MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

DEFINITE INTEGRAL

Example

1. Evaluate $\int_0^1 \frac{1}{3+4x} dx$

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2. Find the value of $\int_{-1}^{1} \frac{d}{dx}\left(\tan^{-1}\left(\frac{1}{x}\right)\right) dx$

A. $\pi/2$

B. $\pi/4$

C. $-\pi/2$

D. None of these

Answer: C

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3. If $I_n = \int (\text{In } x)^n dx, \quad \text{then } I_n + nI_{n-1} =$

A. $\frac{1}{e}$

B. e

C. $e - 1$

D. None of these

Answer: B

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4. All the values of a for which $\int_1^2 \left[a^2 + (4 - 4a)x + 4x^3\right] dx \leq 12$ are given by (A) $a = 3$ (B) $a \leq 4$ (C) $0 \leq a < 3$ (D) none of these

A. $a = 3$

B. $a \leq 4$

C. $0 \leq a < 3$

D. None of these

Answer: A

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5. Evaluate $\int_0^3 |(x-1)(x-2)| dx.$

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6. Evaluate: $\int_0^{\pi/2} \frac{1}{\left(a^2\cos^2 x + b^2\sin^2 x\right)^2} dx$

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7. Evaluate $\int_{-2}^{2} \frac{dx}{4+x^2}$ directly as well as by

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8. Evaluate: $\int_{0}^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

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9. For any $n > 1$, evaluate the integral

$$\int_{0}^{\infty} \frac{1}{\left(x+\sqrt{x^2+1}\right)^n} dx$$

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10. Evaluate: $\int_{0}^{e-1} \frac{\frac{x^2+2x-1}{2}}{x+1} dx + \int_{1}^{e} x \log x e^{\frac{x^2-2}{2}} dx$

A. $\left(\sqrt{e}\right)^{\left(e^2+1\right)}$

B. $\left(\sqrt{e}\right)^{e^{2-1}}$

C. 0

D. $\left(\sqrt{e}\right)^{e^{2-2}}$

Answer: D

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11. Let $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f. Then, the value of $g'(0)$ is

A. 1

B. 17

C. $\sqrt{17}$

D. None of these

Answer: C

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12. Let $a_n = \int_0^{\pi/2} (1 - \sin t)^n \sin 2t,$

then $\lim_{n \to \infty} \sum_{n=1}^{n} \frac{a_n}{n}$ is equal to

A. $1/2$

B. 1

C. $4/3$

D. $3/2$

Answer: A

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13. The value of $x > 1$ satisfying the equation

$\int_1^x t \ln t dt = \frac{1}{4}$ is

A. $\sqrt{e}$

B. e

C. e^2

D. $e-1$

Answer: A

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14. If $\lim_{a \to \infty} \frac{1}{a}\int_0^{\infty} \frac{x^2+ax+1}{1+x^4}.\tan^{-1}\left(\frac{1}{x}\right)dx$ is equal to $\frac{\pi^2}{K}$, where K in N, then K equals to

A. 4

B. 8

C. 16

D. 32

Answer: C

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15. If the value of definite integral $\int_1^a x \cdot a^{-[\log_a x]} dx$, where $a > 1$ and [x] denotes the greatest integer, is, $\frac{e-1}{2}$ then the value of equal to

A. $\sqrt{e}$

B. e

C. $\sqrt{e+1}$

D. $e - 1$

Answer: A

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16. Show that (i) $\int_0^{\pi/2} f(\sin x) dx = \int_0^{\pi/2} f(\cos x) dx$ (ii) $\int_0^{\pi/2} f(\tan x) dx = \int_0^{\pi/2} f(\cot x) dx$ (iii) $\int_0^{\pi/2} f(\sin 2x) \sin x dx = \int_o^{\pi/2} f(\sin 2x). \cos x dx$

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17. If f and g are continuous functions on $[0, a]$ satisfying $f(x) = (a - x)$ and $g(x) + g(a - x) = 2$, then show that $\int_0^a f(x)g(x)dx = \int_0^a f(x)dx.$

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18. Evaluate

(i) $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$

(ii) $\int_0^{\pi/2} \log(\tan x)dx$

(iii) $\int_0^{\pi/4} \log(1 + \tan x)dx$

(iv) $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}dx$

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19. The value of $\int_0^a \log(\cot a + \tan x)dx,$

where $a \in (0, \pi/2)$ is equal to

A. $a \log(\sin a)$

B. $-a \log(\sin a)$

C. $-a \log(\cos a)$

D. None of these

Answer: B

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20. Evaluate $\int \frac{\pi}{6} \frac{\pi}{3} \frac{dx}{1+\sqrt{\tan x}}$

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21. Prove that: $\int_a^b \frac{f(x)}{f(x) = f(a+b-x)} dx = \frac{b-a}{2}$

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22. Solve $l = \int_{\cos^4 t}^{-\sin^4 t} \frac{\sqrt{f(z)}dz}{\sqrt{f(\cos 2t - z)} + \sqrt{f(z)}}$

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23. Let the function f satisfies f(x).f ' (−x)=f(−x).f ' (x) for all x and f(0)=3 The value of $f(x).\ f(-x)$ for all x is

A. 4

B. 9

C. 12

D. 16

Answer: B

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24. $\int_{-51}^{51} \frac{dx}{3 + f(x)}$ has the value equal to

A. 17

B. 34

C. 102

D. 0

Answer: A

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25. Let $f(x) = |x|$ Then number of solutions of $f(x) = 0 \ \text{in}[-2, 2]$ is

A. 0

B. 1

C. 2

D. 4

Answer: A

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26. Given fuction, $\begin{cases} x^2 & \text{for} 0 \leq x < 1 \\ \sqrt{x} & \text{for} 1 \leq x \leq 2 \end{cases}$

Evaluate $\int_0^2 f(x)dx.$

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27. Evaluate the integral $l = \int_0^2 |1 - x|dx.$

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28. Evaluate (i) $\int_0^\pi |\cos x|dx$ (ii) $\int_0^2 |x^2 + 2x - 3|dx$

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29. Evaluate $\int_{-1}^1 (x - [x])dx$, where $[.]$

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30. Evaluate $\int_0^2 \{x\}dx$, where $\{x\}$ denotes the fractional part of x.

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31. $\int_0^9 \{\sqrt{x}\}dx$, where $\{x\}$ denotes the fractional part of x, is 5 (b) 6 (c) 4 (d) 3

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32. If for a real number y, $[y]$ is the greatest integral function less, then or equal to y, then the value of the integral $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2\sin x]dx$ is $-\pi$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

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33. The value of $\int_0^{100} [\tan^{-1} x]dx$ is equal to (where $[.]$ denotes the greatest integer function)

A. $\tan 1 - 100$

B. $\pi / 2 - \tan 1$

C. $100 - \tan 1$

D. None of these

Answer: C

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34. Expand $\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}$

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35. The value of $\int_1^2 \left(x^{[x^2]} + [x^2]^x\right) dx$ is equal to where $[.]$ denotes the greatest integer function

A. $\frac{5}{4}\sqrt{3} + \left(2^{\sqrt{3}} - 2^{\sqrt{2}}\right) + \frac{1}{\log 3}\left(9 - 3^{\sqrt{3}}\right)$

B. $\frac{5}{4} + \sqrt{3} + \frac{\sqrt{2}}{3} + \frac{1}{\log 2}\left(2^{\sqrt{3}} - 2^{\sqrt{2}}\right) + \frac{1}{\log 3}\left(9 - 3^{\sqrt{3}}\right)$

C. $\frac{5}{4}+\frac{\sqrt{2}}{2}+\frac{1}{\log 2}\left(2^{\sqrt{3}}-2^{\sqrt{2}}\right)+\frac{1}{\log 3}\left(9-3^{\sqrt{3}}\right)$

D. None of the above

Answer: B

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36. The value of $\int_0^{2\pi}[|\sin x|+|\cos x|]dx$ is equal to

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. 2π

Answer: D

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37. The value of $\int_0^{\pi/2} \sin|2x - \alpha| dx$, where $a\varepsilon[0, \pi]$ is

A. 1

B. $\cos\alpha$

C. $\frac{1 + \cos\alpha}{2}$

D. $\frac{1 - \cos\alpha}{2}$

Answer: A

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38. Let f be a continuous function satisfying $f'(lnx) = [1$ for $0 < x \leq 1, x$ for $x > 1$ and $f(0) = 0$ then $f(x)$ can be defined as

A. $f(x) = \begin{cases} 1 & \text{if} x \leq 0 \\ 1 - e^x & \text{if} x > 0 \end{cases}$

B. $f(x) = \begin{cases} 1 & \text{if} x \leq 0 \\ e^x - 1 & \text{if} x > 0 \end{cases}$

C. $f(x) = \begin{cases} x & \text{if} x < 0 \\ e^x & \text{if} x > 0 \end{cases}$

D. $f(x) = \begin{cases} x & \text{if} x \leq 0 \\ e^x - 1 & \text{if} x > 0 \end{cases}$

Answer: D

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39. The integral $\int_{\frac{\pi}{4}}^{5\frac{\pi}{4}} (|\cos t|\sin t + |\sin t|\cos t)$ has the value equal to

A. 0

B. $1/2$

C. $1/\sqrt{2}$

D. 1

Answer: A

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40. $$Iff(x) = \{0, wherex = \frac{n}{n+1}, n = 1, 2, 31, elsewhere$$

$thenthevalueof \int_0^2 f(x)dxis$ 1 (b) 0 (c) 2 (d) ∞

A. 1

B. 2

C. 3

D. None of thses

Answer: B

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41. Evaluate $\int_{-1}^{1} (x^3 + 5x + \sin x) dx$

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42. Evaluate: $\int_{\pi/3}^{\pi/4} x^3 \sin^4 x dx$ (ii) $\int_a^a \sqrt{\frac{a-x}{a+x}} dx$

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43. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$

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44. The value of $\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$ is equal to

A. $\frac{1}{2}$

B. 1

C. -1

D. 0

Answer: D

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45. Evaluate: $\int_0^{\pi}\left(x\sin 2x\frac{\sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi}dx\right.$

A. $\frac{8}{\pi}$

B. $\frac{\pi}{8}$

C. $\frac{8}{\pi^2}$

D. $\frac{\pi^2}{8}$

Answer: C

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46. Let $f(x)=\begin{vmatrix}\cos x & e^{x^2} & 2x\frac{\cos^2 x}{2}\\ x^2 & \sec x & \sin x+x^3\\ 1 & 2 & x+\tan x\end{vmatrix}$ then the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x^2+1)(f(x)+f''(x))dx$

A. (a)1

B. (b)-1

C. (c)2

D. (d)None of these

Answer: D

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47. The value of $\int_{-1}^{1} \frac{x}{\sqrt{1-x^2}} . \sin^{-1}\left(2x\sqrt{1-x^2}\right) dx$ is equal to

A. $4\sqrt{2}$

B. $4\left(\sqrt{2}-1\right)$

C. $4\left(\sqrt{2}+1\right)$

D. `None of these

Answer: B

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48. Suppose the function $g_n(x) = x^{2n+1} + a_n x + b_n (N \in N)$ satisfes the equation $\int_{-1}^{1} (px+q) g_n(x) dx = 0$ for all linear functions $(px+q)$ then

A. $a_n = b_n = 0$

B. $b_n = 0, a_n = -\frac{3}{2n+3}$

C. $a_n = 0, b_n = -\frac{3}{2n+3}$

D. $a_n = \frac{3}{2n+3}, b_n = -\frac{3}{2n+3}$

Answer: B

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49. Evaluate $\int_0^{\pi} \frac{x}{1+\cos^2 x} dx.$

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50. Prove that $\int_0^{\pi/2} \log(\sin x)dx = \int_0^{\pi/2} \log(\cos x)dx = -\frac{\pi}{2}\log 2.$

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51. If $f(x) = -\int_0^x \log(\cos t)dt,$ then the value of $f(x) - 2f\left(\frac{\pi}{4} + \frac{x}{2}\right) + 2f\left(\frac{\pi}{4} - \frac{x}{2}\right)$ is equal to

A. $-x\log 2$

B. $\frac{x}{2}\log 2$

C. $\frac{x}{3}\log 2$

D. None of these

Answer: A

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52. If $\int_0^{\pi}\left(\frac{x}{1+\sin x}\right)^2 dx = A$, then the value for $\int_0^{\pi}\frac{2x^2.\cos^2 x/2}{(1+\sin x^2)}dx$ is equal to

A. $A+2\pi-\pi^2$

B. $A-2\pi+\pi^2$

C. $2\pi-A-\pi^2$

D. None of these

Answer: A

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53. about to only mathematics

A. $\pi a-A$

B. $\pi a+2A$

C. $\pi a-2A$

D. $\pi a + A$

Answer: C

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54. Evaluate: $\int_{-4}^{-5} e^{(x+5)^2} dx + 3\int_{\frac{1}{3}}^{\frac{2}{3}} e^{9\left(x-\frac{2}{3}\right)^2} dx$

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55. If $f(x) = \int_0^x \log\left(\frac{1-t}{1+t}\right) dt$, then discuss whether even or odd?

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56. Evaluate $\int_0^{4\pi} |\cos x| dx.$

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57. Prove that $\int_0^{25} e^{x - [x]} dx = 25(e-1)$.

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58. The value of $\int_0^{2n\pi} [\sin x \cos x] dx$ is equal to

A. $-n\pi$

B. $n\pi$

C. $-2n\pi$

D. None of these

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59. The value of $\int_{-5}^{5} f(x) dx$, where $f(x) = \text{minimum}(\{x+1\}, \{x-1\}), Aax \in R \text{ and } \{.\}$ denotes fractional part of x, is equal to

A. 3

B. 4

C. 5

D. 6

Answer: C

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60. Show that $\int_0^{n\pi+v}|\sin x|dx = 2n+1-\cos v$, where n is a positive integer and $0 \leq v < \pi$.

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61. The value of $\int_{-2\pi}^{5\pi}\cot^{-1}(\tan x)dx$ is equal to

A. $\frac{7\pi}{2}$

B. $-(7\pi^2)$

C. $\frac{3\pi}{2}$

D. $21\frac{\pi^2}{4}$

Answer: B

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62. Let g(x) be a continuous and differentiable function such that $\int_0^2 \left\{\int_{\sqrt{2}}^{\frac{\sqrt{5}}{2}} \left[2x^2 - 3\right] dx\right\}. g(x) dx = 0,$ then g(x) = 0 when $x \in (0, 2)$ has (where [*] denote greatest integer function)

A. exactly one real root

B. atleat one real root

C. no real root None of these

D.

Answer: B

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63. The value of x satisfying $\int_0^{2[x+14]} \left\{\frac{x}{2}\right\} dx = \int_0^{\{x\}} [x+14] dx$ is equal to (where, $[.]$ and $\{.\}$ denotes the greates integer and fractional part of x)

A. $[-14, -13)$

B. $(0, 1)$

C. $(-15, -14]$

D. None of these

Answer: A

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64. Find the derivative of the following with respect to x.

(i) $\int_0^x \cos t dt$

(ii) $\int_0^{x^2} \cos^2 dt$

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65. Evaluate $\frac{d}{dx}\left(\int_{1/x}^{\sqrt{x}} \cos t^2 dt\right)$

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66. If $\frac{d}{dx}\left(\int_0^y e^{-t^2} dt + \int_0^{x^2} \sin^2 t dt\right) = 0$, find $\frac{dy}{dx}$.

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67. The points of extremum of $\int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$ are

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68. If $y = \int_{x^2}^{x^3} \frac{1}{\log t} dt (x > 0)$, then find $\frac{dy}{dx}$

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69. If $y=\int_0^x f(t)\sin\{k(x-t)\}dt$, then prove that $\frac{d^2y}{dx^2}+k^2y=kf(x)$.

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70. $If\int_{\frac{\pi}{3}}^{x}\sqrt{(3-\sin^2 t)}dt+\int_0^y \cos t dt=0, thenevaluate\frac{dy}{dx}$

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71. Let $\frac{d}{dx}F(x)=\frac{e^{\sin x}}{x}.\ x<0.$ If $\int_1^4 \frac{2e^{\sin x^2}}{x}dx=F(k)-F(1)$ then find the possible value of k.

A. 10

B. 14

C. 16

D. 18

Answer: C

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72. The function $f(x)=\int_0^x \log_{|\sin t|}\left(\sin t+\frac{1}{2}\right)dt$, where $x\in(0, 2\pi)$, then $f(x)$ strictly increases in the interval

A. $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

B. $\left(\frac{5\pi}{6}, 2\pi\right)$

C. $\left(\frac{\pi}{6}, \frac{7\pi}{6}\right)$

D. $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$

Answer: D

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73. $f:(0, \infty)\rightarrow R$ and $F(x)=\int_0^x tf(t)dt$

If $F(x^2)=x^4+x^5$, $\text{then}\Sigma_{r=1}^{12} f(r^2)$ is equal to S

A. 216

B. 219

C. 221

D. 223

Answer: B

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74. A function f(x) satisfies

$f(x) = \sin x + \int_0^x f'(t)\left(2\sin t - \sin^2 t\right)dt$ Then

A. $\dfrac{x}{1-\sin x}$

B. $\dfrac{\sin x}{1-\sin x}$

C. $\dfrac{1-\cos x}{\cos x}$

D. $\dfrac{\tan x}{1-\sin x}$

Answer: B

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75. If $F(x) = \int_1^x f(t)dt$,where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u} du$, then the value of F''(2) equals to

A. $\frac{7}{4\sqrt{17}}$

B. $\frac{15}{\sqrt{17}}$

C. $\sqrt{257}$

D. $\frac{15\sqrt{17}}{68}$

Answer: C

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76. Evaluate $I(b) = \int_0^1 (x^b) dx = \int_0^1 \frac{x^b - 1}{\ln x} dx, b \geq 0.$

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77. Prove that $\int_0^{\pi/2} \frac{dx}{\left(a^2 \sin^2 x + b^2 \cos^2 x\right)^2} = \frac{\pi\left(a^2 + b^2\right)}{4a^3b^3}$.

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78. The value of

$\int_0^{\pi/2} \frac{\log\left(1 + x\sin^2\theta\right)}{\sin^2\theta} d\theta, x \geq 0$ is equal to

A. $\pi\left(\sqrt{1+x-1}\right)$

B. $\pi\left(\sqrt{1+x-2}\right)$

C. $\sqrt{\pi}\left(\sqrt{1+x-1}\right)$

D. None of these

Answer: A

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79. Let $f(x)$ be a continuous function for all x, which is not identically zero such that $\{f(x)\}^2 = \int_0^x f(t) \frac{2\sec^2 t}{4+\tan t} dt$ and $f(0) = \ln 4$, then

A. $f\left(\frac{\pi}{4}\right) = \log(5)$

B. $f\left(\frac{\pi}{4}\right) = \frac{3}{4}$

C. $f\left(\frac{\pi}{2}\right) = 2$

D. None of these

Answer: A

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80. Evaluate the following

(i) $\lim_{n \to \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} \ldots . + \frac{n-1}{n^2}\right)$

(ii) $\lim_{n \to \infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \ldots . + \frac{1}{2n}\right)$

(iii) $\lim_{n \to \infty} \left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \ldots . + \frac{n}{2n^2}\right)$

(iv) $\lim_{n \to \infty} \frac{(1^p + 2^p + \ldots . . + n^p)}{n^{p+1}}, p > 0$

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81. Evaluate $S = \sum_{r=0}^{n-1} \frac{1}{\sqrt{4n^2 - r^2}} asn \to \infty.$

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82. Evaluate $\lim_{n \to \infty} \left(\frac{1}{2n+1} + \frac{1}{2n+2} + \ldots\ldots + \frac{1}{6n} \right).$

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83. Evaluate $\int_1^4 (ax^2 + bx + c) dx$.

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84. The value of $\lim_{n \to \infty} \left(\sin\frac{\pi}{2n}. \sin\frac{2\pi}{2n}. \sin\frac{3\pi}{2n} \ldots \sin\frac{(n-1)\pi}{2n} \right)^{1/n}$ is equal to

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. None of these

Answer: C

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85. The interval $[0, 4]$ isdivided into n equal sub-intervals by the points $x_0, x_1, x_2, \ldots, x_{n-1}, x_n \text{where} 0 = x_0 < x_1 < x_2 < x_3 < \ldots < x_n = 4$ If $\delta x = x_i - x_{i-1} \text{for} i = 1, 2, 3, \ldots, n, \text{ then } \lim_{\delta x \to 0} \sum_{i-1}^{n} x_i \delta x$ is equal to

A. (a)4

B. (b)8

C. (c)$\frac{32}{3}$

D. (d)16

Answer: B

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86. The value of $\lim_{n \to \infty} \frac{1}{n} . \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ is equal to

A. $1 + \sqrt{5}$

B. $-1 + \sqrt{5}$

C. $-1 + \sqrt{2}$

D. $1 + \sqrt{2}$

Answer: B

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87. Estimate the absolute value of the integral $\int_{10}^{19} \frac{\sin x}{1 + x^8} dx$

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88. The minimum odd value of 'a' $(a > 1)$ for which $\int_{10}^{19} \frac{\sin x}{1+x^a} dx < \frac{1}{9}$, is equal to

A. 1

B. 3

C. 5

D. 9

Answer: B

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89. Prove that $\int_0^1 \sqrt{(1+x)(1+x^3)}\,dx$ cannot exceed $\sqrt{15/8}$.

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90. If $f(x)$ is a continous function such that $f(x) \mid 0, \forall x \in [2, 10]$ and $\int_4^8 f(x) dx = 0$, then find f (6)

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91. Prove that `pi/6

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92. Prove that $4 \leq \int_1^3 \sqrt{3 + x^3} dx \leq 2\sqrt{30}.$

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93. Prove that 1 $\leq \int_0^1 e^{x^2} dx \leq e$

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94. Evaluate

(i) $\Gamma 1$

(ii) $\Gamma 2$

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95. Evaluate $\int_0^\infty e^{-x} x^3 dx.$

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96. Evaluate $\int_0^1 \left(\left(\log\left(\frac{1}{x}\right)\right)^{n-1} dx.\right.$

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97. Evaluate $\int_0^1 x^6 \sqrt{1-x^2}$dx

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98. Evaluate $\int_0^{\pi/2} \sin^4 x . \cos^6 x dx.$

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99. The value of $\int_0^{\infty} e^{-a^2x^2} dx$ is equal to

A. (a) $\frac{\sqrt{\pi}}{2a}$

B. (b) $\frac{\pi}{2a}$

C. (c) $\frac{\pi}{\sqrt{2a}}$

D. (d)`None of these

Answer: A

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100. about to only mathematics

A. n

B. n!

C. $(n+1)!$

D. $n.\,n!$

Answer: D

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101. The true set values of 'a' for which the inequality $\int_a^0 (3^{-2x} - 2.3^{-x})dx \geq 0$ is true, is

A. $[0, 1]$

B. $(-\infty, -1]$

C. $[0, \infty)$

D. $(-\infty, -1] \cup [0, \infty)$

Answer: D

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102. The value of the definite integral $\int_0^{2n\pi} \max\left(\sin x, \sin^{-1}(\sin x)\right) dx$ equals to (where, n in I)

A. $\frac{n\left(\pi^2 - 4\right)}{2}$

B. $\frac{n\left(\pi^2 - 4\right)}{4}$

C. $\frac{n\left(\pi^2 - 8\right)}{4}$

D. $\frac{n\left(\pi^2 - 2\right)}{4}$

Answer: C

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103. $\int \frac{\sqrt{a^2 - x^2}}{x^2} dx$

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104. $\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}}$ is equal to

A. $\frac{8}{\pi} f(2)$

B. $\frac{2}{\pi} f(2)$

C. $\frac{2}{\pi} f\left(\frac{1}{2}\right)$

D. $4f(2)$

Answer: A

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105. Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t)dt, 0 \leq x \leq 1, and\ f(0) = 0$, then

A. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

B. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

C. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

D. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Answer: C

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106. The value of $\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t \ln(1+t)}{t^4+4} dt$

A. 0

B. $\frac{1}{12}$

C. $\frac{1}{24}$

D. $\frac{1}{64}$

Answer: B

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107. The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^4} dx$ is (are)

A. $\dfrac{22}{7} - \pi$

B. $\dfrac{2}{105}$

C. 0

D. $\dfrac{71}{15} - \dfrac{3\pi}{2}$

Answer: A

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108. $\int_{\sin\theta}^{\cos\theta} f(x\tan\theta)dx$ (where $\theta \neq \dfrac{n\pi}{2}, n \in I$) is equal to

A. $-\cos\theta \int_{1}^{\tan\theta} f(x\sin\theta)dx$

B. $-\tan\theta \int_{\cos\theta}^{\sin\theta} f(x)dx$

C. $-\tan\theta \int_{1}^{\sin\theta} f(x\cos\theta)dx$

D. $\dfrac{1}{\tan\theta} \int_{\sin\theta}^{\sin\theta\tan\theta} f(x)dx$

Answer: A

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109. Evaluate: $\int \frac{1}{x\sqrt{1+x^3}}\,dx$

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110. Find $\frac{dy}{dx}$ if $x^4 - y^4 = \sin x$

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111. Statement I $Iff(x) = \int_0^1 (xf(t)+1)dt,$ then $\int_0^3 f(x)dx = 12$

Statement II $f(x) = 3x + 1$

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement 1.

B. Statement I is true, Statement II is also true , Statement II is not the correct explanation of Statement II.

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: c

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112. Statement I The function $f(x) = \int_0^x \sqrt{1 + t^2 dt}$ is an odd function and STATEMENT 2 :$g(x) = f'(x)$ is an even function , then $f(x)$ is an odd function.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement 1.

B. Statement I is true, Statement II is also true , Statement II is not the correct explanation of Statement II.

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: A

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113. Given, $f(x) = \sin^3 x$ and $P(x)$ is a quadratic polynomial with coefficient unity.

Statement I $\int_0^{2\pi} P(x).\, f''(x)dx$ vanishes.

Statement II $\int_0^{2\pi} f(x)dx$ vanishes.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement 1.

B. Statement I is true, Statement II is also true , Statement II is not the correct explanation of Statement II.

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: a

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114. Find the second order derivative if $y = x^3$

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115. Suppose we define integral using the following formula $\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b))$, for more accurate result for $c \in (a, b)$, $F(c) = \frac{c-a}{2}(f(a)+f(c)) + \frac{b-c}{2}(f(b)+f(c))$.

When $c = \frac{a+b}{2}$, then $\int_a^b f(x)dx = \frac{b-a}{4}(f(a)+f(b)+2f(c))$.

$\lim_{t \to a} \frac{\int_a^t f(x)dx - \frac{(t-a)}{2}(f(t)+f(a))}{(t-a)^3} = 0 \forall a$ Then the degree of $f(x)$ can at most be

A. 1

B. 2

C. 3

D. 4

Answer: A

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116. Let $f(\alpha, \beta) = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$

The value of $l = \int_0^{\pi/2} e^{\beta}\left(f(0,0) + f\left(\frac{\pi}{2}, \beta\right) + f\left(\frac{3\pi}{2}, \frac{\pi}{2} - \beta\right)\right)d\beta$ is

A. (a) $e^{\pi/2}$

B. (b) 0

C. (c) $2\left(2e^{\pi/2} - 2\right)$

D. (d) None of these

Answer: C

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117. Let $f(\alpha, \beta) = \; = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$

if $i = \int_{-\pi/2}^{\pi/2} \cos^2\beta\left(f(0,\beta) + f\left(0, \frac{\pi}{2} - \beta\right)\right)d\beta$ then i is

A. $e^{\pi/2}$

B. 3

C. $2\left(2e^{\pi/2}-1\right)$

D. `None of these

Answer: B

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118. Match the following

	Column I		Column II
(A)	$\int_{-1}^{1}\frac{dx}{1+x^2}$	(p)	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B)	$\int_{0}^{1}\frac{dx}{\sqrt{1-x^2}}$	(q)	$\frac{1}{2}\log\left(\frac{3}{2}\right)$
(C)	$\int_{2}^{3}\frac{dx}{1-x^2}$	(r)	$\frac{\pi}{3}$
(D)	$\int_{1}^{2}\frac{dx}{x\sqrt{x^2-1}}$	(s)	$\frac{\pi}{2}$

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119. Let $f: R \to R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of $f(1n5)$ is_____

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120. For any real number x, let $[x]$ denote the largest integer less than or equal to x, Let f be a real-valued function defined on the interval $[-10, 10]$ be $f(x) = \{x - [x], \quad \text{if} \quad [x]$ is odd, $1 + [x] - x, \quad \text{if} \quad [x]$ is even Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x dx$ is____

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121. Find the error in steps to evaluate the following integral $\int_0^{\pi} \frac{dx}{1 + 2\sin^2 x} = \int_0^{\pi} \frac{\sec^2 x dx}{\sec^2 x + 2\tan^2 x} = \int_0^{\pi} \frac{\sec^2 x dx}{1 + 3\tan^2 x}$
$= \frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\sqrt{3}\tan x\right)\right]_0^{\pi} = 0$

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122. If $\int_a^b |\sin x| dx = 8$ and $\int_0^{a+b} |\cos x| dx = 9$, then

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123. Evaluate $\int_{\cos(\cos^{-1}\alpha)}^{\sin(\sin^{-1}\beta)} \left| \left(\frac{\cos(\cos^{-1}x)}{\sin(\sin^{-1}x)} \right) \right| dx$

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124. $\int_\alpha^\beta \sqrt{\frac{x-\alpha}{\beta-x}}$ dx is equal to

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125. Evaluate: $\int_{\sqrt{\frac{3a^2+b^2}{2}}}^{\sqrt{\frac{a^2+b^2}{2}}} \frac{x \cdot dx}{\sqrt{(x^2-a^2)(b^2-x^2)}}$

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126. Evaluate $\int_0^{\pi/4} \frac{e^{\sec x}\left[\sin\left(x+\frac{\pi}{4}\right)\right]}{\cos x(1-\sin x)}dx$

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127. $\int_0^{\pi/4} \frac{x^2(\sin 2x - \cos 2x)}{(1+\sin 2x)\cos^2 x}dx$

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128. Evaluate $\int_0^{\pi} x^2\left\{(1+\sin x)^{-2}\cos x\right\}dx$

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129. Compute the following integrals.

(i) $\int_0^{\infty} f\left(x^n + x^{-n}\right)\ln x\frac{dx}{x} = 0$

(ii) $\int_0^{\infty} f\left(x^n + x^{-n}\right)\ln x\frac{dx}{1+x^2} = 0$

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130. Show that

(a) $\int_0^{\infty} \sin x dx = 1$

(b) $\int_0^{\infty} \cos x dx = 0$

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131. Find a function $g: R \to R$, continuous in $[0, \infty)$ and positive in $(0, \infty)$ satisfying $g(1) = 1$ and $\frac{1}{2}\int_0^x g^2(t)dt = \frac{1}{x}\left(\int_0^x g(t)dt\right)^2$

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132. If $I_n = \int_0^{\pi/4} \tan^n x dx$, $(n > 1$ and is an integer), then (a) $I_n + I_{n-2} = \frac{1}{n+1}$ (b) $I_n + I_{n-2} = \frac{1}{n-1}$ (c) $I_2 + I_4, I_4 + I_6, I_6 + I_8$ are in HP (d) $\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}$

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133. If $U_n = \int_0^{\pi} \frac{1 - \cos nx}{1 - \cos x} dx$ where n is positive integer of zero, then

The value of $\int_0^{\pi/2} \frac{\sin^2 n\theta}{\sin^2 \theta} d\theta$ is

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134. Prove that for any positive integer k, $\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + + \cos(2k-1)x]\cdot$ Hence, prove that $\int_0^{\frac{\pi}{2}} \sin 2xk \cot x dx = \frac{\pi}{2}\cdot$

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135. Evaluate $\int_0^{\sqrt{3}} \left(\frac{1}{1+x^2}\right) \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx.$

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136. Prove that $\int_0^x e^{xt} e^{-t^2} dt = e^{x^2/4} \int_0^x e^{-t^2/4} dt.$

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137. If $f(x) = e^x + \int_0^1 (e^x + te^{-x}) f(t) dt,$ find $f(x)$.

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138. $If|a| < 1,$ show that $\int_0^\pi \frac{\log(1 + a\cos x)}{\cos x} dx = \pi \sin^{-1} a$

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139. Evaluate $\int_0^{\pi/2} \cos\theta \tan^{-1}(\sin\theta) d\theta.$

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140. Evaluate $\int_0^{\pi/2} \cos\theta \tan^{-1}(\sin\theta) d\theta.$

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141. Let f be a continuous function on $[a, b]\cdot$ Prove that there exists a number $x \in [a, b]$ such that $\int_a^x f(t)dx = \int_x^b f(t)dt\cdot$

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142. $Iff(x) = x + \int_0^1 (xy^2 + x^2y)(f(y))dy, \text{find} f(x)$ if x and y are independent.

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143. $\int_0^x \left\{ \int_0^u f(t)dx \right\} du$ is equal to (a) $\int_0^x (x - u)f(u)du$ (b) $\int_0^x uf(x - u)du$ (c) $x\int_0^x f(u)du$ (d) $x\int_0^x uf(u - x)du$

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144. Evaluate: $\int_0^{\frac{3\pi}{2}} (\ln|\sin x|)\cos(2nx)dx, n \in N$

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145. Evaluate $\int_0^\infty e^{-x}\sin^n x dx$, if n is an even integer.

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146. Evaluate $\int_0^1 (tx+1-x)^n dx$, where n is a positive integer and t is a parameter independent of x . Hence , show that ∫ 0 1 x k (1−x) n−k dx= [n C k (n+1)] P fork=0,1,......n, then P=

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147. Given a real valued fuction f(x) which is monotonic and differentiable , prove that for any real number a and b, $\int_a^b \{f^2(x)-f^2(a)\}dx = \int_{f(a)}^{f(b)} 2x\{b-f^{-1}(x)\}dx$

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148. $\int_0^1 \frac{\sin\theta\left(\cos^2\theta - \cos^2\pi/5\right)\left(\cos^2\theta - \cos^2 2\pi/5\right)}{\sin 5\theta} d\theta$

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149. Show that $\lim_{n \to \infty} \sum_{k=0}^{n} \frac{{}^{n}Ck}{n^k(k+3)} = e - 2$

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150. Let I= $\int_0^{\pi/2} \frac{\cos x}{a\cos x + b\sin x} dx$ and $J = \int_0^{\pi/2} \frac{\sin x}{a\cos x + b\sin x} dx$, where a > 0 and $b > 0$ Compute the values of I and j

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151. Evaluate $\int_0^{\infty} \frac{\ln x dx}{x^2 + 2x + 4}$

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152. Find a function f, continous for all x(and not zero everywhere) such that $f^2(x) = \int_0^x \frac{f(t)\sin t}{2+\cos t}dt$

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153. Evaluate $\int_0^\infty \frac{\tan^{-1}ax - \tan^{-1}x}{x}dx$, where a is a parameter.

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154. Evaluate $\int_0^\infty \frac{\tan^{-1}ax - \tan^{-1}x}{x}dx$, where a is a parameter.

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155. Evaluate $\int_0^{\pi/2} \cos x dx$

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Exercise For Session 1

1. $\int_0^{\pi/4} \cos^2 x dx$

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2. Evaluate $\int_0^{\pi/2} \frac{dx}{1+\cos x}$

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3. Evaluate $\int_0^{\pi/2} \sqrt{1+\cos x dx}$

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4. $\int_0^{\frac{\pi}{6}} \sin 2x . \cos x dx$

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5. $\int_1^2 \frac{dx}{\sqrt{x} - \sqrt{x-1}}$

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6. $\int_0^1 x dx$

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7. $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x}$

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8. The value of $\int \frac{dx}{\sqrt{(x-a)(b-x)}}$, is

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9. $\int_a^b \sqrt{\frac{x-a}{b-x}} dx$ is equal to

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10. $\int_0^{\frac{\pi}{4}} \sqrt{\tan x} dx$

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11. $\int_0^{\pi} \cos 2x . \log(\sin x) dx$

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12. $\int_0^{\pi/4} e^{\sin x} \left(\frac{(x \cos^3 x - \sin x)}{\cos^2 x} \right) dx$

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13. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right)+x^2 f(x)=0$ for all nonzero x , then evaluate $\int_{\sin\theta}^{\cos ec\theta} f(x)dx$

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14. about to only mathematics

A. n

B. $n!$

C. $(n+1)!$

D. $n. n!$

Answer: D

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15. The true set values of 'a' for which the inequality $\int_a^0 \left(3^{-2x} - 2.3^{-x}\right)dx \geq 0$ is true, is

A. $[0, 1]$

B. $[-\infty, -1]$

C. $[0, \infty]$

D. $[-\infty, -1] \cup [1, \infty]$

Answer: D

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Exercise For Session 2

1. Evaluate $\int_0^{\pi/4} \log(1 + \tan\theta)d\theta$

A. $\frac{\pi}{2}\log 2$

B. $-\frac{\pi}{4}\log 2$

C. $\frac{\pi}{8}\log 2$

D. None of these

Answer: C

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2. Q. $\int_0^{\pi} e^{\cos^2 x}\left(\cos^3(2n+1)x dx, n \in I\right.$

A. 0

B. 1

C. -1

D. None of these

Answer: A

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3. The value of $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ is ______

A. $1/2$

B. $1/3$

C. $1/4$

D. None of these

Answer: A

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4. Evaluate: $\int_0^2 \frac{dx}{(17+8x-4x^2)\left[e^{6(1-x)}+1\right)}$

A. $-\frac{1}{8\sqrt{21}}\log\left|\frac{2-\sqrt{21}}{2+\sqrt{21}}\right|$

B. $-\frac{1}{8\sqrt{21}}\log\left|\frac{2-\sqrt{21}}{\sqrt{21}-2}\right|$

C. $-\frac{1}{8\sqrt{21}}\left\{\log\left|\frac{2-\sqrt{21}}{2+\sqrt{21}}\right|-\log\left|\frac{2+\sqrt{21}}{\sqrt{21}-2}\right|\right\}$

D. None of these

Answer: C

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5. If f is an odd function, then evaluate $I=\int_{-a}^{a}\frac{f(\sin x)dx}{f(\cos x)+f(\sin^2 x)}$

A. 0

B. $f(\cos x)+f(\sin x)$

C. 1

D. None of these

Answer: A

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6. If [x] stands for the greatest integer function, the value of $\int_{4}^{10}\frac{[x^2]}{[x^2-28x+196]+[x^2]}dx$, is

A. 1

B. 2

C. 3

D. 4

Answer: C

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7. Evaluate $\int_0^{\pi} \frac{x\,dx}{1+\cos\alpha\sin x}$,where $0 < \alpha < \pi$.

A. $\frac{\pi}{\sin\alpha}$

B. $\frac{\pi\alpha}{\sin\alpha}$

C. $\frac{\alpha}{\sin\alpha}$

D. $\frac{\sin\alpha}{\alpha}$

Answer: B

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8. $f, g, h,$ are continuous in $[0, a], f(a-x) = f(x), g(a-x) = -g(x), 3h(x) - 4h(a-x) = 5.$

Then prove that $\int_0^a f(x)g(x)h(x)dx = 0$

A. 0

B. 1

C. a

D. 2a

Answer: A

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9. If $2f(x) + f(-x) = \frac{1}{x}\sin\left(x - \frac{1}{x}\right)$ then the value of $\int_{\frac{1}{e}}^{e} f(x)dx$ is

A. 0

B. e

C. $1/e$

D. $e+1/e$

Answer: A

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10. Show that $\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2} \int_0^{\pi} \text{fr}(\sin x) dx.$

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11. Evaluate $\int_0^{\pi} \frac{x \sin 2x \sin\left(\frac{\pi}{2} \cos x\right)}{2x - \pi} dx.$

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12. The numbers of possible continuous $f(x)$ defined in $[0, 1]$ for which

$I_1 = \int_0^1 f(x) dx = 1, I_2 = \int_0^1 x f(x) dx - a, I_3 = \int_0^1 x^2 f(x) dx = a^2 is / a$

1 (b) ∞ (c) 2 (d) 0

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13. Let $l_1 = \int_0^1 \frac{e^x}{1+x} dx$ and $l_2 = \int_0^1 \frac{x^2}{e^{x^3}(2-x^3)} dx$. Then $\frac{l_1}{l_2}$ is equal to

A. $\frac{3}{e}$

B. $\frac{3}{e}$

C. 3e

D. $\frac{1}{3e}$

Answer: C

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14. If $f(x) = \frac{e^x}{1+e^x}, I_1 = \int_{f(-a)}^{f(a)} xg(x(1-x))dx$, and $I_2 = \int_{f(-a)}^{f(a)} g(x(1-x))dx$, then the value of $\frac{I_2}{I_1}$ is

A. 1

B. -3

C. -1

D. 2

Answer: D

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Exercise For Session 3

1. The value of $\int_{-1}^{3}\{|x-2|+[x]\}dx$, where [.] denotes the greatest integer function, is equal to

A. 5

B. 6

C. 3

D. None of these

Answer: D

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2. The value of $\int_{-1}^{3}(|x|+|x-1|)dx$ is equal to

A. 9

B. 6

C. 3

D. None of these

Answer: A

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3. Let $f(x) = x - [x]$, for every real number x, where [x] is integral part of x. Then $\int_{-1}^{1} f(x)dx$ is

A. 0

B. 1

C. 2

D. None of these

Answer: B

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4. The value of $\int_0^2 [x + [x + [x]]]dx$ (where, [.] denotes the greatest integer function)is equal to

A. 2

B. 3

C. -3

D. None of these

Answer: B

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5. The value of $\int_0^{[x]} \frac{2^x}{2^{[x]}} dx$ is equal to (where, [.] denotes the greatest integer function)

A. $\frac{[x]}{\log 2}$

B. $\frac{[x]}{2 \log 2}$

C. $\frac{[x]}{4 \log 2}$

D. None of these

Answer: A

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6. The value of $\int_0^4 \{x\}dx$ (where , {.} denotes fractional part of x) is equal to

A. $\frac{4}{3}$

B. $\frac{5}{3}$

C. $\frac{7}{3}$

D. None of these

Answer: D

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7. The value of $\int_1^4 \{x\}^{[x]} dx$ (where , [.] and {.} denotes the greatest integer and fractional part of x) is equal to

A. (a) $\frac{11}{12}$

B. (b) $\frac{13}{12}$

C. (c) $\frac{70}{12}$

D. (d) $\frac{19}{12}$

Answer: B

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8. The value of $\int_0^x [t+1]^3 dt$ (where, [.] denotes the greatest integer function of x) is qeual to

A. $\left(\frac{[x]([x]+1)}{2}\right)^2 + ([x]+1)^3\{x\}$

B. $\left(\frac{[x]([x]+1)}{2}\right)^3 + ([x]+1)^3\{x\}$

C. $\left(\frac{[x]([x]+1)}{2}\right)^3 + ([x]+1)^2\{x\}$

D. None of these

Answer: D

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9. The value of $\int_0^{10\pi} [\tan^{-1} x] dx$ (where, [.] denotes the greatest integer functionof x) is equal to

A. $\tan 1$

B. 10π

C. $10\pi - \tan 1$

D. None of these

Answer: C

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10. If $f(x) = \min \{|x-1|, |x|, |x+1|$, then the value of $\int_{-1}^{1} f(x) dx$ is equal to

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Answer: c

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11. The value of $\int_0^{\infty}\left[2e^{-x}\right]dx$ (where ,[.] denotes the greatest integer function of x) is equal to

A. 1

B. $\log_e 2$

C. 0

D. $\frac{1}{e}$

Answer: B

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12. The value of $\int_{1}^{10\pi}\left(\left[\sec^{-1}x\right]\right)dx$ (where ,[.] denotes the greatest integer function) is equal to

A. (a)$(\sec 1)-10\pi$

B. (b)$10\pi-\sec 1$

C. (c)$\pi-\sec 1$

D. (d)None of these

Answer: B

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13. The value of $\int_{-\pi/2}^{\pi/2}\left[\cot^{-1}x\right]dx$ (where ,[.] denotes greatest integer function) is equal to

A. (a)$\pi+\cot 1$

B. (b)$\pi+\cot 2$

C. (c)$\pi + \cot 1 + \cot 2$

D. (d)$\cot 1 + \cot 2$

Answer: C

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14. The value of $\int_0^{\frac{\pi}{4}} \left(\tan^n(x - [x]) + \tan^{n-2}(x - [x])\right) dx$ (where, [*] denote(d) cot 1+ cot2X-X)))dx (where, - denotes greatest integer function) is equal to

A. (a)$\frac{1}{n}$

B. (b)$\frac{1}{n-1}$

C. (c)$\frac{1}{n(n-1)}$

D. (d)$\frac{1}{n(+1)}$

Answer: B

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15. The value of $\int_0^2 [x^2 - x + 1] dx$ (where , [.] denotes the greatest integer function) is equal to

A. $\frac{5+\sqrt{5}}{2}$

B. $\frac{1+\sqrt{5}}{2}$

C. $\frac{1-\sqrt{5}}{2}$

D. $\frac{5-\sqrt{5}}{2}$

Answer: D

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16. Evaluate $\int_0^a [x^n] dx$, (where,[*] denotes the greatest integer function).

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17. Prove that $\int_0^x [t]dt = \frac{[x]([x]-1)}{2} + [x](x-[x])$, where [.] denotes the greatest integer function.

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18. If $f(n) = \frac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$(where,[*] and {*} denotes greatest integer and fractional part of x and $n \in N$). Then, the value of $f(4)$ is...

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19. $\int_0^x [\cos t]dt, wherex \in \left(2n\pi, 2n\pi + \frac{\pi}{2}\right), n \in N, and[.]$ denotes the greatest integer function . then the value of$f\left(\frac{1}{\pi}\right)is$

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20. If $\int_0^x [x]dx = \int_0^{[x]} xdx, x \notin$ integer (where, [*] and {*} denotes the greatest integer and fractional parts respectively,then the value of $4\{x\}$

is equal to ...

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Exercise For Session 4

1. Let $f : R \overrightarrow{a} ndg : R\overrightarrow{R}$ be continuous function. Then the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) + f(-x)][g(x) - g(-x)]dxis$ (a)π (b) 1 (c) -1 (d) 0

A. -1

B. 0

C. 1

D. None of these

Answer: B

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2. The value of $\int_{-1}^{1}(x|x|)dx$ is equal to

A. 1

B. $\frac{1}{2}$

C. 0

D. None of these

Answer: C

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3. The value of $\int_{-1}^{1}\left(\frac{x^2+\sin x}{1+x^2}\right)dx$ is equal to

A. 2π

B. $\pi - 2$

C. $2 - \frac{\pi}{2}$

D. None of these

Answer: C

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4. If f is an odd function, then evaluate $I = \int_{-a}^{a} \frac{f(\sin x)dx}{f(\cos x) + f(\sin^2 x)}$

A. 0

B. $f(\cos x) + f(\sin)$

C. 1

D. None of these

Answer: A

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5. Evaluate: $\int_{-\left(\frac{1}{\sqrt{3}}\right)}^{\frac{1}{\sqrt{3}}} \frac{\cos^{-1}\left(\frac{2x}{1+x^2}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x + 1} dx$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{\sqrt{3}}$

C. $\frac{\pi}{2\sqrt{3}}$

D. $\frac{\pi}{3\sqrt{3}}$

Answer: C

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6. Find the value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx, a > 0.$

A. π

B. $a\pi$

C. 2π

D. $\frac{\pi}{2}$

Answer: A

Watch Video Solution

7. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([x] + 1n\left(\frac{1+x}{1-x}\right)\right)dx$ is equal to (where [.] represents the greatest integer function)

A. $\frac{-1}{2}$

B. 0

C. 1

D. $2\frac{\log(1)}{2}$

Answer: A

Watch Video Solution

8. Evaluate: $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}dx$

A. 0

B. 1

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{2}$

Answer: C

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9. If [*] denots the greatest integer function then the value of the integeral $\int_{-\pi/2}^{\pi/2}\left(\left[\frac{x}{\pi}\right]+0.5\right)dx$, is

A. π

B. $\frac{\pi}{2}$

C. 0

D. $-\frac{\pi}{2}$

Answer: C

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10. The equation $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left\{ a|\sin x| + \frac{b \sin x}{1 + \cos^2 x} + c \right\} dx = 0$ where a, b, c are constants gives a relation between

A. a, b and c

B. a and c

C. a and b

D. b and c

Answer: B

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11. The value of $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$ where [.] denotes greatest integer function , is

A. 1

B. 0

C. $4 \sin 4$

D. None of these

Answer: B

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12. Let $f(x)$ be a contiuous function such a $\int_{n}^{n+1} f(x)dx = n^3, n \in Z.$ Then, the value of the intergral $\int_{-3}^{3} f(x)dx$ (A) 9 (B) -27 (C) -9 (D) none of these

A. 9

B. -27

C. -9

D. 27

Answer: B

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13. Let $f(x)=\frac{e^x+1}{e^x-1}$ and $\int_0^1 x^3 . \frac{e^x+1}{e^x-1}dx=\alpha \text{Then,} \int_{-1}^{1} t^3 f(t)dt$ is equal to

A. 0

B. α

C. 2α

D. None of these

Answer: C

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14. Let $f: R \rightarrow R$be a continuous function given by $f(x+y)=f(x)+f(y)$ for all $x, y, \in R$, if $\int_0^2 f(x)dx=\alpha$, then $\int_{-2}^{2} f(x)dx$ is equal to

A. 2α

B. α

C. 0

D. None of these

Answer: C

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15. The value of $\int_{-2}^{2} |[x]| dx$ is equal to

A. 1

B. 2

C. 3

D. 4

Answer: D

Watch Video Solution

16. Find the second order derivative if $y = e^{2x}$

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17. Let

$f(x) = \{1 - |x|, |x| \leq 1 \text{ and } 0, |x| < 1 \text{ and } g(x) = f(x -) + f(x + 1)$

, for all $x \in R$.Then,the value of $\int_{-3}^{3} g(x)dx$ is

A. 2

B. 3

C. 4

D. 5

Answer: A

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18. If $\int_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots\ldots\ldots$ then

A. $n\pi$

B. π

C. $-\pi$

D. 0

Answer: D

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19. If $\int_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots\ldots\ldots$ then

A. 0

B. 5π

C. 10π

D. 0

Answer: C

Watch Video Solution

20. If $\int_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots \ldots \ldots$ then

A. 0

B. 5π

C. 10π

D. None of these

Answer: A

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Exercise For Session 5

1. The value of $\int_{-1}^{10} sgn(x-[x])dx$ is equal to (where, [:] denotes the greatest integer function

A. 9

B. 10

C. 11

D. 12

Answer: C

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2. the value of $\int_{0}^{[x]} dx$ (where , [.] denotes the greatest integer function)

A. [x]

B. $\frac{[x]}{2}$

C. $x[x]$

D. None of these

Answer: A

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3. Evaluate: $\int_{-\frac{\pi}{4}}^{n\pi-\frac{\pi}{4}}|\sin x+\cos x|dx$

A. $2\sqrt{2n}$

B. $\sqrt{2}n$

C. $\frac{1}{2\sqrt{2}}n$

D. None of these

Answer: A

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4. Let $f(x) = x - [x]$, for ever real number $x, where[x]$ is the greatest integer less than or equal to x. Then, evaluate $\int_{-1}^{1} f(x)dx$.

A. 1

B. 2

C. 0

D. $\frac{1}{2}$

Answer: A

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5. $f(x) = \int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of $f(1)$ is

A. $\frac{1}{2}$

B. 0

C. 1

D. $-\frac{1}{2}$

Answer: A

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6. The least value of the function

$$\phi(x) = \int_{5\pi/4}^{x} (3\sin t + 4\cos t)dt$$

on the integral $[5\pi/4, 4\pi/3]$, is

A. $\sqrt{3} + \frac{3}{2}$

B. $1 - 2\sqrt{3} + \frac{3}{2} + \frac{1}{\sqrt{2}}$

C. $\frac{3}{2} + \frac{1}{\sqrt{2}}$

D. None of these

Answer: B

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7. The poins of extremum of $\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)\,dt$ are

A. $x = 1, \ -1$

B. $x = \ -1, 2$

C. $x = 2, 1$

D. $x = \ -2, 1$

Answer: A

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8. If f(x) is periodic function with period, T, then

A. $\int_a^b f(x)dx = \int_a^{b+T} f(x)dx$

B. $\int_a^b f(x) = \int_{a+T}^{b+T} f(x)dx$

C. $\int_a^b f(x)dx = \int_{a+T}^{b} f(x)dx$

D. $\int_a^b f(x)dx = \int_{a+T}^{b+2T} f(x)dx$

Answer: A

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9. Let $\frac{d}{dx}F(x)=\frac{e^{\sin x}}{x}.\ x<0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x}dx=F(k)-F(1)$ then find the possible value of k.

A. 4

B. 8

C. 16

D. 32

Answer: C

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10. Let $f:(0,\infty)\rightarrow R$ and $F(x)=\int_0^x f(t)dt$. If $F(x^2)=x^2(1+x)$, then

f(4) equals

A. $\frac{5}{4}$

B. 7

C. 4

D. 2

Answer: C

Watch Video Solution

11. Let $T > 0$ be a fixed real number. Suppose f is continuous function such that for all $x \in R$, $f(x+T) = f(x)$. If $I = \int_0^T f(x)dx$, then the value of $\int_3^{3+3T} f(2x)dx$ is $\frac{3}{2}I$ (b) $2I$ (c) $3I$ (d) $6I$

A. $\frac{3}{2}l$

B. $2l$

C. $3l$

D. $6l$

Answer: C

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12. Let $f(x) = \int_1^x \sqrt{2-t^2}dt\cdot$ Then the real roots of the equation , $x^2 - f'(x) = 0$ are: ± 1 b. $\pm\frac{1}{\sqrt{2}}$ c.$\pm\frac{1}{2}$ d. $0\&1$

A. ± 1

B. $\pm\frac{1}{\sqrt{2}}$

C. $\pm\frac{1}{2}$

D. $\pm\sqrt{2}$

Answer: A

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13. Let f(x) be an odd continuous function which is periodic with period 2.

if $g(x) = \int_0^x f(t)dt$, then

A. $g(x)$ is an odd function

B. $g(n) = 0 \text{for all} n \in N$

C. $g(2n) = 0 \text{for all} n \in N$

D. $g(x)$ is non-peridic

Answer: C

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14. Let f(x) be a function defined by $f(x) = \int_1^x t(t^2 - 3t + 2)dt, 1 \le x \le 3$ Then the range of f(x) is

A. $[0, 2]$

B. $\left[-\frac{1}{4}, 4\right]$

C. $\left[-\frac{1}{4}, 2\right]$

D. None of these

Answer: C

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15. The value of $\lim_{x \to 0} \frac{2\int_0^{\cos x} \cos^{-1}(t)}{2x - \sin 2x} dx$ is

A. 0

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. $\frac{2}{3}$

Answer: C

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16. If $\int_0^x \frac{bt\cos 4t - a\sin 4t}{t^2} dt = \frac{a\sin 4x}{x}$ for all $x \neq 0$, then a and b are given by

A. $a = \frac{1}{4}, b = 1$

B. $a = 2, b = 2$

C. $a = -1b = 4$

D. $a = 2, b = 4$

Answer: A

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17. If $f(x) = \int_0^x \{f(t)\}^{-1} dt$ and $\int_0^1 \{f(t)\}^{-1} = \sqrt{2}$, then

A. $f(x) = \sqrt{2x}$

B. $f(x) = \sqrt{2\log_e x}$

C. $f(x) = \sqrt{3x - 1}$

D. None of these

Answer: A

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18. Let f be a real-valued function defined on the inverval $(-1, 1)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4 + 1}dt,$ for all, $x \in (-1, 1) and let f^{-1}$ be the inverse function of f. Then $\left(f^{-1}\right)'(2)$ is equal to (a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{e}$

A. 1

B. $1/3$

C. $1/2$

D. $1/e$

Answer: B

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19. Consider the function defined on $[0, 1] \rightarrow R$, $f(x) = \dfrac{\sin x - x \cos x}{x^2}$, if $x \neq 0$ and $f(0) = 0$

$\int_0^1 f(x)$ is equal to

A. A. $1 - \sin(1)$

B. B. $\sin(1) - 1$

C. C. $\sin(1)$

D. D. $-\sin(1)$

Answer: A

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20. Evaluate $\int_0^2 x^3 dx$

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Exercise For Session 6

1. The value of

$$\int_0^{\pi/2} \frac{\log(1 + x\sin^2\theta)}{\sin^2\theta} d\theta, x \geq 0 \text{ is equal to}$$

A. $\frac{1}{\pi}(\sqrt{1+x-1})$

B. $\sqrt{\pi}(\sqrt{1+x-1})$

C. $\pi(\sqrt{1+x-1})$

D. None of these

Answer: C

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2. The value of $\lim_{n \to \infty} \frac{1}{2} \sum_{r-1}^{n} \left(\frac{r}{n+r}\right)$ is equal to

A. $1 - \log 2$

B. $\log 4 - 1$

C. $\log 2$

D. None of these

Answer: A

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3. The value of $\lim_{n \to \infty} \frac{1}{n}\{(n+1)(n+2)(n+3)...(n+n)\}^{1/n}$ is equal to

A. $4e$

B. $\frac{e}{4}$

C. $\frac{4}{e}$

D. None of these

Answer: C

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4. If $m, n \in N,$ then the value of $\int_a^b (x-a)^m (b-x)^n dx$ is equal to

A. $\frac{(b-a)^{m+n} . M!n!}{(m+n)}!$

B. $\frac{(b-a)^{m+n+1} . m!n!}{(m+n+1)!}$

C. $\frac{(b-a)^m . m!}{m!}$

D. None of these

Answer: B

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5. The value of $\lim_{n \to \infty} \left(\frac{n!}{n^n}\right)^{\frac{2n^4+1}{5n^5+1}}$ is equal

A. e

B. $\frac{2}{e}$

C. $\left(\frac{1}{e}\right)^{\frac{2}{5}}$

D. None of these

Answer: C

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6. The value of $\lim_{n \to \infty} n\left\{\frac{1}{3n^2+8n+4}+\frac{1}{3n^2+16n+16}+\ldots+n\text{terms}\right\}$ is equal to

A. $\frac{1}{2}\log\left(\frac{9}{5}\right)$

B. $\frac{1}{3}\log\left(\frac{9}{5}\right)$

C. $\frac{1}{4}\log\left(\frac{9}{5}\right)$

D. None of these

Answer: C

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7. Evaluate $\int_1^3 x^2 dx$

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8. The value of $f(k) = \int_0^{\pi/2} \log(\sin^2\theta + k^2\cos^2\theta)\,d\theta$ is equal to

A. $\pi\log(1+k) - \pi\log^2$

B. $\pi\log 2 - \log(1+k)$

C. $\log(1+k) - \pi\log 2$

D. None of these

Answer: A

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9. If $I(mn) = \int_0^1 x^{m-1}(1-x)^{n-1}dx, (m, n\varepsilon I, m, n \geq 0)$, then

A. $\dfrac{m!n!}{(m+n+2)!}$

B. $\dfrac{2m!n!}{(m+n+1)!}$

C. $\dfrac{m!n!}{(m+n+1)!}$

D. None of tese

Answer: C

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10. The value of $I(n) = \int_0^{\pi} \frac{\sin^2 n\theta}{\sin^2 \theta} d\theta$ is $(\forall n \in N)$

A. (a)$n\pi$

B. (b)$\dfrac{n\pi}{2}$

C. (c)$\dfrac{n\pi}{4}$

D. (d)None of these

Answer: A

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Exercise (Single Option Correct Type Questions)

1. $\int_0^4 \frac{(y^2 - 4y + 5)\sin(y - 2)dy}{[2y^2 - 8y + 1]}$ is equal to

A. 0

B. 2

C. -2

D. 2

Answer: A

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2. Let $f(x) = x^2 + ax + b$ and the only solution of the equation $f(x) = \min (f(x)) is x = 0$ and $f(x) = 0$ has root α and β, then $\int_\alpha^\beta x^3 d$

is equal to

A. $\frac{1}{4}(\beta^4 + \alpha^4)$

B. $\frac{1}{4}\left(a^2 - b^2\right)$

C. 0

D. None of these

Answer: C

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3. If $\int_{\pi/3}^{x}\sqrt{\left(3-\sin^2 t\right)}dt+\int_0^y \cos t dt = 0$ then evaluate $\frac{dy}{dx}$.

A. $\sqrt{3}$

B. $-\sqrt{2}$

C. $-\sqrt{3}$

D. None of these

Answer: A

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4. $\int_{-4}^{4} \frac{\sin^{-1}(\sin x) + \cos^{-1}(\cos x)}{(1+x^2)\left(1+\left[\frac{x^2}{17}\right]\right)} dx = \log\left(\frac{(1+\pi^2)}{\sqrt{a}}\right)$

$b\pi \ \ \tan^{-1}\left(\frac{c-\pi}{1+c\pi}\right)$ (where, [.] denotes greatest inteher function), then the number of ways in which $a-(2b+c)$ distinct object can distributed among $\frac{a-5}{c}$ persons equally , is

A. $\frac{9!}{(3!)^3}$

B. $\frac{12!}{(140^3)}$

C. $\frac{15!}{(5!)^3}$

D. $\frac{10!}{(6!)^3}$

Answer: A

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5. The value of the definite integral $\int_0^{\infty} \frac{dx}{(1+x^a)(1+x^2)}(a>0)$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. some function of a

Answer: A

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6. The value of the definite integral $\int_0^{3\pi/4} (1+x)\sin x + (1-x)\cos x \Big) dx$

is

A. $2\frac{\tan(3\pi)}{8}$

B. $2\frac{\tan(\pi)}{4}$

C. $2\frac{\tan(\pi)}{8}$

D. 0

Answer: A

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7. Let $C_n = \int_{1/n+1}^{1/n} \frac{\tan^{-1}(nx)}{\sin^{-1}(nx)} dx$, then $\lim_{n \to \infty} n^n . C_n$ is equal to

A. 1

B. 0

C. -1

D. $\frac{1}{2}$

Answer: D

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8. If

$$\left[\int_0^1 \frac{dt}{t^2 + 2t\cos\alpha + 1}\right]x^2 - \left[\int_{-3}^3 \frac{t^2 \sin 2t}{t^2 + 1} dt\right]x - 2 = 0 (0 < \alpha < \pi)$$

then the value of x is

A. (a)$\pm\left(\frac{\sqrt{\alpha}}{2\sin\alpha}\right)$

B. (b)$\pm\sqrt{\frac{2\sin\alpha}{\alpha}}$

C. (c)$\pm\sqrt{\frac{\alpha}{\sin\alpha}}$

D. (d)$\pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$

Answer: D

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9. $Iff(x) = e^{g(x)} andg(x) = \int_2^x \frac{tdt}{1+t^4}$, then find the value of $f'(2)$

A. $2/17$

B. 0

C. 1

D. Cannot be determined

Answer: A

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10. If a, b and c are real numbers, then the value of $\lim_{t \to 0} \left(\frac{1}{t} \int_0^t (1 + a \sin bx)^{c/x} dx \right)$ equals

A. abc

B. $\frac{ab}{c}$

C. $\frac{bc}{a}$

D. $\frac{ca}{b}$

Answer: A

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11. The value of ("lim")_(nvecoo)sum_(r=1)^(4n)(sqrt(n))/(sqrt(r)(3sqrt(r)+sqrt(n))^2) is equal to 1/(35) (b) 1/4 (c) 1/(10) (d) 1/5

A. $\frac{1}{35}$

B. $\frac{1}{14}$

C. $\frac{1}{10}$

D. $\frac{1}{5}$

Answer: C

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12. Let $f(x)=\int_{-1}^{x} e^{t^2} dt$ and $h(x)=f(1+g(x))$, where $g(x)$ is defined for all x, $g'(x)$ exists for all x, and $g(x)<0f$ or $x>0$. If $h'(1)=e$ and $g'(1)=1$, then the possible values which $g(1)$ can take

A. 0

B. -1

C. -2

D. -4

Answer: C

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13. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be the function satisfying $f(x) + g(x) = x^2$. Then the value of integral $\int_0^1 f(x)g(x)dx$ is equal to (A) $\frac{e-2}{4}$ (B) $\frac{e-3}{2}$ (C) $\frac{e-4}{2}$ (D) none of these

A. $e - \frac{1}{2}e^2 - \frac{5}{2}$

B. $e - e^2 - 3$

C. $\frac{1}{2}(e-3)$

D. $e - \frac{1}{2}e^2 - \frac{3}{2}$

Answer: D

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14. Let $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^2}}$ where $g(x) = \int_0^{\cos x} (1 + \sin t^2) dt$. Also $h(x) = e^{-|x|}$ and $l(x) = x^2 \sin\left(\frac{1}{x}\right)$ if $x \neq 0$ and $l(0) = 0$ then $f'\left(\frac{\pi}{2}\right)$ equals:

A. (a) $l'(0)$

B. (b) $h'(0^{-})$

C. (c) $h'(0^{+})$

D. (d) $\lim_{x \to 0} \frac{1 - \cos x}{x \sin x}$

Answer: C

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15. For $f(x) = x^4 + |x|, \leq tI_1 = \int_0^{\pi} f(\cos x) dx$ and $I_2 = \int_0^{\pi/2} f(\sin x) dx \text{then} \frac{I_1}{I_2}$ has the value equal to

A. 1

B. $1/2$

C. 2

D. 4

Answer: C

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16. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} x f[x(1-x)]dx$, $I_2 = \int_{1-k}^{k} f[x(1-x)]dx$, where $2k-1>0$. Then $\frac{I_1}{I_2}$ is

A. k

B. $1/2$

C. 1

D. 2

Answer: D

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17. Suppose that the quadratic function $f(x) = ax^2 + bx + c$ is non-negative on the interval [-1,1] . Then, the area under tha graph of f over the interval [-1,1] and the X - axis is given by the formula

A. $A = f(-1) + f(1)$

B. $A = f\left(-\frac{1}{2}\right) + f\left(\frac{1}{2}\right)$

C. $A = \frac{1}{2}[f(-1) + 2f(0) + f(1)]$

D. $A = \frac{1}{3}[f(-1) + 4f(0) + f(1)]$

Answer: D

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18. Let $I(a) = \int_0^{\pi} \left(\frac{x}{a} + a\sin x\right)^2 dx$ where a is positive real.

The value of a for which $I(a)$ attains its minimum value is

A. (a) $\sqrt{\pi\sqrt{\frac{2}{3}}}$

B. (b) $\sqrt{\pi\sqrt{\frac{3}{2}}}$

C. (c) $\sqrt{\frac{\pi}{16}}$

D. (d) $\sqrt{\frac{\pi}{13}}$

Answer: A

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19. The set of value of 'a' which satisfy the equation $\int_0^2 (t - \log_2 a)dt = \log_2\left(\frac{4}{a^2}\right)$ is

A. $a \in R$

B. $a \in R^+$

C. $a < 2$

D. $a > 2$

Answer: B

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20. $\lim_{x \to \infty} \left(x^3 \int_{-1/x}^{1/x} \frac{\text{In}(1+t^2)}{1+e^t} dt\right)$ is equal to `

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. 0

Answer: D

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21. The value of $\sqrt{\pi\left(\int_0^{2008} x|\sin \pi x|dx\right)}$ is equal to

A. $\sqrt{2008}$

B. $\pi\sqrt{2008}$

C. 1004

D. 2008

Answer: D

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22. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{n}{n^2 + k^2 x^2}, x > 0$ is equal to

A. $x \tan^{-1}(x)$

B. $\tan^{-1}(x)$

C. $\frac{\tan^{-1}(x)}{x}$

D. $\frac{\tan^{-1}(x)}{x}$

Answer: C

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23. Let $a > 0$ and $f(x)$ is monotonic increase such that $f(0) = 0$ and $f(a) = b$, then $\int_0^a f(x)dx + \int_0^b f^{-1}(x)dx$ is equal to

A. (a)$a + b$

B. (b)$ab + b$

C. (c)$ab + a$

D. (d)ab

Answer: D

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24. Evaluate the definite integral: $\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}}\left(\frac{x^4}{1-x^4}\right)\cos^{01}\left(\frac{2x}{1+x^2}\right)dx.$

A. π

B. 2π

C. 2

D. 1

Answer: A

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25. $\int_0^{\infty} f\left(x+\frac{1}{x}\right)\frac{\ln x}{x}dx$ is equal to:

A. (a) 0

B. (b) 1

C. (c) $\frac{1}{2}$

D. (d) cannot be evaluated

Answer: A

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26. $\lim_{\lambda \to 0} \left(\int_0^1 (1+x)^{\lambda} dx \right)^{1/\lambda}$ Is equal to

A. (a) $2In2$

B. (b) $\frac{4}{e}$

C. (c) $In\frac{4}{e}$

D. (d) 4

Answer: B

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27. If g(x) is the inverse of f(x) and f(x) has domain $x \in [1, 5]$, where f(1) = 2 and f(5) = 10 then the value of $\int_1^5 f(x)dx + \int_2^{10} g(y)dy$ is

A. 48

B. 64

C. 71

D. 52

Answer: A

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28. The value of the definite integral $\int_0^{\pi/2} \sin x \sin 2x \sin 3x dx$ is equal to

A. $\frac{1}{3}$

B. $-\frac{2}{3}$

C. $-\frac{1}{6}$

D. $\frac{1}{6}$

Answer: D

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29. If $f(x)=\int_0^x (f(t))^2 dt, f: R \rightarrow R$ be differentiable function and $f(g(x))$ is differentiable at $x=a$, then

A. g (x) must be differentiable at $x=a$

B. g(x) may be non-differentiable at $x=a$

C. g(x) may be discontinous at $x=a$

D. None of the above

Answer: A

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30. The number of integral solutions of the equation $4\int_0^{\infty}\frac{In\ \ tdt}{x^2+t^2}-\pi In2=0, x>0,$ is

A. 0

B. 1

C. 2

D. 3

Answer: C

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31. $\int_0^{16n^2/\pi}\cos\frac{\pi}{2}\left[\frac{x\pi}{n}\right]dx$ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: A

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32. If $\int_{-2}^{-1}\left(ax^2-5\right)dx=0 \text{ and } 5+\int_{1}^{2}(bx+c)dx=0$, then

A. $ax^2-bx+x=0$ has aleast one root in $(1, 2)$

B. $ax^2-bx+c=0$ has aleast one root in $(-2, -1)$

C. $ax^2+bx+c=0$ has atleast one root in $(-2, -1)$

D. None of these

Answer: B

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33. The value of $\int_3^6 \left(\sqrt{x+\sqrt{12x-36}}+\sqrt{x-\sqrt{12x-36}}\right)dx$ is equal to

A. $6\sqrt{3}$

B. $4\sqrt{3}$

C. $12\sqrt{3}$

D. None of these

Answer: A

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34. Let $I_n = \int_{-n}^{n} \left(\{x+1\}\cdot\{x^2+2\}+\{x^2+2\}\{x^3+4\}\right)dx$, where {*} denotet the fractional part of x. Find I_1.

A. $-\frac{1}{3}$

B. $-\frac{2}{3}$

C. $\frac{1}{3}$

D. None of these

Answer: B

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Definite intefral Exercise 1 : Single Option Correct Type Questions

1. Find the locus of a point which moves such that its distance from x axis is five times its distance from y axis.

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Exercise (More Than One Correct Option Type Questions)

1. If $f(x)=\left[\sin^{-1}(\sin 2x)\right]$ (where, [] denotes the greatest integer function), then

A. $\int_0^{\pi/2} f(x)dx = \frac{\pi}{2} - \sin^{-1}(\sin 1)$

B. $f(x)$ is periodic with period π

C. $\lim_{x \to \frac{\pi}{2}} f(x) = -1$

D. None of thses

Answer: A::B::C

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2. Which of the following definite integral (s) vanishes ?

A. $\int_0^{\pi/2} In(\cot x)dx$

B. $\int_0^{\pi 2} \sin^3 x dx$

C. $\int_{1/e}^{e} \frac{dx}{x(Inx)^{1/3}}$

D. $\int_0^{\pi} \sqrt{\frac{1+\cos 2x}{2}} dx$

Answer: A::C

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3. The equation $x^3 - 3x + 1 = 0$ has

A. atleast one root in $(-1, 0)$

B. alteast one root in $(0, 1)$

C. alteast two roots in $(-1, 1)$

D. no roots in $(-1, 1)$

Answer: A

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4. Suppose $I_1 = \int_0^{\frac{\pi}{2}} \cos(\pi \sin^2 x) dx$ and $I_2 = \int_0^{\frac{\pi}{2}} \cos(2\pi \sin^2 x) dx$ and $I_3 = \int_0^{\frac{\pi}{2}} \text{co}$

, then

A. (a) $I_1 = 0$

B. (b) $I_2 + I_3 = 0$

C. (c) $I_1 + I_2 + I_3 = 0$

D. (d) $I_2 = I_3$

Answer: A::B::C

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5. Evaluate $\int_0^{\frac{\pi}{4}} \cos x dx$

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6. The function f is continuous and has the property $f(f(x)) = 1 - x$ for all $x \in [0, 1]$ and $J = \int_0^1 f(x) dx$. Then which of the following is/are true?

(A) $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) = 1$

(B) $f\left(\frac{1}{3}\right) \cdot f\left(\frac{2}{3}\right) = 1$

(C) the value of J equals to $\frac{1}{2}$

(D) $\int_0^{\frac{\pi}{2}} \frac{\sin x dx}{(\sin x + \cos x)^3}$ has the same value as J

A. $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) = 1$

B. $f\left(\frac{1}{3}\right) . f\left(\frac{2}{3}\right) = 1$

C. the value of J equals to $1/2$

D. $\int_0^{\pi/2} \frac{\sin x dx}{(\sin x + \cos x)^3}$ has the value of as J

Answer: A::C::D

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7. Let $f(x)$ is a real valued function defined by $f(x) = x^2 + x^2\int_{-1}^{1} tf(t)dt + x^3\int_{-1}^{1} f(t)dt$ then which of the following hold (s) good?

A. $\int_{-1}^{1} tf(t)dt = \frac{10}{11}$

B. $f(1) + f(-1) = \frac{30}{11}$

C. $\int_{-1}^{1} tf(t)dt > \int_{-1}^{1} f(t)dt$

D. $f(1) - f(-1) = \frac{20}{11}$

Answer: B::D

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8. Let $f(x)$ and $g(x)$ be differentiable functions such that $f(x) + \int_0^x g(t)dt = \sin x(\cos x - \sin x)$ and $(f'(x))^2 + (g(x))^2 = 1$, then respectively , can be

A. $\frac{1}{2}\sin 2x, \sin 2x$

B. $\frac{\cos 2x}{2}, \cos 2x$

C. $\frac{1}{2}\sin 2x, -\sin 2x$

D. $-\sin^2 x, \cos 2x$

Answer: C

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9. Let $f(x) = \int_{-x}^{x} (t \sin at + bt + c) dt$, where a,b,c are non-zero real numbers , then $\lim_{x \to 0} \frac{f(x)}{x}$ is

A. independent of a

B. independent of a and b , and has the value equals to c

C. independent a, b and c

D. dependent only on c

Answer: A,D

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10. Let $L = \lim_{n \to \infty} \int_{a}^{\infty} \frac{n dx}{1 + n^2 x^2}$, where $a \in R$, then L can be

A. π

B. $\pi / 2$

C. 0

D. 1

Answer: C

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Exercise (Passage Based Questions)

1. *Suppose* $\lim_{x \to 0} \frac{\int_0^x \frac{t^2 dt}{(a+t^r)^{1/p}}}{bx - \sin x} = l,$

$p \in N, p \geq 2, a > \ > 0, r > 0$ and $b \neq 0$

If l exists and is non- zero, then

A. $b > 1$

B. $0 < b < 1$

C. $b < 0$

D. $b = 1$

Answer: D

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2. Suppose $\lim_{x \to 0} \frac{\int_0^x \frac{t^2 dt}{(a+t^r)^{1/p}}}{bx - \sin x} = l,$

$p \in N, p \geq 2, a > \; > 0, r > 0 \text{ and } b \neq 0$

If $p = 3 \text{ and } l = 1,$ then the value of 'a' is equal to

A. 8

B. 3

C. 6

D. $3/2$

Answer: A

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3. Suppose $\lim_{x \to 0} \frac{\int_0^x \frac{t^2 dt}{(a+t^r)^{1/p}}}{bx - \sin x} = l,$

$p \in N, p \geq 2, a > \; > 0, r > 0 \text{ and } b \neq 0$

If $p=2$ and $a=9$ and l exists , then the value of l is equal to

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $\frac{7}{9}$

Answer: B

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4. Suppose $f(x)$ and $g(x)$ are two continuous functions defined for $0 \leq x \leq 1$.Given,

$f(x)=\int_0^1 e^{x+1}. f(t)dt$ and $g(x)=\int_0^1 e^{x+1}\cdot g(t)dt+x$ The value of $f(1)$ equals

A. 0

B. 1

C. e^{-1}

D. e

Answer: A

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5. Suppose $f(x)$ and $g(x)$ are two continuous function defined for $0 \leq x \leq 1$. Given,

$$f(x) = \int_0^1 e^{x+1}. F(t)dt \text{ and } g(x) = \int_0^1 e^{x+t}. G(t)dt + x,$$

The value of $f(1)$ equals

A. (a) $\dfrac{2}{3-e^2}$

B. (b) $\dfrac{3}{e^2-2}$

C. (c) $\dfrac{1}{e^2-1}$

D. (d) 0

Answer: A

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6. Suppose $f(x)$ and $g(x)$ are two continuous function defined for $0 \leq x \leq 1$. Given,

$$f(x) = \int_0^1 e^{x+1}. F(t)dt \text{ and } g(x) = \int_0^1 e^{x+t}. G(t)dt + x,$$

The value of $f(1)$ equals

A. 0

B. $\frac{1}{3}$

C. $\frac{1}{e^2}$

D. $\frac{2}{e^2}$

Answer: B

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7. We are given the curvers $y = \int_{-\infty}^{x} f(t)dt$ through the point $\left(0, \frac{1}{2}\right)$ any $y = f(x)$, where $f(x) > 0$ and $f(x)$ is differentiable ,$\forall x \in$ R through $(0, 1)$ Tangents drawn to both the curves at the points with equal abscissae intersect on the same point on the X- axists

The number of solutions $f(x) = 2ex$ is equal to

A. 0

B. 1

C. 2

D. None of these

Answer: B

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8. $f(x) = \int_0^x (4t^4 - at^3)dt$ and $g(x)$ is quadratic satifying $g(0) + 6 = g'(0) - c = g''(c) + 2b = 0$. $y = h(x)$ and y=g(x) intersect in 4 distinct points with abscissae $x_i, i = 1, 2, 3, 4$ such that $\sum \frac{i}{x_i} = 8, a, b, c \in R^+$ and h (x)=f' (x)`. 'c' is equal to

A. 25

B. $25/2$

C. $25/4$

D. $25/8$

Answer: A

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9. Let $y = \int_{u(x)}^{y(x)} f(t)dt,$ let us define $\frac{dy}{dx} as \frac{dy}{dx} = v'(x)f^2(v(x)) - u'(x)f^2(u(x))$ and the equation of the tangent at (a, b) and $y - b = \left(\frac{dy}{dx}\right)(a, b)(x - a)$.
If $f(x) = \int_1^x e^{t^2/2}(1 - t^2)dt$ then $\frac{d}{dx} f(x) at x = 1$ is

A. $x + y = 1$

B. $y = x - 1$

C. $y = x$

D. $y = x + 1$

Answer: B

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10. Let $y=\int_{u(x)}^{y(x)} f(t)dt,$ let us define $\frac{dy}{dx} as \frac{dy}{dx} = v'(x)f(v(x)) - u'(x)f(u(x))$ and the equation of the tangent at (a, b) and $y-b=\left(\frac{dy}{dx}\right)(a, b)(x-a).$

If $y=\int_{x^2}^{x^4} (Int)dt$, then $\lim_{x\to 0^+}\frac{dy}{dx}$ is equal to

A. 0

B. 1

C. 2

D. -1

Answer: A

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11. If $y = \int\limits_{u(x)}^{v(x)} f(t)dt$, let us define $\frac{dy}{dx}$ in a different manner as $\frac{dy}{dx} = v'(x)f^2(v(x)) - u'(x)f^2(u(x))$ alnd the equation of the tangent at (a, b) as $y - b = \left(\frac{dy}{dx}\right)_{a\ \ b}(x - a)$

If $F(x) = \int\limits_1^x e^{t^2/2}(1 - t^2)dt$, then $\frac{d}{dx}F(x)$ at $x = 1$ is

A. 0

B. 1

C. 2

D. -1

Answer: A

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12. The value of $\int_0^{\infty}\left[\tan^{-1}x\right]dx$ is equal to (where ,[.] denotes the greatest integer function)

A. $-\frac{\pi}{2}$

B. $\frac{\pi}{2}$

C. ∞

D. 1

Answer: C

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Exercise (Matching Type Questions)

1. Let $\lim_{T\to\infty}\frac{1}{T}\int_0^T(\sin x+\sin ax)^2dx=L$, then

Column I		Column II	
(A)	For $a=0$, the value of L is	(p)	0
(B)	For $a=1$, the value of L is	(q)	1/2
(C)	For $a=-1$, the value of L is	(r)	1
(D)	For $a\in R-\{-1, 0, 1\}$, the value of L is	(s)	2

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2. Let $f(\theta) = \int_0^1 (x + \sin\theta)^2 dx$ and $g(\theta) = \int_0^1 (x + \cos\theta)^2 dx$ where $\theta \in [0, 2\pi]$. The quantity $f(\theta) - g(\theta)$, $\forall\theta$ in the interval given in column I, is

	Column I		Column II
(A)	$\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$	(p)	negative
(B)	$\left(\frac{3\pi}{4}, \pi\right]$	(q)	positive
(C)	$\left[\frac{3\pi}{2}, \frac{7\pi}{4}\right)$	(r)	non-negative
(D)	$\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$	(s)	non-positive

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3. Match the following

	Column I		Column II
(A)	$\int_0^1 (1 + 2008\, x^{2008})\, e^{x^{2008}}\, dx$ equals	(p)	e^{-1}

Column I	Column II
(B) The value of the definite integral $\int_0^1 e^{-x^2}\,dx + \int_1^{1/e} \sqrt{-\ln x}\,dx$ is equal to	(q) $e^{-1/4}$
(C) $\lim_{n\to\infty}\left(\dfrac{1^1 \cdot 2^2 \cdot 3^3 \ldots (n-1)^{n-1} \cdot n^n}{n^{1+2+3+\ldots+n}}\right)^{\frac{1}{n^2}}$ equals	(r) $e^{1/2}$
	(s) e

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4. Match the following

Column I	Column II
(A) If $f(x)=\int_0^{g(x)}\frac{dt}{\sqrt{1+t^3}}$, where $g(x)=\int_0^{\cos x}(1+\sin t^2)\,dt$, then value of $f'\left(\frac{\pi}{2}\right)$ is	(p) -2
(B) If $f(x)$ is a non-zero differentiable function such that $\int_0^x f(t)\,dt=\{f(x)\}^2$, $\forall x\in R$, then $f(2)$ is equal to	(q) 2
(C) If $\int_a^b(2+x-x^2)\,dx$ is maximum, then $a+b$ is equal to	(r) 1
(D) If $\lim_{x\to 0}\left(\frac{\sin 2x}{x^3}+a+\frac{b}{x^2}\right)=0$, then $3a+b$ has the value	(s) -1

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Exercise (Single Integer Answer Type Questions)

1. If $f(x) = \sum_{n=1}^{\infty} \frac{\sin nx}{4^n}$ and $\int_0^{\pi} f(x)dx = \log\left(\frac{m}{n}\right)$, then the value of $(m+n)$ is ....

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2. The value of $I = \int_{-\pi/2}^{\pi/2} \frac{\cos x dx}{1 + 2\left[\sin^{-1}(\sin x)\right]}$ (where ,[.] denotes greatest integer function) is ...

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3. If $f(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{\sin^2(n\theta)}{\sin^2\theta} d(\theta)$ then evaluate $\frac{f(15) + f(3)}{f(15) - f(9)}$

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4. Let $f(x) = \int_{-2}^{x} e^{(1+t)^2} dt$ and $g(x) = f(h(x))$,where $h(x)$ is defined for all $x \in R$. If $g'(2) = e^4$ and $h'(2) = 1$ then absolute value of sum of all possible values of $h(2)$, is ___

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5. $If = \int_0^{\pi/2} \sin x. \log(\sin x) dx = \log\left(\frac{K}{e}\right)$. Then, the value of K is ....

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Exercise (Questions Asked In Previous 13 Years Exam)

1. Evaluate: $\int_{-\pi/2}^{\pi/2} \frac{x^2 \cos x}{1+e^x} dx$

A. $\frac{\pi^2}{4} - 2$

B. $\frac{\pi^2}{4} + 2$

C. $\pi^2 - e^{-\pi/2}$

D. $\pi^2 + e^{\pi/2}$

Answer: A

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2. The total number for distinct $x\varepsilon[0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is ________.

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3. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression (s) is (are) (a) $\int_0^{\frac{\pi}{4}} x f(x) dx = \frac{1}{12}$ (b) $\int_0^{\frac{\pi}{4}} f(x) dx = 0$ (c) $\int_0^{\frac{\pi}{4}} x f(x) = \frac{1}{6}$ (d) $\int_0^{\frac{\pi}{4}} f(x) dx = \frac{1}{12}$

A. $\int_0^{\pi/4} x f(x) dx = \frac{1}{12}$

B. $\int_0^{\pi/4} f(x) dx = 0$

C. $\int_0^{\pi/4} x f(x) dx = \frac{1}{6}$

D. $\int_0^{\pi/4} f(x) dx = \frac{1}{12}$

Answer: A::B

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4. Let $f'(x) = \frac{192x^3}{2 + \sin^4 \pi x}$ for all $x \varepsilon R$ with $f\left(\frac{1}{2}\right) = 0$. If $m \leq \int_{1/2}^{1} f(x)dx \leq M$, then the possible values of m and M are

A. $m = 13, M = 24$

B. $m = \frac{1}{4}, M = \frac{1}{2}$

C. $m = -11, M = 0$

D. $m = 1, M12$

Answer: B

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5. The option(s) with the values of $aandL$ that satisfy the following equation is (are) $\frac{\int_0^{4\pi} e^t(\sin^6 at + \cos^4 at)dt}{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)dt} = L$ (a) $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$ (b) $a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$ $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$ (d) $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

A. $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi}}$

B. $a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi + 1}}$

C. $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi + 1}}$

D. $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi + 1}}$

Answer: A::C

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6. Let $F : R \to R$ be a thrice differntiable function. Suppose that $F(1) = 0, F(3) = -4$ and $F'(x) < 0$ for all $x\varepsilon(1/2, 3)$. Let $f(x) = xF(x)$ for all $x \in R$. Then the correct statement(s) is (are)

A. $f'(1) < 0$

B. $f(2) < 0$

C. $f'(x) \neq 0$ for any $x \in (1, 3)$

D. $f'(x) = 0$ for some $x \in (1, 3)$

Answer: A::B::C

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7. Let F : R $\rightarrow$ R be a thrice differentiable function . Suppose that $F(1) = 0$, $F(3) = -4$ and $F(x) < 0$ for all $x \in (1, 3)$. $f(x) = xF(x)$ for all $x \in R$.

If $\int_1^3 x^2 F'(x)dx = -12$ and $\int_1^3 x^3 F''(x)dx = 40$, then the correct expression (s) is //are

A. (a) $9f'(3) + f'(1) - 32 = 0$

B. (b) $\int_1^3 f(x)dx = 12$

C. (c) $9f'(3) - f'(1) + 32 = 0$

D. (d) $\int_1^3 f(x)dx = -12$

Answer: C::D

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8. Let $f: R \to R$ be a function defined by $f(x) = \begin{cases} [x] & x \le 2 \\ 0 & x > 2 \end{cases}$ where $[x]$ is th greatest integer less than or equal to x. If $I = \int_{-1}^{2} \frac{xf(x^2)}{2 + f(x+1)} dx$, then the value of $(4I - 1)$ is

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9. If $\alpha = \int_0^1 \left(e^{9x + 3\tan^{(-1)x}}\right)\left(\frac{12 + 9x^2}{1 + x^2}\right) dxwhereηn^{-1}$ takes only principal values, then the value of $\left((\log)_e |1 + \alpha| - \frac{3\pi}{4}\right) is$

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10. The integral $\int_{\pi/4}^{\pi/2} (2\cos ecx)^{17} dx$ is equal to

A. $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$

B. $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17} du$

C. $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17} du$

D. $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$

Answer: a

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11. Let $f:[0,2]\overrightarrow{R}$ be a function which is continuous on [0,2] and is differentiable on (0,2) with $f(0)=1$ $Let: F(x) = \int_0^{x^2} f(\sqrt{t})\,dtf$ or $x \in [0,2]\dot{I}fF'(x) = f'(x)$. for all $x \in (0,2)$, then $F(2)$ equals $e^2 - 1$ (b) $e^4 - 1$ $e - 1$ (d) e^4

A. $e^2 - 1$

B. $e^4 - 1$

C. $e - 1$

D. e^4

Answer: B

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12. Match the conditions/ expressions in Column I with statement in Column II

	Column I		Column II
A.	$\int_{-1}^{1} \frac{dx}{1+x^2}$	p.	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
B.	$\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$	q.	$2\log\left(\frac{2}{3}\right)$
C.	$\int_{2}^{3} \frac{dx}{1-x^2}$	r.	$\frac{\pi}{3}$
D.	$\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$	s.	$\frac{\pi}{2}$

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13. Match List I with List II and select the correct answer using codes given below the lists

	List I		List II
A.	The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)\,dx = 1$, is	p.	8
B.	The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	q.	2

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14. The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}\left(1-x^2\right)^5\right\}dx$ is

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15. The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2+\log\frac{\pi-x}{\pi+x}\right)\cos x\,dx$

A. 0

B. $\frac{\pi^2}{2} - 4$

C. $\frac{\pi^2}{2} + 4$

D. $\frac{\pi^2}{2}$

Answer: B

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16. The valued of $\int_{\sqrt{In2}}^{\sqrt{In3}} \frac{x \sin x^2}{\sin x^2 + \sin(In6 - x^2)} dx$ is

A. $\frac{1}{4} \frac{\log(3)}{2}$

B. $\frac{1}{2} \frac{\log(3)}{2}$

C. $\frac{\log(3)}{2}$

D. $\frac{1}{6} \frac{\log(3)}{2}$

Answer: B::D

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17. Let $f:[1,\infty]$ be a differentiable function such that $f(1)=2$. If $\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is

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18. The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^4}dx$ is (are)

A. $\frac{22}{7} - \pi$

B. $\frac{2}{105}$

C. 0

D. $\frac{71}{15} - \frac{3\pi}{2}$

Answer: A

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19. For $a \varepsilon R$ (the set of all real numbers)

$$a \neq -1, \lim_{n \to \infty} \frac{(1^a + 2^a + \dots\dots\dots + n^a)}{(n+1)^{a-1}[(na+1)+(n+2)+\dots\dots\dots\dots + (na+n)}$$

Then $a =$

A. 5

B. 7

C. $\frac{-15}{2}$

D. $\frac{-17}{2}$

Answer: B::D

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20. Let $f: [0, 1] \to R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable,f(0)=f(1)=0 and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1]$ Consider the statements.

I. There exists some $x \in R$ such that, $f(x) + 2x = 2(1 + x^2)$

(II) There exists some $x \in R$ such that, 2f(x)+1=2x(1+x)

A. both P and Q are true

B. P is true and Q is false

C. P is false and Q is true

D. both P and Q are false

Answer: C

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21. Which of the following is true?

A. g is increasing on $(1, \infty)$

B. g is decreasing on $(1, \infty)$

C. g is increasing on $(1, 2)$ and decreasing on$(2, \infty)$

D. g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

Answer: B

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22. For any real number $x, let[x]$ denote the largest integer less than or equal to $x, Let f$ be a real-valued function defined on the interval $[-10, 10]$ be $f(x) = \{x - [x], \quad \text{if } [x] is odd 1 + [x] - x, \quad \text{if } [x] is even$ Then the value of $\frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi x dx is\_\_$

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23. Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt, 0 \le x \le 1, and\ f(0) = 0$, then

A. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

B. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

C. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

D. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Answer: C

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24. If $\int_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots\ldots\ldots$ then

A. $I_n = I_{n+2}$

B. $\sum_{k=0}^{10} I_{2m+1} = 10\pi$

C. $\sum_{m=1}^{n} I_{2m} = 0$

D. $I_n = I_{n+1}$

Answer: A::B::C

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25. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$ for $n = 1, 2, 3,$ .... Then

A. $S_n < \frac{\pi}{3\sqrt{3}}$

B. $S_n > \frac{\pi}{3\sqrt{3}}$

C. $T_n < \frac{\pi}{3\sqrt{3}}$

D. $T_n > \frac{\pi}{3\sqrt{3}}$

Answer: D

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26. Then integral $\int_{\pi/4}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to

A. -1

B. -2

C. 2

D. 4

Answer: C

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27.

Let $I_n = \int \tan^n x dx, (n > 1). I_4 + I_6 = a \tan^5 x + bx^5 + C,$ where C is a

A. $\left(-\frac{1}{5}, 0\right)$

B. $\left(-\frac{1}{5}, 1\right)$

C. $\left(\frac{1}{5}, 0\right)$

D. $\left(\frac{1}{5}, -1\right)$

Answer: C

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28. $\lim_{n \to \infty} \left(\frac{(n+1)(n+2)(n+3).......2n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to

A. $\frac{18}{e^4}$

B. $\frac{27}{e^2}$

C. $\frac{9}{e^2}$

D. None of these

Answer: B

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29. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$ is equal to

A. 2

B. 4

C. 1

D. 6

Answer: C

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30. The integral $\int_0^{\pi} \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}dx$ is equals to (a)$\pi - 4$ (b) $\frac{2\pi}{3} - 4 - \sqrt{3}$ (c)$\frac{2\pi}{3} - 4 - \sqrt{3}$ (d)$4\sqrt{3} - 4 - \frac{\pi}{3}$

A. $\pi - 4$

B. $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

C. $4\sqrt{3} - 4$

D. $4\sqrt{3} - 4 - \pi/3$

Answer: D

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31. Statement I The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} is \frac{\pi}{6}$

Statement II $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

A. Statement In is true, Statement II is true, Statement II is a true explanation for Statement I

B. Statement I is true , Statement II is true' Statement II is not a true explanation for Statement I

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: D

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32. The intercepts on x- axis made by tangents to the curve, $y = \int_0^x |t| dt, x \varepsilon R$ which are parallel to the line $y = 2x$, are equal to

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: A

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33. If $g(x)=\int_0^x \cos^4 t dt$, then $g(x+\pi)$ equals to (a) $\frac{g(x)}{g(\pi)}$ (b) $g(x)+g(\pi)$ (c) $g(x)-g(\pi)$ (d) $g(x).\ g(\pi)$

A. $\frac{g(x)}{g(\pi)}$

B. $g(x)+g(\pi)$

C. $g(x)-g(\pi)$

D. $g(x).\ g(\pi)$

Answer: B

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34. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is

A. $\frac{\pi}{8}\log 2$

B. $\frac{\pi}{2}\log 2$

C. $\log 2$

D. $\pi \log 2$

Answer: D

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35. For $x \varepsilon \left(0, \frac{5\pi}{2}\right)$, definite $f(x) = \int_0^x \sqrt{t} \sin t dt$. Then f has

A. local minimum at $\pi \text{ and } 2\pi$

B. local minimum at π and local minimum at 2π

C. local minimum at π and local minimum at 2π

D. local maximum at $\pi \text{ and } 2\pi$

Answer: C

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36. Let $p(x)$ be a function defined on R such that $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1, p'(x) = p'(1-x)$, for all $x \in [0, 1], p(0) = 1$ and $p(1)$ equals

A. $\sqrt{41}$

B. 21

C. 41

D. 42

Answer: B

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37. $\int_0^{\pi} [\cos x] dx$, [] denotes the greatest integer function , is equal to

A. $\frac{\pi}{2}$

B. 1

C. (-1)

D. $-\frac{\pi}{2}$

Answer: D

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38. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx \text{ and } f = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$

Then , which one of the following is true ?

A. $I > \frac{2}{3} \text{ and } f > 2$

B. $I < \frac{2}{3} \text{ and } f < 2$

C. $I < \frac{2}{3} \text{ and } f > 2$

D. $I > \frac{2}{3} \text{ and } f < 2$

Answer: B

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