3^{100}$ is.

A. 2000

B. 4000

C. 6000

D. 8000

Answer: d

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22. If 5^{99} is divided by 13, the remainder is

A. 2

B. 4

C. 6

D. 8

Answer: d

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23. Find the value of $\{3^{2003}/28\}, where\{.\}$ denotes the fractional part.

A. $17/28$

B. 19/28

C. 23/28

D. $2/28$

Answer: b

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24. The value of $\sum_{r=0}^{20} r(20-r)(\hat{\ }(20)C_r)^2$ is equal to $400^{39}C_{20}$ b. $400^{40}C_{19}$ c. $400^{39}C_{19}$ d. $400^{38}C_{20}$

A. $400^{37}C_{20}$

B. $400^{40}C_{19}$

C. $400^{38}C_{19}$

D. $400^{38}C_{20}$

Answer: d

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25. If $(3+x^{2008}+x^{2009})^{2010}=a_0+a_1x+a_2x^2++a_nx^n$, then the value of $a_0-\frac{1}{2}a_1-\frac{1}{2}a_2+a_3-\frac{1}{2}a_4-\frac{1}{2}a_5+a_6-$ is a.3^{2010} b. 1 c. 2^{2010} d. none of these

A. 1

B. 2^{2010}

C. 5^{2010}

D. 3^{2010}

Answer: b

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26. The total number of terms which are dependent on the value of x in the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^n$ is equal to $2n + 1$ b. $2n$ c. n d. $n + 1$

A. $2n + 1$

B. 2n

C. $n + 1$

D. n

Answer: b

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27. The coefficient of x^{10} in the expansion of $\left(1+x^2-x^3\right)^8$ is 476 b. 496

c. 506 d. 528

A. 420

B. 476

C. 532

D. 588

Answer: b

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28. The number of real negative terms in the binomial expansion of $(1+ix)^{4n-2}, n \in N, x > 0$ is n b. $n+1$ c. $n-1$ d. $2n$

A. n

B. $n+1$

C. $n-1$

D. 2n

Answer: a

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29. $\sum_{p=1}^{n} \sum_{m=p}^{n} \binom{n}{m} \binom{m}{p}$ is equal to

A. (a)3^n

B. (b)2^n

C. (c)$3^2 + 2^n$

D. (d)$3^n - 2^n$

Answer: d

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30. The largest real value of x, such that

$$\sum_{r=0}^{4}\left(\frac{5^{4-r}}{(4-r)!}\right)\left(\frac{x^r}{r!}\right)=\frac{8}{3} \text{ is}$$

A. $2\sqrt{2}-5$

B. $2\sqrt{2}+5$

C. $-2\sqrt{3}-5$

D. $-2\sqrt{2}+5$

Answer: a

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Exercise More Than One Correct Option Type Questions

1. If in the expansion of $(1+x)^m(1-x)^n$, the coefficients of x and x^2 are 3 and - 6 respectively, the value of m and n are

A. 3

B. 6

C. 9

D. 12

Answer: c,d

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2. If the coefficients of rth, $(r+1)th, and(r+2)th$ terms in the expansion of $(1+x)^{14}$ are in A.P., then r is/are a. 5 b. 11 c. 10 d. 9

A. 5

B. 9

C. 10

D. 12

Answer: a,b

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3. If n is a positive integer and $(3\sqrt{3}+5)^{2n+1} = l + f$ where l is an integer annd $0 < f < 1$, then

A. α is an even integer

B. $(\alpha+\beta)^2$ is divisible by 2^{2n+1}

C. the integer just below $(3\sqrt{3}+5)^{2n+1}$ divisible by 3

D. α is divisible by 10

Answer: a,d

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4. If $(8+3\sqrt{7})^n = P + F$, where P is an integer and F is a proper fraction , then

A. P is a odd integer

B. P is an even integer

C. $F(P+F)=1$

D. $(1-F)(P+F)=1$

Answer: a,d

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5. The value of x for which the sixth term in the expansion of $\left[2^{\log_2\sqrt{9^{x-1}+7}}+\frac{1}{2^{\frac{1}{5}}(\log)_2\left(3^{(x-1)}+1\right)}\right]^7$ is 84 is a. 4 b. 1 or 2 c. 0 or 1 d. 3

A. 4

B. 3

C. 2

D. 1

Answer: c,d

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6. Consider the binomial expansion of $\left(\sqrt{x}+\left(\frac{1}{2x^{\frac{1}{4}}}\right)\right)^{n}$ $n\in\mathbb{N}$, where the terms of the expansion are written in decreasing powers of x. If the coefficients of the first three terms form an arithmetic progression then the statement(s) which hold good is(are) (A) total number of terms in the expansion of the binomial is 8 (B) number of terms in the expansion with integral power of x is 3 (C) there is no term in the expansion which is independent of x (D) fourth and fifth are the middle terms of the expansion

A. Total number of terms in the expansion of the binomial is 8

B. Number of terms in the expansion with integral power of x is 3

C. There is no term in the expansion which in independant of x

D. Fourth and fifth are the middle terms of the expansion

Answer: b,c

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7. Let $(1+x^2)^2(1+x)^n = a_0 + a_1x + a_2x^2 + \ldots$ if

a_1, a_2 and a_3 are in A.P , the value of n is

A. 2

B. 3

C. 4

D. 7

Answer: b,c

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8. The 10th term of $\left(3-\sqrt{\frac{17}{4}+3\sqrt{2}}\right)^{20}$ is (a) a irrational number (b) a rational number (c) a positive integer (d) a negative integer

A. an irrational number

B. a rational number

C. a positive integer

D. a negative integer

Answer: a,d

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9. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots.. + C_nx^n$, then $C_0 - (C_0 + C_1) + (C_0 + C_1 + C_2) - (C_0 + C_1 + C_2 + C_3) + \ldots..$ $(-1)^{n-1}(C_0 + C_1 + \ldots\ldots + C_{n-1})$ is (where n is even integer and $C_r = .^n C_r$)

A. a positive value

B. a negative value

C. divisivle by 2^{n-1}

D. divisible by 2^n

Answer: b,c

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10. If $f(m) = \sum_{i=0}^{m} (30(\hat{\ })30 - i)(20(\hat{\ })m - i) where (pq) =^p C_q, then$ (a) maximum value of $f(m) is^{50}C_{25}$ (b)$f(0) + f(1) + ...f(50) = 2^{50}$ (c)$f(m)$ is always divisible by $50(1 \leq m \leq 49)$ (d)The value of $\sum_{m=0}^{50} (f(m))^2 =^{100} C_{50}$

A. maximum value of (n) is ${}^{50}C_{25}$

B. $f(0) + f(1) + f(2) + \ldots + f(50) = 2^{50}$

C. f (n) is always divisible by 50

D. $f^2(0) + f^2(1) + f^2(2) + \ldots + f^2(50) = {}^{100}C_{50}$

Answer: a,b,d

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11. Find the value (s) of r satisfying the equation $\hat{} \ 69C_{3r-1} -^{69} C_{r^2} =^{69} C_{r^2-1} -^{69} C_{3r}.$

A. 1

B. 2

C. 3

D. 7

Answer: c,d

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12. If the middle term of $\left(x + \frac{1}{x}\sin^{-1}x\right)^8 isequal \rightarrow \frac{630}{16}$,

the value of x is/are

A. $-\frac{\pi}{3}$

B. $-\frac{\pi}{6}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer: a,d

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13. If $ac < b^2$ then the sum of the coefficient in the expansion of $\left(a\alpha^2x^2 + 2b\alpha x + c\right)^n$ is $(a, b, c, \alpha \in R$ and $n \in N)$

A. (a)$+$ve , if $a > 0$

B. (b)$+$ve, if c > 0

C. (c)$-$ ve, if a < 0, n is odd

D. (d)$+$ ve , if $c < 0$, n is even

Answer: a,b,c,d

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14. In the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n, n \in N,$

A. number of term = 2n + 1

B. term independant of $x = 2^{n-1}$

C. coefficient of $x^{2n-2} = n$

D. coefficient of $x^2 = n$

Answer: a,c,

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15. The coefficient of the (r +1)th term of $\left(x+\frac{1}{x}\right)^{20}$, when expanded in the descending power of x , is equal to the coefficient of the 6th term of $\left(x^2+2+\frac{1}{x^2}\right)$ when expanded in ascending power of x . The value of r is

A. 5

B. 6

C. 14

D. 15

Answer: ad

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Exercise Passage Based Questions

1. Consider $(1+x+x^2)^n = \sum_{r=0}^{n} a_r x^r$, where $a_0, a_1, a_2, \ldots, a_{2n}$ are real number and n is positive integer.

The value of $\sum_{r=0}^{n-1} a_r$ is

A. $\frac{-3^n - a_n}{2}$

B. $\frac{3^n - a_n}{2}$

C. $\frac{a_n - 3^n}{2}$

D. $\frac{3^n + a_n}{2}$

Answer: b

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2. Consider $\left(1+x+x^2\right)^n = \sum_{r=0}^{n} a_r x^r$, where $a_0, a_1, a_2, \ldots, a_{2n}$ are real number and n is positive integer.

If n is even, the value of $\sum_{r=0}^{n/2-1} a_{2r}$ is

A. $\dfrac{3^n - 1 + a_n}{2}$

B. $\dfrac{3^n - 1 - a_n}{4}$

C. $\dfrac{3^n + 1 + a_n}{2}$

D. $\dfrac{3^n + 1 - 2a_n}{4}$

Answer: d

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3. Consider $\left(1+x+x^2\right)^n = \sum_{r=0}^{n} a_r x^r$, where $a_0, a_1, a_2, \ldots, a_{2n}$ are real number and n is positive integer.

The value of $\sum_{r=0}^{n-1} a_r$ is

A. $\dfrac{3^n - 1 + 2a_n}{2}$

B. $\dfrac{3^n - 1 + 2a_n}{4}$

C. $\dfrac{3^n + 1 + 2n_n}{2}$

D. $\dfrac{3^n + 1 - 2a_n}{4}$

Answer: b

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4. If $\left(1+x+2x^2\right)^{20} = a_0 + a_1x^2 \ldots\ldots + a_{40}x^{40}$, then following questions.

The value of $a_0 + a_2 + a_4 + \ldots\ldots + a_{38}$ is

A. $2^{19}\left(2^{19} - 1\right)$

B. $2^{20}(2^{19}-1)$

C. $2^{19}(2^{20}-1)$

D. $2^{20}(2^{20}-19)$

Answer: c

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5. If $(1+x+2x^2)^{20}=a_0+a_1x^2......+a_{40}x^{40}$, then following questions.

The value of $a_0+a_2+a_4+......+a_{38}$ is

A. $2^{19}(2^{19-20}$

B. $2^{19}(2^{20}-21)$

C. $2^{19}(2^{19}-21)$

D. $2^{19}(2^{19}-19)$

Answer: b

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6. If $\left(1+x+2x^2\right)^{20} = a_0 + a_1x + a_2x^2 + \ldots + a_{40}x^{40}$.

The value of $\frac{a_{39}}{a_{40}}$, is

A. 2^{20}

B.

C. 10

D. 1

Answer: c

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7. Suppose ,m divided by n , then quotient q and remainder r $\frac{n)m(q}{\frac{-}{r}}$ or

$m = nq + r, \forall m, n, q, r \in 1$ and $n \neq 0$ If a is the remainder when 5^{40}

us divided by 11 and b is the remainder when 2^{2011} is divided by 17 , the value of a + b is

A. 7

B. 8

C. 9

D. 10

Answer: c

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8. Suppose ,m divided by n , then quotient q and remainder r

or $m = nq + r, \forall m, n, q, r \in 1 \text{ and } n \neq 0$

If 13^{99} is divided by 81 , the remainder is

A. (a)13

B. (b)23

C. (c)39

D. (d)55

Answer: d

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9. Suppose ,m divided by n , then quotient q and remainder r

or $m = nq + r, \forall m, n, q, r \in 1 \text{ and } n \neq 0$

If 13^{99} is divided by 81 , the remainder is

A. 13

B. 23

C. 39

D. 55

Answer: d

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10. Consider the binomial expansion of $R = (1+2x)^n = I + f$, where I is the integral part of R and f is the fractional part of R , n $\in$ N .

Also , the sum of coefficient of R is 2187.

The value of $(n + Rf)\text{for } \text{x} = \frac{1}{\sqrt{2}}$ is

A. 7

B. 8

C. 9

D. 10

Answer: b

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11. Consider the binomial expansion of $R = (1+2x)^n = I = f$, where I is the integral part of R and f is the fractional part of R , n $\in$ N .

Also , the sum of coefficient of R is 2187.

If ith term is the geratest term for x= 1/3, then i equal

A. 4

B. 5

C. 6

D. 7

Answer: a

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12. Consider the binomial expansion of $R = (1+2x)^n = I = f$, where I is the integral part of R and f is the fractional part of R , n $\in$ N .

Also , the sum of coefficient of R is 2187.

If kth term is having greatest coefficient , the sum of all possible value of k, is

A. 7

B. 9

C. 11

D. 13

Answer: b

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13. If

$(x + a_1)(x + a_2)(x + a_3)\ldots(x + a_n) = x^n + S_1 x^{n-1} + S_2 x^{n-2} + \ldots + S$

where ,$S_1 = \sum_{i=0}^{n} a_i$, $S_2 = \sum\sum_{1 \le i < j \le n} a_i a_j$, $S_3 \sum\sum\sum_{1 \le i < k \le n} a_i a_j a_k$

and so on .

Coefficient of x^7 in the expansion of

$(1 + x)^2 (3 + x)^3 (5 + x)^4$ is

A. $n \cdot 2^n$

B. $(n + 1) \cdot 2^n$

C. $n \cdot 2^{n+1}$

D. $n \cdot 2^n + 1$

Answer: b

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14. If

$(x+a_1)(x+a_2)(x+a_3)\ldots(x+a_n)=x^n+S_1x^{n-1}+S_2x^{n-2}+\ldots+S$

where ,

$S_1=\sum_{i=0}^{n}a_i,\ S_2=\left(\sum\sum\right)_{1\le i<j\le n}a_ia_j,\ S_3\left(\sum\sum\sum\right)_{1\le i<k\le n}a_ia_ja$

and so on .

If $(1+x)^n=C_0+C_1x+C_2x^2+\ldots+C_nx^n$ the

cefficient of x^n in the expansion of

$(x+C_0)(x+C_1)(x+C_2)\ldots(x+C_n)$ is

A. $2^{2n-1}-\frac{1}{2}{}^{2n}C_{n-1}$

B. $2^{2n-1}-\frac{1}{2}{}^{2n}C_n$

C. $2^{2n-1}-\frac{1}{2}{}^{2n+1}C_n$

D. $2^{2n-1}-\frac{1}{2}{}^{2n+1}C_{n-1}$

Answer: b

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15. If

$(x+a_1)(x+a_2)(x+a_3)\ldots(x+a_n) = x^n + S_1x^{n-1} + S_2x^{n-2} + \ldots + S$

where ,

$S_1 = \sum_{i=0}^{n} a_i,\ S_2 = \left(\sum\sum\right)_{1 \le i < j \le n} a_ia_j,\ S_3\left(\sum\sum\sum\right)_{1 \le i < k \le n} a_ia_ja$

and so on .

Coefficient of x^7 in the expansion of

$(1+x)^2(3+x)^3(5+x)^4$ is

A. 112

B. 224

C. 342

D. 416

Answer: d

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16. $A=\left(\frac{5}{2}+\frac{x}{2}\right)^{n}, B=(1+3x)^{m}$

Sum of coefficients of expansion of B is 6561 . The difference of

the coefficient of third to the second term in the expansion of A is equal

to 117 .

The value of m is

A. 4

B. 5

C. 6

D. 7

Answer: c

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17. Sum of coefficients of expansion of B is 6561 . The difference of the coefficient of third to the second term in the expansion of A is equal to 117 .

If n^m is divided by 7 , the remainder is

A. 1

B. 2

C. 3

D. 5

Answer: a

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18. Sum of coefficients of expansion of B is 6561 . The difference of the coefficient of third to the second term in the expansion of A is equal to 117 .

The ratio of the coefficient of second term from the

beginning and the end in the expansion of B , is

A. 125

B. 625

C. 3125

D. 15625

Answer: d

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19. Let us consider the binomial expansion $(1+x)^n = \sum_{r=0}^{n} a_r x^r$

where $a_4, a_5 \text{and } a_6$ are in AP , (n $<$ 10). Consider another

binomial expansion of $A = \sqrt[3]{2} + \left(\sqrt[4]{3}\right)^{13n}$, the expansion of A

contains some rational terms $T_{a1}, T_{a2}, T_{a3}, \dots, T_{am}$

$(a_1 < a_2 < a_3 < \dots < a_m)$

The value of $\sum_{i=1}^{n} a_i$ is

A. 63

B. 127

C. 255

D. 511

Answer: b

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20. Let us consider the binomial expansion $(1+x)^n = \sum_{r=0}^{n} a_r x^r$ where $a_4, a_5 \text{and } a_6$ are in AP , (n $<$ 10). Consider another binomial expansion of $A = \sqrt[3]{2} + \left(\sqrt[4]{3}\right)^{13n}$, the expansion of A contains some rational terms $T_{a1}, T_{a2}, T_{a3}, \ldots, T_{am}$

$(a_1 < a_2 < a_3 < \ldots < a_m)$

The value of a_m is

A. 87

B. 88

C. 89

D. 90

Answer: c

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21. Let us consider the binomial expansion $(1+x)^n = \sum_{r=0}^{n} a_r x^r$

where $a_4, a_5 \text{and} \ a_6$ are in AP , (n $<$ 10). Consider another

binomial expansion of $A = \sqrt[3]{2} + \left(\sqrt[4]{3}\right)^{13n}$, the expansion of A

contains some rational terms $T_{a1}, T_{a2}, T_{a3}, \ldots, T_{am}$

$(a_1 < a_2 < a_3 < \ldots < a_m)$

The value of a_m is

A. 6

B. 8

C. 10

D. 12

Answer: d

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Binomial Theorem Exerciese 4 Single Integer Answer Type Questions

1. For integer $n > 1$, the digit at unit's place in the number

$$\sum_{r=0}^{100} r! + 2^{2^n}$$ I

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Exercise Single Integer Answer Type Questions

1. If $(1 + x + x^2 + x^3)^n = \sum_{r=0}^{3n} b_r x^r \text{ and } \sum_{r=0}^{3n} b_r = k, \text{then} \sum_{r=0}^{3n} r b_r$ is

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2. The last two digits of the number 19^{9^4} is

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3. If $\dfrac{[{}^nC_r + 4 \cdot {}^nC_{r+1} + 6 \cdot {}^nC_{r+2} + 4 \cdot {}^nC_{r+3} + {}^nC_{r+4}]}{[{}^nC_r + 3. {}^nC_{r+1} + 3 \cdot {}^nC_{r+2} + {}^nC_{r+3}]} = \dfrac{n+\lambda}{r+\lambda}$

the value of λ is

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4. The value of $99^{50} - 99.98^{50} + \dfrac{99 \cdot 98}{1 \cdot 2}(97)^{50} - \ldots + 99$ is

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5. If the greatest term in the expansion of $(1+x)^2 n$ has the greatest coefficient if and only if $x \varepsilon \left(\dfrac{10}{11}, \dfrac{11}{10}\right)$ and the fourth term in the expansion of $\left(kx + \dfrac{1}{x}\right)^m is \dfrac{n}{4}$ then find the value off mk.

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6. If the value of

$$(n+2).\,{}^nC_0\cdot 2^{n+1}-(n+1)\cdot{}^nC_1\cdot 2^n+n\cdot{}^nC_2\cdot 2^{n-1}-\ldots$$

is equal to $k(n+1)$, the value of k is .

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7. If $\left(1+x+x^2+\ldots+x^9\right)^4\left(x+x^2+x^3+\ldots+x^9\right)$

$$=\sum_{r=1}^{45}a_rx^r \text{ and the value of } a_2+a_6+a_{10}+\ldots+a_{42} \text{ is } \lambda$$

the sum of all digits of λ is .

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Binomial Theorem Exerciese 5 Matching Type Questions

1. Match the following Column I to Column II

	Column I		Column II
(A)	If $\binom{18}{r-2} + 2\binom{18}{r-1} + \binom{18}{r} \geq \binom{20}{13}$, then the values of r is /are	(p)	5
(B)	The digit in the unit's place of the number $183! + 3^{183}$ is less than	(q)	6
(C)	If the 4th term in the expansion of $\left(ax + \frac{1}{x}\right)^n$ is $5/2$, then na is less than	(r)	7
		(s)	8
		(t)	9

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	Column I		Column II
(A)	The sum of binomial coefficients of terms containing power of x more than x^{30} in $(1+x)^{61}$ is divisible by	(p)	2^{57}

2.

(B)	The sum of binomial coefficients of rational terms in the expansion of $(1+\sqrt{3})^{62}$ is divisible by	(q)	2^{58}
(C)	If $\left(x+\frac{1}{x}+x^2+\frac{1}{x^2}\right)^{31} = a_0 x^{-62} + a_1 x^{-61} + a_2 x^{-60} + \ldots + a_{124} x^{62}$, then $a_1 + a_3 + a_5 + \ldots + a_{123}$ is divisible by	(r)	2^{59}
		(s)	2^{60}
		(t)	2^{61}

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3. Match the following Column I to Column II

	Column I		Column II
(A)	If $11^n + 21^n$ is divisible by 16, then n can be	(p)	4
(B)	The remainder, when 3^{37} is divided by 80, is less than	(q)	5
(C)	In the expansion of $(1+x)^{29}$ coefficient of $(r+1)$th term is equal to that of $(r+k)$th term, then the value of k cannot be	(r)	6
(D)	If the ratio of 2nd and 3rd terms in the expansion of $(a+b)^n$ is equal to ratio of 3rd and 4th terms in the expansion of $(a+b)^{n+3}$, then n is less than	(s)	7

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4. Match the Column - I with the Column-II to form the correct pair

	Column I		Column II
(A)	If number of dissimilar terms in the expansion of $(x+2y+3z)^n$ $(n \in N)$ is an^2+bn+c, then	(p)	$a+b+c=3$
(B)	If number of dissimilar terms in the expansion of $(x+y+z)^{2n+1}-(x+y-z)^{2n+1}$ $(n \in N)$ is an^2+bn+c, then	(q)	$a+b+c=4$
(C)	If number of dissimilar terms in the expansion of $(x-y+z)^n+(x+y-z)^n$ $(n \in$ is even natural number) is an^2+bn+c, then	(r)	$a+b=2c$
(D)	If number of dissimilar terms in the expansion of $\left(\frac{x^2+1+x^4}{x^2}\right)^{\Sigma n}$ $(n \in N)$ is an^2+bn+c, then	(s)	$b+c=8a$

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Exercise Statement I And Ii Type Questions

1. Statement-1 Greatest coefficient in the expansion of

$(1+3x)^6$ is ${}^6C_3 \cdot 3^3$.

Statement-2 Greatest coefficient in the expansion of

$(1+x)^{2n}$ is the middle term .

A. Statement I is True, Statement II is True, Statement II is a correct explanation for statement I

B. Statement I is True, Statement II is True, Statement II is NOT a correct explanation for Statement I

C. Statement I is True, Statement II is False

D. Statement I is False, Statement II is True.

Answer: d

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2. Statement-1 The term independant of x in the expansion of $\left(x^2+\frac{1}{x^2}+2\right)^{25}$ is ${}^{50}C_{25}$.

Statement-2 In a binomial expansion middle term is indepandnet of x.

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3. Statement-I : In the expansion of $(1+x)^n$ ifcoefficient of 31^{st} and 32^{nd} terms are equal then n = 61 Statement -II : Middle term in the expansion of $(1+x)^n$ has greatest coefficient.

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4. Statement-1 The number of terms in the expansion of

$\left(x+\frac{1}{x}+1\right)^n$ is $(2n+1)$

Statement-2 The number of terms in the expansion of

$(x_1+x_2+x_3+\ldots+x_m)^n$is ${}^{n+m-1}C_{m-1}$.

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5. Statement-1 4^{101} when divided by 101 leaves the remainder4.

Statement-2 (n^p-n) when divided by 'p' leaves

remainder zero when $n\geq 2, n\in N$ is a prime number .

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6. Statement-1: $11^{25} + 12^{25}$ when divided by 23 leaves the remainder zero.Statement-2: $(a+b)^n$ is divisible by $(a+b)$ for all values of $n \varepsilon$N

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7. Statement- 1 The maximum value of the term independant of x in the expansion of $\left(ax^{1/6} + bx^{1/3}\right)^9$ is 84

Statement- 2$a^2 + b = 2$

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Exercise Subjective Type Questions

1. If the third term in the expansion of $\left(\frac{1}{x} + {}_{\text{x}}(\log)_{10x}\right)^5$ is $1000,$ then find $x\cdot$

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2. Find the value of

$$\frac{18^3 + 7^3 + 3 \times 18 \times 7 \times 25}{3^6 + 6 \times 243 \times 2 + 15 \times 18 \times 4 + 20 \times 27 \times 8 + 15 \times 9 \times 16}$$

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3. Determine the term independent of a in the expansion of

$$\left(\frac{a+1}{a^{\frac{2}{3}} - a^{\frac{1}{3}} + 1} - \frac{a-1}{a - a^{\frac{1}{2}}}\right)^{10}.$$

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4. If in the expansion of $(1+x)^n, a, b, c$ are three consecutive coefficients, then $n =$ a. $\frac{ac+ab+bc}{b^2+ac}$ b. $\frac{2ac+ab+bc}{b^2-ac}$ c. $\frac{ab+ac}{b^2-ac}$ d. none of these

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5. In $\left(33+\frac{1}{33}\right)^{n}$ if the ratio of 7th term from the beginning to the 7th term from the end is 1/6, then find the value of n.

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6. if $S_n = C_0C_1 + C_1C_2 + \ldots + C_{n-1}C_n$ and $\frac{S_{n+1}}{S_n} = \frac{15}{4}$ then n is

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7. $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots\ldots\ldots\ldots + n\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$

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8. Find the term in $\left(3\sqrt{\left(\frac{a}{\sqrt{b}}\right)} + \left(\sqrt{\frac{b}{\hat{}}3\sqrt{a}}\right)\right)^{21}$ which has the same power of a and b.

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9. The coefficient of $x^r[0 \leq r \leq (n-1)]$ in the expansion of $(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+\ldots.$ $+(x+2)^{n-1}$ is $a.^n C_r(3^r-2^n)$ $b.^n C_r\left(3^{n-r}-2^{n-r}\right)$ $c.^n C_r\left(3^r+2^{n-r}\right)$

d. none of these

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10. Prove that if p is a prime number greater than 2, then the difference $\left[(2+\sqrt{5})^p\right]-2^{p+1}$ is divisible by p, where [.] denotes greatest integer.

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11. Integer just greater tehn $\left(\sqrt{3}+1\right)^{2n}$ is necessarily divisible by (A) $n+2$ (B) 2^{n+3} (C) 2^n (D) 2^{n+1}

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12. Solve the equation

$$ {}^{11}C_1x^{10} - {}^{11}C_3x^8 + {}^{11}C_5x^6 - {}^{11}C_7x^4 + {}^{11}C_9x^2 - {}^{11}C_{11} = 0 $$

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13. If $(1+x)^n = C_0 = C_1x + C_2x^2 + \ldots + C_nx^n$,

find the values of the following

$$ \sum\sum_{0 \le i \le j \le n} (i+j)\left(C_i \pm C_j\right)^2 $$

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14. Evaluate $\sum_{0 \le i \le j \le 10} {}^{21}C_i \cdot {}^{21}C_j$.

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15. Find the coefficient of x^4 in the expansion of $\left(1+x+x^2+x^3\right)^{11}$.

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16. Find the coefficient of x^4 in the expansion of $(2 - x + 3x^2)^6$.

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17. If for z as real or complex,

$(1 + z^2 + z^4)^8 = C_0 + C1z2 + C2z4 + \ + C_{16}z^{32} then$

$C_0 - C_1 + C_2 - C_3 + \ + C_{16} = 1$

$C_0 + C_3 + C_6 + C_9 + C_{12} + C_{15} = 3^7$

$C_2 + C_5 + C_6 + C_{11} + C_{14} = 3^6$

$C_1 + C_4 + C_7 + C_{10} + C_{13} + C_{16} = 3^7$

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18. If for z as real or complex such that

$(1 + z^2 + z^4)^8 = C_0 + C_1z^2 + C_2z^4 + \ldots + C_{16}z^{32}$,

prove that

$C_0 + C_3 + C_6 + C_9 + C_{12} + C_{15}$

$+ (C_2 + C_5 + C_8 + C_{11} + C_{14})\omega$

$+ (C_1 + C_4 + C_7 + C_{10} + C_{16})\omega^2 = 0$,

where ω is a cube root of unity .

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19. If $a_0, a_1, a_2, \ldots.$ be the coefficients in the expansion of $(1 + x + x^2)^n$ in ascending powers of x. prove that : $(i) a_0 a_1 - a_1 a_2 + a_2 a_3 - \ldots. = 0$

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20. If $a_0, a_1, a_2, \ldots$ are the coefficients in the expansion of $(1 + x + x^2)^n$ in ascending powers of x, prove that

$a_0 a_2 - a_1 a_3 + a_2 a_4 - \ldots + a_{2n-2} a_{2n} = a_{n+1}$.

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21. If $a_0, a_1, a_2, \ldots$ are the coefficients in the expansion of $(1+x+x^2)^n$ in ascending powers of x, prove that if $E_1 = a_0 + a_3 + a_6 + \ldots, E_2 = a_1 + a_4 + a_7 + \ldots$ and $E_3 = a_2 + a_5 + a_8 + \ldots$ then $E_1 = E_2 = E_3 = 3^{n-1}$

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Binomial Theorem Exerciese 7 Subjective Type Questions

1. Show that there will ve a term independant of x in the expansion of $(x^a + a^{-b})$ only , if an is multiple of (a+b) .

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2. If g (x) $= \sum_{r=0}^{200} \alpha_r . x^r$ and f(x) $= \sum_{r=10}^{200} \beta_r x^r, \beta_r = 1$ for $r \geq 100$ and g (x) = f (1 + x) , show that the greatest coefficient in the expansion of $(1+x)^{201}$ is α_{100} .

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3. Prove the identity

$$\frac{1}{{}^{2n+1}C_r} + \frac{1}{{}^{2n+1}C_{r+1}} = \frac{2n+2}{2n+1} \cdot \frac{1}{{}^{2n}C_r}$$

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4. Let $a_0, a_1, a_2, \ldots$ are the coefficients in the expansion of $(1 + x + x^2)^n$ aranged order of x. Find the value of $a_r - {}^nC_1 a_{r-1} + {}^nC_r a_{r-2} - \ldots + (-1)^r {}^nC_r a_0$, where r is not divisible by 3.

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5. Prove that $(n-1)^2 C_1 + (n-3)^2 C_3 + (n-5)^2 C_5 .$ $+ \ldots. = n(n+1)2^{n-3}$, where C_r stands for ${}^nC_r$.

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6. Show that $\frac{C_0}{1}-\frac{C_1}{4}+\frac{C_2}{7}-\ldots+(-1)^n\frac{C_n}{3n+1}$ $=\frac{3^n\cdot n!}{1\cdot4\cdot7\ldots(3n+1)}$, where C_r stands for ${}^nC_r$.

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Exercise Questions Asked In Previous 13 Years Exam

1. The value of $\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+(302)(3012)++(3020)(3030)=$ a. $\hat{}\ 60C20$ b. $\hat{}\ 30C10$ c. $\hat{}\ 60C30$ d. $\hat{}\ 40C30$

A. ${}^{60}C_{20}$

B. ${}^{30}C_{10}$

C. ${}^{60}C_{30}$

D. ${}^{40}C_{30}$

Answer: B

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2. If the coefficient of the rth, $(r+1)th$ and $(r+2)th$ terms in the expansion of $(1+x)^n$ are in A.P., prove that $n^2 - n(4r+1) + 4r^2 - 2 = 0.$

A. $n^2 - 2np + 4p^2 = 0$

B. $n^2 - n(4p+1) + 4p^2 - 2 = 0$

C. $n^2 - n(4p+1) + 4p^2 = 0$

D. None of the above

Answer: B

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3. If the coefficient of x^7in $\left[ax^2 + \left(\frac{1}{b}x\right)\right]^{11}$ equals the coefficient of x^{-7}in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$ then a and b satisfy the relation

A. 1

B. $1/2$

C. 2

D. 3

Answer: A

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4. For natural numbers m, n, if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots$, and $a_1 = a_2 = 10$, t

a. $m < n$ b. $m > n$ c. $m + n = 80$ d. $m - n = 20$

A. (20,45)

B. (35,20)

C. (45,35)

D. (35,45)

Answer: D

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5. In the binomial expansion of $(a-b)^n, n \geq 5$ the sum of the 5th and 6th term is zero , then find $\frac{a}{b}$

A. $\frac{5}{n-4}$

B. $\frac{6}{n-5}$

C. $\frac{n-5}{6}$

D. $\frac{n-4}{5}$

Answer: D

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6. The sum of series $\wedge(20)C0 - \wedge(20)C1 + \wedge(20)C2 - \wedge(20)C3 + + \wedge(20)C10$ is $\frac{1}{2}$ $\wedge(20)C10$ b. 0 c. $\wedge(20)C10$ d. $-\wedge(20)C10$

A. $-{}^{20}C_{10}$

B. $\frac{1}{2}{}^{20}C_{10}$

C. 0

D. ${}^{20}C_{10}$

Answer: B

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7. Statement-1: $\sum_{r=0}^{n}(r+1)^{n}C_{r}=(n+2)2^{n-1}$

Statement -2: $\sum_{r=0}^{n}(r+1)^{n}C_{r}x^{r}=(1+x)^{n}+nx(1+x)^{n-1}$

A. Statement-1 is ture ,Statement-2 is treu, Statement-2 is a correct explanation for Statement-1

B. Statement-1 is ture ,Statement-2 is treu, Statement-2 is not a correct explanation for Statement-1

C. Statement-1 is true ,Statement-2 is false

D. Statement-1 is true ,Statement-2 is ture

Answer: A

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8. The reamainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is

A. 8

B. 0

C. 2

D. 7

Answer: C

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9. For $r = 0, 1, \ldots\ldots, 10$, let A_r, B_r, and C_r denote, respectively, the coefficient of x^r in the expansion of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to

A. $B_{10} - C_{10}$

B. $A_{10}(B_{10} - C_{10}A_{10})$

C. 0

D. $C_{10} - B_{10}$

Answer: D

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10. Let $S_1 = \sum_{j=1}^{10} j(j-1).\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j.\,{}^{10}C_j$, and $S_3 = \sum_{j=1}^{10} j^2.\,{}^{10}C_j$

.

Statement 1 : $S_3 = 55 \times 2^9$.

Statement 2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

A. Statement-1 is ture ,Statement-2 is treu, Statement-2 is a correct explanation for Statement-1

B. Statement-1 is ture ,Statement-2 is treu, Statement-2 is not a correct explanation for Statement-1

C. Statement-1 is true ,Statement-2 is false

D. Statement-1 is true ,Statement-2 is ture

Answer: B

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11. Find the coefficient of x^7 in the expansion of $\left(1 - x - x^2 + x^3\right)^6$.

A. -132

B. -144

C. 132

D. 144

Answer: B

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12. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is (1) an irrational number (2) an odd positive integer (3) an even positive integer (4) a rational number other than positive integers

A. an odd positive integer

B. an even positive integer

C. a rational number other then positive integer

D. an irrational number

Answer: D

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13. The term independent of x in expansion of $\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$ is (1) 120 (2) 210 (3) 310 (4) 4

A. 120

B. 210

C. 310

D. 4

Answer: B

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14. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5:10:14. Then $n =$ __________.

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15. If the coefficient of x^3 and x^4 in the expansion of $(1+ax+bx^2)(1-2x)^{18}$ in power of x are both zero, then (a, b) is equal to

A. $\left(14, \frac{272}{3}\right)$

B. $\left(16, \frac{272}{3}\right)$

C. $\left(14, \frac{251}{3}\right)$

D. $\left(16, \frac{251}{3}\right)$

Answer: B

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16. Coefficient of x^{11} in the expansion of $(1+x^2)(1+x^3)^7(1+x^4)^{12}$ is 1051 b. 1106 c. 1113 d. 1120

A. 1051

B. 1106

C. 1113

D. 1120

Answer: C

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17. The sum of coefficient of integral powers of x in the binomial expansion of $\left(1 - 2\sqrt{x}\right)^{50}$ is

A. $\frac{1}{2}\left(2^{50} + 1\right)$

B. $\frac{1}{2}\left(2^{50} + 1\right)$

C. $\frac{1}{2}\left(3^{50}\right)$

D. $\frac{1}{2}\left(3^{50} - 1\right)$

Answer: B

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18. The coefficient of x^9 in the expansion of $(1+x)(16x^2)(1+x^3)(1+x^{100})$ is

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19. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)x \neq 0$, is 28, then the sum of coefficient of all the terms in this expansion, is

A. 243

B. 729

C. 64

D. 2187

Answer: B

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20. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2+(1+x)^3+\ldots\ldots+(1+x)^{49}+(1+mx)^{50}$ is $(3n+1).^{51}C_3$ for some positive integer n, then the value of n is ______.

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21. The value of $(.^{21}C_1-.^{10}C_1)+(.^{21}C_2-.^{10}C_2)+(.^{21}C_3-.^{10}C_3)+(.^{21}C_4-.^{10}C_4$ is

A. $2^{20}-2^{10}$

B. $2^{21}-2^{11}$

C. $2^{21}-2^{10}$

D. $2^{20}-2^{9}$

Answer: A

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