MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

COMPLEX NUMBERS

Examples

1. Is the following computation correct ? If not give the correct compuatation:

$$\sqrt{(-2)}\sqrt{(-3)} = \sqrt{(-2)(-3)} = 6$$

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2. A student writes the formula $\sqrt{ab} = \sqrt{a}\sqrt{b}$. Then he substitutes $a = -1 \, and \, b = -1 \, and$ finds $1 = -1$. Explain where is he wrong?

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3. Evaluate.

(i) i^{1998}

(ii)i^{-9999}

(iii) $(-\sqrt{-1})^{4n=3}$,n $\neq$ N

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4. Find the value of $1+i^2+i^4+i^6+\ldots+i^{2n}$,

where$i=\sqrt{-1}$ and n in N.

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5. If $a=\frac{1+i}{\sqrt{2}}, \quad \text{where} \quad i=\sqrt{-1},$ then find the value of a^{1929}.

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6. The value of $sum \sum_{n=1}^{13}\left(i^{n}+i^{n+1}\right), w h e r e i=\sqrt{-1} e q u a l s\ i$ (b) $i-1$ (c) $-i$ (d) 0

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7. The value of $\sum_{n=0}^{100} i^{n!}$ equals (where $i=\sqrt{-1}$)

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8. Find he value of $\sum_{r=1}^{4n+7} i^{r}$ where, $i=\sqrt{-1}$.

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9. Show that the polynomial $x^{4p}+x^{4q+1}+x^{4r+2}+x^{4s+3}$ is divisible by $x^{3}+x^{2}+x+1, wherep, q, r, s \in n\cdot$

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10. What is the digit in the unit's place of

$(5172)^{11327}$?

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11. What is the digit in the unit's place of

$(143)^{86}$?

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12. What is the digit in unit's place of

$(1354)^{22222}$?

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13. What is the digit in the unit's place of

$(13057)^{941120579}$?

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14. What is the digit in the unit's place of $(1008)^{786}$?

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15. What is the digit in the unit's place of $(2419)^{111213}$?

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16. If $\frac{x-3}{3+i}+\frac{y-3}{3-i}=i$ where $x, y \in R$ then find values of x and y

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17. If $(x+iy)^5=p+iq,$ then prove that $(y+ix)^5=q+ip$.

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18. Find the least positive integral value of

n, for which $\left(\frac{1-i}{1+i}\right)^n$, where $i=\sqrt{-1}$, is purely

imaginary with positive imaginary part.

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19. If the multicative inverse of a comlex number is $\left(\sqrt{3}+4i\right)\backslash 19$, where

$i=\sqrt{-1}$, find the complex number.

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20. Find the value of θ if $(3+2i\sin\theta)/(1-2i\sin\theta)$ is purely real or

purely imaginary.

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21. Find real value of $x and y$ for which the complex numbers

$-3+ix^2 y and x^2+y+4i$ k are conjugate of each other.

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22. If $x = -5 + 2\sqrt{-4}$, find the value of $x^4 + 9x^3 + 35x^2 - x + 4.$

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23. Let z be a complex number satisfying the equation $z^2 - (3+i)z + \lambda + 2i = 0$, whre $\lambda \in$ R and suppose the equation has a real root , then find the non -real root.

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24. Findthe principal value of the arguments of $z_1 = 2 + 2i, z_2 = -3 + 3i, z^3 = -4 - 4i$ and $z_4 = 5 - 5i, where i = \sqrt{}$

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25. Find the modulus and argument of the complex number $\frac{2+i}{4i+(1+i)^2}$

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26. 1. If $|z-2+i| \leq 2$ then find the greatest and least value of $|z|$

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27. If z is any complex number such that $|z+4| \leq 3$, then find the greatest value of $|z+1|$.

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28. If $|z_1|=1, |z_2|=2, |z_3|=3,$ and $|9z_1z_2+4z_1z_3|=12,$ then find the value of $|z_1+z_2+z_3|$.

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29. For any two complex numbers, z_1, z_2

$\left|\frac{1}{2}(z_1 + z_2) + \sqrt{z_1 z_2}\right| + \left|\frac{1}{2}(z_1 + z_2) - \sqrt{z_1 z_2}\right|$ is equal to

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30. A complex number z is said to be unimodular if $|z| = 1$ Suppose z_1 and z_2 are complex number such that $(z_{91}) - 2z_2 \frac{)}{2 - z_1 \bar{z}_2}$ is unimodular and z_2 is non-unimodular. Then the poit z_1 lies on a.

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31. If arg $(z_1) = \frac{17\pi}{18}$ and arg $(z_2) = \frac{7\pi}{18}$, find the principal argument of $z_1 z_2$ and(z_1 / z_2).

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32. If z_1 and z_2 are conjugate to each other , find the principal argument of $(-z_1z_2)$.

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33. Write the value of $arg(z) + arg(z)$.

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34. Write the polar form of $-\frac{1}{2} - \frac{i\sqrt{3}}{2}$ (Where, $i = \sqrt{-1}$).

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35. If $argz = \alpha$ and given that $|z-1| = 1$, where z is a point on the argand plane , show that $\left|\frac{z-2}{z}\right|$ = $|\tan \alpha|$,

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36. Let z be a non-real complex number

lying on $|z|=1$, prove that $z=\dfrac{1+i\tan\left(\frac{arg(z)}{2}\right)}{1-i\tan\left(\frac{arg(z)}{2}\right)}$ (where $i=\sqrt{-1}$.)

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37. Prove that $tan\left(i(\log)_e\left(\frac{a-ib}{a+ib}\right)\right)=\frac{2ab}{a^2-b^2}(wherea, b\in R^+)$

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38. If m and x are two real numbers where $m\in I$, then $e^{2mi\cot^{-1}x}\left(\frac{x\cdot i+1}{x\cdot i-1}\right)^m$

(A) $\cos x+i\sin x$ (B) $\frac{m}{2}$ (C) 1 (D) $\frac{m+1}{2}$

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39. Express $(1+i)^{-1}$,where, i=$\sqrt{-1}$ in the

form A+iB.

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40. If $\sin\left(\log_e i^i\right) = a + ib,$ where $i = \sqrt{-1},$

find a and b, hence and find $\cos\left(\log_e i^i\right)$.

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41. Find the general value of $\log_2(5i)$,

where $i = \sqrt{-1}$.

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42. If $|z_1| = |z_2|$ and arg $(z_1 / z_2) = \pi,$ then

find the of $z_1 z_2$.

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43. Let z and w are two non zero complex number such that $|z| = |w|$, and $Arg(z) + Arg(w) = \pi$ then (a) $z = w$ (b) $z = \overline{w}$ (c) $\bar{z} = \overline{w}$ (d) $\bar{z} = -\overline{w}$

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44. Find the square root of $X + \sqrt{(-X^4 - X^2 - 1)}$.

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45. Solve that equation $z^2 + |z| = 0$, where z is a complex number.

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46. Number of solutions of the equation $z^2 + |z|^2 = 0$, where $z \in C$, is

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47. Find the all complex numbers satisying the equation $2|z|^2 + z^2 - 5 + i\sqrt{3} = 0, wherei = \sqrt{-1}.$

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48. If $z_r = \cos\left(\frac{\pi}{3_r}\right) + i\sin\left(\frac{\pi}{3_r}\right), r = 1, 2, 3, ,$ prove that $z_1 z_2 z_3 z_\infty = i.$

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50. Find all roots of $X^5 - 1 = 0$.

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51. Find all roots of the equation

$X^6 - X^5 + X^4 - X^3 + X^2 - X + 1 = 0$.

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52. if $\alpha, \beta, \gamma$are the roots of $x^3 - 3x^2 + 3x + 7 = 0$ then $\frac{\alpha - 1}{\beta - 1} + \frac{\beta - 1}{\gamma - 1} + \frac{\gamma - 1}{\alpha - 1}$

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53. If $z = \frac{\sqrt{3 + i}}{2}$ (where $i = \sqrt{-1}$) then $\left(z^{101} + i^{103}\right)^{105}$ is equal to

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54. If $\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)^{50}=3^{25}(x+iy)$, where x and y are reals, then the ordered pair (x,y) is given by

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55. If the polynomial $7x^3+ax+b$ is divisible by x^2-x+1, find the value of $2a+b$.

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56. If $1, \omega, \omega^2, \ldots \omega^{n-1}$ are n, nth roots of unity, find the value of $(9-\omega)(9-\omega^2)\ldots(9-\omega^{n-1})$.

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57. F $a=\cos(2\pi/7)+i\sin(2\pi/7)$, then find the quadratic equation whose roots are $\alpha=a+a^2+a^4 and \beta=a^3+a^5+a^7$.

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58. Find the value of

$$\sum_{k=1}^{10}\left[\sin\left(\frac{2\pi k}{11}\right)-i\cos\left(\frac{2\pi k}{11}\right)\right], wherei=\sqrt{-1}.$$

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59. If $n\geq 3$ and $1,\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_{n-1}$ are the n,nth roots of unity, then find value of $\left(\sum\sum\right)_{1\leq i<j\leq n-1}\alpha_i\alpha_j$

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60. Complex numbers z_1, z_2 and z_3 are the vertices A,B,C respectivelt of an isosceles right angled triangle with right angle at C. show that $(z_1-z_2)^2=2(z_1-z_3)(z_3-z_2)$.

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61. Complex numbers z_1, z_2, z_3 are the vertices of A,B,C respectively of an equilteral triangle. Show that $z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1.$

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62. If $z_1, z_2 \text{ and } z_3$ are the vertices of an equilasteral triangle with z_0 as its circumcentre , then changing origin to z^0 ,show that $z_1^2+z_2^2+z_3^2=0,$ where$z_1, z_2, z_3,$ are new complex numbers of the vertices.

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63. Show that inverse of a point a with respect to the circle $|z-c|=R(a \text{ and } c$ are complex numbers and center respectively and R is the radius) is the point $c+\dfrac{R^2}{\bar{a}-\bar{c}}$,

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64. Find the perpendicular bisector of $3 + 4i$ and $-5 + 6i, where i = \sqrt{-1}$.

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65. If z_1, z_2 and z_3 are the affixes of the vertices of a triangle having its circumcentre at the

origin. If zis the affix of its orthocentre, prove that

$Z_1 + Z_2 + Z_3 - Z = 0.$

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66. Let $z_1 z_2$ and z_3 be three complex numbers and $a, b, c \in R,$ such that $a + b + c = 0$ and $az_1 + bz_2 + cz_3 = 0$ then show that $z_1 z_2$ and z_3 are collinear.

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67. Show that the area of the triangle on the Argand diagram formed by the complex numbers z, zi and $z+zi$ is $=\frac{1}{2}|z|^2$

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68. Show that the point a' is the reflection of the point a in the line $z\bar{b}+\bar{z}b+c=0$, If $a'\bar{b}+\bar{a}b+c=0.$

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69. Find the center and radius of the circle $2z\bar{z}+(3-i)z+(3+i)z-7=0, \quad \text{where} \quad i=\sqrt{-1}.$

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70. Find all circles which are orthogonal to $|z|=1 \text{ and } |z-1|=4.$

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71. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that the argument of $\frac{(z - z_1)}{(z - z_2)}$ is $\frac{\pi}{4}$, then prove that $|z - 7 - 9i| = 3\sqrt{2}$.

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72. 1. If $|z - 2 + i| \leq 2$ then find the greatest and least value of$|z|$

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73. In the argand plane, the vector $z = 4 - 3i, wherei = \sqrt{-1},$ is turned in the clockwise sense by 180°. Find the complex number represented by the new vector .

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74. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$.If he point D and M represent the complex numbers $1 + i$ and $2 - i$ respectively, then A represents the complex number......or......

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75. Find the maximum and minimum values of $|z|$ satisfying $\left|z + \frac{1}{z}\right| = 2$

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76. If $\left|z + \frac{4}{z}\right| = 2$, find the maximum and minimum values of $|z|$.

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77. If $|z| \geq 3$, then determine the least value of $\left|z + \frac{1}{z}\right|$.

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A. $4k+1$

B. $4k+2$

C. $4k+3$

D. $4k$

Answer: d

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79. If $|z|=1$ and $w=\frac{z-1}{z+1}$ (where $z\neq-1$), then $Re(w)$ is 0 (b) $\frac{1}{|z+1|^2}\left|\frac{1}{z+1}\right|, \frac{1}{|z+1|^2}$ (d) $\frac{\sqrt{2}}{|z|1|^2}$

A. 0

B. $\frac{-1}{|z+1|^2}$

C. $\left|\frac{z}{z=1}\right|\cdot\frac{1}{|z+1|^2}$

D. $\dfrac{\sqrt{2}}{|z+1|^2}$

Answer: a

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80. if a, b, c, a_1, b_1 and c_1 are non-zero complexnumbers satisfying $\dfrac{a}{a_1}+\dfrac{b}{b_1}+\dfrac{c}{c_1}=1+i$ and $\dfrac{a_1}{a}+\dfrac{b_1}{b}+\dfrac{c_1}{c}=0,$ where $i=\sqrt{-1},$ the value of $\dfrac{a^2}{a_1^2}+\dfrac{b^2}{b_1^2}+\dfrac{c^2}{c_1^2}$ is

$(a)2i(b)2+2i(c)2$ (d)None of these

A. 2i

B. 2+2i

C. 2

D. None of these

Answer: a

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81. Let z and ω be complex numbers. If $Re(z) = |z - 2|, Re(\omega) = |\omega - 2|$ and $arg(z - \omega) = \frac{\pi}{3}$, then the value of $Im(z + w)$, is

A. $\frac{1}{\sqrt{3}}$

B. $\frac{2}{\sqrt{3}}$

C. $\sqrt{3}$

D. $\frac{4}{\sqrt{3}}$

Answer: d

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82. The mirror image of the curve $arg\left(\frac{z-3}{z-i}\right) = \frac{\pi}{6}, i = \sqrt{-1}$ in the real axis

A. $arg\left(\frac{z+3}{z+i}\right) = \frac{\pi}{6}$

B. $arg\left(\frac{z-3}{z+i}\right)=\frac{\pi}{6}$

C. $arg\left(\frac{z+i}{z+3}\right)=\frac{\pi}{6}$

D. $arg\left(\frac{z+i}{z-3}\right)=\frac{\pi}{6}$

Answer: d

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83. Expand $\begin{vmatrix}3 & x\\ x & 1\end{vmatrix}$

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84. If $z+\frac{1}{z}=1$ and $a=z^{2017}+\frac{1}{z^{2017}}$ and b is the lastdigit of the number $2^{2^n}-1$,, when the integer $n>1$, the value of a^2+b^2 is

A. 23

B. 24

C. 26

D. 27

Answer: c

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85. if $\omega and \omega^2$ are the nonreal cube roots of unity and $[1/(a+\omega)] + [1/(b+\omega)] + [1/(c+\omega)] = 2\omega^2$ and $\left[1/(a+\omega)^2\right] + \left[1/(b+\omega)^2\right] + \left[1/(c+\omega)^2\right] = 2\omega$, then find the value of $[1/(a+1)] + [1/(b+1)] + [1/(c+1)]$.

A. -2

B. -1

C. 1

D. 2

Answer: d

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86. If a,b,c are distinct integers and $\omega(\neq 1)$ is a cube root of unity, then the minimum value of $\left|a+b\omega+c\omega^2\right|+\left|a+b\omega^2+c\omega\right|$ is

A. (a)$\sqrt{3}$

B. (b)3

C. (c)$6\sqrt{2}$

D. (d)2

Answer: a

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87. If $|z-2i|\leq\sqrt{2}$, where $i=\sqrt{-1}$, then the maximum value of $|3-i(z-1)|$, is

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $2+\sqrt{2}$

D. $3+2\sqrt{2}$

Answer: C

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88. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $Re(z_1 \bar{z}_2) = 0$, then the pair ofcomplex nunmbers $\omega_1 = a + ic$ and $\omega_2 = b + id$ satisfies

A. $|\omega_1| = 1$

B. $|\omega_2| = 1$

C. $Re(\omega_1 \overline{\omega}_2) = -0$

D. None of these

Answer: a,b,c

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89. The complex numbers z_1, z_2, z_3 stisfying $(z_2 - z_3) = (1 + i)(z_1 - z_3). where i = \sqrt{-1},$ are vertices of a triangle which is

A. equilateral

B. isosceles

C. right angled

D. scalene

Answer: b,c

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90. Find the imaginary part of the complex number if $z = (2 - i)(5 + i)$

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91. The equation $z^2 - i|z - 1|^2 = 0,$ where $i = \sqrt{-1},$ has.

A. no real root

B. no purely imaginary root

C. all roots inside $|z| = 1$

D. atleast two roots

Answer: a,b,c

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92. Let z_1, z_2 be two complex numbers represented by points on the circle $|z_1| =$ and and $|z_2| = 2$ are then

A. $\max . |2z_1 + z_2| = 4$

B. $\min . |z_1 + z_2| = 1$

C. $\left|z_2 + \frac{1}{z_1}\right| \leq 3$

D. $\left|z_1 + \frac{2}{z_2}\right| \leq 2$

Answer: a,b,c,d

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93. Consider a quadratic equation $az^2bz + c = 0$, where a,b and c are complex numbers.

The condition that the equation has one purely real root, is

A. `(abarb-barac)(b barc+barbc)+(cbara-barca)^(2)=0

B. $(a\bar{b} + \bar{a}b)(b\bar{c} + \bar{b}c) + (c\bar{a} - \bar{c}a)^2 = 0$

C. $(a\bar{b} - \bar{a}b)(b\bar{c} - \bar{b}c) + (c\bar{a} + \bar{c}a)^2 = 0$

D. $(a\bar{b} + \bar{a}b)(b\bar{c} - \bar{b}c) + (c\bar{a} - \bar{c}a)^2 = 0$

Answer: b

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94. Consider a quadratic equaiton $az^2 + bz + c = 0$, where a,b,c are complex number.

The condition that the equaiton has one purely real roots is

A. $\left(a\bar{b}+\bar{a}b\right)\left(b\bar{c}-\bar{b}c\right)=\left(c\bar{a}+\bar{c}a\right)^2$

B. $\left(a\bar{b}-\bar{a}b\right)\left(b\bar{c}+\bar{b}c\right)=\left(c\bar{a}+\bar{c}a\right)^2$

C. $\left(a\bar{b}-\bar{a}b\right)\left(\bar{b}c-\bar{b}c\right)=\left(c\bar{a}-\bar{c}a\right)^2$

D. $\left(a\bar{b}-\bar{a}b\right)\left(b\bar{c}-\bar{b}c\right)=\left(c\bar{a}+\bar{c}a\right)^2$

Answer: c

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95. Consider the quadratic equation $az^2+bz+c=0$ where a, b, c are non-zero complex numbers. Now answer the following.

The condition that the equation has both roots purely imaginary is

A. $\frac{a}{a}=\frac{b}{b}=\frac{c}{c}$

B. $\frac{a}{a}=\frac{b}{b}=\frac{c}{c}$

C. $\frac{a}{a}=\frac{b}{b}=-\frac{c}{c}$

D. $\frac{a}{a}=-\frac{b}{b}=\frac{c}{c}$

Answer: d

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96. Let Papoint denoting a comples number z on the complex plane.

$$i.\,e. \qquad z = Re(z) + iIm(z), \quad \text{where} \quad i = \sqrt{-1}$$

$$\text{if} \qquad Re(z) = x \ \text{and} \ Im(z) = y, thenz = x + iy$$

If Pmovew such that

$|Re(z)| + \ |\ Im(z) = a\left(a \in R^{+}\right)$

The locus of P is

A. a parallelogram which is not arhombus

B. a rhombus which is not a square

C. a rectangle which is not a square

D. a square

Answer: d

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97. Let P point denoting a complex number z on the complex plane. $i.e.\ z = Re(z) + iIm(z), \quad \text{where} \quad i = \sqrt{-1}$

if $Re(z) = x$ and $Im(z) = y, thenz = x + iy$.The area of the circle inscribed in the region denoted by $|Re(z)| + |Im(z)| = 10$ equal to

A. $50\pi \quad \text{sq units}$

B. $100\pi \quad \text{sq units}$

C. $55 \quad \text{sq units}$

D. $110 \quad \text{sq units}$

Answer: a

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98. Let P point denoting a complex number z on the complex plane. $i.e.\ z = Re(z) + iIm(z), \quad \text{where} \quad i = \sqrt{-1}$

if $Re(z) = x$ and $Im(z) = y, thenz = x + iy$ Number of integral solutions satisfying the eniquality$|Re(z)| + |Im(z)| < 21, . is$

A. 841

B. 839

C. 840

D. 842

Answer: c

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99. If$z_1, z_2 \in C, z_1^2 + z_2^2 \in R, z_1(z_1^2 - 3z_2^2) = 2$ and $z_2(3z_1^2 - z_2^2) = 11,$ the value of $z_1^2 + z_2^2$ is

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100. Consider four complex numbers $z_1 = 2 + 2i$, ,

$z_2 = 2 - 2i, z_3 = -2 - 2i \text{ and } z_4 = -2 + 2i), where i = \sqrt{-1},$

Statement -1 $z_1, z_2, z_3 \text{ and } z_4$

constitute the vertices of a

square on the complex plane because

Statement -2 The non-zero complex numbers $z, \bar{z}, -z, -\bar{z}$

always constitute the vertices of a square.

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101. Consider $z_1 \text{ and } z_2$ are two complex numbers

such that $|z_1 + z_2| = |z_1| + |z_2|$

Statement -1 $amp(z_1) - amp(z_2) = 0$

Statement -2 The complex numbers z_1 and z_2 are collinear.

Check for the above statements.

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102. If $|z - iRe(z)| = |z - Im(z)|$, then prove that z lies on the bisectors of the quadrants, where $i = \sqrt{-1}$.

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103. Find the gratest and the least values of $|z_1 + z_2|$, if $z_1 = 24 + 7i$ and $|z_2| = 6$, where $i = \sqrt{-1}$

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104. If $|z - 1| = 1$, where z is a point on the argand plane, show that $\frac{z-2}{z} = i\tan(argz), wherei = \sqrt{-1}$.

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105. If $arg\left(z^{1/2}\right) = \frac{1}{2}arg\left(z^2 + \bar{z}z^{1/3}\right)$, find the value of $|z|$.

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106. C is the complex numbers $f: C \to R$ is defined by $f(z) = \left|z^3 - z + 2\right|$. Find the maximum value of $f(z)$, $If|z| = 1$.

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107. Prove that the complex numbers z_1 and z_2 and the origin form an isosceles triangle with vertical angle $\frac{2\pi}{3}$, if $z_1^2 + z_2^2 + z_1 z_2 = 0$.

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108. If $\alpha = e^{i2\pi/7} and f(x) = a_0 + \sum_{k=0}^{20} a_k x^k$, then prove that the value of $f(x) + f(\alpha x) + \ldots. + f(\alpha^6 x)$ is independent of α.

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109. Show that all the roots of the equation $a_1 z^3 + a_2 z^2 + a_3 z + a_4 = 3$,

$(where|a_i| \leq 1, i = 1, 2, 3, 4,)$ lie

outside the circle with centre at origin and radius $2/3$.

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110. The points A, B, C represent the complex numbers z_1, z_2, z_3 respectively on a complex plane & the angle $B \& C$ of the triangle ABC are each equal to $\frac{1}{2}(\pi - \alpha)$. If $(z_2 - z_3)^2 = \lambda(z_3 - z_1)(z_1 - z_2)\sin^2\left(\frac{\alpha}{2}\right)$ then determine λ.

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111. If $z = x + iy$ is a complex number with $x, y \in Q and |z| = 1,$ then show that $\left|z^{2n} - 1\right|$ is a rational number for every $n \in N$.

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112. If a is a complex number such that $|a| = 1$, then find the value of a, so that equation $az^2 + z + 1 = 0$ has one purely imaginary root.

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113. If $n \in N > 1$, then the sum of real part of roots of $z^n = (z+1)^n$ is equal to

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114. Among the complex numbers z which satisfies $|z - 25i| \leq 15$, find the complex numbers z having least modulas ?

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115. Two different non-parallel lines cut the circle $|z| = r$ at points a, b, c and d, respectively. Prove that these lines meet at the point z

given by $\dfrac{a^{-1}+b^{-1}-c^{-1}-d^{-1}}{a^{-1}b^{-1}-c^{-1}d^{-1}}$

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116. Find $\dfrac{dy}{dx}$ if $x-y=\cos x$

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117. Show that the triangle whose vertices are $z_1z_2z_3$ and $z_1'z_2'z_3'$ are directly similar , if $\begin{vmatrix} z_1 & z'_1 & 1 \\ z_2 & z'_2 & 1 \\ z_3 & z'_3 & 1 \end{vmatrix}=0$

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118. if ω is the nth root of unity and Z_1, Z_2 are any two complex numbers , then prove that .

$\Sigma_{k=0}^{n-1}\left|z_1+\omega^k z_2\right|^2=n\left\{\left|z_1\right|^2+\left|z_2\right|^2\right\}$ where $n\in N$

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119. If $z_1 + z_2 + z_3 + z_4 = 0$ where $b_i \in R$ such that the sum of no two values being zero and $b_1 z_1 + b_2 z_2 + b_3 z_3 + b_4 z_4 = 0$ where z_1, z_2, z_3, z_4 are arbitrary complex numbers such that no three of them are collinear, prove that the four complex numbers would be concyclic if $|b_1 b_2||z_1 - z_2|^2 = |b_3 b_4||z_3 - z_4|^2$.

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Example

1. about to only mathematics

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2. If

$\theta_i \in [0, \pi/6], i = 1, 2, 3, 4, 5,$ and $\sin\theta_1 z^4 + \sin\theta_2 z^3 + \sin\theta_3 z^2 + \sin\theta_4 z$

show that $\frac{3}{4} < |Z| < 1$.

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3. If z and w are two non-zero complex

numbers such that z=-w.

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4. Find the square roots of the following

(i)$4+3i$

(ii) $-5+12i$

(iii) $8-15i$

(iv) $7-24i\left(\text{ where }, i=\sqrt{-1}\right)$

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5. If ω is a non-real complex cube root of unity, find the values of the following.

(i)ω^{1999}

(ii) ω^{998}

(iii) $\left(\frac{-1+i\sqrt{3}}{2}\right)^{3n+2}, n \in N \text{ and } i = \sqrt{-1}$

(iv) $(1+\omega)(1+\omega)^2(1+\omega)^4(1+\omega)^8$...upto $2n$ factors

(v) $\left(\frac{\alpha+\beta\omega+\gamma\omega^2+\delta\omega^2}{\beta+\alpha\omega^2+\gamma\omega+\delta\omega}\right), \quad \text{where } \alpha, \beta, \gamma, \delta, \in R$

(vi)

$1\cdot(2-\omega)(2-\omega^2)+2\cdot(3-\omega)(3-\omega^2)+3\cdot(4-\omega)(4-\omega^2)+\ldots+\ldots$

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6. If $\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_{n-1}$ are the n, nth

roots of the unity , then find the value of $\sum_{i=0}^{n-1} \frac{\alpha_i}{2-a_i}$.

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7. Let z_1 and z_2 be the roots of the equation $z^2+pz+q=0$, where the coefficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane, respectively. If

$\angle AOB = \theta \neq 0$ and $OA = OB, where\ O$ is the origin, prove that $p^2 = 4q\cos^2(\theta/2)$.

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8. Find the multiplicative inverse of $z = 4 - 3i$

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9. If $z_1 = 2 + 5i,\ z_2 = 3 - i,\ where \quad i = \sqrt{-1}$, find

(i)$Z_1 \cdot Z_2$

(ii) $Z_1 \times Z_2$

(iii) $Z_2 \cdot Z_1$

(iv) $Z_2 \times Z_1$

(v) acute angle between Z_1 and Z_2.

(vi) projection of $Z_1 on Z_2$.

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10. Express in a complex number if $z = (2 - i)(5 + i)$

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Example Single Integer Answer Type Questions

1. The number of solutions of the equations $|z - (4 + 8i)| = \sqrt{10}$ and $|z - (3 + 5i)| + |z - (5 + 11i)| = 4\sqrt{5}$, where$i = \sqrt{-1}$.

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Example Matching Type Questions

1. Express in the complex form if $z = i^{19}$

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Subjective Type Examples

1. Express in the form of complex number $z = (5 - 3i)(2 + i)$

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2. Express in the complex number if $3(7 + 7i) + (i(7 + 7i)$

Watch Video Solution

3. Find the multiplicative inverse of $z = 6 - 3i$

Watch Video Solution

4. Find $\frac{dy}{dx}$ if $ax - by^2 = \cos x$

Watch Video Solution

5. if α and β the roots of $z+\frac{1}{z}=2(\cos\theta+I\sin\theta)$ where $0<\theta<\pi$ and $i=\sqrt{-1}$ show that $|\alpha-i|=|\beta-i|$

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Exercise For Session 1

1. If$(1+i)^{2n}+(1-i)^{2n}=-2^{n+1}(where, i=\sqrt{-1}$ for all those n, which are

A. even

B. odd

C. multiple of 3

D. None of these

Answer:

Watch Video Solution

2. If $i = \sqrt{-1}$, the number of values of i^{-n} for a different $n \in I$ is

A. 1

B. 2

C. 3

D. 4

Answer:

Watch Video Solution

3. If$a > 0 \text{ and } b < 0, then\sqrt{a}\sqrt{b}$ is equal to (where, $i = \sqrt{-1}$)

A. $-\sqrt{a \cdot |b|}$

B. $\sqrt{a \cdot |b|i}$

C. $\sqrt{a \cdot |b|}$

D. none of these

Answer:

Watch Video Solution

4. The value of $\sum_{r=-3}^{1003} i^r (where i = \sqrt{-1})$ is

A. 1

B. -1

C. i

D. $-i$

Answer:

Watch Video Solution

5. The digit in the unit's place of $(153)^{98}$ is

A. 1

B. 3

C. 7

D. 9

Answer:

Watch Video Solution

6. The digit in the unit's place of $(141414)^{12121}$ is

A. 4

B. 6

C. 3

D. 1

Answer:

Watch Video Solution

Exercise For Session 4

1. Find $\frac{dy}{dx}$ if $x^2 + xy = \tan x$

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Exercise For Session 5

1. Express in the complex number $(-5i)\left(\frac{i}{8}\right)$

Watch Video Solution

Exercise For Session 2

1. If $\frac{1-ix}{1+ix} = a - ib$ and $a^2 + b^2 = 1$, $where\ a, b \in R$ and $i = \sqrt{-1}$,

then xis equal to

A. $\frac{2a}{(1+a)^2 + b^2}$

B. $\dfrac{2b}{(1+a)^2+b^2}$

C. $\dfrac{2a}{(1+b)^2+a^2}$

D. $\dfrac{2b}{(1+b)^2+a^2}$

Answer:

Watch Video Solution

2. The least positive integer n for which $\left(\dfrac{1+i}{1-i}\right)^n = \dfrac{2}{\pi}\left(\sec^{-1}\dfrac{1}{x}+\sin^{-1}x\right)$ (where, $X \neq 0, -1 \leq X \leq 1$ and $i = \sqrt{-1}$, is

A. 2

B. 4

C. 6

D. 8

Answer:

Watch Video Solution

3. If $z = (3+4i)^6 + (3-4i)^6$, where $i = \sqrt{-1}$, then Find the value of Im(z) .

A. -6

B. 0

C. 6

D. none of these

Answer:

Watch Video Solution

4. If $(x+iy)^{1/3} = a+ib$, where $i = \sqrt{-1}, then\left(\frac{x}{a} + \frac{y}{b}\right)$ is equal to

A. $4a^2b^2$

B. $4(a^2 - b^2)$

C. $4a^2 - b^2$

D. $a^2 + b^2$

Answer:

Watch Video Solution

5. If

$$\frac{3}{2+\cos\theta+i\sin\theta} = a+ib \text{ where } i=\sqrt{-1} \text{ and } a^2+b^2=\lambda a-3, \text{ th}$$

A. 3

B. 4

C. 5

D. 6

Answer:

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6. about to only mathematics

A. $\frac{1}{2}$

B. 1

C. $\sqrt{2}$

D. 2

Answer:

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7. The complex numbers $\sin x + i \sin 2x$ and $\cos x - i \sin 2x$ are conjugate to each other, for

A. $x = n\pi, n \in I$

B. $x = 0$

C. $x = \left(n + \frac{1}{2}\right), n \in I$

D. 2

Answer:

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8. If $\alpha \text{ and } \beta$ are different complex numbers with $|\beta| = 1, \ f \in d \left| \frac{\beta - \alpha}{1 - \alpha\beta} \right|$

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

Answer:

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9. If $x = 3 + 4i$ find the value of $x^4 - 12x^3 - 70x^2 - 204x + 225$

A. -45

B. 0

C. 35

D. 15

Answer:

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10. If $|z_1 - 1| \leq , |z_2 - 2| \leq 2, |z_3 - 3| \leq 3,$ then find the greatest value of $|z_1 + z_2 + z_3|$

A. 6

B. 12

C. 17

D. 23

Answer:

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11. The principal value of $arg(z)$, where $z = 1 + \cos\left(\frac{8\pi}{5}\right) + i\sin\left(\frac{8\pi}{5}\right)$ (where, $i = \sqrt{-1}$) is given by

A. $-\frac{\pi}{5}$

B. $-\frac{4\pi}{5}$

C. $\frac{\pi}{5}$

D. $\frac{4\pi}{5}$

Answer:

Watch Video Solution

12. If $|z_1| = 2, |z_2| = 3, |z_3| = 4 \text{ and } |z_1 + z_2 + z_3| = 5. \text{ then} |4z_2z_3 + 9z_3z_1 + 1$

is

A. 24

B. 60

C. 120

D. 240

Answer:

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13. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, the find the value of $arg\left(\frac{z_1}{z_4}\right) + arg(z_2 / z_3)$.

A. 0

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$

Answer:

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Exercise For Session 3

1. Find the real part of $(1-i)^{-i}$.

A. $e^{-\pi/4}\cos\left(\frac{1}{2}\log_e 2\right)$

B. $-e^{-\pi/4}\sin\left(\frac{1}{2}\log_e 2\right)$

C. $e^{-\pi/4}\cos\left(\frac{1}{2}\log_e 2\right)$

D. $e^{-\pi/4}\sin\left(\frac{1}{2}\log_e 2\right)$

Answer:

Watch Video Solution

2. The amplitude of $e^{e^{-(i\theta)}}$, where $\theta \in R$ and $i=\sqrt{-1}$, is

A. $\sin\theta$

B. $-\sin\theta$

C. $e^{\cos\theta}$

D. $e^{\sin\theta}$

Answer:

Watch Video Solution

3. If $z = i\log_e\left(2-\sqrt{3}\right)$, where $i = \sqrt{-1}$ then the cos z is equal to

A. i

B. 2i

C. 1

D. 2

Answer:

Watch Video Solution

4. If $z = (i)^{i} \hat{} (((i)))wherei = \sqrt{-1}, then|z|$ is equal to 1 b. $e^{-\pi/2}$ c. $e^{-\pi}$ d. none of these

A. 1

B. $e^{-\pi/2}$

C. $e^{-\pi}$

D. e^{π}

Answer:

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5. $\sqrt{(-8-6i)}$ is equal to (where, $i = \sqrt{-1}$

A. (a)$1 \pm 3i$

B. (b)$\pm(1-3i)$

C. (c)$\pm(1+3i)$

D. (d)$\pm(3-i)$

Answer:

Watch Video Solution

6. Simplify: $\frac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$

A. $-\frac{3}{2}i$

B. $\frac{3}{4}i$

C. $-\frac{3}{4}i$

D. $-\frac{3}{2}$

Answer:

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7. If $0 < amp(z) < \pi, 'then' amp(z) - amp(-z)$` is equal to

A. 0

B. $2amp(z)$

C. π

D. $-\pi$

Answer:

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8. If $|z_1| = |z_2|$ and $amp(z_1) + amp(z_2) = 0,$ then

A. $z_1 = z_2$

B. $\bar{z}_1 = z_2$

C. $z_1 + z_2 = -0$

D. $\bar{z}_1 = \bar{z}_2$

Answer: B

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9. Solve the equation $|z| = z + 1 + 2i$.

A. $2 - \frac{3}{2}i$

B. $\frac{3}{2} + 2i$

C. $\frac{3}{2} - 2i$

D. $-2 + \frac{3}{2}i$

Answer: C

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10. The number of solutions of the equation $z^2 + \bar{z} = 0$ is .

A. 1

B. 2

C. 3

D. 4

Answer: D

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11. If $z_r = \cos\left(\frac{r\alpha}{n^2}\right) + i\sin\left(\frac{r\alpha}{n^2}\right)$, where $r = 1, 2, 3, \ldots, n$ and $i = \sqrt{-1}$, then

is equal to

A. $e^{i\alpha}$

B. $e^{-i\alpha/2}$

C. $e^{i\alpha/2}$

D. $\sqrt[3]{e^{i\alpha}}$

Answer:

Watch Video Solution

12. If $\theta \in R$ and $i = \sqrt{-1}$, then $\left(\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n$ is equal to

A. $\cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$

B. $\cos\left(\frac{n\pi}{2} + n\theta\right) + i\sin\left(\frac{n\pi}{2} + n\theta\right)$

C. $\sin\left(\frac{n\pi}{2} - n\theta\right) + i\cos\left(\frac{n\pi}{2} - n\theta\right)$

D. $\cos\left(n\left(\frac{\pi}{2} + 2\theta\right)\right) + i\sin\left(n\left(\frac{\pi}{2} + 2\theta\right)\right)$

Answer:

Watch Video Solution

13. If $iz^4 + 1 = 0,$ then prove that z can take the value $\cos\pi/8 + is \in \pi/8.$

A. $\frac{1+i}{\sqrt{2}}$

B. $\cos\left(\frac{\pi}{8}\right) + i\sin\left(\frac{\pi}{8}\right)$

C. $\frac{1}{4i}$

D. i

Answer:

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14. If $\omega(\neq 1)$ is a cube root of unity, then $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)$...upto $2n$ is factors, is

A. 2^n

B. 2^{2n}

C. 0

D. 1

Answer:

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15. If α, β, γ are the cube roots of p, then for any x,y,z $\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}$ =

A. $\frac{1}{2}(-1-i\sqrt{3}), i=\sqrt{-1}$

B. $\frac{1}{2}(1+i\sqrt{3}), i=\sqrt{-1}$

C. $\frac{1}{2}(1 - i\sqrt{3}), i = \sqrt{-1}$

D. none of these

Answer:

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Exercise For Session 4

1. If z_1, z_2, z_3 and z_4 are the roots of the equation $z^4 = 1$, the value of $\sum_{i=1}^{4} z_i^3$ is

A. 0

B. 1

C. $i, i = \sqrt{-1}$

D. $1 + i, i = \sqrt{-1}$

Answer: A

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2. If $z_1, z_2, z_3, \ldots\ldots\ldots\ldots\ldots, z_n$ are n nth roots of unity, then for $k = 1, 2, , \ldots\ldots\ldots, n$

A. (a)$|z_k| = k \mid z_{k+1|}$

B. (b)$|z_{k+1}| = k \mid z_{k1|}$

C. (c)$|z_{k+1}| = |zk| + |z_{k-1}|$

D. (d)$|z_k| = |z_{k+1}|$

Answer: D

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3. If $1, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{n-1}$ are n, nth roots of unity, then $(1-\alpha_1)(1-\alpha_2)(1-\alpha_3)\ldots(1-\alpha_{n-1})$ equals to

A. 0

B. 1

C. n

D. n^2

Answer: C

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4. about to only mathematics

A. -1

B. 0

C. $-i$

D. i

Answer:

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5. If α is the nth root of unity then prove that $1+2\alpha+3\alpha^2+\ldots.$ upto n terms $=\frac{-n}{1-\alpha}$

A. $\frac{2n}{1-\alpha}$

B. $-\frac{2n}{1-\alpha}$

C. $\frac{n}{1-\alpha}$

D. $-\frac{n}{1-\alpha}$

Answer:

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6. a and b are real numbers between 0 and 1 such that the points $Z_1=a+i, Z_2=1+bi, Z_3=0$ form an equilateral triangle, then a and b are equal to

A. $a=b=2+\sqrt{3}$

B. $a=b=2-\sqrt{3}$

C. $a = b = -2 - \sqrt{3}$

D. none of these

Answer: B

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7. If $|z| = 2,$ the points representing the complex numbers $-1 + 5z$ will lie on

A. a circle

B. a straight line

C. a parabola

D. an ellipse

Answer:

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8. If $|(z-2)/(z-3)| = 2$ represents a circle, then find its radius.

A. 1

B. $\frac{1}{3}$

C. $\frac{3}{4}$

D. $\frac{2}{3}$

Answer:

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9. If center of a regular hexagon is at the origin and one of the vertices on the Argand diagram is $1+2i$, then its perimeter is $2\sqrt{5}$ b. $6\sqrt{2}$ c. $4\sqrt{5}$ d. $6\sqrt{5}$

A. $2\sqrt{5}$

B. $6\sqrt{2}$

C. $4\sqrt{5}$

D. $6\sqrt{5}$

Answer:

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10. If z is a comlex number in the argand plane, the equation $|z-2|+|z+2|=8$ represents

A. a parabola

B. an ellipse

C. a hyperbola

D. a circle

Answer: D

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11. If $|z-2-3i|+|z+2-6i|=4$ where $i=\sqrt{-1}$ then find the locus of $P(z)$

A. an ellipse

B. ϕ

C. line segment of points $2+3i \text{ and } -26i$

D. none of these

Answer:

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12. locus of the point z satisfying the equation $|z-1|+|z-i|=2$ is

A. a straight line

B. a circle

C. an ellipse

D. a pair of straight lines

Answer:

Watch Video Solution

13. If z, iz and $z + iz$ are the vertices of a triangle whose area is 2units, the value of $|z|$is

A. 1

B. 2

C. 4

D. 8

Answer:

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14. If $\left|z - \frac{4}{z}\right| = 2$ then the greatest value of $|z|$ is:

A. (A) $\sqrt{5}-1$

B. (B) $\sqrt{5}+1$

C. (C) $\sqrt{5}$

D. (D) 2

Answer:

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Exercise Single Option Correct Type Questions

1. if cos (1-i) = a+ib, where a , b $\in$ R and $i=\sqrt{-1}$, then

A. $a=\frac{1}{2}\left(e-\frac{1}{e}\right)\cos 1, b=\frac{1}{2}\left(e+\frac{1}{e}\right)\sin 1$

B. $a=\frac{1}{2}\left(e+\frac{1}{e}\right)\cos 1, b=\frac{1}{2}\left(e-\frac{1}{e}\right)\sin 1$

C. $a=\frac{1}{2}\left(e+\frac{1}{e}\right)\cos 1, b=\frac{1}{2}\left(e+\frac{1}{e}\right)\sin 1$

D. $a=\frac{1}{2}\left(e-\frac{1}{e}\right)\cos 1, b=\frac{1}{2}\left(e-\frac{1}{e}\right)\sin 1$

Answer: B

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2. Number of roots of the equation $z^{10} - z^5 - 992 = 0$ with negative real part is

A. 3

B. 4

C. 5

D. 6

Answer: C

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3. If z and $\bar{z}$ represent adjacent vertices of a regular polygon of n sides where centre is origin and if $\frac{Im(z)}{Re(z)} = \sqrt{2} - 1$, then n is equal to:

A. (A) 8

B. (B) 16

C. (C) 24

D. (D) 32

Answer: D

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4. If $\prod_{p=1}^{r} e^{ip\theta} = 1$, where $\prod$ denotes the continued product and $i = \sqrt{-1}$, the most general value of θ is (where, n is an integer)

A. (a) $\frac{2n\pi}{r(r-1)}, n \in I$

B. (b) $\frac{2n\pi}{r(r+1)}, n \in I$

C. (c) $\frac{4n\pi}{r(r-1)}, n \in I$

D. (d) $\frac{4n\pi}{r(r+1)}, n \in I$

Answer: D

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5. If $(3+i)(z+\bar{z})-(2+i)(z-\bar{z})+14i=0$, where $i=\sqrt{-1}$, then z $\bar{z}$ is equal to

A. (a)10

B. (b)8

C. (c)-9

D. (d)-10

Answer: A

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6. The centre of a square ABCD is at z=0, A is z_1. Then, the centroid of $\triangle ABC$ is (where, $i=\sqrt{-1}$)

A. (a)$z_1(\cos\pi \pm i\sin\pi)$

B. (b) $\frac{z_1}{3}(\cos\pi \pm i\sin\pi)$

C. (c) $z_1\left(\cos\left(\frac{\pi}{2}\right) \pm i\sin\left(\frac{\pi}{2}\right)\right)$

D. (d) $\frac{z_1}{3}\left(\cos\left(\frac{\pi}{2}\right) \pm i\sin\left(\frac{\pi}{2}\right)\right)$

Answer: D

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7. If $z = \frac{\sqrt{3} - i}{2}$, where $i = \sqrt{-1}$, then $\left(i^{101} + z^{101}\right)^{103}$ equals to

A. iz

B. z

C. $\bar{z}$

D. None of these

Answer: B

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8. Let α and β be two fixed non-zero complex numbers and 'z' a variable complex number. If the lines $\alpha\bar{z} + \bar{a}z + 1 = 0$ and $\beta\bar{z} + \bar{\beta}z - 1 = 0$ are mutually perpendicular, then

A. $ab + \bar{a}\bar{b} = 0$

B. $ab - \bar{a}\bar{b} = 0$

C. $\bar{a}b - a\bar{b} = 0$

D. $a\bar{b} + \bar{a}b = 0$

Answer: D

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9. If $\alpha = \cos\left(\frac{8\pi}{11}\right) + i\sin\left(\frac{8\pi}{11}\right)$ then $Re\left(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5\right)$ is

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 0

D. None of these

Answer: B

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10. The set of points in an Argand diagram which satisfy both $|z| \leq 4$ and $0 \leq arg(z) \leq \frac{\pi}{3}$, is

A. (a)a circle and a line

B. (b)a radius of a circle

C. (c)a sector of a circle

D. (d)an infinite part line

Answer: C

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11. If $f(x) = g(x^3) + xh(x^3)$ is divisiblel by $x^2 + x + 1$, then

A. g(x) is divisible by (x-1) but not h(x) but not h(x)

B. h(x) is divisible by (x-1) but not g(x)

C. both g(x) and h(x) are divisible by (x-1)

D. None of above

Answer: C

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12. If the points represented by complex numbers $z_1 = a + ib, z_2 = c + id$ and $z_1 - z_2$ are collinear, where $i = \sqrt{-1}$, then

A. ad+bc=0

B. ad-bc=0

C. ab+cd=0

D. ab-cd=0

Answer: B

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13. Let C and R denote the set of all complex numbers and all real numbers respectively. Then show that $f: C \to R$ given by $f(z) = |z|$ for all $z \in C$ is neither one-one nor onto.

A. f is injective but not surjective

B. f is surjective but not injective

C. f is nither injective nor surjective

D. f is both injective and surjective

Answer: C

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14. Let α and β be two distinct complex numbers, such that $|\alpha| = |\beta|$. If real part of α is positive and imaginary part of β is negative, then the complex number $(\alpha + \beta)/(\alpha - \beta)$ may be

A. zero

B. real and negative

C. real and positive

D. purely imaginary

Answer: D

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15. The complex number z satisfies thc condition $\left|z - \frac{25}{z}\right| = 24$. The maximum distance from the origin of co-ordinates to the points z is

A. 25

B. 30

C. 32

D. None of these

Answer: A

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16. The points A,B and C represent the complex numbers $z_1, z_2, (1-i)z_1 + iz_2$ respectively, on the complex plane (where, $i = \sqrt{-1}$). The $\triangle ABC$, is

A. isosceles but not right angled

B. right angled but not isosceles

C. isosceles and right angled

D. None of the above

Answer: C

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17. The system of equations $|z+1-i|=\sqrt{2}$ and $|z|=3$ has how many solutions?

A. no solution

B. one solution

C. two solution

D. None of these

Answer: A

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18. Dividing f(z) by z-i, we obtain the remainder 1-i and dividing it by z+i, we get the remainder 1+i. Then, the remainder upon the division of f(z) by z^2+1, is

A. i+z

B. 1+z

C. 1-z

D. None of these

Answer: C

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19. The centre of circle represented by $|z+1| = 2|z-1|$ in the complex plane is

A. 0

B. $\frac{5}{3}$

C. $\frac{1}{3}$

D. None of these

Answer: B

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20. If $x = 9^{\frac{1}{3}} 9^{\frac{1}{9}} 9^{\frac{1}{27}} \ldots\ldots ad$ inf $y = 4^{\frac{1}{3}} 4^{-\frac{1}{9}} 4^{\frac{1}{27}} \ldots\ldots ad$ inf and $z = \sum_{r=1}^{\infty} (1+i)^{-r}$ then , the argument of the complex number $w = x + yz$ is

A. 0

B. $-\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

C. $-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

D. $\pi - \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

Answer: B

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21. If center of a regular hexagon is at the origin and one of the vertices on the Argand diagram is $1 + 2i$, then its perimeter is $2\sqrt{5}$ b. $6\sqrt{2}$ c. $4\sqrt{5}$ d. $6\sqrt{5}$

A. $2\sqrt{5}$

B. $4\sqrt{5}$

C. $6\sqrt{5}$

D. $8\sqrt{5}$

Answer: C

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22. Let $|Z_r - r| \leq r, Aar = 1, 2, 3. \dots, n$. Then $\left|\sum_{r=1}^{n} z_r\right|$ is less than

A. n

B. 2n

C. n(n+1)

D. $\dfrac{n(n+1)}{2}$

Answer: C

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23. If $\arg\left(\dfrac{z_1 - \frac{z}{|z|}}{\frac{z}{|z|}}\right) = \dfrac{\pi}{2}$ and $\left|\dfrac{z}{|z|} - z_1\right| = 3$, then $|z_1|$ equals to a. $\sqrt{3}$ b. $2\sqrt{2}$ c. $\sqrt{10}$ d. $\sqrt{26}$

A. $\sqrt{3}$

B. $2\sqrt{2}$

C. $\sqrt{10}$

D. $\sqrt{26}$

Answer: C

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24. about to only mathematics

A. a pair of straight lines

B. circle

C. parabola

D. ellipse

Answer: C

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25. about to only mathematics

A. $\frac{n \cdot 3^{n-1}}{3^n - 1} + \frac{1}{2}$

B. $\frac{n \cdot 3^{n-1}}{3^n - 1} - 1$

C. $\frac{n \cdot 3^{n-1}}{3^n - 1} + 1$

D. None of these

Answer: D

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26. If $z = (3+7i)(\lambda + i\mu)$, when $\lambda, \mu \in I - \{0\}$ and $i = \sqrt{-1}$, is purely imaginary then minimum value of $|z|^2$ is

A. 0

B. 58

C. $\frac{3364}{3}$

D. 3364

Answer: D

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27. Given $z = f(x) + ig(x)$ where $f, g: (0, 1) \to (0, 1)$ are real valued functions. Then which of the following does not hold good?

a. $z = \frac{1}{1-ix} + i\frac{1}{1+ix}$

b. $z = \frac{1}{1+ix} + i\frac{1}{1-ix}$

c. $z = \frac{1}{1+ix} + i\frac{1}{1+ix}$

d. $z = \frac{1}{1-ix} + i\frac{1}{1-ix}$

A. $z=\frac{1}{1-ix}+i\left(\frac{1}{1+ix}\right)$

B. $z=\frac{1}{1+ix}+i\left(\frac{1}{1-ix}\right)$

C. $z=\frac{1}{1+ix}+i\left(\frac{1}{1+ix}\right)$

D. $z=\frac{1}{1-ix}+i\left(\frac{1}{1-ix}\right)$

Answer: B

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28. If $z^3+(3+2i)z+(-1+ia)=0$ has one real roots, then the value of a lies in the interval $(a\in R)$ $(-2,1)$ b. $(-1,0)$ c. $(0,1)$ d. $(-2,3)$

A. (-2,-1)

B. (-1,0)

C. (0,1)

D. (1,2)

Answer: B

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29. If m and n are the smallest positive integers satisfying the relation $\left(2CiS\frac{\pi}{6}\right)^m = \left(4CiS\frac{\pi}{4}\right)^n$, where $i = \sqrt{-1}$, $(m+n)$ equals to

A. (a)60

B. (b)72

C. (c)96

D. (d)36

Answer: B

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30. Number of imaginergy complex numbers satisfying the equation, $z^2 = \bar{z} \cdot 2^{1-|z|}$ is

A. 0

B. 1

C. 2

D. 3

Answer: C

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Exercise More Than One Correct Option Type Questions

1. If $\frac{z+1}{z+i}$ is a purely imaginary number (where$\left(i=\sqrt{-1}\right)$, then z lies on a

A. straight line

B. circle

C. circle with radius = $\frac{1}{\sqrt{2}}$

D. circle passing through the origin

Answer: B::C::D

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2. Find the multiplicative inverse of $z = 2 - 3i$

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3. If the complex numbers is $(1 + ri)^3 = \lambda(1 + i)$, when $i = \sqrt{-1}$, for some real λ, the value of r can be

A. $\cos\frac{\pi}{5}$

B. $\cos ec\frac{3\pi}{2}$

C. $\cot\frac{\pi}{12}$

D. $\tan\frac{\pi}{12}$

Answer: B::C::D

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4. If z $\in$ C, which of the following relation(s) represents a circle on an Argand diagram? (where,$i = \sqrt{-1}$)

A. $|z-1| + |z+1| = 3$

B. $|z-3| = 2$

C. $|z-2+i| = \frac{7}{3}$

D. $(z-3+i)(\bar{z}-3-i) = 5$

Answer: B::C::D

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5. about to only mathematics

A. 1+ω

B. -1

C. 0

D. 1

Answer: A::C::D

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6. If z is a complex number which simultaneously satisfies the equations $3|z-12| = 5|z-8i|$ and $|z-4| = |z-8|$, where $i = \sqrt{-1}$, then Im(z) can be

A. 8

B. 17

C. 7

D. 15

Answer: A::B

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7. If $P(z_1), Q(z_2), R(z_3)$ and $S(z_4)$ are four complex numbers representing the vertices of a rhombus taken in order on the complex plane, which one of the following is held good?

A. $\dfrac{z_1 - z_4}{z_2 - z_3}$ is purely real

B. $\dfrac{z_1 - z_3}{z_2 - z_4}$ is purely imaginary

C. $|z_1 - z_3| \neq |z_2 - z_4|$

D. $amp\left(\dfrac{z_1 - z_4}{z_2 - z_4}\right) \neq amp\left(\dfrac{z_2 - z_4}{z_3 - z_4}\right)$

Answer: A::B::C

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8. If $a|z - 3| = \min\{|z1, |z - 5|\}, thenRe(z)$ equals to 2 b. $\dfrac{5}{2}$ c. $\dfrac{7}{2}$ d. 4

A. 2

B. 2.5

C. 3.5

D. 4

Answer: A::D

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9. about to only mathematics

A. $|z| = a$

B. $|z| = 2a$

C. $arg(z) = \frac{\pi}{3}$

D. $arg(z) = \frac{\pi}{2}$

Answer: A::C

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10. If z=x+iy, where $i = \sqrt{-1}$, then the equation $\left|\left(\frac{2z - i}{z + 1}\right)\right| = m$ represents a circle, then m can be

A. $\frac{1}{2}$

B. 1

C. 2

D. $\in (3, 2\sqrt{3})$

Answer: A::B::D

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11. Equation of tangent drawn to circle $|z| = r$ at the point $A(z_0)$, is

A. $Re\left(\frac{z}{z_0}\right) = 1$

B. $Im\left(\frac{z}{z_0}\right) = 1$

C. $Im\left(\frac{z_0}{z}\right) = 1$

D. $z\overline{z_0} + z_0\bar{z} = 2r^2$

Answer: A::D

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12. z_1 and z_2 are the roots of the equaiton $z^2 - az + b = 0$ where $|z_1| = |z_2| = 1$ and a,b are nonzero complex numbers, then

A. (a)$|a| \leq 1$

B. (b)$|a| \leq 2$

C. (c)$arg(a) = arg(b^2)$

D. (d)$arg(a^2) = arg(b)$

Answer: B::D

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13. If α is a complex constant such that $az^2 + z + \alpha = 0$ has a ral root, then $\alpha + \alpha = 1\ \alpha + \alpha = 0\ \alpha + \alpha = \ -1$ the absolute value of the real root is 1

A. $\alpha + \overline{\alpha} = 1$

B. $\alpha + \overline{\alpha} = 0$

C. $\alpha + \overline{\alpha} = \ -1$

D. the absolute value of real root is 1

Answer: A::C::D

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14. If the equation $z^3 + (3+i)z^2 - 3z - (m+i) = 0$, where $i = \sqrt{-1}$ and $m \in R$, has atleast one real root, value of m is

A. 1

B. 2

C. 3

D. 5

Answer: A::D

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15. If $z^3+(3+2i)z+(-1+ia)=0$ has one real roots, then the value of a lies in the interval $(a \in R)$ $(-2,1)$ b. $(-1,0)$ c. $(0,1)$ d. $(-2,3)$

A. (-2,1)

B. (-1,0)

C. (0,1)

D. (-2,3)

Answer: A::B::D

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Exercise Passage Based Questions

1.

$$arg(\bar{z}) + arg(-z) = \begin{cases} \pi, & \text{if } \arg(\text{z}) < 0 \\ -\pi, & \text{if } \arg(\text{z}) > 0 \end{cases}, \text{where } -\pi < arg(z) \leq \pi$$

.

If $arg(z) > 0$, then arg (-z)-arg(z) is equal to

A. $-\pi$

B. $-\frac{\pi}{2}$

C. $\frac{\pi}{2}$

D. π

Answer: A

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2. Find $\frac{dy}{dx}$ if $x + y^2 = \tan x + y$

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3.

$$arg(\bar{z}) + arg(-z) = \begin{cases} \pi, & \text{if arg (z)} & < 0 \\ -\pi, & \text{if arg (z)} & > 0 \end{cases}, \text{ where } -\pi < arg(z) \leq \pi$$

.

If $arga(4z_1) - arg(5z_2) = \pi, \quad \text{then } \left|\frac{z_1}{z_2}\right|$ is equal to

A. 1

B. 1.25

C. 1.5

D. 2.5

Answer: B

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4. Sum of four consecutive powers of i(iota) is zero.

i.e.,$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \ \forall n \in I.$

If $\sum_{n=1}^{25} i^{n!} = a + ib, \quad \text{where} \quad i = \sqrt{-1}$, then a-b, is

A. prime number

B. even number

C. composite number

D. perfect number

Answer: A

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5. Sum of four consecutive powers of i(iota) is zero.

i.e.,$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \ \forall n \in I.$

If $\sum_{r=-2}^{95} i^r + \sum_{r=0}^{50} i^{r!} = a + ib, \quad \text{where} \quad i = \sqrt{-1},$ the unit digit of $a^{2011} + b^{2012}$, is

A. (a)2

B. (b)3

C. (c)5

D. (d)6

Answer: C

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6. Sum of four consecutive powers of i(iota) is zero.

i.e.,$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \forall n \in I.$

If $\sum_{r=4}^{100} i^{r!} + \prod_{r=1}^{101} i^r = a + ib, \quad \text{where} \quad i = \sqrt{-1}$, then a+75b, is

A. 11

B. 22

C. 33

D. 44

Answer: B

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7. For any two complex numbers $z_1 \text{and} z_2$,

$$|z_1 - z_2| \geq \begin{cases} |z_1| - |z_2| \\ |z_2| - |z_1| \end{cases}$$

and equality holds iff origin $z_1 \text{ and } z_2$ are collinear and z_1, z_2 lie on the same side of the origin .

If $\left|z - \frac{1}{z}\right| = 2$ and sum of greatest and least values of $|z|$ is λ, then λ^2, is

A. 2

B. 4

C. 6

D. 8

Answer: D

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8. For any two complex numbers $z_1 \text{and} z_2$, $|z_1 - z_2| \geq \begin{cases} |z_1| - |z_2| \\ |z_2| - |z_1| \end{cases}$ and equality holds iff origin $z_1 \text{ and } z_2$ are collinear and z_1, z_2 lie on the same side of the origin . If $\left|z-\frac{2}{z}\right|=4$ and sum of greatest and least values of $|z|$ is λ, then λ^2, is

A. 12

B. 18

C. 24

D. 30

Answer: C

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9. For any two complex numbers $z_1 \text{ and } z_2$,

$$|z_1-z_2| \geq \begin{cases} |z_1|-|z_2| \\ |z_2|-|z_1| \end{cases}$$

and equality holds iff origin z_1 and z_2 are collinear and z_1, z_2 lie on the same side of the origin .

If $\left|z-\frac{3}{z}\right|=6$ and sum of greatest and least values of $|z|$ is 2λ, then λ^2, is

A. 12

B. 18

C. 24

D. 30

Answer: A

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10. Consider the two complex numbers z and w, such that

$$w = \frac{z-1}{z+2} = a + ib, \quad \text{where} \quad a, b \in R \quad \text{and} \quad i = \sqrt{-1}.$$

If $z = CiS\theta$, which of the following does hold good?

A. $\sin\theta = \dfrac{9b}{1-4a}$

B. $\cos\theta = \dfrac{1-5a}{1+4a}$

C. $(1+5a)^2 + (3b)^2 = (1-4a)^2$

D. All of these

Answer: C

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11. Express in the complex form $z = (7 - i)(2 + i)$

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12. Express in the complax form if $z = (4 - 3i)(2 + i)$

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Exercise Single Integer Answer Type Questions

1. The number of values of z (real or complex) e simultaneously satisfying the system of equations $1 + z + z^2 + z^3 + \dots z^{17} = 0$ and $1 + z + z^2 + z^3 + .. + z^{13} = 0$ is

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2. Number of complex numbers satisfying $z^3 = \bar{z}$ is

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3. Let z=9+ai, where $i = \sqrt{-1}$ and a be non-zero real.

If $Im(z^2) = Im(z^3)$, sum of the digits of a^2 is

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4. Numbers of complex numbers z, such that $|z| = 1$

and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$ is

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5. If $x = a + bi$ is a complex number such that $x^2 = 3 + 4i$ and $x^3 = 2 + 1i, where i = \sqrt{-1}, then (a + b)$ equal to

_____.

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6. If $z=\frac{\pi}{4}(1+i)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i}+\frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right), then\left(\frac{|z|}{amp(z)}\right)$ equal

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7. Suppose A is a complex number and $n \in N$, such that $A^n=(A+1)^n=1,$ then the least value of n is 3 b. 6 c. 9 d. 12

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8. Let $z_r, r=1, 2, 3, \ldots, 50$ be the roots of the equation $\sum_{r=0}^{50}(z)^r=0$. If $\sum_{r=1}^{50}\frac{1}{z_r-1}=-5\lambda$, then λ equals to

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9. If $p=\sum_{p=1}^{32}(3p+2)\left(\sum_{q=1}^{10}\left(\sin\frac{2q\pi}{11}-i\cos\frac{2q\pi}{11}\right)\right)^{p}$, where $i=\sqrt{-1}$

and if (1+i)P=n(n!),n $\in$ N, then the value of n is

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10. Find the least positive integer n for which $\left(\frac{1+i}{1-i}\right)^{n}$

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Complex Number Exercise 5

1.

	Column I		Column II
(A)	If $\left\lvert z-\frac{1}{z}\right\rvert=2$ and if greatest and least values of $\lvert z\rvert$ are G and L respectively, then $G-L$, is	(p)	natural number
(B)	If $\left\lvert z+\frac{2}{z}\right\rvert=4$ and if greatest and least values of $\lvert z\rvert$ are G and L respectively, then $G-L$, is	(q)	prime number
(C)	If $\left\lvert z-\frac{3}{z}\right\rvert=6$ and if greatest and least values of $\lvert z\rvert$ are G and L respectively, then $G-L$, is	(r)	composite number
		(s)	perfect number

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	Column I		Column II
(A)	If $\sqrt{(6+8i)}+\sqrt{(-6+8i)}=z_1, z_2, z_3, z_4$ (where $i=\sqrt{-1}$), then $\|z_1\|^2+\|z_2\|^2\|+\|z_3\|^2+\|z_4\|^2$ is divisible by	(p)	7
(B)	If $\sqrt{(5-12i)}+\sqrt{(-5-12i)}=z_1, z_2, z_3, z_4$ (where $i=\sqrt{-1}$), then $\|z_1\|^2+\|z_2\|^2+\|z_3\|^2+\|z_4\|^2$ is divisible by	(q)	8
(C)	If $\sqrt{(8+15i)}+\sqrt{(-8-15i)}=z_1, z_2, z_3, z_4$(where $i=\sqrt{-1}$), then $\|z_1\|^2+\|z_2\|^2+\|z_3\|^2+\|z_4\|^2$ is divisible by	(r)	13
		(s)	17

2.

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	Column I		Column II
(A)	If λ and μ are the unit's place digits of $(143)^{861}$ and $(5273)^{1358}$ respectively, then $\lambda+\mu$ is divisible by	(p)	2
(B)	If λ and μ are the unit's place digits of $(212)^{7820}$ and $(1322)^{1594}$ respectively, then $\lambda+\mu$ is divisible by	(q)	3
(C)	If λ and μ are the unit's place digits of $(136)^{786}$ and $(7138)^{13491}$ respectively, then $\lambda+\mu$ is divisible by	(r)	4
		(s)	5
		(t)	6

3.

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Exercise Statement I And Ii Type Questions

1. **Statement-1** $3+7i > 2+4i, \quad \text{where} \quad i=\sqrt{-1}.$

Statement-2 $3 > 2 \quad \text{and} \quad 7 > 4$

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2. Which statement is correct.?

statement-1 $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta, i=\sqrt{-1}$

statement-2 $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^2 = i$

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3. **statement-1** Locus of z satisfying the equation $|z-1|+|z-8|=5$ is an ellipse.

statement-2 Sum of focal distances of any point on ellipse is constant for an ellipse.

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4. Let z_1, z_2 and z_3 be three complex numbers in AP.

Statement-1 Points representing z_1, z_2 and z_3 are collinear

Statement-2 Three numbers a,b and c are in AP, if b-a=c-b

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5. Statement-1 If the principal argument of a complex numbere z is 0, the principal argument of z^2 is 2θ.

Statement-2 $arg(z^2) = 2arg(z)$

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Complex Number Exercise 6

1. statement-1 Let z_1, z_2 and z_3 be htree complex numbers, such that

$|3z_1+1|=|3z_2+1|=|3z_3+1|$ and $1+z_1+z_2+z_3=0,$ then z_1, z_2, z_3

will represent vertices of an equilateral triangle on the complex plane.

statement-2 z_1, z_2, z_3 represent vertices of an triangle, if

$z_1^2+z_2^2+z_3^2+z_1z_2+z_2z_3+z_3z_1=0$

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Exercise Subjective Type Questions

1. If z_1, z_2, z_3 are any three complex numbers on Argand plane, then $z_1(Im(\bar{z}_2z_3))+z_2(Im\bar{z}_3z_1))+z_3(Im\bar{z}_1z_2))$ is equal to

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2. The roots z_1, z_2, z_3 of the equation $x^3+3ax^2+3bx+c=0$ in which a, b, c are complex numbers correspond to points A, B, C. Show triangle will be an equilateral triangle if $a^2=b$.

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3. If $1, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots $x^5 - 1 = 0,$ then value of $\frac{\omega - \alpha_1}{\omega^2 - \alpha_1} \cdot \frac{\omega - \alpha_2}{\omega^2 - \alpha_2} \cdot \frac{\omega - \alpha_3}{\omega^2 - \alpha_3} \cdot \frac{\omega - \alpha_4}{\omega^2 - \alpha_4}$ is (where ω is imaginary cube root of unity)

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4. If $z_1 and z_2$ both satisfy $z + \bar{z}r = 2|z - 1|$ and $arg(z_1 - z_2) = \frac{\pi}{4}$, then find I m(z_1+z_2) .

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5. For every real number c $\geq$ 0, find all complex numbers z which satisfy the equation $|z|^2 - 2iz + 2c(1 + i) = 0$, where $i = \sqrt{-1}$ and passing through (-1,4).

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6. Express in the complex form if $(5i)\left(\dfrac{-3i}{5}\right)$

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7. Find the point of intersection of the curves $arg(z-3i)=\dfrac{3\pi}{4} and arg(2z+1-2i)=\pi/4.$

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8. Show that if a and b are real, the principal value of arg a is 0 or π according as a is positive or negative and that of bi is $\dfrac{\pi}{2}$ or $-\dfrac{\pi}{2}$ according as b is positive or negative.

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9. If $|z|\leq 1$ and $|\omega|\leq 1$, show that $|z-\omega|^2\leq(|z|-|\omega|)^2+(argz-arg\omega)^2$

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10. If $z_1 and z_2$ are two complex numbers and $c > 0$, then prove that $|z_1 + z_2|^2 \leq (1 + c)|z_1|^2 + (1 + c^{-1})|z_2|^2$.

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11. Find the circumstance of the triangle whose vertices are given by the complex numbers z_1, z_2 and z_3.

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12. Find the circumstance of the triangle whose vertices are given by the complex numbers z_1, z_2 and z_3.

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Complex Number Exercise 7

1. Find $\frac{dy}{dx}$ if $y = \cos(\sin x)$

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2. Express in the form of complax number $z = (2 - i)(3 + i)$

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3. Two different non-parallel lines meet the circle $|z| = r$. One of them at points a and b and the other which is tangent to the circle at c. Show that the point of intersection of two lines is $\frac{2c^{-1} - a^{-1} - b^{-1}}{c^{-2} - a^{-1}b^{-1}}$.

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4. A,B and C are the points respectively the complex numbers z_1, z_2 and z_3 respectivley, on the complex plane and the circumcentre of $\triangle ABC$ lies at the origin. If the altitude of the triangle through the vertex. A meets the circumcircle again at P, prove that P represents the complex number $\left(-\frac{z_2 z_3}{z_1}\right)$.

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5. Let z, z_0 be two complex numbers. It is given that $|z| = 1$ and the numbers $z, z_0, z = (0), 1$ and 0 are represented in an Argand diagram by the points P,P_0,Q,A and the origin, respectively. Show that $\triangle POP_0$ and $\triangle AOQ$ are congruent. Hence, or otherwise, prove that

$|z - z_0| = |z\overline{z_0} - 1| = |z\overline{z_0} - 1|$.

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6. Express in a complex form if $z = i^7$

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7. Let a, b and c be any three nonzero complex number. If $|z| = 1$ and $'z'$ satisfies the equation $az^2 + bz + c = 0$, prove that $a.\,\bar{a} = c.\,\bar{c}$ and |a||b|=$\sqrt{ac(\bar{b})^2}$

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8. Let z_1, z_2 and z_3 be three non-zero complex numbers and $z_1 \neq z_2$. If $\begin{vmatrix} |z_1| & |z_2| & |z_3| \\ |z_2| & |z_3| & |z_1| \\ |z_3| & |z_1| & |z_2| \end{vmatrix} = 0$, prove that

(i) z_1, z_2, z_3 lie on a circle with the centre at origin.

(ii)$arg\left(\frac{z_3}{z_2}\right) = arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)^2$.

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9. Prove that the roots of the equation

$8x^3 - 4x^2 - 4x + 1 = 0$ are $\cos\frac{\pi}{7}, \cos\frac{3\pi}{7}$ and $\cos\frac{5\pi}{7}$.

Evaluate $\sec\frac{\pi}{7} + \sec\frac{3\pi}{7} + \sec\frac{5\pi}{7}$

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10. If the complex number z is to satisfy $|z|=3, |z-\{a(1+i)-i\}| \leq 3$ and $|z+2a-(a+1)i| > 3,$ where $i=\sqrt{-1}$ simultaneously for atleast one z, then find all $a \in R.$

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11. Find $\frac{dy}{dx}$ if $x-5y=\tan y$

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Exercise Questions Asked In Previous 13 Years Exam

1. If ω is a cube root of unity but not equal to 1, then minimum value of $|a+b\omega+c\omega^2|$, (where a,b and c are integers but not all equal), is

A. 0

B. $\frac{\sqrt{3}}{2}$

C. 1

D. 2

Answer: C

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2. If one of the vertices of the square circumscribing the circle $|z-1|=\sqrt{2}$ is $2+\sqrt{3}\iota$. Find the other vertices of square

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3. If $z_1 and z_2$ are two nonzero complex numbers such that = $|z_1+z_2|=|z_1|+|z_2|$, then $arg z_1 - arg z_2$ is equal to $-\pi$ b. $\frac{\pi}{2}$ c. 0 d. $\frac{\pi}{2}$ e. π

A. $-\pi$

B. $-\pi/2$

C. $\pi/2$

D. 0

Answer: D

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4. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x-1)^3+8=0$ are a. $-1, 1+2\omega, 1+2\omega^2$ b. $-1, 1-2\omega, 1-2\omega^2$ c. $-1, -1, -1$ d. none of these

A. $-1, 1+2\omega, 1+2\omega^2$

B. $-1, 1-2\omega, 1-2\omega^2$

C. $-1-1-1$

D. None of these

Answer: B

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5. If $\omega = z/[z - (1/3)i]$ and $|\omega| = 1$, then find the locus of z.

A. a straight line

B. a parabola

C. an ellipse

D. a circle

Answer: A

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6. If $w = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w - \overline{w}z}{1 - z}\right)$ is a purely real, then the set of values of z is $|z| = 1, z \neq 2$ (b) $|z| = 1 and z \neq 1$ (c)$z = \bar{z}$ (d) None of these

A. $\{z : |z| = 1\}$

B. $\{z : z = \bar{z}\}$

C. $\{z : z \neq 1\}$

D. $\{z : |z| = 1, z \neq 1\}$

Answer: D

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7. Find the value of $\sum_{k=1}^{10}\left[\sin\left(\frac{2\pi k}{11}\right) - i\cos\left(\frac{2\pi k}{11}\right)\right], where i = \sqrt{-1}.$

A. i

B. 1

C. -1

D. $-i$

Answer: D

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8. If $z^2+z+1=0$ where z is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\ldots.+\left(z^6+\frac{1}{z^6}\right)^2$ is

A. 18

B. 54

C. 6

D. 12

Answer: D

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9. A man walks a distance of 3 units from the origin towards the North-East $(N45^0E)$ direction.From there, he walks a distance of 4 units towards the North-West $(N45^0W)$ direction to reach a point P. Then, the position of P in the Argand plane is (a) $3e^{\frac{i\pi}{4}}+4i$ (b) $(3-4i)e^{\frac{i\pi}{4}}$ $(4+3i)e^{\frac{i\pi}{4}}$ (d) $(3+4i)e^{\frac{i\pi}{4}}$

A. $3e^{i\pi/4} + 4i$

B. $(3 - 4i)e^{i\pi/4}$

C. $(4 + 3i)e^{i\pi/4}$

D. $(3 + 4i)e^{i\pi/4}$

Answer: D

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10. If $|z| = 1 and z \neq \pm 1,$ then all the values of $\frac{z}{1 - z^2}$ lie on a line not passing through the origin $|z| = \sqrt{2}$ the x-axis (d) the y-axis

A. a line not passing through the origin

B. $|z| = \sqrt{2}$

C. the X-axis

D. the Y-axis

Answer: D

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11. If $|z+4| \leq 3$, the maximum value of $|z+1|$ is

A. 4

B. 10

C. 6

D. 0

Answer: C

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12. Let A, B, C be three sets of complex number as defined below:

$A = \{z : Im \geq 1\}, B = \{z : |z-2-i| = 3\}, C : \{z : Re((1-i)z) = \sqrt{2}\}$

The number of elements in the set $A \cap B \cap C$ is

A. 0

B. 1

C. 2

D. ∞

Answer: B

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13. Let A,B and C be three sets of complex numbers as defined below:

$$A = \{z: Im(z) \geq 1\}$$
$$B = \{z: |z - 2 - i| = 3\}$$
$$C = \{z: Re(1 - i)z) = 3\sqrt{2} \text{where} i = \sqrt{-1}$$

Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

A. 25 and 29

B. 30 and 34

C. 35 and 39

D. 40 and 44

Answer: C

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14. Express in the form of complex number $i^9 + i^{19}$

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15. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by $6 + 7i$ (b) $-7 + 6i$ $7 + 6i$ (d) $-6 + 7i$

A. 6+7i

B. $-7 + 6i$

C. 7+6i

D. $-6+7i$

Answer: D

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16. If the conjugate of a complex numbers is $\frac{1}{i-1}$, where $i=\sqrt{-1}$. Then, the complex number is

A. $\frac{-1}{i-1}$

B. $\frac{1}{i+1}$

C. $\frac{-1}{i+1}$

D. $\frac{1}{i-1}$

Answer: C

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17. Let $z = x + iy$ be a complex number where x and y are integers. Then ther area of the rectangle whose vertices are the roots of the equaiton $\bar{z}z^3 + z\bar{z}^3 = 350.$

A. 48

B. 32

C. 40

D. 80

Answer: A

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18. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m \to 1-15} Img(z^{2m-1})$ at $\theta = 2^\circ$ is:

A. $\dfrac{1}{\sin 2^\circ}$

B. $\dfrac{1}{3\sin 2^\circ}$

C. $\dfrac{1}{2\sin 2^{\circ}}$

D. $\dfrac{1}{4\sin 2^{\circ}}$

Answer: D

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19. If $\left|z - \dfrac{4}{z}\right| = 2$ then the greatest value of $|z|$ is:

A. $2 + \sqrt{2}$

B. $\sqrt{3} + 1$

C. $\sqrt{5} + 1$

D. 2

Answer: C

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20. Let z_1 and z_2 be two distinct complex numbers and $z = (1-t)z_1 + tz_2$, for some real number t with $0 < t < 1$ and $i = \sqrt{-1}$. If arg(w) denotes the principal argument of a non-zero compolex number w, then

A. $|z - z_1| + |z - z_2| = |z_1 - z_2|$

B. $arg(z - z_1) = arg(z - z_2)$

C. $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

D. $arg(z - z_1) = arg(z_2 - z_1)$

Answer: A:B:C::D

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21. about to only mathematics

A. 0

B. 1

C. 2

D. 3

Answer: B

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22. If α and β are the roots of the equation x^2-$x+1=0$, then $\alpha^{2009}+\beta^{2009}=$ (1) 4 (2) 3 (3) 2 (4) 1

A. -1

B. 1

C. 2

D. -2

Answer: B

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23. The number of complex numbers z such that $|z-1|=|z+1|=|z-i|$ is

A. 1

B. 2

C. ∞

D. 0

Answer: A

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24. If z is any complex number satisfying $|z-3-2i| \leq 2$, where $i=\sqrt{-1}$, then the minimum value of $|2z-6+5i|$, is

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25. The set $\left\{Re\left(\frac{2iz}{1-z^2}\right): zisacomplex\nu mber, |z|=1, z=\pm 1\right\}$ is________.

A. $(-\infty, -1] \cap [1, \infty)$

B. $(-\infty, 0) \cup (0, \infty)$

C. $(-\infty, -1] \cup [1, \infty)$

D. $[2, \infty)$

Answer: A

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26. The maximum value of $\left|arg\left(\frac{1}{1-z}\right)\right| f$ or $|z|=1, z\neq 1$ is given by.

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer: C

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27. about to only mathematics

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28. Let α and β be real numbers and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct non-real roots with Re(z)=1, then it is necessary that

A. $\beta \in (-1, 0)$

B. $|\beta| = 1$

C. $\beta \in (1, \infty)$

D. $\beta \in (0, 1)$

Answer: C

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29. If ω is a cube root of unity and $(1+\omega)^7 = A + B\omega$ then find the values of A and B`

A. (1,1)

B. (1,0)

C. (-1,1)

D. (0,1)

Answer: A

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30. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + z + 1$ is real. Then a cannot take the value.

A. -1

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Answer: D

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31. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

A. on a circle with centre at the origin

B. either on the real axis or on a circle not passing through the origin

C. on the imaginary axis

D. either on the real axis or on a circle passing through the origin

Answer: D

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32. If z is complex number of unit modulus and argument θ then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals

A. $\frac{\pi}{2}-\theta$

B. θ

C. $\pi-\theta$

D. $-\theta$

Answer: B

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33. Let complex numbers α and $\frac{1}{\alpha}$ lies on circle $(x-x_0)^2+(y-y_0)^2=r^2$ and $(x-x_0)^2+(y-y_0)^2=4r^2$ respectively. If $z_0=x_0+iy_0$ satisfies the equation $2|z_0|^2=r^2+2$ then $|\alpha|$ is equal to

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{7}}$

D. $\frac{1}{3}$

Answer: C

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34. Let w = $\left(\frac{\sqrt{3}}{2} + \frac{\iota}{2}\right)$ and $P = \{w^n : n = 1, 2, 3, \ldots..\}$, Further $H_1 = \left\{z \in C : Re(z) > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in c : Re(z) < -\frac{1}{2}\right\}$

Where C is set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represent the origin, then $\angle Z_1 O Z_2$ =

A. $\frac{\pi}{2}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: C

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35. Express in the form of complex number if $z = i^{-39}$

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36. Express in the form of complex number $(1 - i)^4$

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37. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$ (1) is equal to $\frac{5}{2}$ (2) lies in the interval (1, 2) (3) is strictly greater than $\frac{5}{2}$ (4) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

A. is strictly greater than $\frac{5}{2}$

B. is equal to $\frac{5}{2}$

C. is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

D. lies in the interval (1,2)

Answer: D

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38. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1 \bar{z_2}}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a

A. circle of radius z

B. circle of radius $\sqrt{2}$

C. straight line parallel to X-axis

D. straight line parallel to y-axis

Answer: A

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39. Let $\omega \neq 1$ be a complex cube root of unity. If $\left(3-3\omega+2\omega^2\right)^{4n+3}+\left(2+3\omega-3\omega^2\right)^{4n+3}+\left(-3+2\omega+3\omega^2\right)^{4n+3}=0$, then the set of possible value(s) of n is are

A. 1

B. 2

C. 3

D. 4

Answer: A::B::D

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40. For any integer k, let $\alpha_k = \frac{\cos(k\pi)}{7} + i\frac{\sin(k\pi)}{7}, wherei = \sqrt{-1}$.

Value of the expression $\frac{\sum k = 112|\alpha_{k+1} - \alpha_k|}{\sum k = 13|\alpha_{4k-1} - \alpha_{4k-2}|}$ is

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41. A value of θ for which $\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ purely imaginary, is

A. $\frac{\pi}{6}$

B. $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

C. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D. $\frac{\pi}{3}$

Answer: C

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42. Let $0 \neq a, 0 \neq b \in R$. Suppose $S = \left\{ z \in C, z = \frac{1}{a + ibt} t \in R, t \neq 0 \right\}$, where $i = \sqrt{-1}$. If $z = x + iy$ and $z \in S$, then (x, y) lies on

A. the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0, b \neq 0$

B. the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$

C. the X-axis for $a \neq 0, b = 0$

D. the Y-axis for $a = 0, b \neq 0$

Answer: A::C::D

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43. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If $|(1, 1, 1), (1, -\omega^2 - 1, \omega^2), (1, \omega^2, \omega^7)| = 3k$, then k is equal to

A. 1

B. $-z$

C. z

D. -1

Answer: B

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Complex Number Exercise 8

1. Match the statements in **Column I** with those in **Column II**.

	Column I		Column II
(A)	The set of points z satisfying $\lvert z - i\lvert z\rvert\rvert = \lvert z + i\lvert z\rvert\rvert$, where $i = \sqrt{-1}$, is contained in or equal to	(p)	an ellipse with eccentricity 4/5
(B)	The set of points z satisfying $\lvert z+4\rvert + \lvert z-4\rvert = 10$ is contained in or equal to	(q)	the set of points z satisfying $\text{Im}(z) = 0$
(C)	If $\lvert w\rvert = 2$, the set of points $z = w - \frac{1}{w}$ is contained in or equal to	(r)	the set of points z satisfying $\lvert \text{Im}(z)\rvert \le 1$
(D)	If $\lvert w\rvert = 1$, the set of points $z = w + \frac{1}{w}$ is contained in or equal to	(s)	the set of points satisfying $\lvert \text{Re}(z)\rvert \le 2$
		(t)	the set of points z satisfying $\lvert z\rvert \le 3$

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2. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$,k=1,2,...,9. Then match the column

	Column I		Column II
(A)	For each z_k there exists a z_j such that $z_k \cdot z_j = 1$	(1)	True
(B)	There exists a $k \in \{1, 2, \ldots, 9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers	(2)	False
(C)	$\frac{\lvert 1 - z_1 \rvert \lvert 1 - z_2 \rvert \ldots \lvert 1 - z_9 \rvert}{10}$ equals to	(3)	1
(D)	$1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals to	(4)	2

A. $\begin{array}{ccccc} & A & B & C & D \\ (a) & 1 & 2 & 4 & 3 \end{array}$

B. $\begin{array}{ccccc} & A & B & C & D \\ (b) & 2 & 1 & 3 & 4 \end{array}$

C. $\begin{array}{ccccc} & A & B & C & D \\ (b) & 1 & 2 & 3 & 4 \end{array}$

D. $\begin{array}{ccccc} & A & B & C & D \\ (b) & 2 & 1 & 4 & 3 \end{array}$

Answer: C

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